Phenomenology of multi-field inflation

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Based on

- A. Achùcarro, JG, S. Hardeman, G. A. Palma and S. P .Patil, arXiv:1005.3848 [hep-th]
- A. Achùcarro, JG, S. Hardeman, G. A. Palma and S. P .Patil, in preparation
- JG and T. Tanaka, in preparation

Outline

1. Introduction
   - Motivation of inflation
   - Challenges for inflation
   - Signatures to observe
2. Minkoswki space-time
   - (De)coupling of heavy and light fields
   - Modified action
3. Inflationary space-time
   - 2-point correlation function
   - 3-point correlation function
4. Conclusions and outlooks

Why inflation?

Hot big bang	Inflation
Horizon problem	Single causal patch
Flatness problem	Locally flat
Monopole problem	Diluted away
Initial perturbations	Quantum fluctuations

Currently, inflation

1. can provide initial conditions for hot big bang
2. is strongly supported by observations: WMAP, SDSS, etc

Theoretical and observational challenges for inflation

Theoretically

- What is the theory behind inflation?
- How to realize flat potential?
- What is the mechanism of perturbation generation?
- Did inflation ever occur at all?

Observationally

- Can we detect tensor perturbations / higher order correlation functions / $\cdots$?
- How to remove astrophysical contaminations?
- How to control systematics?
- Can we really trust observations?

What to observe?

What to do then?

1. We want to observe distinctive signatures of theories behind inflation
2. Those theories are likely to be beyond the standard model
3. Existence of multi-field is common to beyond SM
4. Rephrase: We want to observe distinctive signatures of multi-field

Q: How to extract information on the multi-field system?

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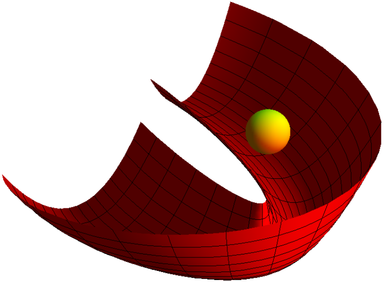
Q: How to extract information on the multi-field system?
field space?
curvature tensor of the field space?

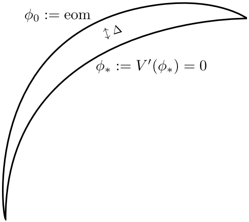
Interaction with mass hierarchies



- Heavy fields: integrated out?
- Time dependent BG: e.g. wild bending
- Light modes receive correction of order

$$\beta = \frac{4\dot{\phi}_0^2}{\kappa^2 M_{\text{heavy}}^2}$$

2-field case: modified gradient term



Shift induced by the inclusion of light fields

$$\Delta = \frac{\dot{\phi}_0^2}{\kappa M_{\rm heavy}^2} = \frac{\kappa}{4}\beta$$

Action and e.o.m. of the light mode

$$S = \int d^4x \frac{1}{2}\left[\dot{\varphi}^2 - e^{-\beta}(\nabla\varphi)^2 - M_{\rm light}^2\varphi^2\right]$$

$$\ddot{\varphi} - e^{-\beta}\Delta\varphi + M_{\rm light}^2\varphi = 0$$

Effect of coupling

From the structure of the e.o.m.

$$\ddot{\varphi} - e^{-\beta}\Delta\varphi + M_{\text{light}}^2\varphi = 0 \to c_s^2 = e^{-\beta}$$

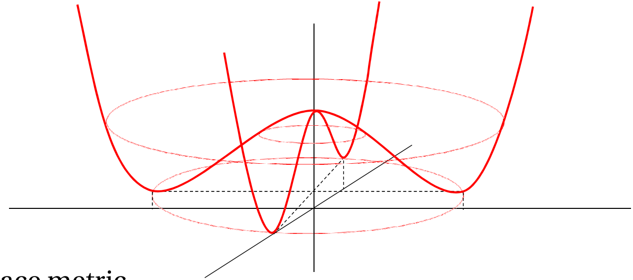
Further, naive expectation says

$$\beta \sim \epsilon\frac{m_{\text{Pl}}^2}{\kappa^2} \sim \epsilon$$

Thus we expect

- Modulation of power spectrum: $\mathscr{P} \sim e^{\beta/2}\mathscr{P}_\star$
- Non-trivial sound velocity c_s: non-Gaussianity (e.g. DBI)

2-field case: elliptic Mexican hat potential



Field space metric

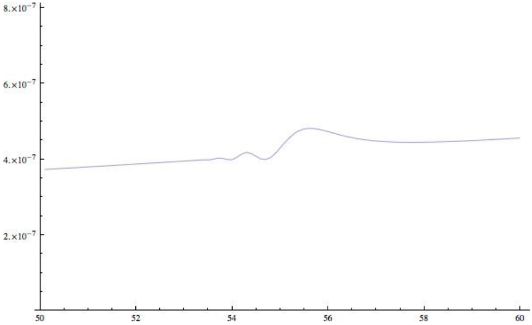
$$G_{IJ} = \frac{l_{\rm loci}^2}{2}\left[\cosh(2\rho) - \cos(2\theta)\right]$$

Potential

$$V(\rho) = \frac{m^2 G_{\rho\rho}|_{\rho=\sqrt{r/2}}}{4r}\left(\frac{r^2}{4} - r\rho^2 + \rho^4\right)$$

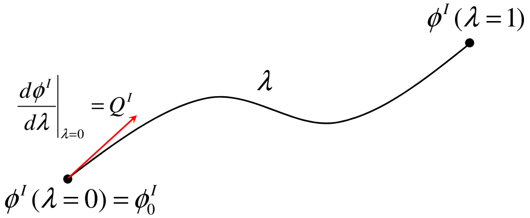
$$V(\theta) = V_0 + l_{\rm loci}^2\delta\left[\frac{1}{2}\cosh(\sqrt{2r})\theta - \frac{1}{4}\sin(2\theta)\right]$$

Resulting power spectrum



- “Bump” at bending
- During bending heavy modes are excited
- Constant curvature → overall modulation

Issue of mapping



With the initial conditions

$$\left.\phi^I\right|_{\lambda=0} = \phi_0^I$$

$$\left.\frac{D\phi^I}{d\lambda}\right|_{\lambda=0} = Q^I$$

Physical field fluctuation $\delta\phi^I$ and infinitesimal vector Q^I

$$\phi^I - \phi_0^I = \delta\phi^I = Q^I - \frac{1}{2}\Gamma^I_{JK}Q^JQ^K + \frac{1}{6}\left(\Gamma^I_{LM}\Gamma^M_{JK} - \Gamma^I_{JK;L}\right)Q^JQ^KQ^L + \cdots$$

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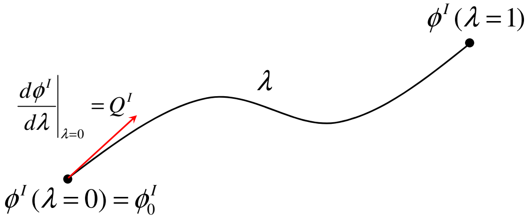

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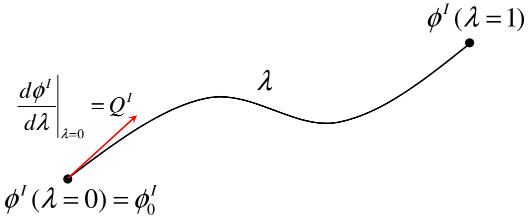

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Quadratic expansion

At 2nd order, from the kinetic term, linear truncation gives

$$S_2 \supset \int d^4x a^3 \frac{-1}{2} \left(G_{IJ} \dot{Q}^I \dot{Q}^J + 2 G_{IJ,K} Q^K \dot{\phi}_0^I \dot{Q}^J + \frac{1}{2} G_{IJ,KL} Q^K Q^L \dot{\phi}_0^I \dot{\phi}_0^J \right)$$

Quadratic truncation gives

$$\begin{aligned} S_2 \supset \int d^4x a^3 \frac{1}{2} \Big[& G_{IJ} \nabla^\mu Q^I \nabla_\mu Q^J - G_{IJ} \nabla^\mu \phi_0^J \nabla_\mu \left(\Gamma^J_{MN} Q^M Q^N \right) \\ & + 2 G_{IJ,K} Q^K \nabla^\mu \phi_0^I \nabla_\mu Q^J + \frac{1}{2} G_{IJ,KL} Q^K Q^L \nabla^\mu \phi_0^I \nabla_\mu \phi_0^J \\ & - \frac{1}{2} G_{IJ,K} \Gamma^K_{AB} Q^A Q^B \nabla^\mu \phi_0^I \nabla_\mu \phi_0^J \Big] \end{aligned}$$

In both cases

$$S_2 \supset \int d^4x a^3 \frac{-1}{2} \left(G_{IJ} D_t Q^I D_t Q^J + R_{IKLJ} Q^K Q^L \dot{\phi}_0^I \dot{\phi}_0^J \right)$$

Expansion up to cubic order: Strategy and result

1. Kinetic function

$$X = \frac{1}{2} G_{IJ} \nabla^\mu \phi^I \nabla_\mu \phi^J$$

2. Expansion with respect to λ

$$X|_{\lambda=\epsilon} = X|_{\lambda=0} + \left.\frac{dX}{d\lambda}\right|_{\lambda=0} \epsilon + \frac{1}{2!} \left.\frac{d^2X}{d\lambda^2}\right|_{\lambda=0} \epsilon^2 + \frac{1}{3!} \left.\frac{d^3X}{d\lambda^3}\right|_{\lambda=0} \epsilon^3 + \cdots$$

3. Being a scalar,

$$\frac{dX}{d\lambda} = \frac{DX}{d\lambda}$$

$$S_3 \supset \int d^4x a^3 \frac{-1}{6} \left(R_{IKLJ;M} Q^K Q^L Q^M \dot{\phi}_0^I \dot{\phi}_0^J + 4 R_{IKLJ} Q^K Q^L D_t Q^J \dot{\phi}_0^I \right)$$

Only terms including R_{IKLJ} appear from the kinetic term

Conclusions and outlooks

1. Multi-field inflation
   - Plausible in beyond SM
   - Distinctive signatures?
2. Minkowski space-time
   - Coupling between heavy and light modes
   - Modification of gradient term
3. Inflationary space-time
   - Modulation of power spectrum
   - Covariant expansion: 3rd order terms with R_{IJKL}
4. Under progress
   - More general matter Lagrangian $P = P(X^{IJ}, \phi^I)$ including gravity
   - Concrete numerical results of 3-point correlation function