Inflationary Observables

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Concordance Parameters

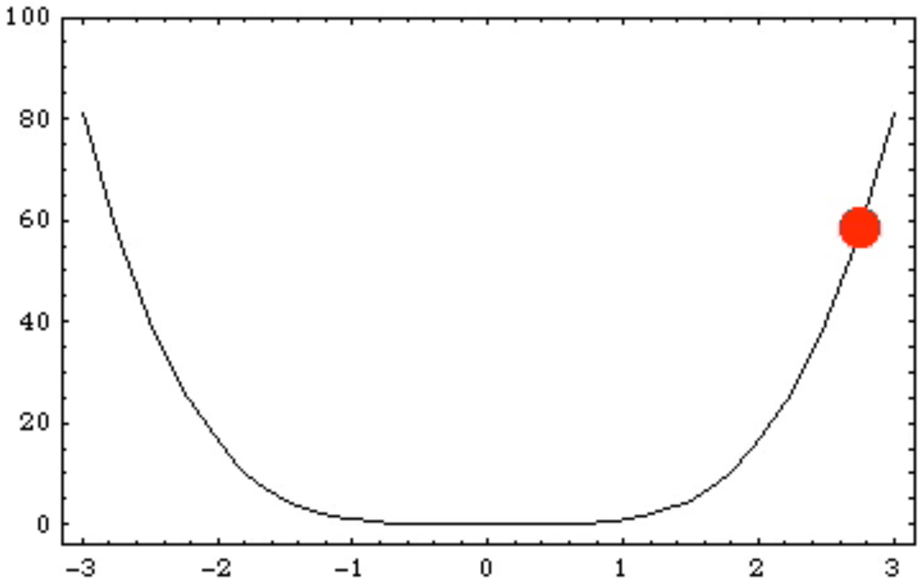
h	Hubble's "constant"	When we are looking
τ	Reionization	First stars (gastrophysics, nuclear physics)
A_{SZ}	Sunyaev-Zeldovich Amplitude	Scattering of photons by hot gas in clusters
Ω_b	Baryon fraction (Mass known, #??)	Baryogenesis (? - GUT, Electroweak?)
Ω_{CDM}	Dark matter (Mass ??, #??)	TeV Scale physics?? Supersymmetry? LHC?
Ω_Λ	Cosmological constant	Quantum Gravity Magic?
A_s, n_s	Primordial Perturbations	Inflation GUT/string physics?

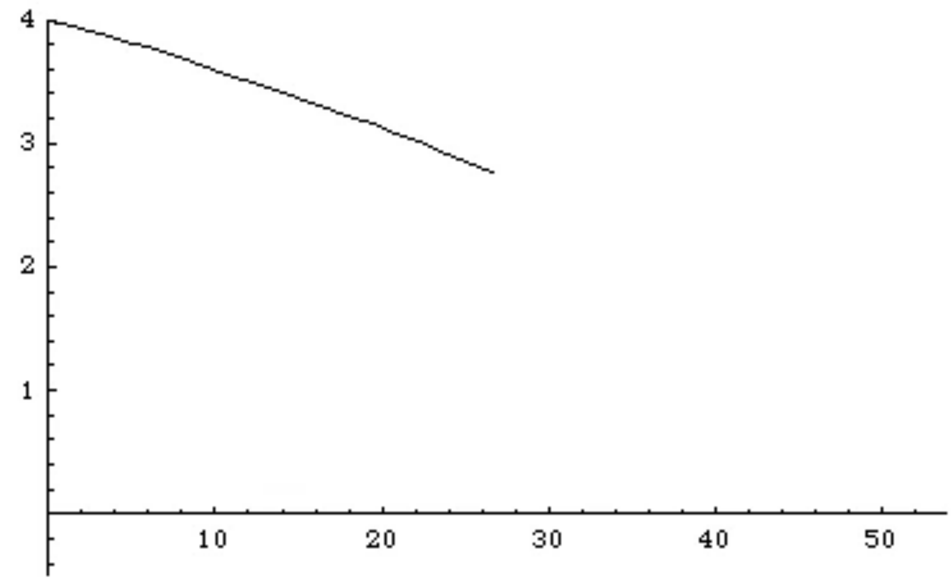
Concordance Cosmology

- Requires initial perturbations
  - Does not say where these perturbations come from
  - Does not explain flatness, homogeneity etc.
- Previously we looked at the tree
  - Now we explore the roots.

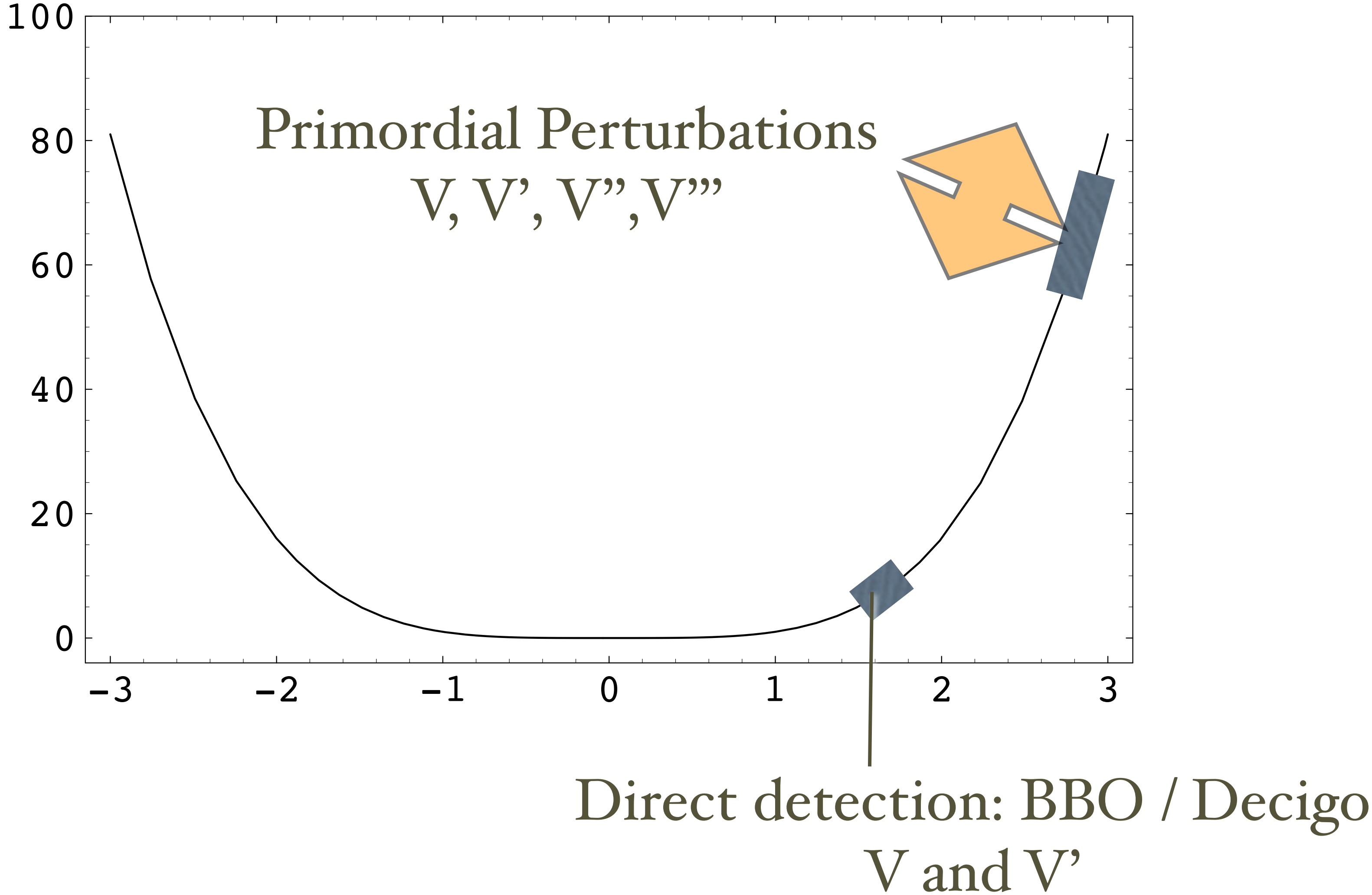
Inflation...

- Standard cosmological paradigm
  - Early universe undergoes accelerated expansion
- High energy physics
  - Inflation not driven by standard model fields
  - Beyond Standard Model (e.g. ~TeV scale or above)
  - GUT scale, in many theories
- Sourced by matter with negative pressure





Inflation: Cartoon Version



Inflation: Cartoon Version

Perturbation Amplitude

- Scalar field in de Sitter space $\delta\phi \approx \frac{H}{2\pi}$ $\quad H^2 = \frac{1}{3M_p^2}\left[\frac{\dot{\phi}}{2} + V(\phi)\right]$
- Klein-Gordon equation in expanding background
  - $\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$ $\qquad H \equiv \frac{\dot{a}}{a} = \frac{d\log a}{dt}$
- Expansion (e-folds) N = log(a)

$$N = \int H dt = \int \frac{H}{\dot{\phi}} d\phi \Rightarrow \delta N \sim \frac{dN}{d\phi}\delta\phi = \frac{1}{2\sqrt{3}\pi M_p^3}\frac{V^{3/2}}{V'}$$

Density Perturbations

- If field jumps "uphill", inflation lasts a little longer
  - Inflation ends a little later, density a little higher: $\delta N \sim \delta\rho/\rho$
  - Observations: $\delta\rho \sim 10^{-5}\rho$ in early universe.
  - Fixes inflationary scale (simple potentials; GUT scale)
- For simple potential $\delta\rho/\rho$ decreases (slowly) with time
- Metric perturbations / gravitational waves ~ H
  - Bound on gravitational wave background limits H (or ρ)

The Story So Far...

- Inflationary perturbations are a function of the potential
  - Minimal inflation: potential defines the model
  - Also kinetic term, coupling to gravity, other fields.
  - MANY inflationary models
- To make predictions we need relevant value of ϕ

Cosmological Horizon

- Key number: Hubble length: 1/H "Hubble horizon"
  - Modes with wavelength larger than 1/H do not evolve GR
  - Comoving wavenumber k
  - Physical wavenumber k/a (decreases with expansion)
- Mode "crosses the horizon" when $k = aH = \dot{a}$
  - Inflation: accelerated expansion - modes *leave* horizon
  - After inflation: modes re-enters horizon
  - Long modes leave before short modes, re-enter later

The duration of inflation

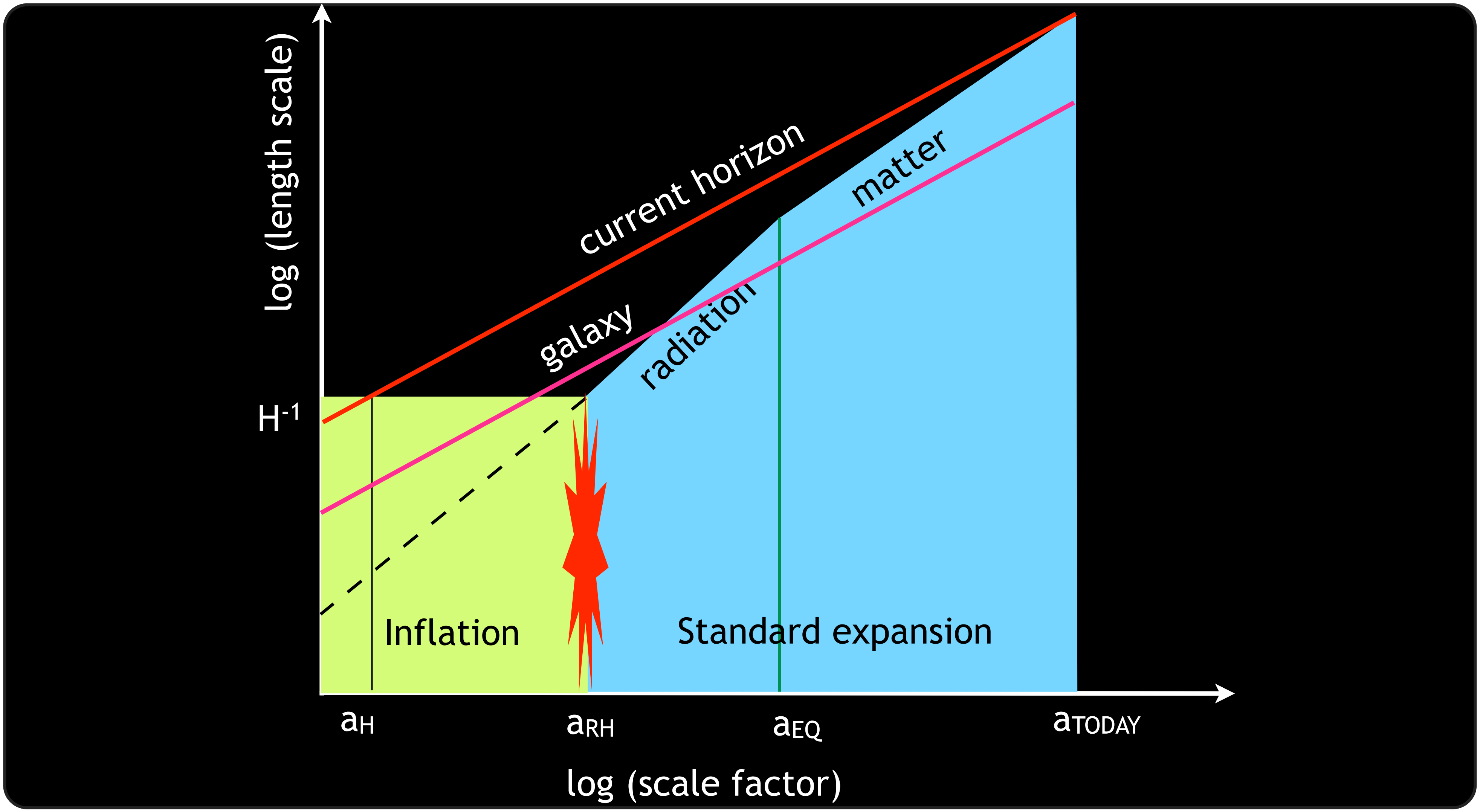
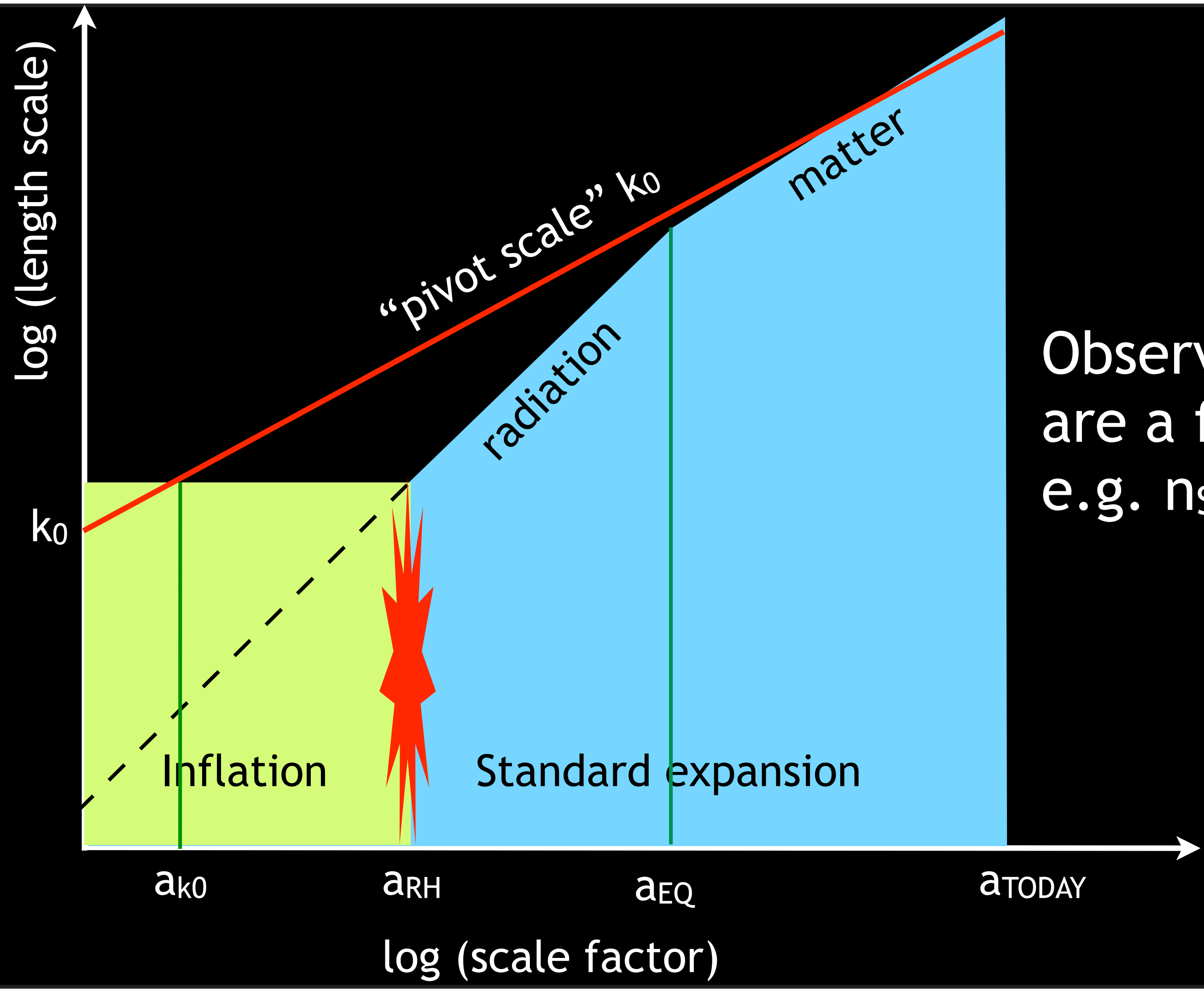


Figure: Peiris

What happens after inflation?

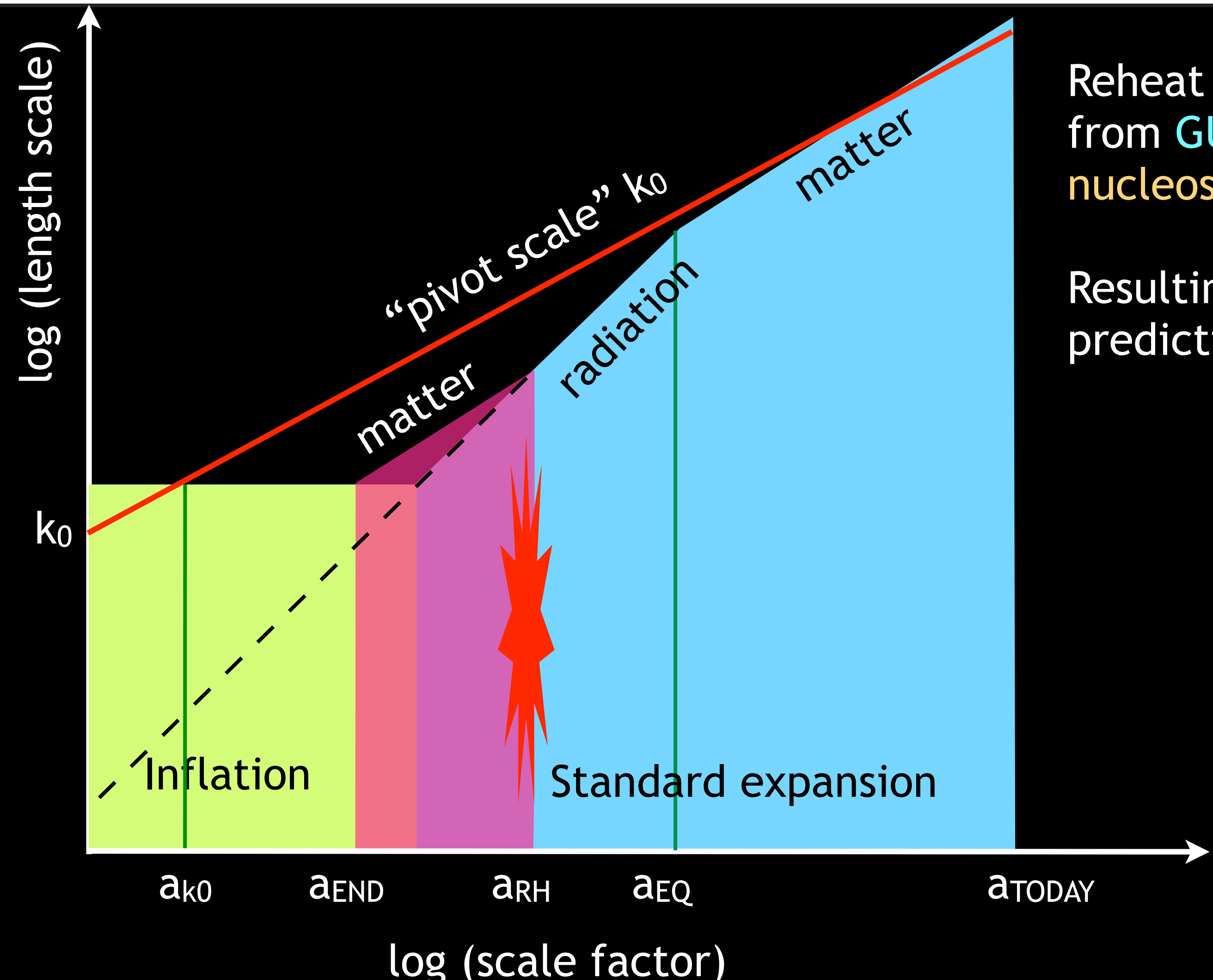
- During inflation universe cold
  - Almost (no) particles
- Successful inflationary model must *reheat*
  - Take energy from inflaton; convert to standard model states
  - Hard limit: must reheat by MeV scales (nucleosynthesis, ν)
  - But inflation (potentially) at GUT scales
  - Huge range of scales; largely unknown particle physics

Pivot Scale



Observable parameters are a function of scale! e.g. $n_S[k(N_{efold})]$

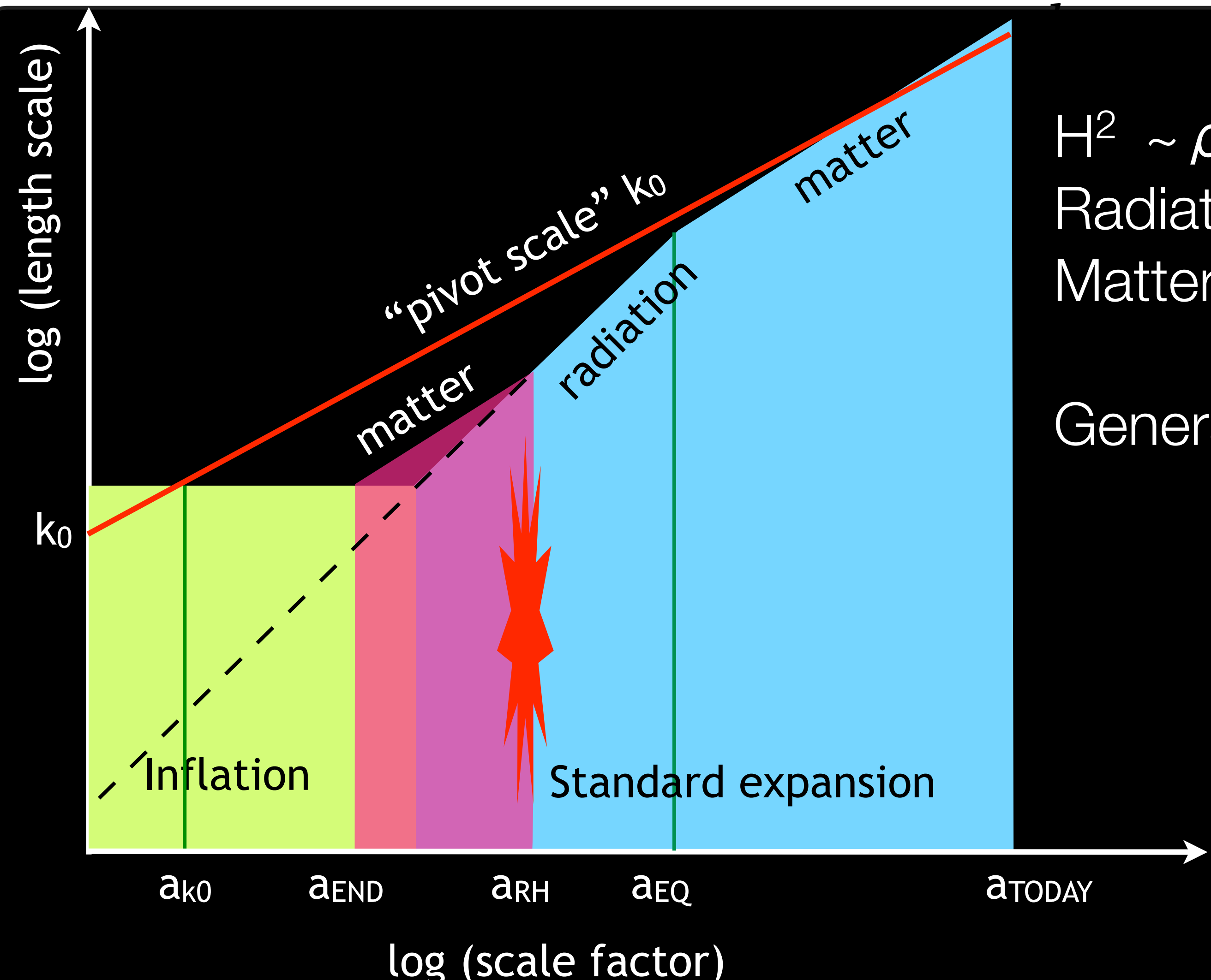
Connecting measurements to model



Reheat temperature can vary from GUT scale (10^{15} GeV) to nucleosynthesis scale (1 MeV)!

Resulting uncertainty in predictions at a given "pivot"

Matching Equation



$H^2 \sim \rho$

Radiation: $\rho \sim a^{-4}$; $H^{-1} \sim a^2$

Matter: $\rho \sim a^{-3}$; $H^{-1} \sim a^{3/2}$

General: $a^{-2} > \rho > a^{-6}$

Matching Equation

- Connects horizon entry and exit $$\frac{k}{a_0 H_0} = \frac{k}{a_\star H_\star}$$

- $$\frac{k}{H_0 a_0} = \frac{a_k H_k}{a_0 H_0} = \frac{a_k}{a_{end}} \frac{a_{end}}{a_{reh}} \frac{a_{reh}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k}{H_0}$$

- $$N = \log\left[\frac{a_{end}}{a_{reh}} \frac{a_{reh}}{a_{eq}} \frac{a_{eq}}{a_0} \frac{H_k}{H_0}\right] - \log\left[\frac{k}{H_0 a_0}\right]$$

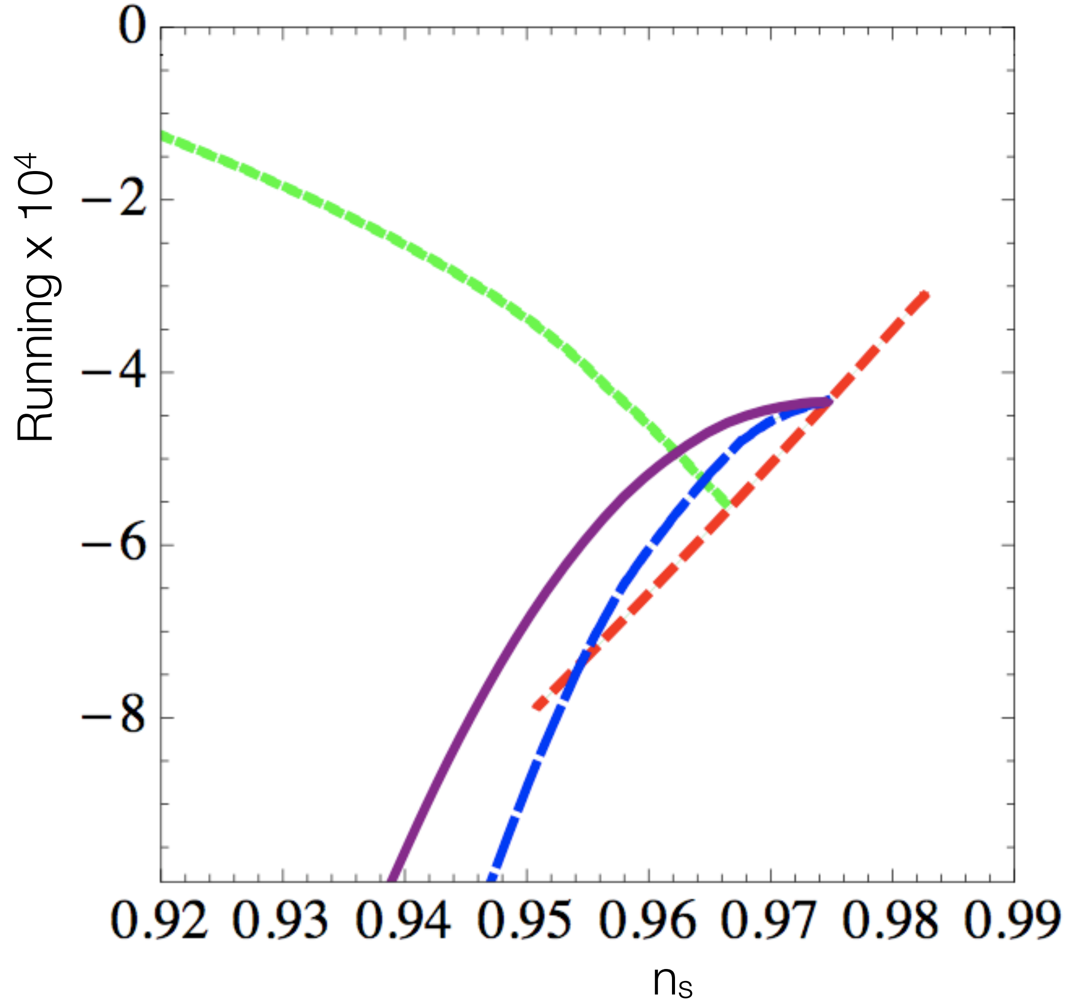
- Assume long matter dominated phase (GUT - TeV)
    - $\Delta N \sim 9$; general equation of state, to MeV scale $\Delta N \sim 30$

Spectral Parameters

- Primordial spectrum specified by empirical parameter
  - $P(k) = A_s \left(\frac{k}{k_0}\right)^{n_s(k)-1}$
  - $n_s(k) = n_s(k_0) + \alpha_s \log\left(\frac{k}{k_0}\right) + \cdots \quad ; \alpha_s \equiv \frac{dn_s(k)}{d\log k}$
- α is the *running*
- Typically $|n_s - 1| \sim N^{-1}$, $\log(k) \sim N$, $\alpha \sim -N^{-2}$, $10^{-3} > |\alpha| > 10^{-4}$
- Can only be detected with very futuristic experiments

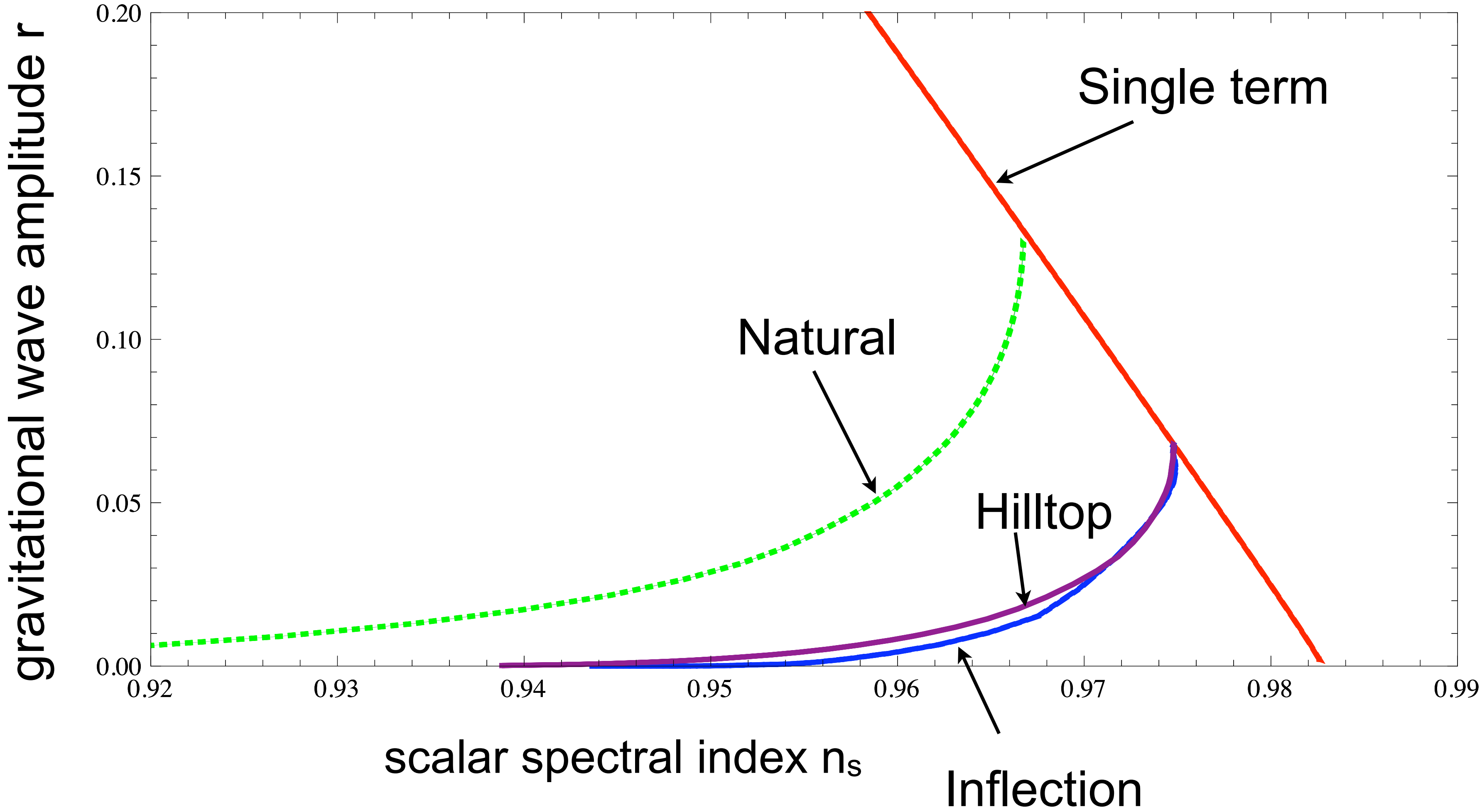
Running: Simple V(ϕ)

- Green: Natural inflation
  - $\Lambda^4 [\cos(\Phi/f) + 1]$
- Red: Single term
  - $\lambda\Phi^p$
- Purple: Hilltop
  - $\Lambda^4 - \lambda\Phi^4$
- Blue: Inflection
  - $\Lambda^4 - \lambda\Phi^3$
- One parameter fixed by A_s



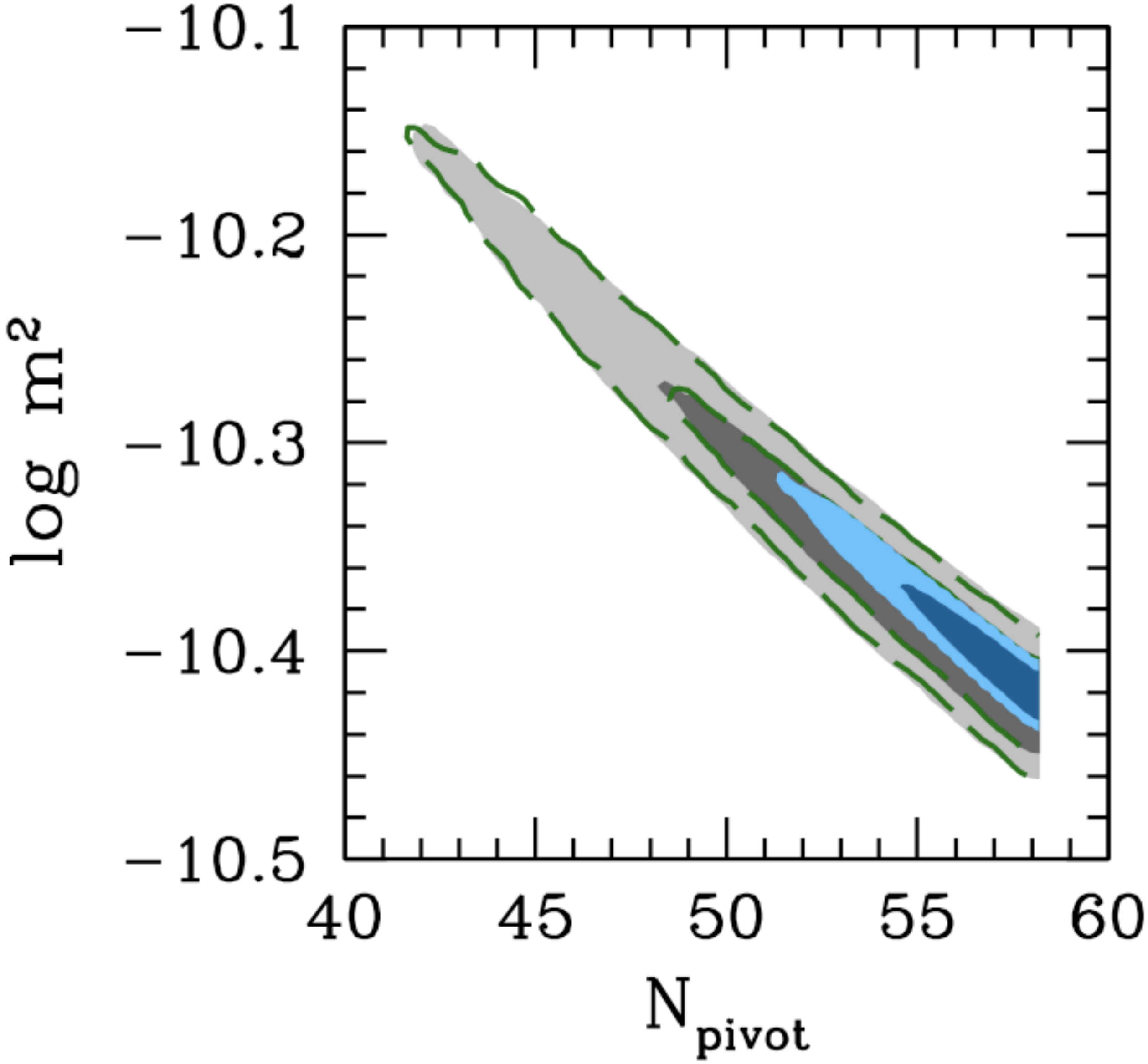
Given that n_s is a function of reheating...

- For *specific* inflationary model
  - Measure n_s and r accurately: $\Delta n_s = \alpha\ \Delta N \sim 0.005$
  - Constrain post-inflationary expansion
  - Constrain physics between TeV and GUT scales
- How well can we do this?
  - Mortonson, Peiris & RE [ModeCode] arXiv:1007.4205
  - Adshead, RE, Pritchard and Loeb arXiv:1007.3748



Spectral Index v.
Tensor amplitude

$$P(k) = A_s\, k^{n-1}$$
$$r = A_{gw}/A_s$$



Constraints for Quadratic Inflation

Peiris, Mortonson, Easther

Grey -- WMAP7 (data)
Blue -- Planck (simulation)

	Natural		ϕ^n	
	N	f	N	n
fiducial values	51	$\sqrt{8\pi}$	51	2
Planck	5.1	-	3.6	-
	-	0.33	-	0.25
	14.5	0.93	19.7	1.4
$+\ \sigma_r = 0.01$	3.5	0.26	8.6	0.41
CIP+Planck	1.69	-	1.2	-
	-	0.11	-	0.09
	13.7	0.87	14.5	1.14
$+\ \sigma_r = 0.01$	2.8	0.18	3.96	0.27
FFTT+Planck	0.41	-	0.29	-
	-	0.027	-	0.024
	7.0	0.45	11.0	0.91
$+\ \sigma_r = 0.01$	2.5	0.17	2.95	0.24

Forecasts for Future Experiments

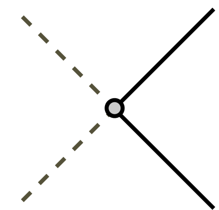
W. Adshead, Pritchard and Loeb

Waiting for Thermalization

- In simple models, thermalization is *naturally* slow
  - Inflaton self-coupling small (to protect slow roll)
- Consequently: inflaton weakly coupled to *other* fields
  - To protect inflaton self-coupling from loop corrections
  - Tree-level particle production inefficient
  - Also need to look at growth of inhomogeneities
- Some models: parametric resonance, rapid thermalization

Parametric Resonance: Quick Sketch

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m^2\phi^2 - \frac{g^2}{2}\phi^2\chi^2$$



- φ is the inflaton field
- χ is a (massless) field coupled to the inflaton
  - Perturbations in χ, canonically quantized

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\phi^2\right)\chi_k = 0$$

$$\hat{\chi}_k = \frac{1}{(2\pi)^{3/2}}\int d^3k\left(\hat{a}_k\chi_k(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_k^\dagger\chi_k^\star(t)e^{i\mathbf{k}\cdot\mathbf{x}}\right)$$

Moduli Domination

- SUSY moduli
  - Weakly coupled, masses ~ 10^9 GeV
  - If universe thermalized early, produced naturally
  - Decouple as universe expands
  - Analogous to dark matter in *present* universe
- Must decay before nucleosynthesis
  - Puts constraints on parameter space
  - Or removed by thermal inflation (Stewart and Lyth)

More Exotic Scenarios

- Temporary cosmic string domination
  - String network has negative pressure
  - Density ~ a^{-2}
- Kination - phase dominated by scalar field kinetic term
  - Density ~ a^{-6}
- Thermal inflation - short period of secondary inflation
- Best approach (?) an effective (average) equation of state
- **KEY MESSAGE: CANNOT *ASSUME* THERMALIZATION**

What Does This Mean...

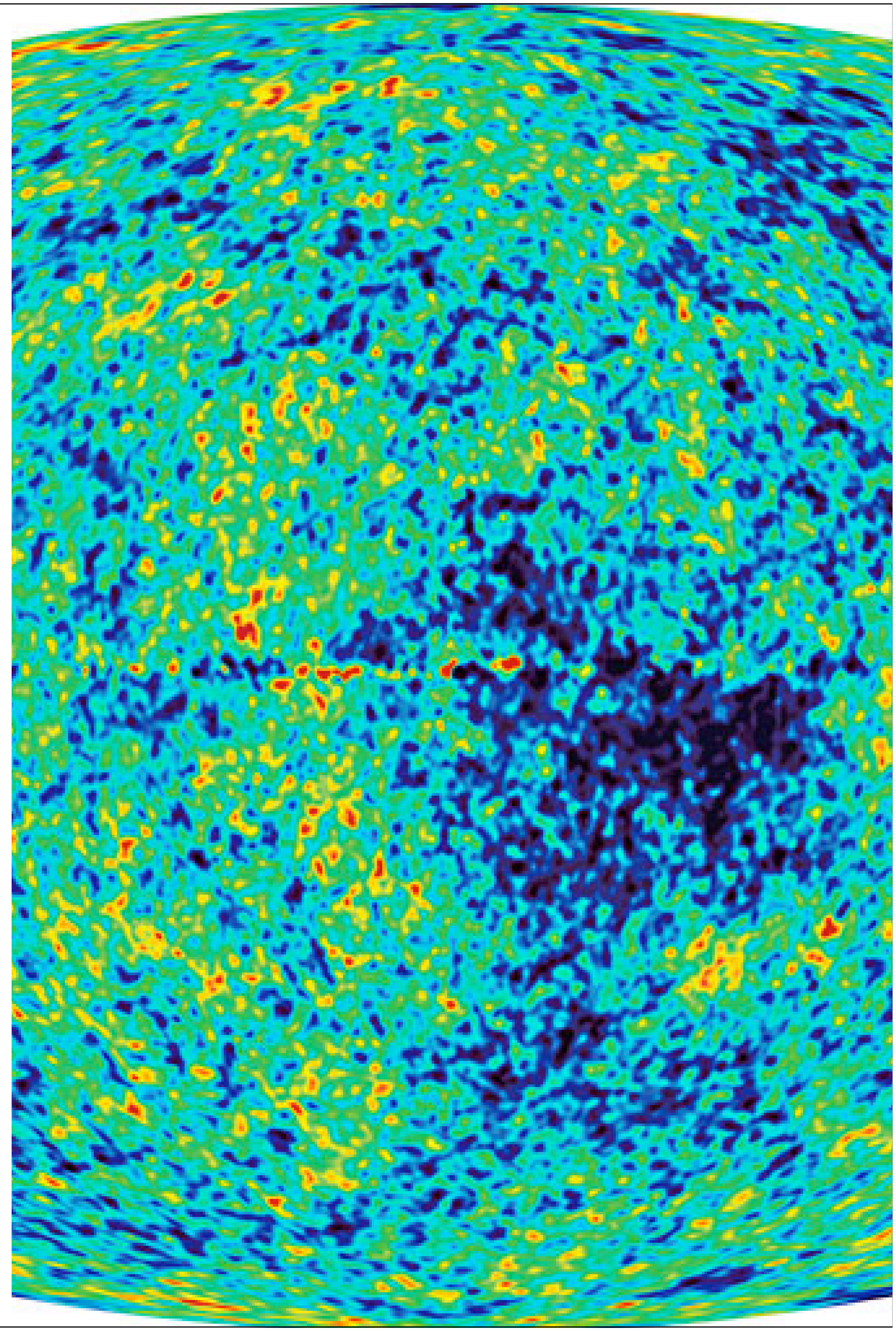
- Interpretation is subtle
  - We do not probe reheating (>TeV scales) on its own
  - We do not probe inflation on its own
  - Inflation and reheating history are now *linked*
- Given assumptions about reheating (e.g. rapid thermalization)
  - We can test *specific* inflationary models (and *vice versa*)
- Different inflation models require different reheating histories
  - *Any* hint about beyond TeV scale physics is worth having!

Other Inflationary Observables

- I have focussed on the power spectrum
  - And simple inflationary models
- Power spectrum is the inflaton 2-point function
  - To lowest order, perturbations are *Gaussian*
  - (2n+1)-point zero, (2n)-point known from 2-point
- But perturbations couple in early universe
  - Gravitationally, and field-field in multi-field models

Non-Gaussianity

- Gaussianity:
  - $<\Delta T> = 0$ (definition)
  - $<\Delta T^2> \sim 10^{-9}$
- Is ΔT Gaussian?
- $\Phi = \Phi + f_{NL}\Phi^2$ (local)
  - $-9 < f_{NL} < 80$ (WMAP7)
- Planck will improve bound
  - $f_{NL} \sim O(10)$ observable?
  - Test for violations of slow roll



Summary

- Have given broad overview of concordance cosmology
  - Focussed on results
  - And implications for particle physics
- Cosmology is becoming a data-driven science
  - Early universe cosmology *driven* by high energy physics
  - Implications for neutrino sector
  - TeV - GUT scale physics
  - Plus dark matter, baryogenesis, dark energy.

Very early this morning...