Galileon and, Modifications of gravity

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Summer Institute 2010 – 10 August 2010

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Tokyo University of Science

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Introduction

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- Motivated different approaches

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 2. Galileon . . .

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- What new degrees of freedom? 1 SF
- Interesting phenomenology [Review by ADF, Tsujikawa '10]

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- Two extra new scalar fields, λ, σ
- Only one on FLRW and $\omega^2 \propto (k/a)^4$
  [ADF, Suyama '09; ADF, Tanaka '10]

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- Φ_3, the ghost, integrated out.

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- Possible example for 4D action coming from ED, ST [de Rham, Tolley '10]

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- Study started with $\mathcal{L}_3$

  [Silva and Koyama '09]

  [Chow and Khoury '09]

  [Kobayashi, Tashiro, Suzuki '10]

  [Kobayashi, Yamaguchi, Yokoyama '10]

  [ADF and Tsujikawa '10]

Generalized Brans-Dicke theory

[ADF, Tsujikawa '10; ADF, Mukohyama, Tsujikawa '10]

- Lagrangian

$$\mathcal{L} = \frac{1}{2}F(\phi)R - \frac{1}{2}B(\phi)\partial\phi^2 + \xi(\phi)\mathcal{L}_3\,.$$

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- Existence of late dS era

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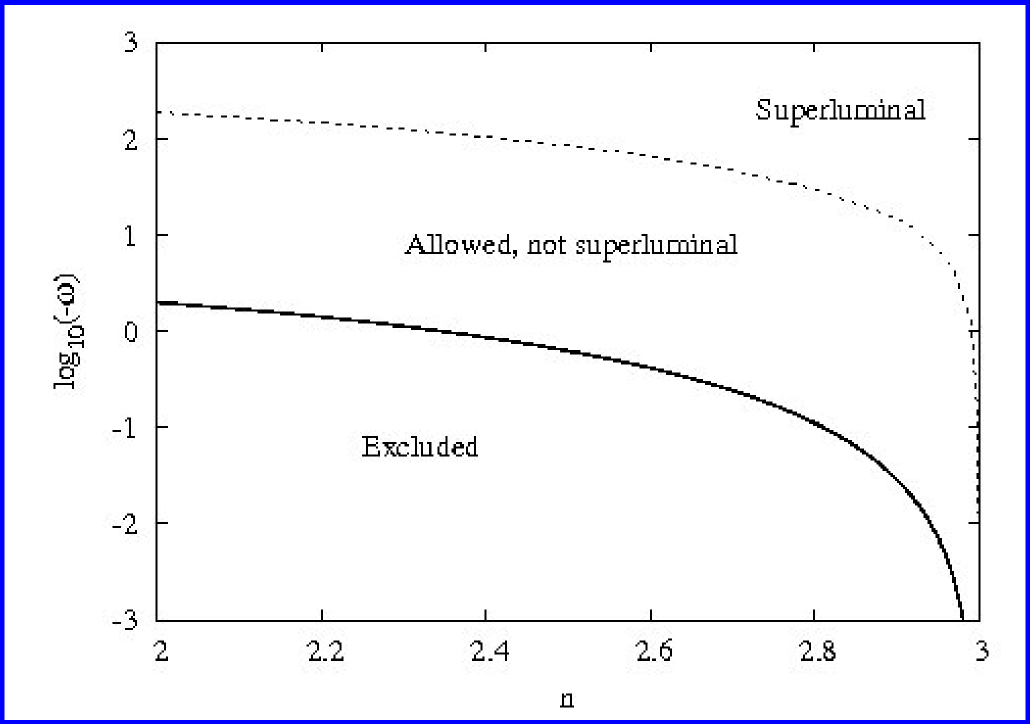
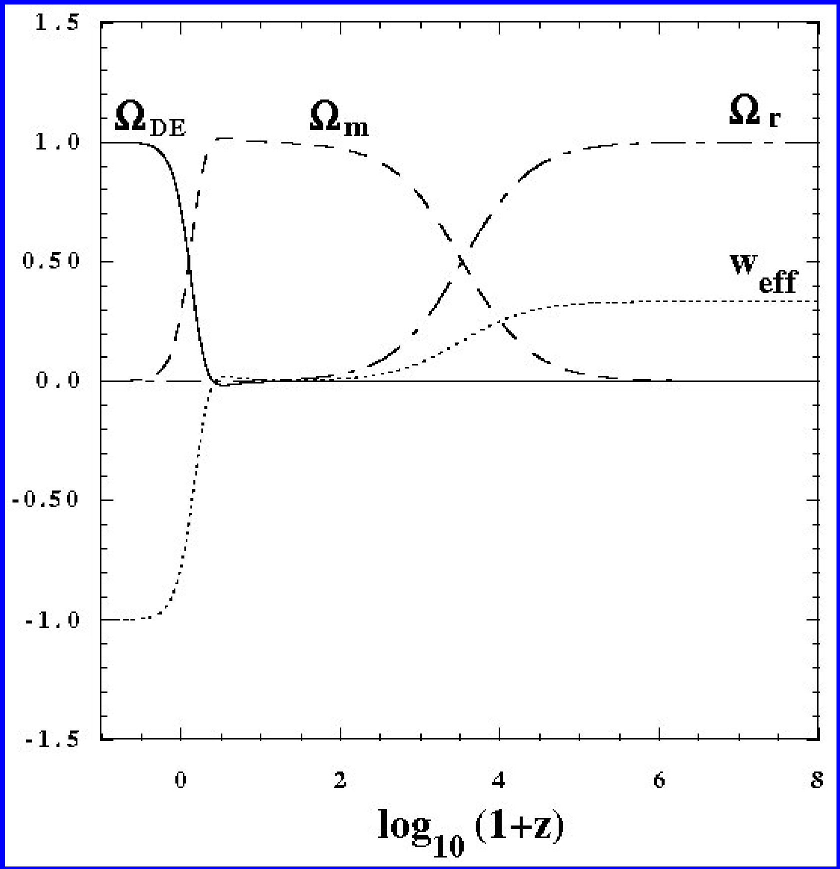
  [ADF, Mukohyama, Tsujikawa '10]
- $2 \leq n \leq 3$, $\dot{\phi}/(H\phi) > 0 \to \omega < -n(n-3)^2, \lambda < 0$

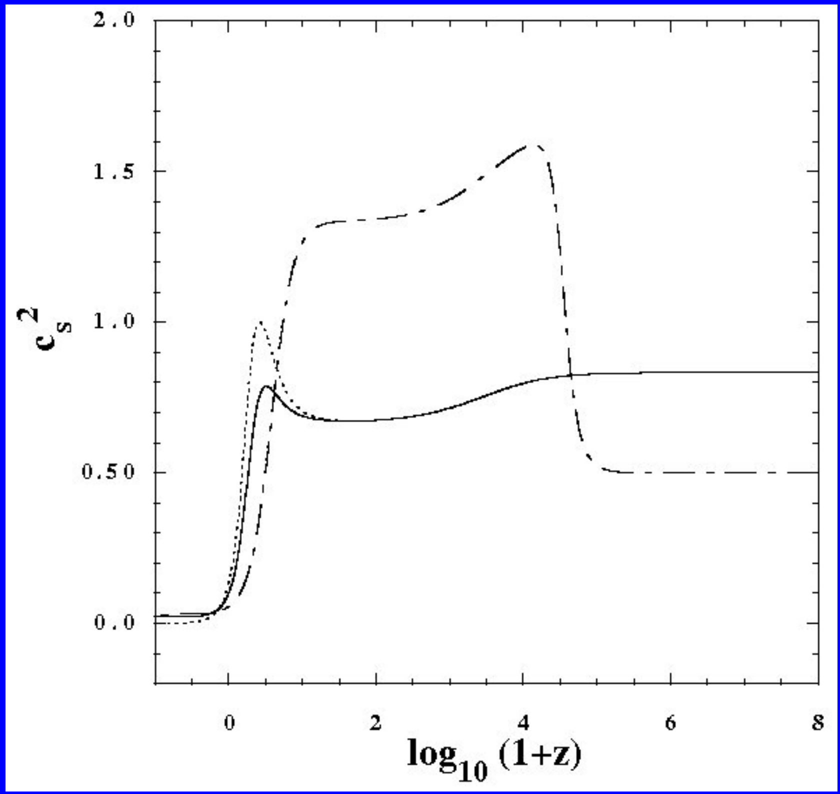
Allowed parameter space



Allowed parameter space in terms of ω vs n. $n < 3$ for DM era, $n > 2$ for $c^2_{s,\mathrm{dS}} > 0$. $\dot{\phi}/\phi > 0$ for no scalar ghost. $F > 0$ for no tensor ghost.

Typical background evolution





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- Studied up to $\mathcal{L}_4$ [Gannouji, Sami '10]

Field equations

- On FLRW:

$$3M_p^2H^2 = \rho_{\rm DE} + \rho_m + \rho_r \, ,$$
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- Admits de Sitter solutions

- Define for convenience

$$r_1 \equiv \frac{\dot{\phi}_{\rm dS}\, H_{\rm dS}}{\dot{\phi}\, H}, \quad r_2 \equiv \frac{\dot{\phi}^4}{r_1\, \dot{\phi}_{\rm dS}^4}, \quad x_{\rm dS} \equiv \frac{\dot{\phi}_{\rm dS}}{H_{\rm dS} M_p}$$

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- From FRD eq

$$\Omega_{\rm DE} \equiv \frac{\rho_{\rm DE}}{3M_p^2 H^2} = -\tfrac{1}{6} c_2 x_{\rm dS}^2 r_1^3 r_2 + c_3 x_{\rm dS}^3 r_1^2 r_2 - \tfrac{15}{2} c_4 x_{\rm dS}^4 r_1 r_2 + 7 c_5 x_{\rm dS}^5 r_2$$

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- Cannot forget $\mathcal{L}_5$

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- EOM for r_1', r_2', and Ω_r', Ω_m from FRD

きれいなGalileon

- EOM for r_1:

$$r_1' = (r_1 - 1)\, f(r_1, r_2, \Omega_r)\,, \quad r_2' = \dots\,, \quad \Omega_r' = \dots$$

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- Small BG perturbations

$$\delta r_1' = -\frac{9 + \Omega_r + 3r_2}{2(1 + r_2)}\, \delta r_1$$

Other variables

- EOM

$$r_2' = \frac{2r_2\left(3 - 3r_2 + \Omega_r\right)}{1 + r_2}, \quad \Omega_r' = \frac{\Omega_r\left(\Omega_r - 1 - 7r_2\right)}{1 + r_2},$$

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- Last is stable, others are saddle as r_2 unstable

Features

- BG Signature

$$w_{\mathrm{DE}} \equiv \frac{p_{\mathrm{DE}}}{\rho_{\mathrm{DE}}} = -\frac{\Omega_r + 6}{3\,(r_2+1)},\;\; w_{\mathrm{eff}} \equiv -1 - \frac{2H'}{3H} = \frac{\Omega_r - 6\,r_2}{3\,(r_2+1)}.$$

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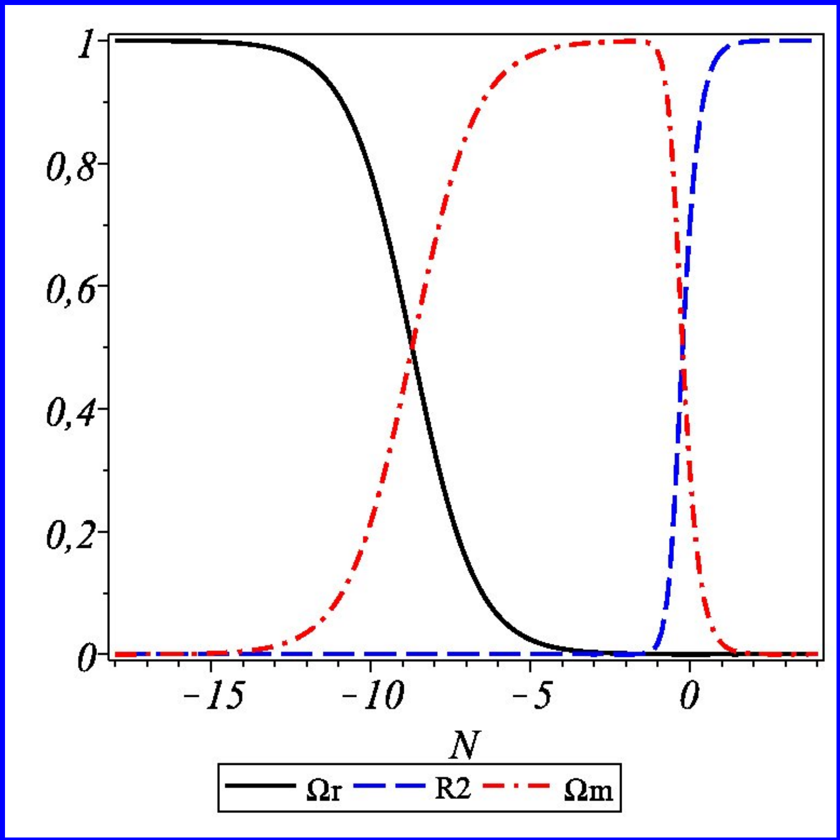
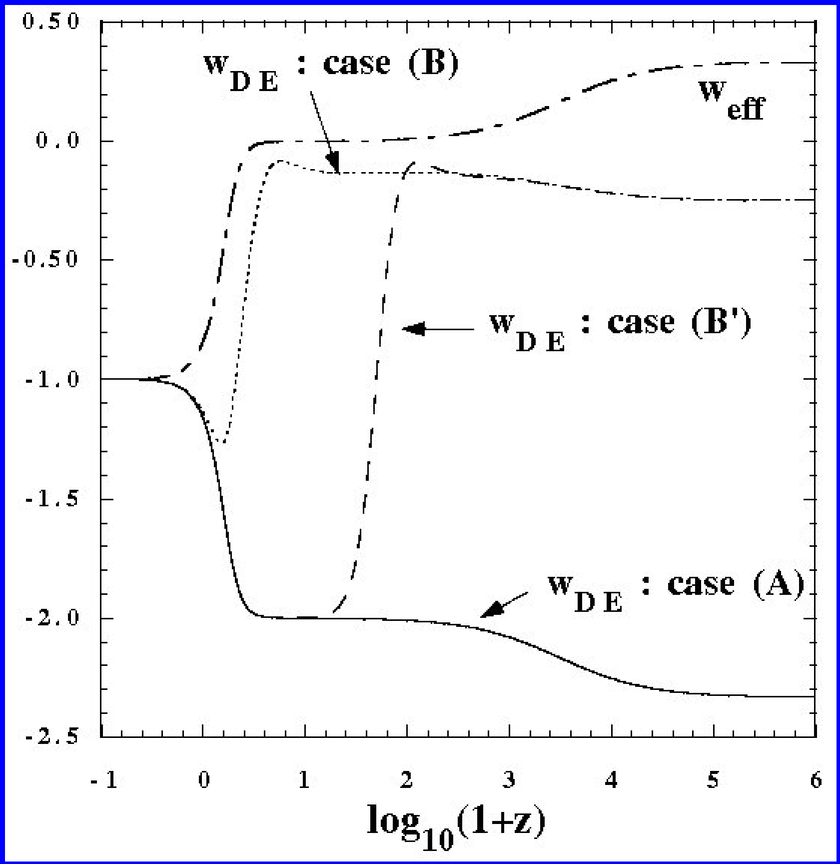
- To be tested against observations

Background



1
0,8
0,6
0,4
0,2
0
-15
-10
-5
0
N
Ωr
R2
Ωm



0.50
0.0
-0.50
-1.0
-1.5
-2.0
-2.5
-1
0
1
2
3
4
5
6
w_DE : case (B)
w_eff
w_DE : case (B')
w_DE : case (A)
log10(1+z)

Removing degeneracy

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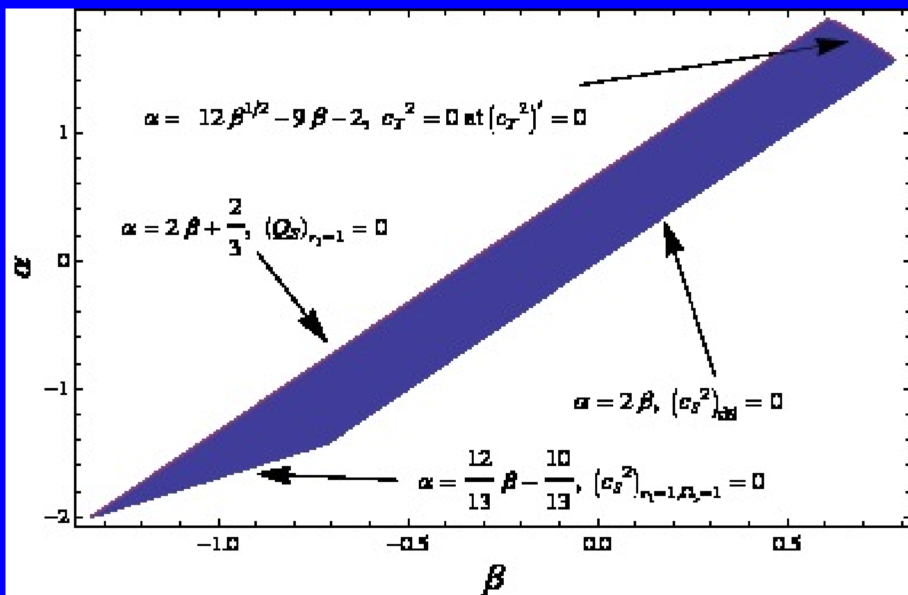
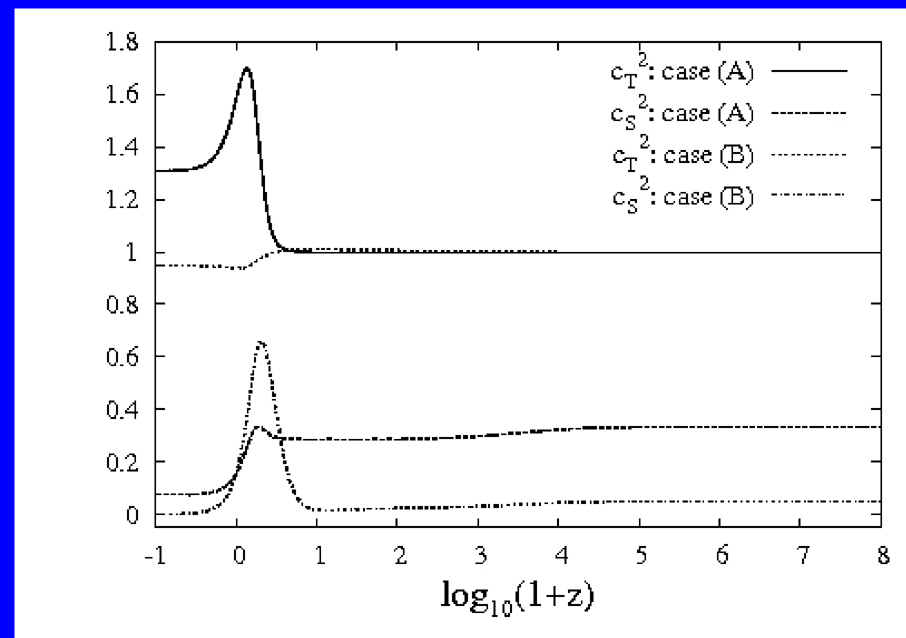
- Use no-ghost and $c_S^2 > 0, c_T^2 > 0$ conditions

LPT results


$\alpha = 12\beta^{1/2} - 9\beta - 2,\ c_T^2 = 0$ at $(c_T^2)' = 0$
$\alpha = 2\beta + \frac{2}{3},\ (Q_S)_{r_1 = 1} = 0$
$\alpha = 2\beta,\ (c_S^2)_{dS} = 0$
$\alpha = \frac{12}{13}\beta - \frac{10}{13},\ (c_S^2)_{r_1=1, \Omega_r=1} = 0$
α
β
1
0
−1
−2
−1.0
−0.5
0.0
0.5


c_T^2: case (A)
c_S^2: case (A)
c_T^2: case (B)
c_S^2: case (B)
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-1
0
1
2
3
4
5
6
7
8
$\log_{10}(1+z)$