The Inhomogeneous Early Universe

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What Inhomogeneity Do you Mean?

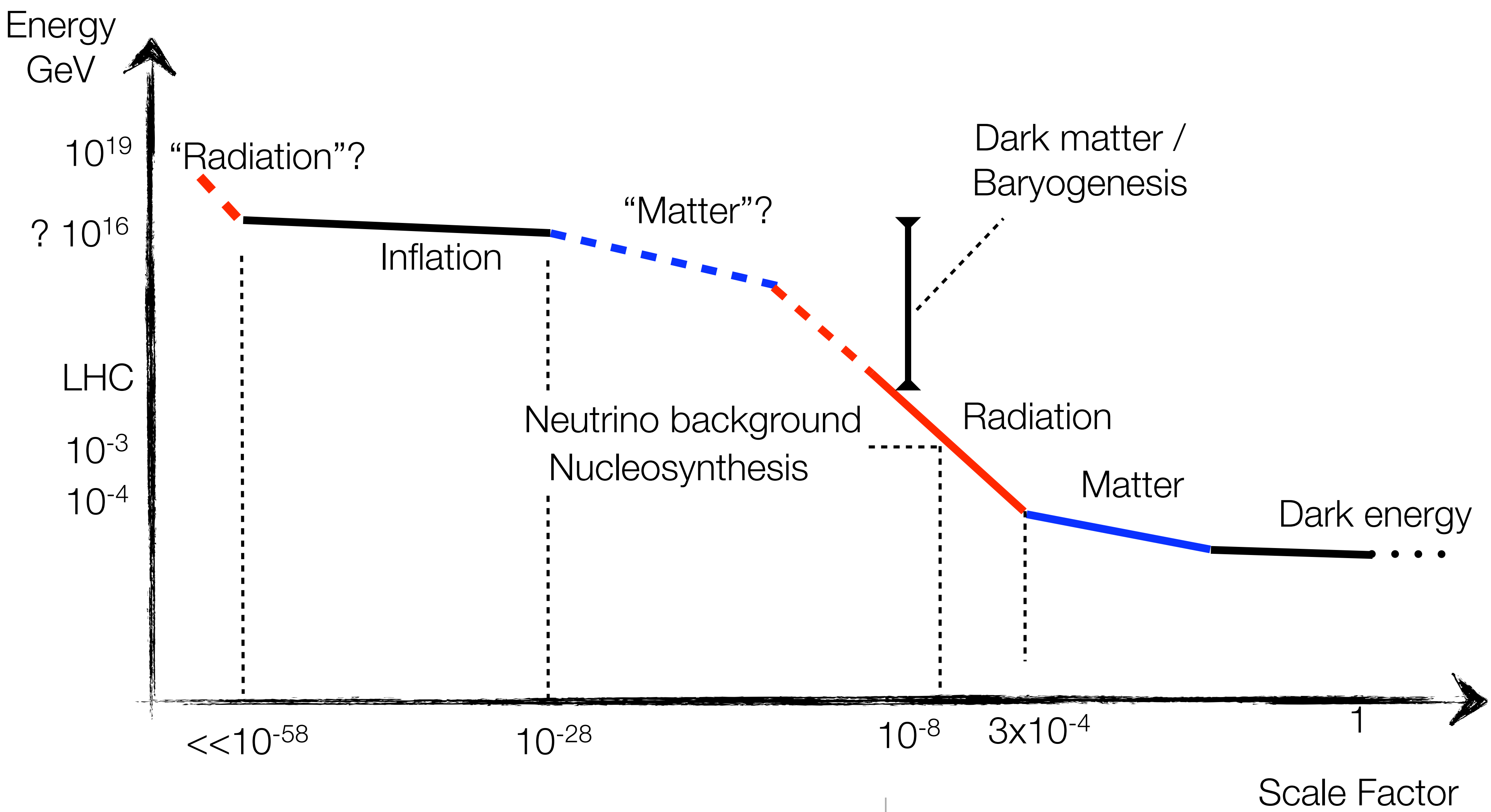
- **Large** inhomogeneities...
  - *Not* the usual 2-point or 3-point functions
  - *Not* loop corrections to the inflationary spectrum
  - *Not* bumps and glitches on the inflaton potential
- Putting the $\nabla^2 \phi$ back into $\Box \phi$!
  - Might also worry about Ψ and Φ as well as ψ and ϕ

But Doesn't Inflation Rule This Out?

- **Yes:** *during* inflation the universe *is* very smooth...
- **Yes but:** Pre-inflation
  - Bubble collisions; remnants of initial conditions
  - Chaotic dynamics in multifield models
- **Yes but:** Post-inflation
  - Smooth on *super horizon* scales
  - What about scales near horizon as inflation ends?
  - Or scales that re-enter horizon soon after inflation ends?

Reasons to Care...

- How does inflation *begin*?
  - Pre-inflationary dynamics in “realistic” potentials
- Impact on present day observables
  - Gravitational waves (although at high frequencies)
  - Connection to inflationary observables (see lectures)
  - Primordial black hole formation (usually bad for a model)
- Post-inflationary expansion history *long* and *unknown*
  - And probes physics above LHC scales



Cosmology: Theory

Two Approaches to the Early Universe

- Build models with new ingredients...
  - Branes / strings / extra dimensions / exotic matter
  - Non-standard gravity (Horava-Lifshitz, Galileon)
- New phenomenology with “standard” ingredients
  - Nonlinearity, inhomogeneity
- This talk: Method 2

Iron Chef: Cosmology

Many Examples [Izakaya-style]

1. Post-inflationary coherent oscillations
   - Just wait long enough without reheating?
2. Parametric resonance; gravity waves; reheating
   - Equation of state; semi-stable states?
3. Oscillons
4. Primordial black holes (and gravitational waves)?
5. Pre-inflation: bubble collisions
   - Mechanisms for exploring landscape?

Tools of the Trade: Klein-Gordon

- Assume universe is made from scalar fields
- Without a potential, in an expanding universe
  - Wave equation with dissipation (weakly nonlinear)
- With a potential
  - Nonlinear wave equation
  - Rich analytics; direct numerical solution
- Standard ingredients; new physics.

Tools of the Trade: Numerical methods

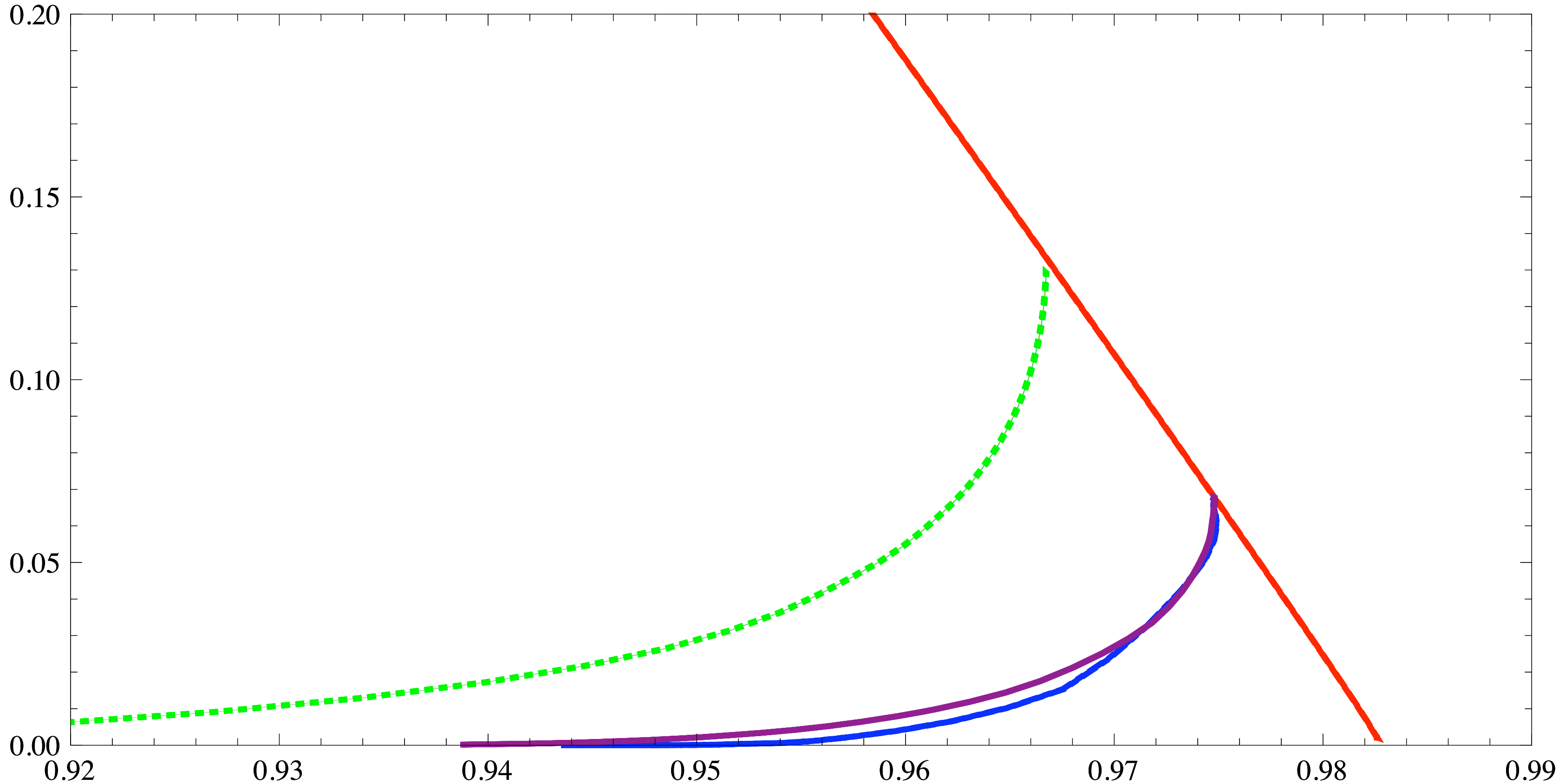
- LatticeEasy (Felder and Tkachev)
  - + Defrost (Frolov)
  - + HLattice (Huang)
- PSPECTRE (Easther, Finkel & Roth) arXiv:1005.1921
  - Pseudo-SPECTral REheating
  - Solves for Fourier components of fields
  - **DOWNLOAD: http://easther.physics.yale.edu/**

1. Delayed Reheating

- Simplest picture of inflation...
  - Universe inflates, then inflation ends
  - Inflaton oscillates at bottom of the potential
  - *Not* a new scenario!
- For a quadratic potential, $a(t) \sim t^{2/3}$ "matter dominated"
  - Naively, perturbations grow ~ a(t)
  - But inflaton weakly coupled; oscillations last a long time...
  - Most potentials quadratic *near minimum*

Summary

- Algebra in paper... [RE, Flauger & Gilmore, arXiv:1003.3011]
  - Two time scales: Hubble time, and (inflaton) oscillation time
  - Modes with frequency > inflation oscillation will *decay*
  - Modes inside horizon, frequency < inflation oscillation *grow*
  - Perturbations nonlinear after universe grows by ~10^5
- Inhomogeneity happens in the *simplest* system!
  - Possible gravitational wave production, nonlinear collapse?
  - See e.g. Assadullahi and Wands



Spectral Index v.
Tensor amplitude

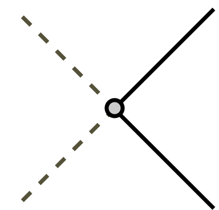
Matching equation [see lecture]

2. Preheating & Gravitational Waves

- Original work: Khlebnikov and Tkachev (1997)
- Revived: RE and Lim (2006) astro-ph/0601617
  - Frequency correlated to inflationary scale
  - RE, Giblin and Lim astro-ph/0612294 arXiv:0712.2991
  - Dufaux et al., Garcia-Bellido et al., Price & Siemans
- Hard numerical problem; hard physical problem

Parametric Resonance: Quick Sketch

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m^2\phi^2 - \frac{g^2}{2}\phi^2\chi^2$$



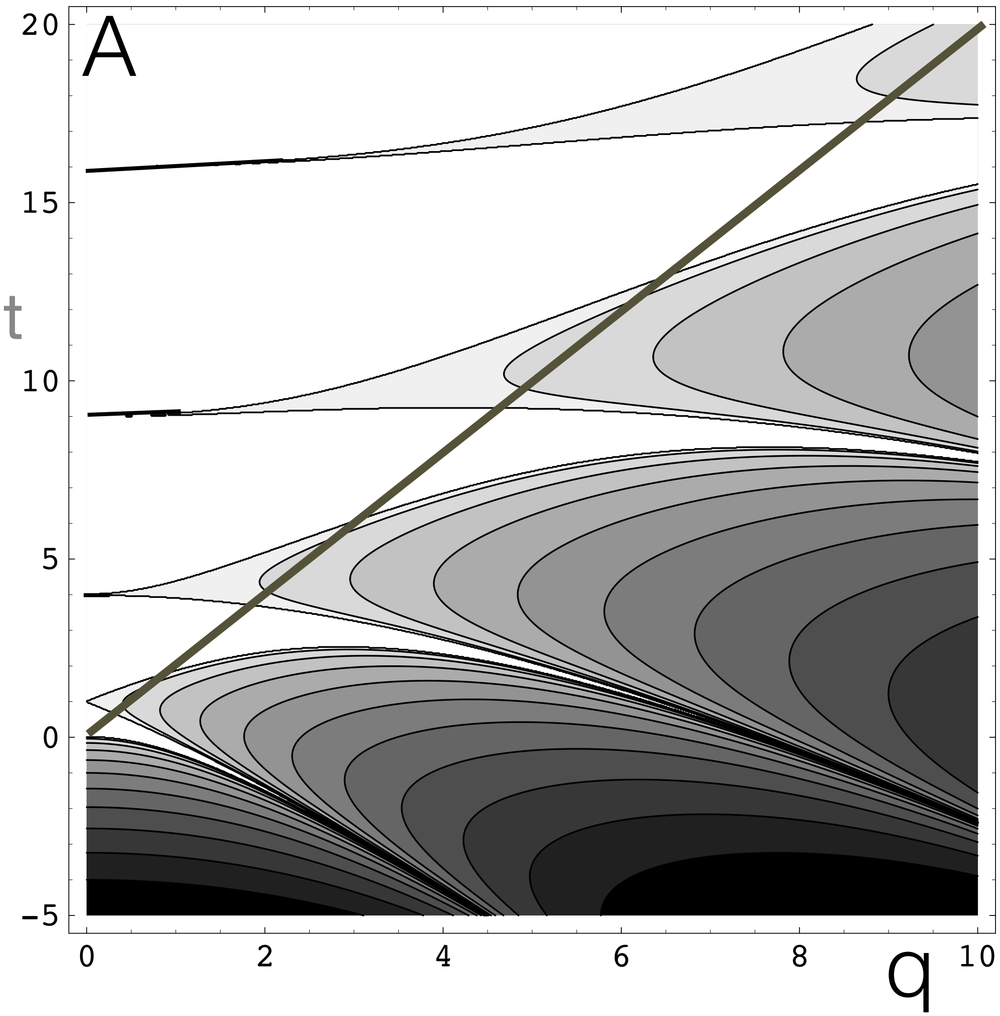
- φ is the inflaton field
- χ is a (massless) field coupled to the inflaton
  - Perturbations in χ, canonically quantized

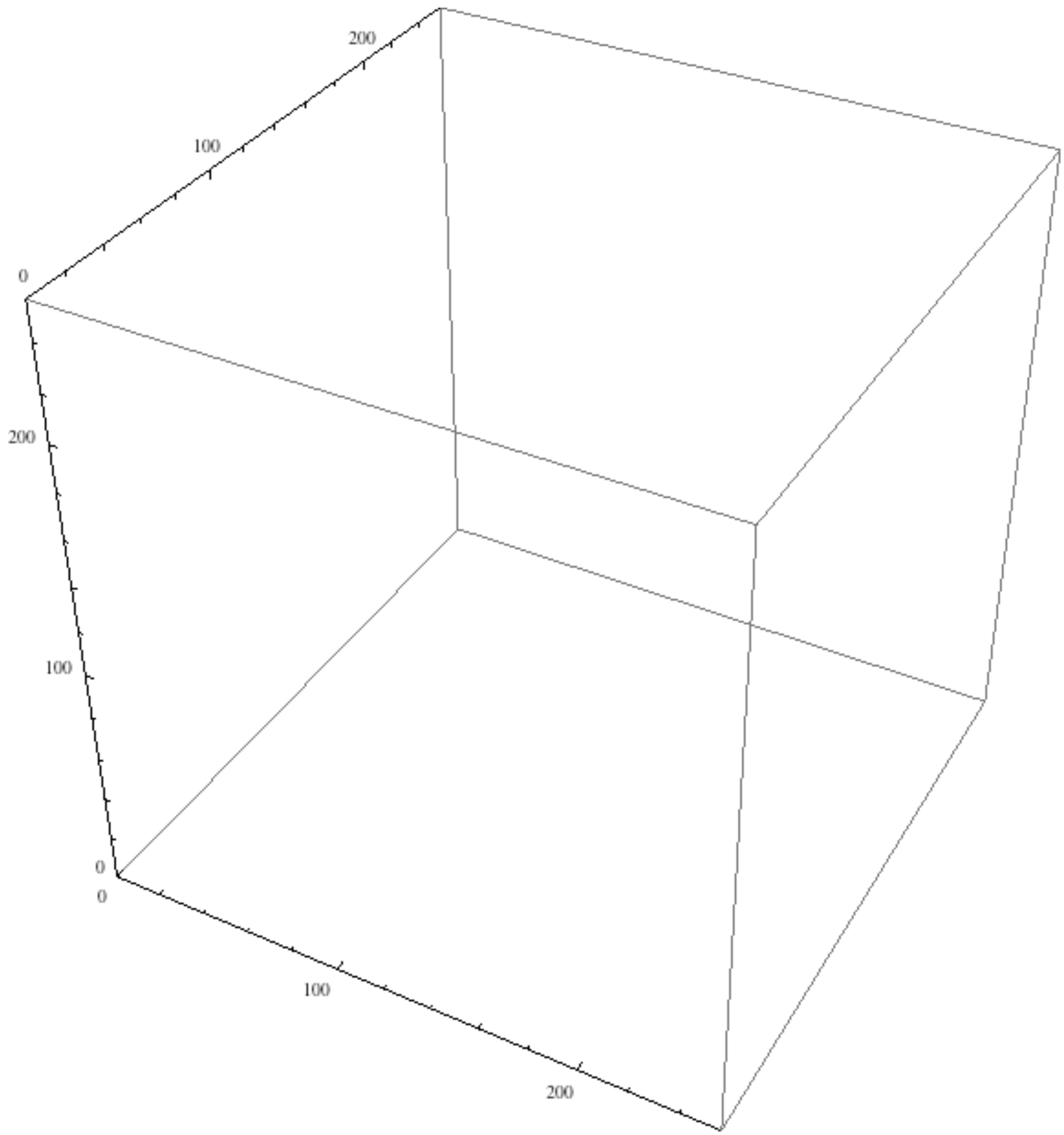
$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0$$

$$\hat{\chi}_k = \frac{1}{(2\pi)^{3/2}}\int d^3k\left(\hat{a}_k\chi_k(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_k^\dagger\chi_k^\star(t)e^{i\mathbf{k}\cdot\mathbf{x}}\right)$$

Parametric Resonance: Quick Sketch

- A > 2q
  - A and q evolve
  - Modes move in and out of resonance
- Pump k-modes
  - Inhomogeneities

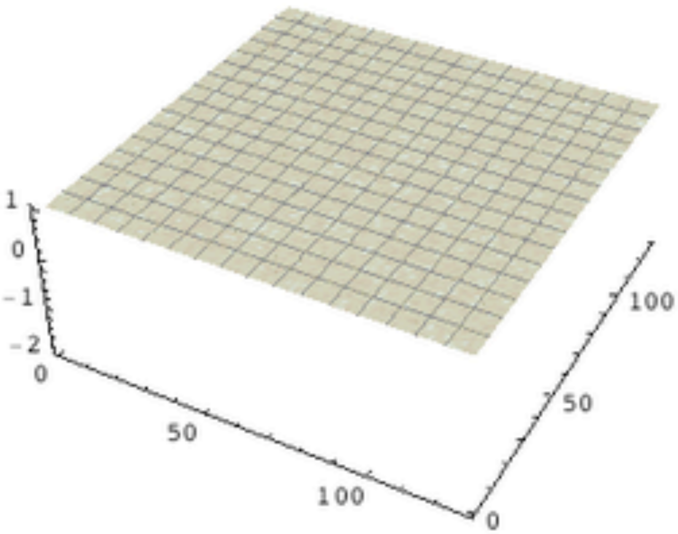


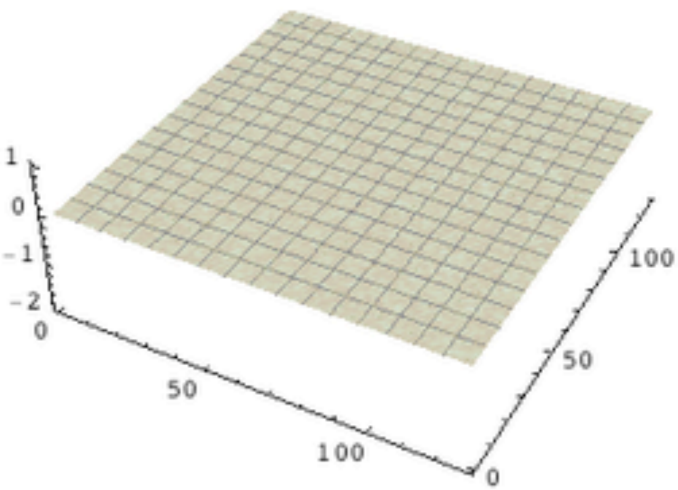


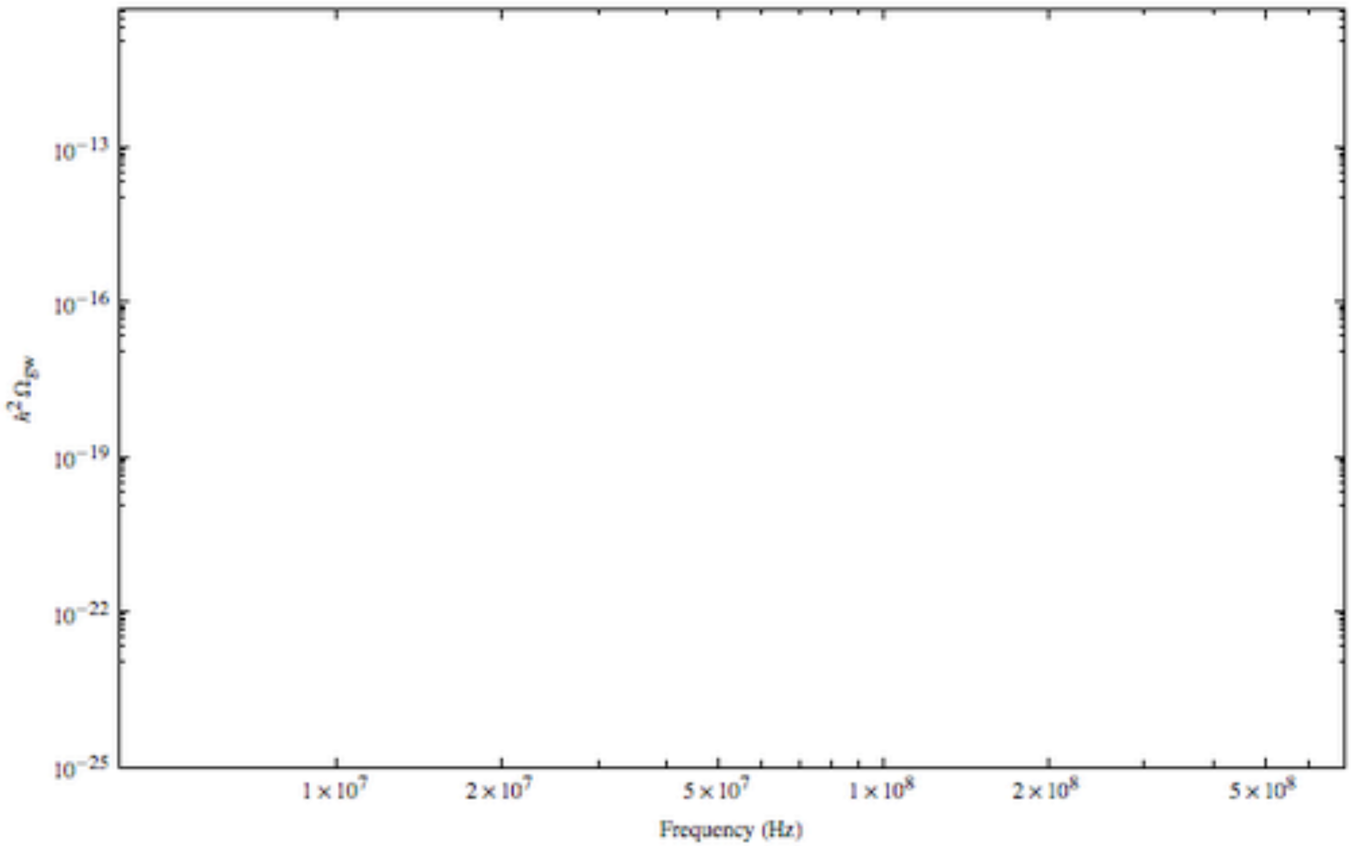
PSpectRe...

Production of Gravitational Waves

- Compute tensor signal:
  - Assume spacetime rigid
  - Solve full nonlinear equations for scalar fields
  - Evolve transverse traceless $h_{\mu\nu}$ (sourced by t-t $T_{\mu\nu}$)
- No back-reaction, but this is small (at least for tensors)
  - $\delta\rho/\rho$ large; metric perturbations small

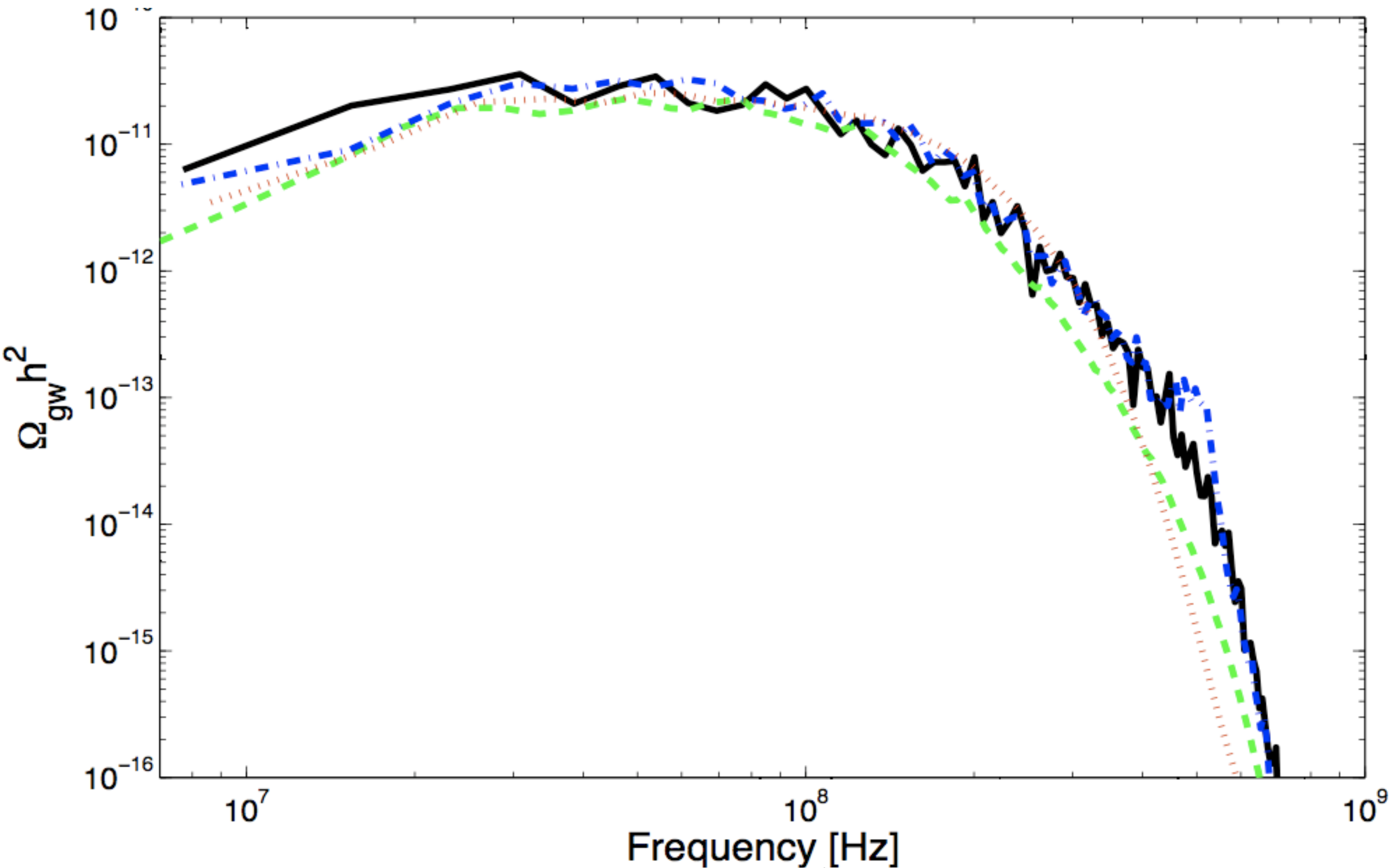






Gravitational wave Background

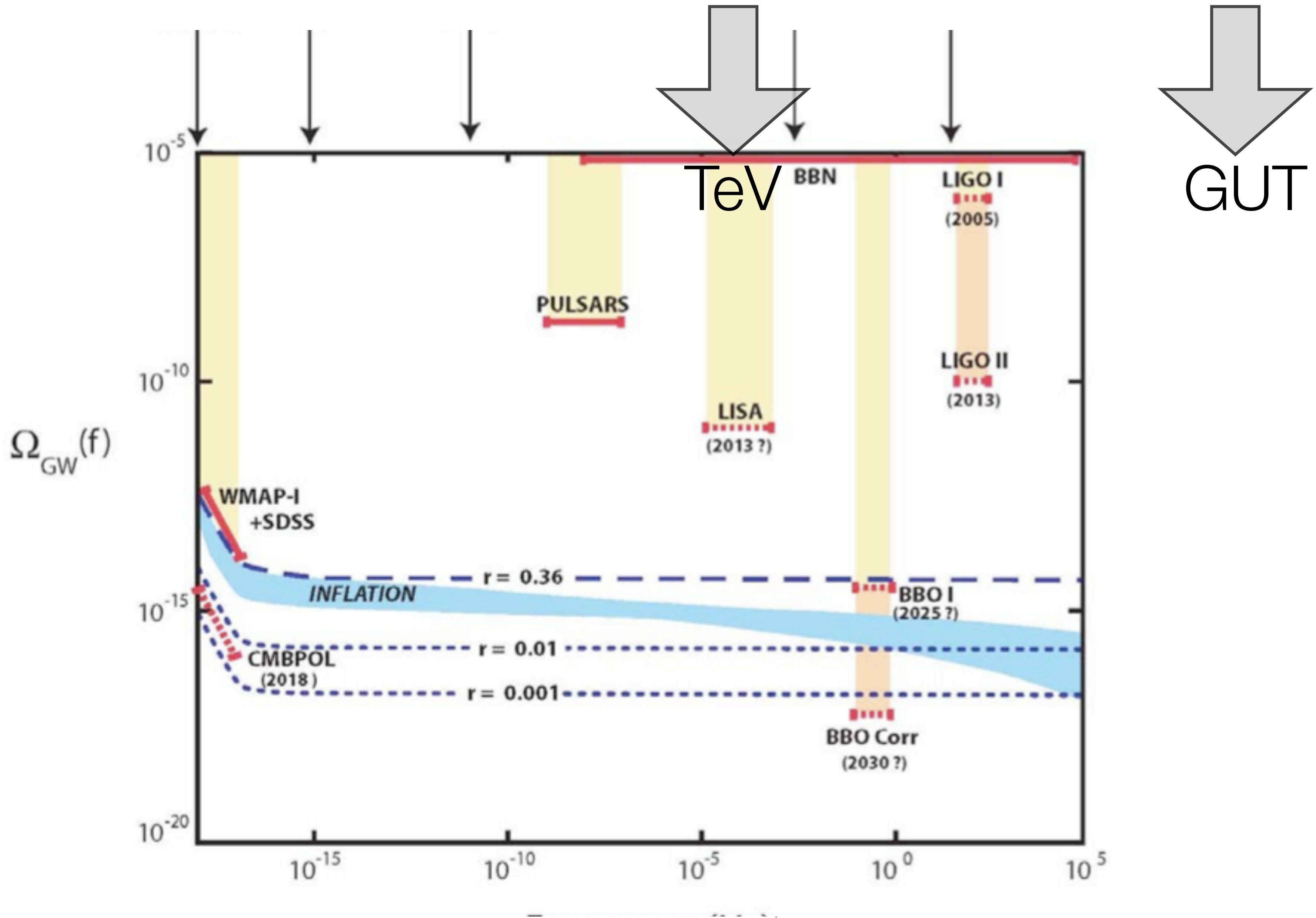
Movie: Giblin



The Spectrum

Location of Peak

- Hubble size at end of inflation: $1/H_{end} \sim (V_{end})^{-1/2}$
  - Instant preheating: $\rho \sim T^4 \sim V_{end}$; $T_{max} \sim (V_{end})^{1/4}$
  - Growth $\sim T_{max} / T_{CMB}$
- Present size of inflationary Hubble patch:
  - $T_{max}/H_{end} \sim (V_{end})^{1/4}$
  - GUT scale inflation: cm - m today; MHz - GHz
  - Inflation at 10 TeV: ~ 10,000,000km10^{-2} Hz
- DECIGO, BBO and advanced ground-based experiments?



Experiments

Height of Peak

- Model dependent
- Gravitational waves require gradient terms
  - Maximal gradient energy less than total density
  - Most models: gradient energy ~ density
- Typically: $d\Omega_{gw}/d\ln k \sim 10^{-5}$ at production, 10^{-10} today.
  - Working on better analytic theory of this now.

Structure of Peak

- Depends on potential and couplings
  - Most power dropped into narrow frequency range
  - Only sampled a small range of models
  - Other preheating mechanisms available
- Very sharp cutoff at high frequencies, k^3 tail
  - Associated with modes that are never in resonance

Current topics...

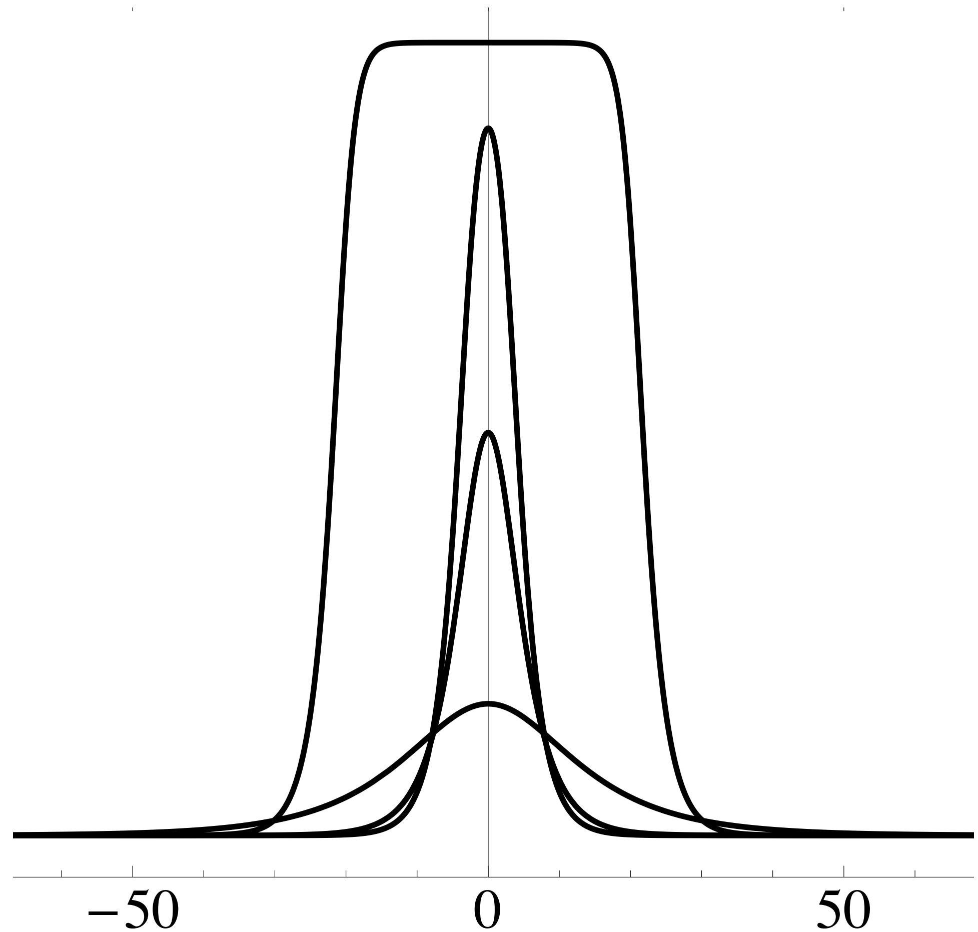
- Achieved "consensus" on overall properties of signal
  - Would like to improve semi-analytic account...
- Non-gaussianity (Bond, Frolov, Huang & Kofman)
- Gravitational wave 3-pt (Adshead & Lim 0912.1615)
- Completion of thermalization?
- Evolution of large local overdensities?
- Key problem: coupling between inflation and "everything else"

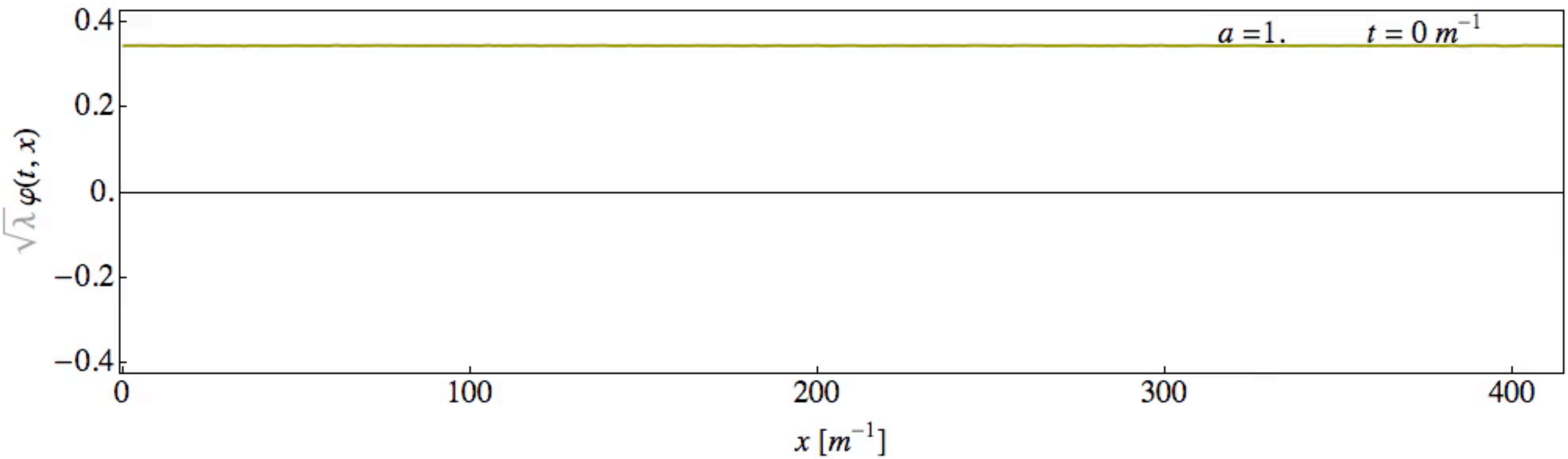
3. Nonlinear Dynamics & Oscillons

- Localized “blobs” of scalar field matter
- Requires a single scalar field with a nonlinear potential
  - Potential must (near origin) increase more slowly than φ^2
  - $V(\varphi^2) = m^2 \varphi^2 - \lambda \varphi^4 + g \varphi^6$
- Related to q-balls and solitons
  - Stationary (≠ soliton), *real* valued field (≠ q-ball)
- Are oscillons relevant to post-inflationary universe?

One Dimensional Oscillons

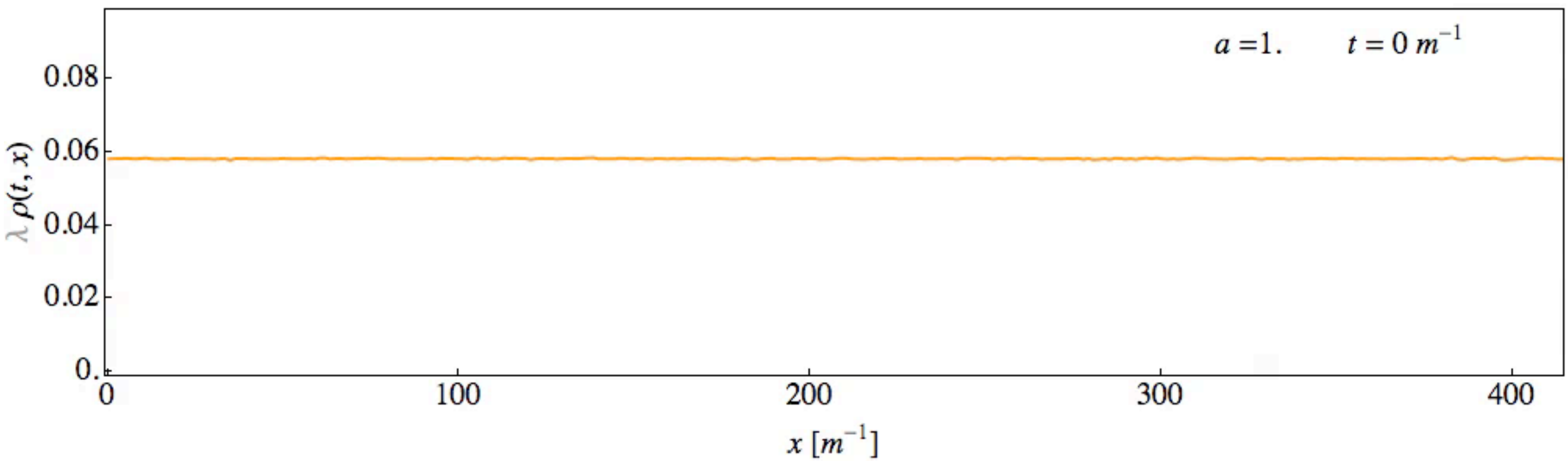
- Semi-analytic theory
  - Amin, Amin & Shirokoff
  - 1002.3380 & 1006.3075
  - “Flat topped” solutions
  - “1/g” expansion
- Formed via resonance





One Dimension: Field

Amin

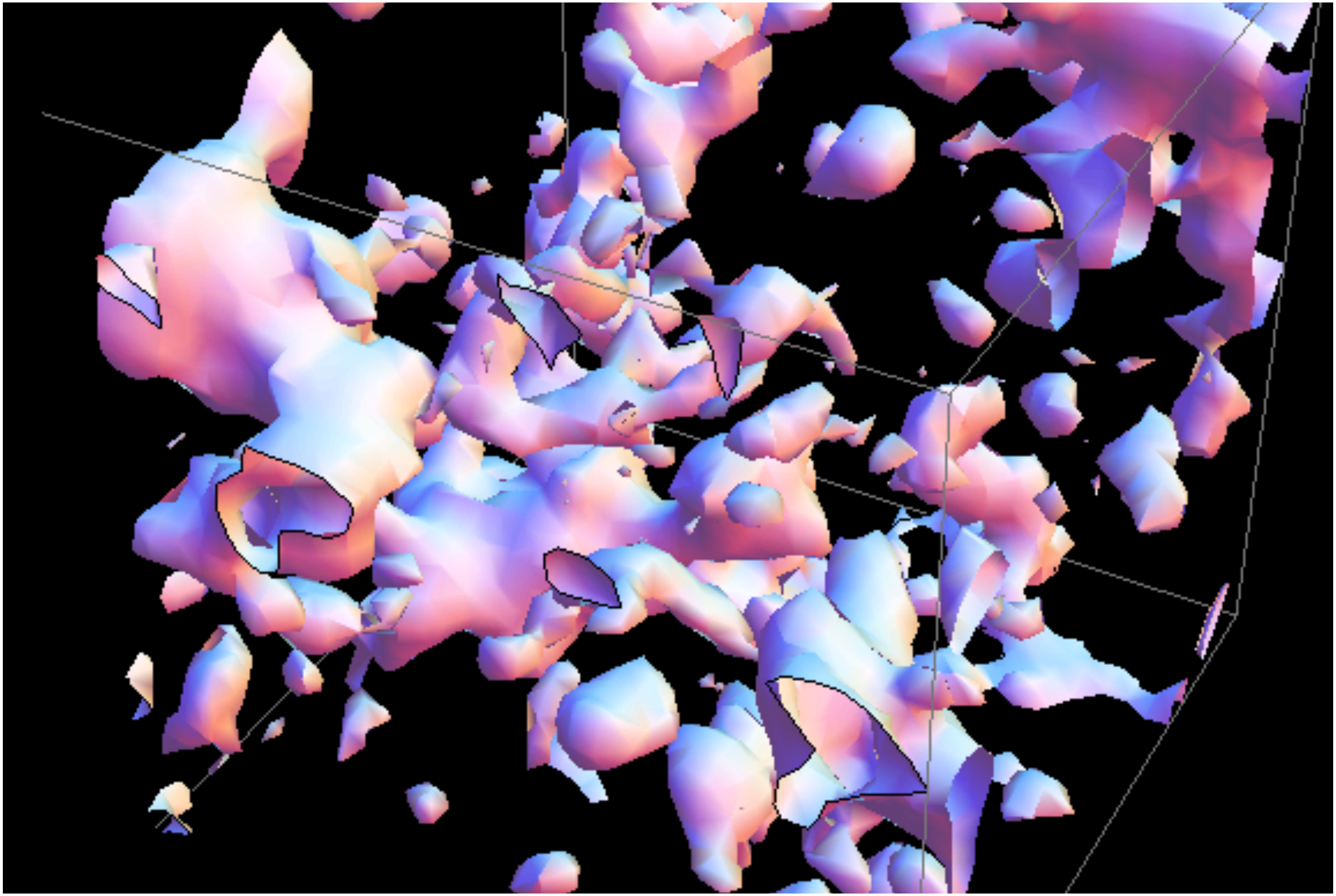


One Dimension: Density

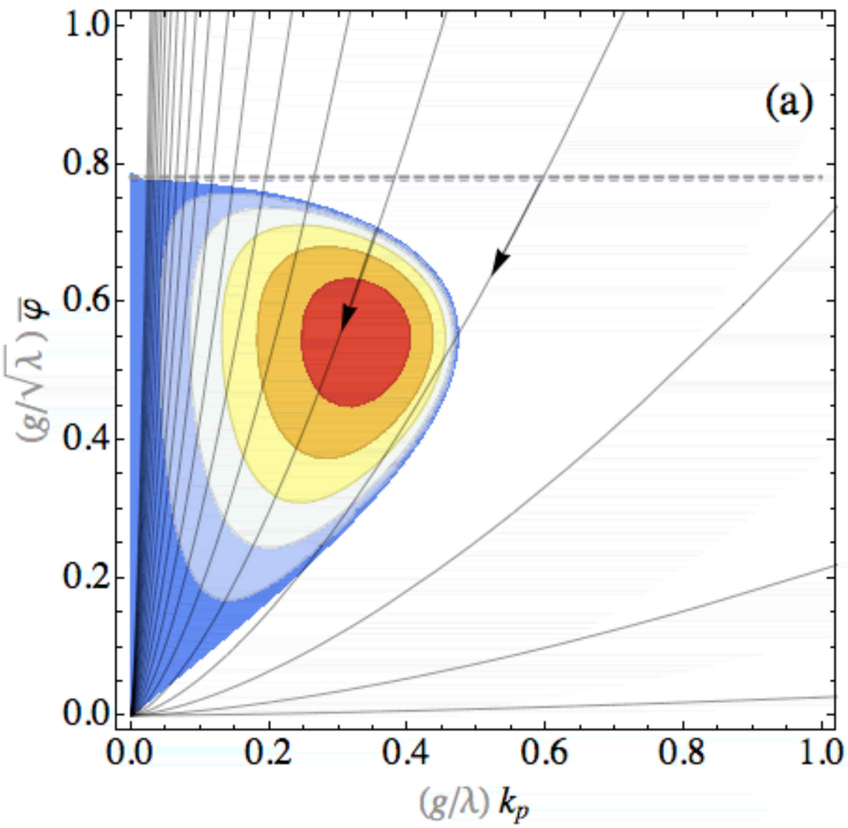
Amin

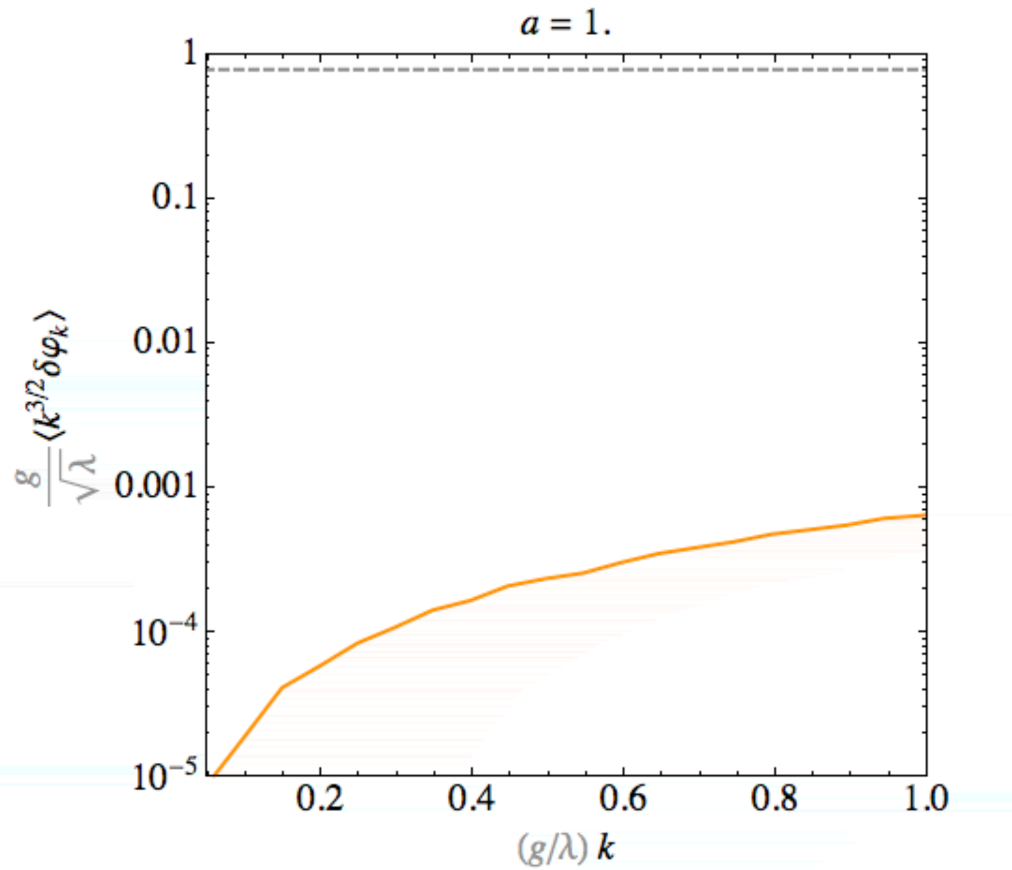
Oscillons in Three Dimensions

- Work in progress: Amin, Easther & Finkel
- 3D approximate generalizations of 1D analytic results
- Questions:
  - Math: do solutions *exist* in 3D, expanding background?
  - Physics: do oscillons *form*, given physical initial conditions?
  - Cosmology: can oscillons *dominate* early universe?
  - Inflation: which inflationary potentials lead to oscillons?
- Answers: YES, YES, YES, WORKING ON IT

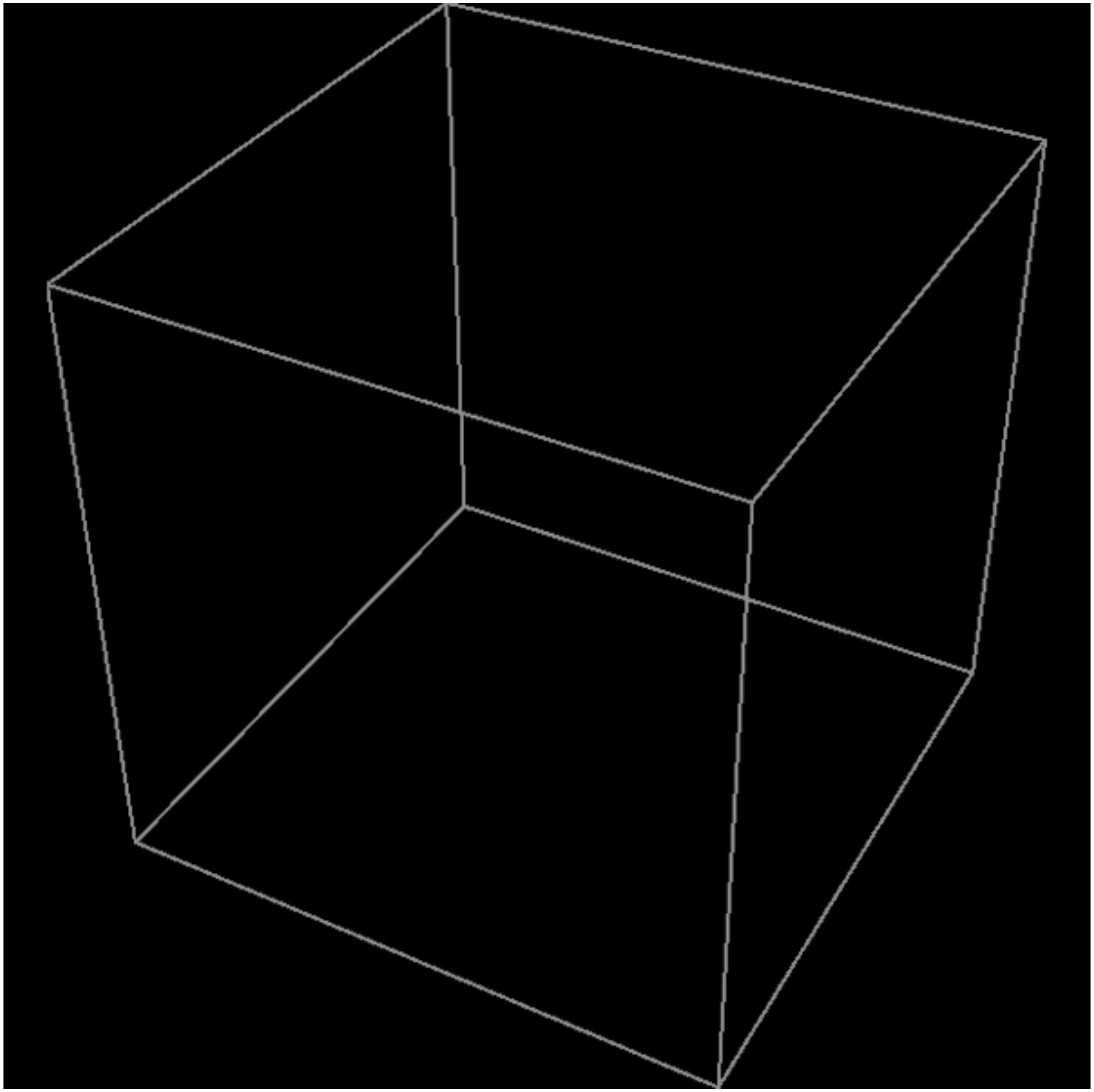


Post-resonance iso-density surface. Standard parametric resonance.

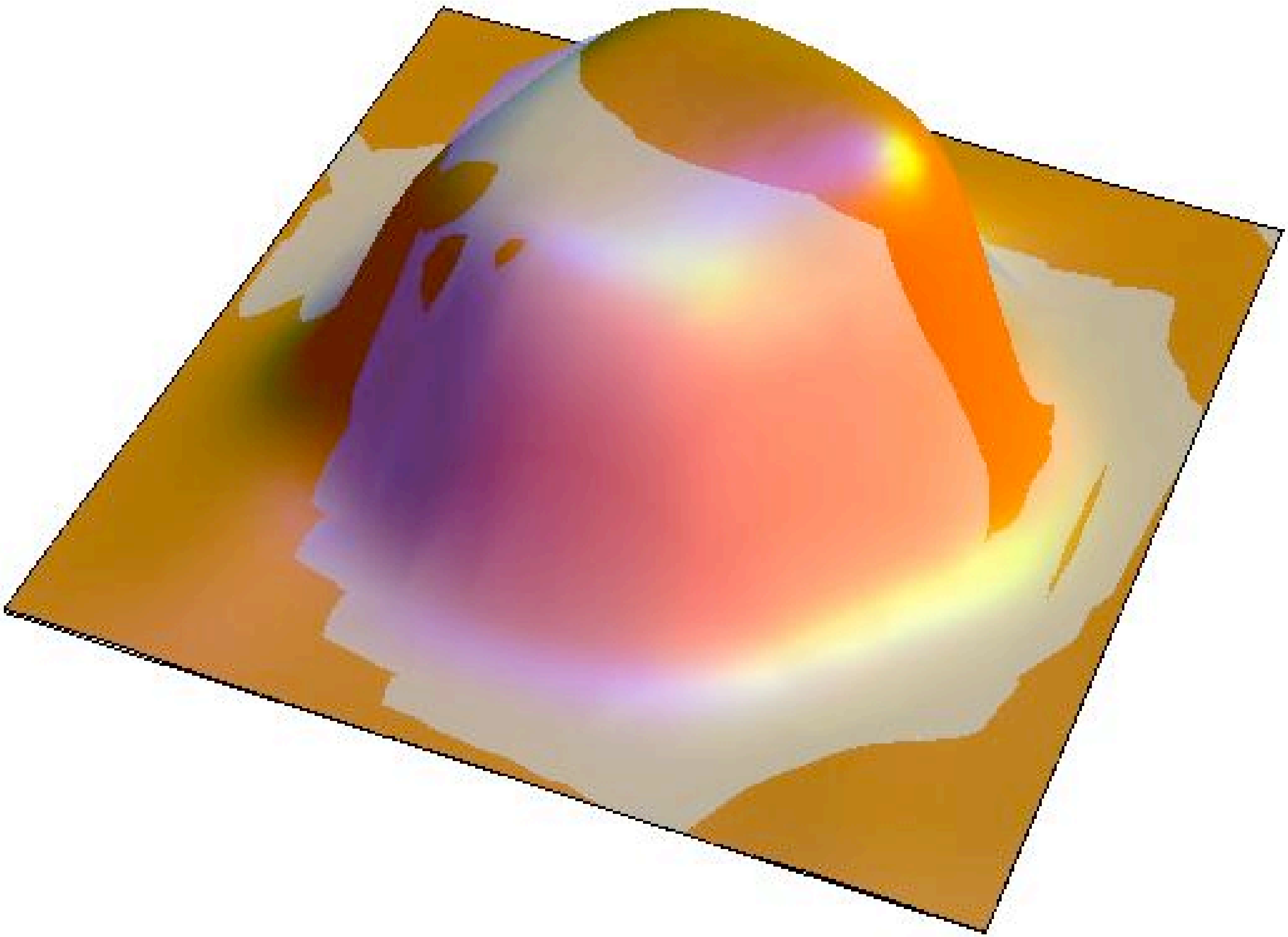




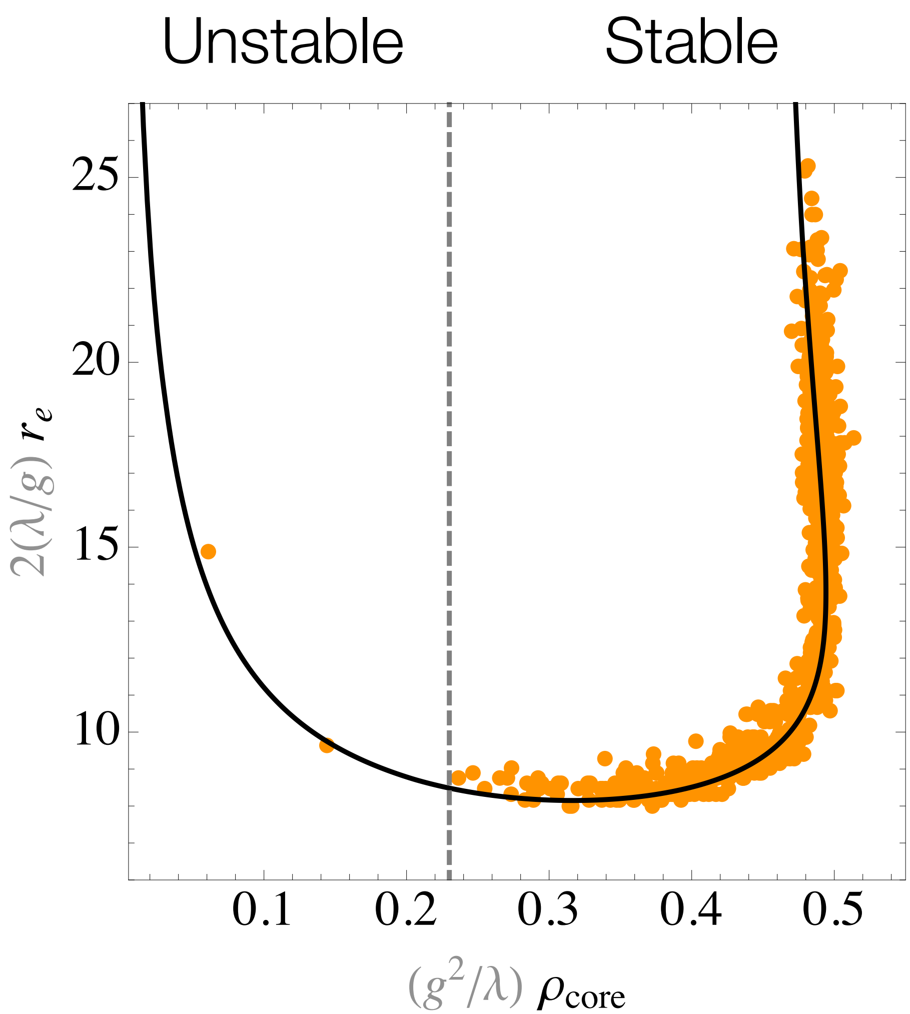
Resonant Growth of Perturbations (3D)

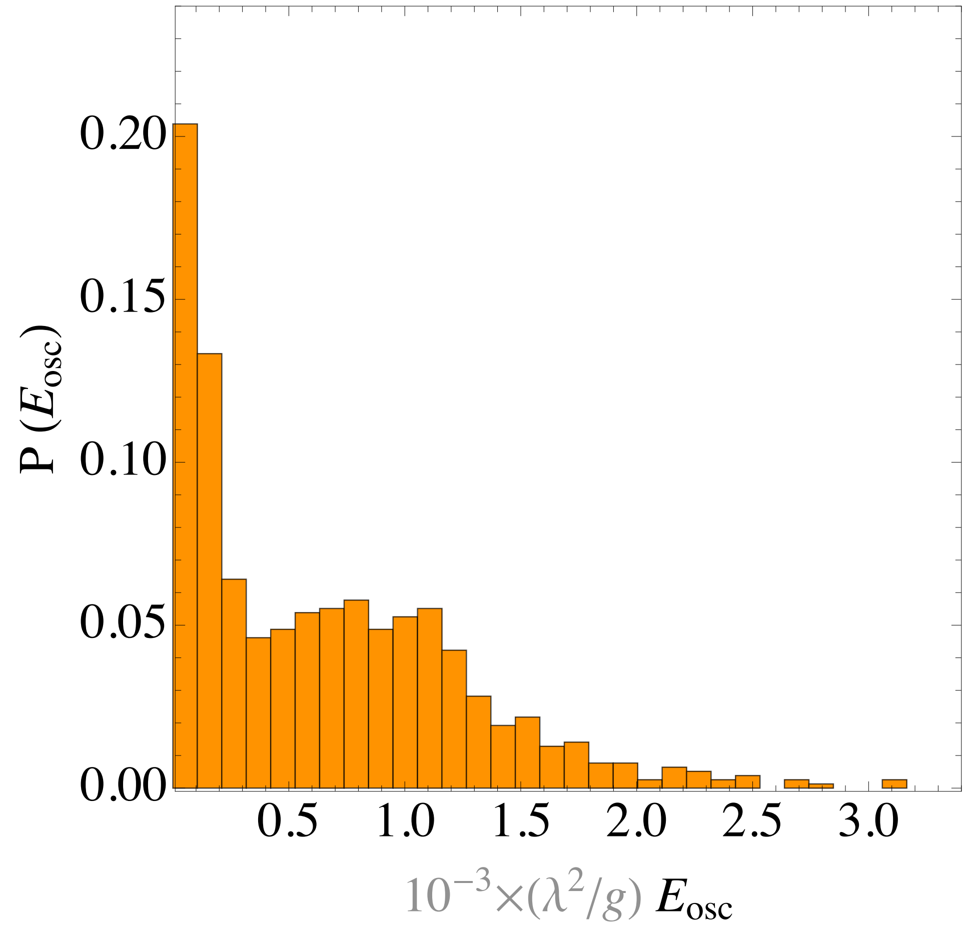


Oscillon Formation (x2 Average Density)
PSpectRe (256^3 Simulation)



Oscillon Density Profile (with semi-analytic fit)





Theory v. numerics

Amin, RE & Finkel

Summary...

- Post-inflation oscillon dominated phase possible...
  - Numerics here for sixth order potential
  - Truncated Taylor series for inflaton potential
  - Realistic scenarios will not match this idealized set up
  - But can produce “blobs” that are stable on timescales ~1/H
- Newtonian potential 0.001-0.01 for these objects
  - Not yet considering *gravitational* backreaction

4. What About Primordial black holes?

- Formed *after* inflation
  - If power spectrum rises at (very) short scales
  - Or nonlinear growth, or resonant growth?
- Contribute *mass* and *radiation* to universe
  - Constraints: BBN, microlensing, x-ray background, dark matter abundance.
- But very small PBH (~1gram) decay *before* BBN
  - Leaves no trace behind??

Anantua, Easther and Giblin

Hawking radiation

- Black hole radiates *all* “massless” states $M << T_{BH}$
  - Including gravitons
  - Quantum source of gravitational waves
  - Rough guess: g states; power in gravitational waves ~ 2/g
- With Anantua and Giblin (PRL, 0812.0825)
  - Grey body corrections in paper (not here).

Computational Strategy

- The evolution equations for the major constituents are:

$$\begin{aligned}\frac{d\rho_{BH}}{dt} &= -3\frac{\dot{a}}{a}\rho_{BH} + \rho_{BH}\dot{M}_{BH} \\ \frac{d\rho_{rad}}{dt} &= -4\frac{\dot{a}}{a}\rho_{rad} - \rho_{BH}\dot{M}_{BH} \\ \frac{\dot{a}}{a} &= \left[\frac{8\pi}{3M_p^2}\left(\rho_{BH} + \rho_{rad}\right)\right]^{1/2}\end{aligned}$$

- Once we know M(t) and a(t), we can calculate

$$\frac{dM_{BH}}{dt} = -\frac{g}{30,720\pi}\frac{M_p^4}{M_{BH}^2}$$

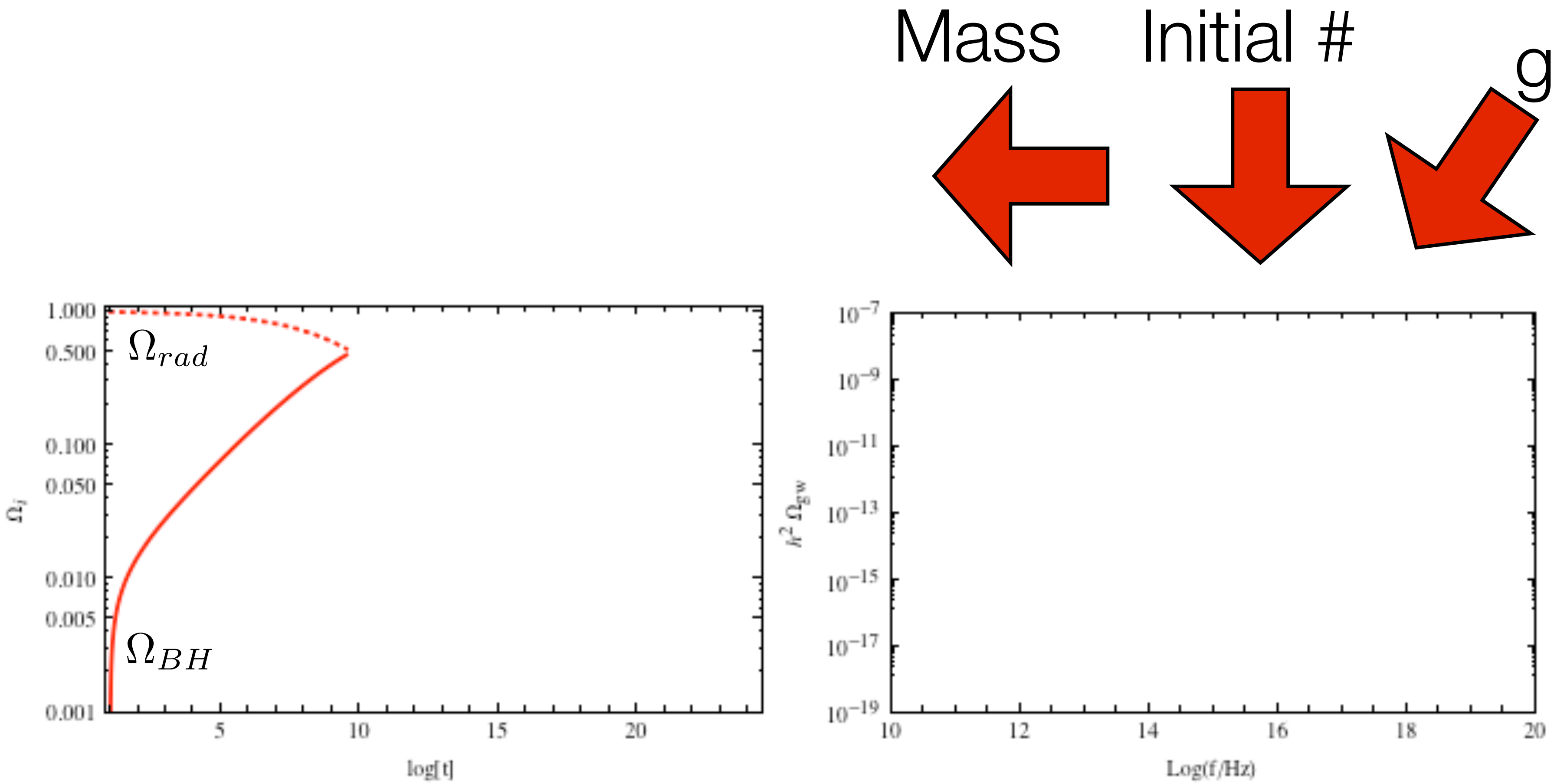
The GW Spectrum

- Number density of gravitons /unit frequency / unit time

$$\frac{dN(k)}{dt} = \frac{2g}{\pi}\frac{M_{BH}^2}{M_P^4}\frac{\tilde{k}^2}{a^2}\frac{1}{e^{\tilde{k}/(aT)}-1}$$

- $\tilde{k}$ is the comoving wavenumber.

$$\Omega_{gw}(f) = \frac{1}{\rho}\frac{d\rho_{gw}}{d\ln f}$$


Mass
Initial #
g
Ω_{rad}
Ω_{BH}
Ω_i
1.000
0.500
0.100
0.050
0.010
0.005
0.001
5
10
15
20
log[t]
$h^2\,\Omega_{gw}$
10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
10^{-19}
10
12
14
16
18
20
Log(f/Hz)

An Example

Matter dominated Phase

- PBH-driven matter dominated phase
  - Matter perturbations will grow
  - PBH live for many Hubble times; can cluster...
- PBH radiate: “Hawking stars”
  - Clustering statistics unknown
  - Open problem
  - Related to inter-oscillon dynamics?

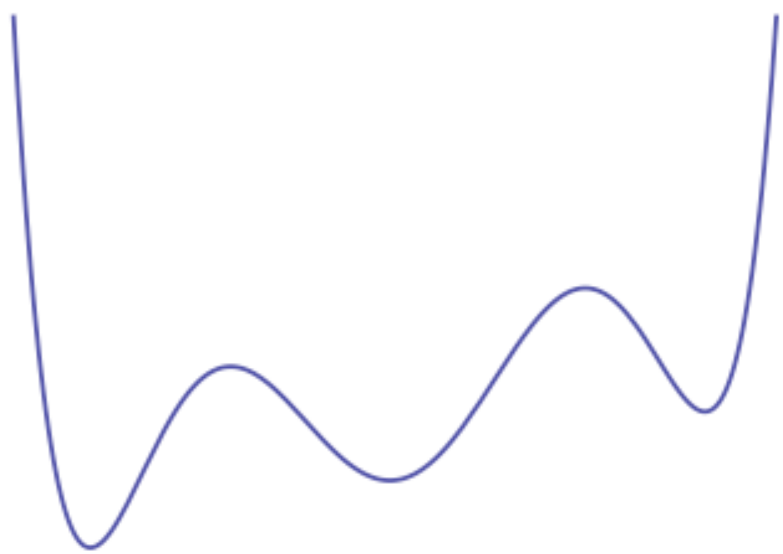
5. Initial conditions for inflation...

- Inflation sets initial conditions for hot big bang...
  - What what sets initial conditions for inflation?
- Worry about this at the homogeneous level:
  - Many fields (chaos), branes in bulk, compact dimensions (topology, # of dimensions)
- But what about at the inhomogeneous level?
  - Colliding bubbles, tuning of initial inhomogeneity

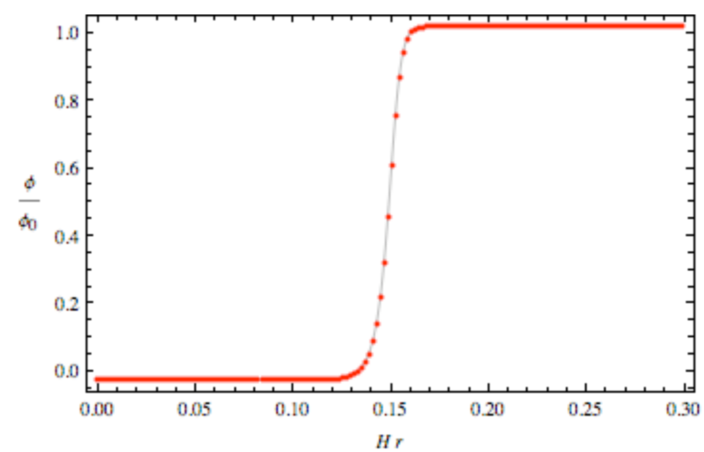
Easther, Giblin, Hui and Lim

New Bubble Collision Mechanism

- Start in highest of three minima
- Nucleate two bubbles in the middle minimum

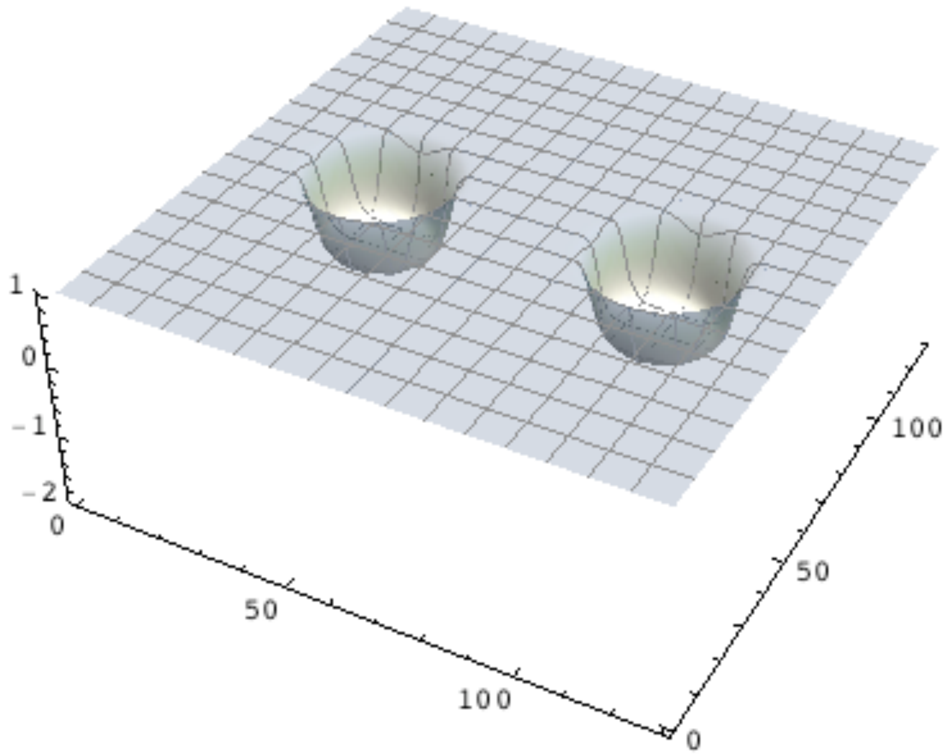


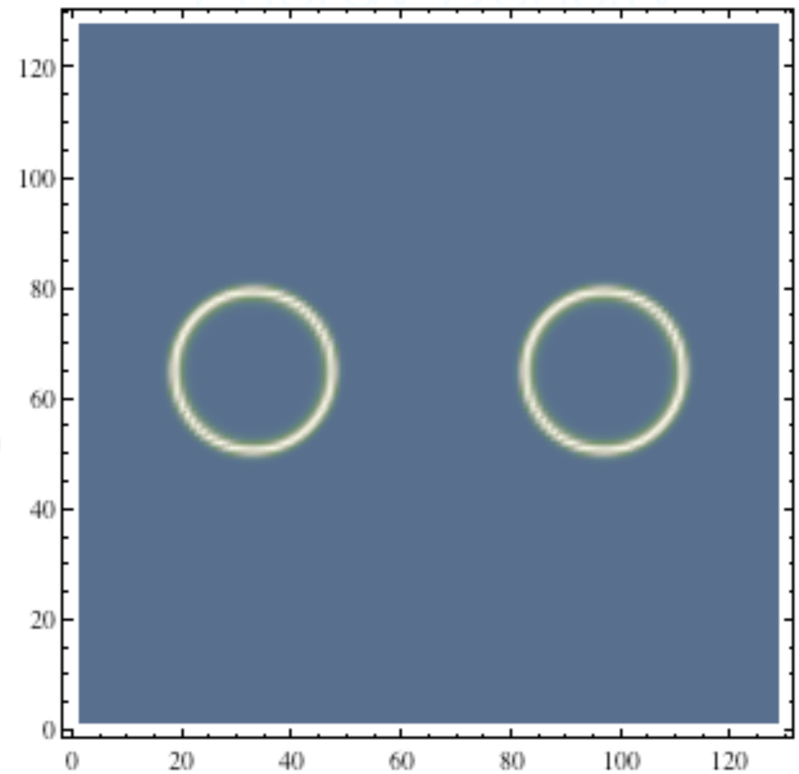
$$\phi = \frac{\phi_0}{\sqrt{1 + 2e^{-\sqrt{2\lambda}\phi_0^2(\rho - R_0)}}}$$



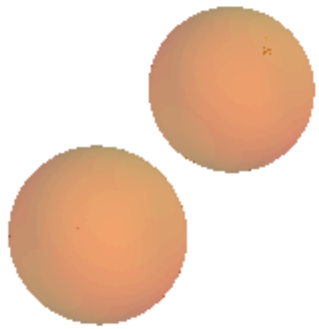
Field Profile (y-z plane)

Energy Density





Two Bubbles



3D View

Consequences...

- Still working on this...
  - *But* new channel for bubble universe production
  - Just solving the Klein-Gordon equation (expanding background, assuming de Sitter for now)
- Implications for “exploring” the landscape?
- Can also worry about pre-inflationary dynamics in general
  - Even without tunneling (especially with coupled fields)
  - Chaos in systems with two or more fields?

6. The Morals of The Story

- Moral #1: Old models with rich new phenomenology
- Moral #2: General program understanding scalar field dynamics (post-inflationary physics, preheating and bubbles)
- Moral #3: Looking at/for signals that are still here today (even if they may be impossible to detect)
- Moral #4: We have to worry about this even in simple models
- Moral #5: Very little thought about *preinflationary* behavior
- Moral #6: Need this in post-inflationary equation of state ("theory error" on any model)