More on Dimension-4 Proton Decay Problem in F-theory

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Based on the work with
Teruhiko Kawano (U. Tokyo), Yoichi Tsuchiya (U. Tokyo)
and Taizan Watari (IPMU) arXiv:1004.3870 [hep-th]

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1. Motivation

◊ A Candidate for Beyond Standard Model
→ Minimal Supersymmetric Standard Model (MSSM)

☆ A Generic Renormalizable Superpotential from Gauge Invariance

$$W_{Generic} = W_{MSSM} + W_{\Delta L=1} + W_{\Delta B=1}$$

$$W_{MSSM} = y_u \bar{u} Q H_u - y_d \bar{d} Q H_d - y_e \bar{e} L H_d + \mu H_u H_d$$

$$W_{\Delta L=1} = \frac{1}{2}\lambda^{ijk} L_i L_j \bar{e}_k + \lambda'^{ijk} L_i Q_j \bar{d}_k + \mu'^{i} L_i H_u$$

$$W_{\Delta B=1} = \frac{1}{2}\lambda''^{ijk} \bar{u}_i \bar{d}_j \bar{d}_k$$

Dangerous Operators!

A Solution: Matter-parity (or R-parity)

Impose $P_M = (-1)^{3(B-L)}$ symmetry

○ Running gauge couplings of MSSM indicate that three gauge couplings will meet at 2×10^{16} GeV

→ Grand Unified Theory (GUT)

☆ Matter Contents

$$10_M \to (3,2)_{\frac{1}{6}} + (\bar{3},1)_{-\frac{2}{3}} + (1,1)_1$$

$$\bar{5}_M \to (\bar{3},1)_{\frac{1}{3}} + (1,2)_{-\frac{1}{2}}$$

$$5_H \to (3,1)_{-\frac{1}{3}} + (1,2)_{\frac{1}{2}} \qquad \bar{5}_H \to (\bar{3},1)_{\frac{1}{3}} + (1,2)_{-\frac{1}{2}}$$

☆ Gauge Invariant Renormalizable Superpotential

$$W_{GUT} \supset y_u 10_M 10_M 5_H + y_d \bar{5}_M 10_M \bar{5}_H + \lambda \bar{5}_M 10_M \bar{5}_M$$

Necessary Yukawa interactions

Dim-4 Proton Decay Operators!

○ For prohibiting dimension-4 proton decay operators, it is important to distinguish …

$$\bar{5}_M \leftrightarrow \bar{5}_H$$

like a matter parity.

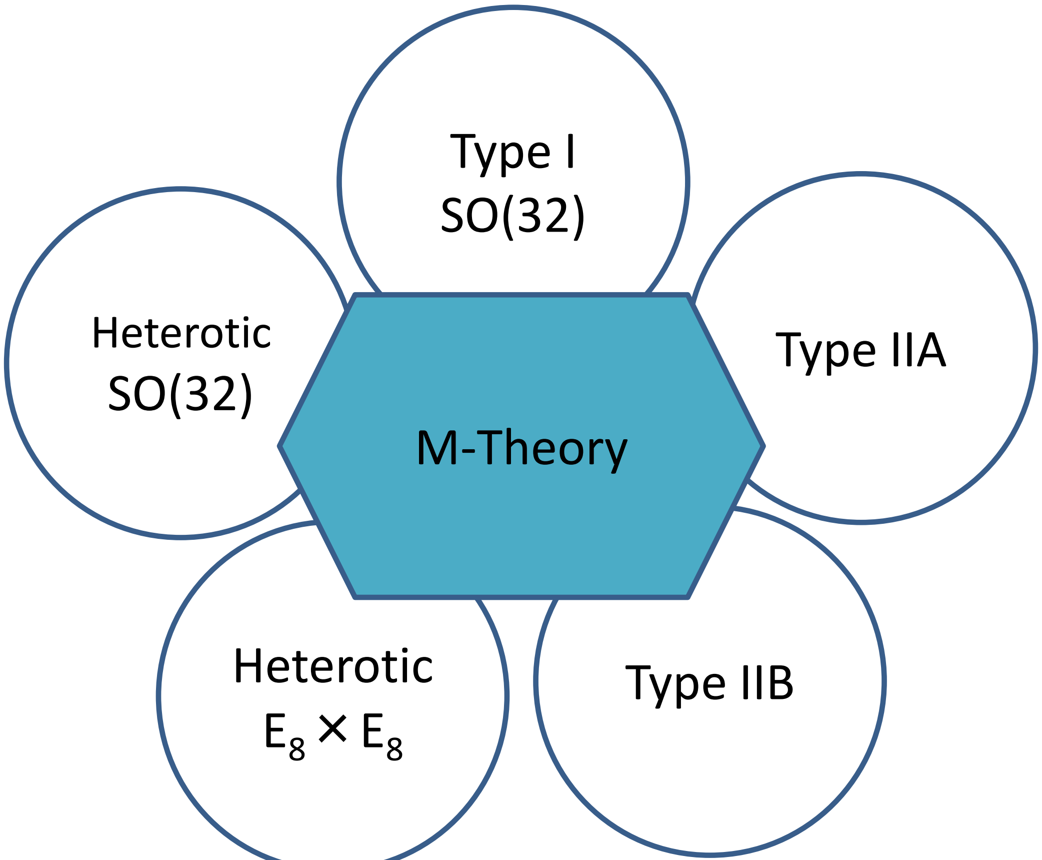
Therefore, it is natural to ask,
"What is the origin of such a kind of symmetry ?"

Since string theory is a candidate to describe high energy physics, we may rephrase it as
"How is that symmetry achieved in string theory?"

2. String Vacua For GUT

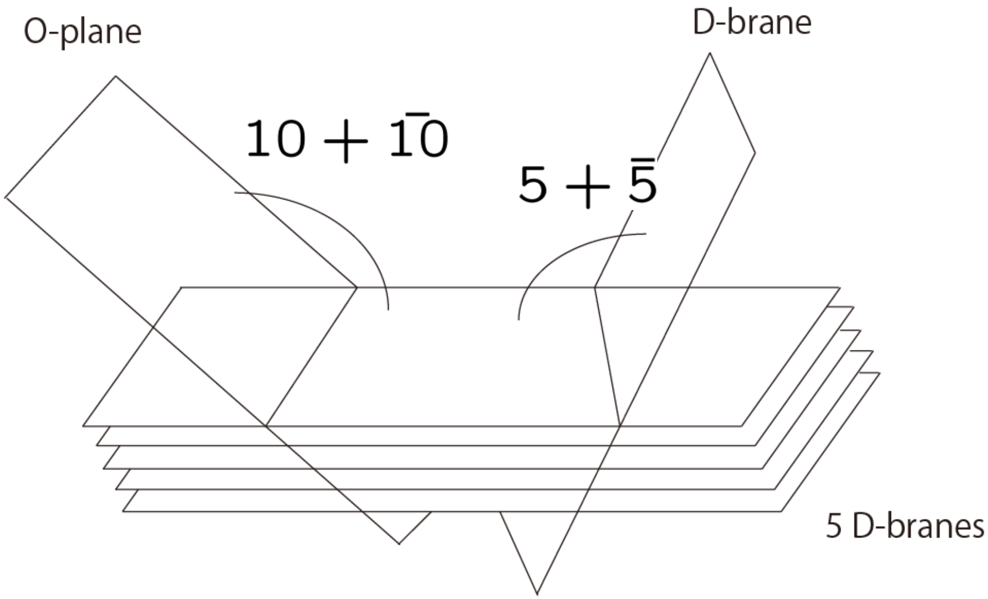
What Vacua is suitable for describing GUT ?

◊ Requirements: (i) $\mathcal{N} = 1$ supersymmetry
(ii) all the necessary Yukawa Couplings
(iii) decoupling gravity from gauge theory



Type IIB/IIA String Vacua

○ Coincident D-branes can be used for the construction of supersymmetric (S)U(5) models



However, we cannot get $y_{u,ij} 10^{ab}_{M,i} 10^{cd}_{M,j} 5^{e}_{H} \epsilon_{abcde}$!

Type I-SO(32) and Heterotic SO(32) string theory also cannot generate the above up-type Yukawa couplings.

Heterotic $E_8 \times E_8$ String Vacua

○ Heterotic string has an $E_8 \times E_8$ gauge symmetry and then we can generate all the necessary Yukawa Couplings.

However, there is another issue.

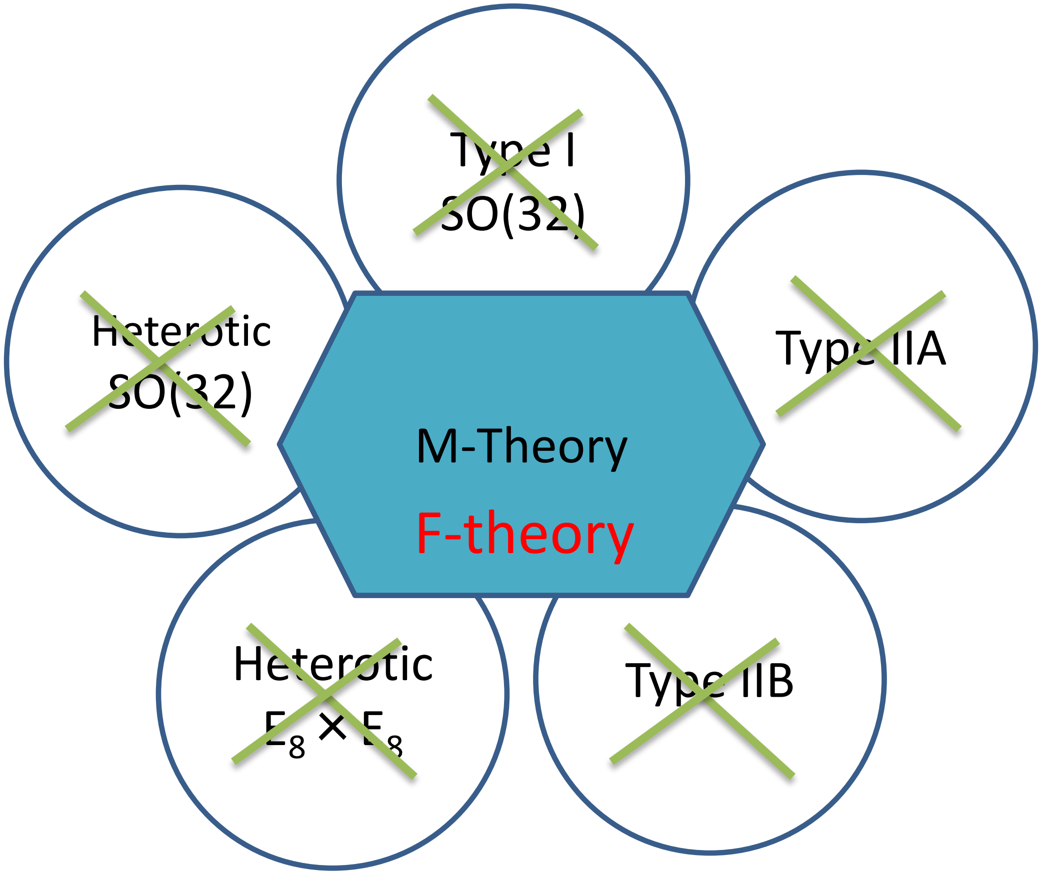
☆ 4d Effective Lagrangian

$$L_{4d} = -\int d^4x \sqrt{-g} e^{-2\phi} V \left(\frac{4}{(\alpha')^4} R + \frac{1}{(\alpha')^3} \mathrm{tr} F^2 + \ldots \right)$$

$$\begin{cases} G_N = \dfrac{e^{2\phi}(\alpha')^4}{64\pi V} \\ \alpha_{GUT} = \dfrac{e^{2\phi}(\alpha')^3}{16\pi V} \end{cases} \Rightarrow \quad G_N = \frac{\alpha_{GUT}\alpha'}{4}$$

We cannot decouple gravity with the GUT gauge coupling constant fixed.

+ There is a no known mechanism to stabilize all the moduli in Heterotic String , although this is a technical difficulty.



In fact, F-theory can generate all the Yukawa couplings and also decouple gravity from gauge theory.

3. F-theory Model Building

What is F-theory?

F theory $\sim$ Strong Coupling region of Type IIB string theory

(Vafa '96)

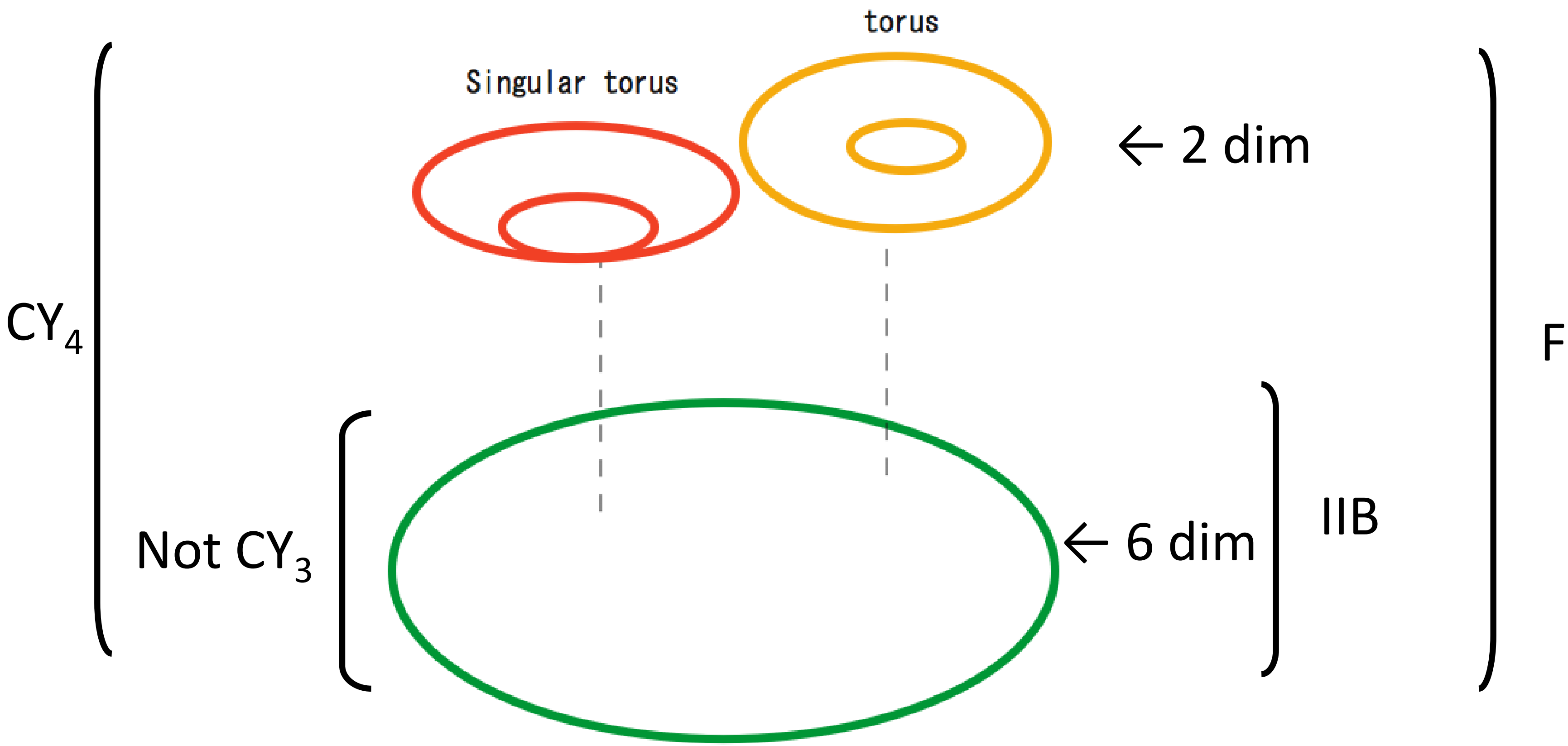
We compactify Type IIB string theory on a background where complex coupling constant $\tau = C_0 + ie^{-\phi}$ is NOT constant, but is holomorphic.

⇒ We get ***N***=1 4d effective theory.
(NOT ***N***=2 like CY_3 compacitifcations)

τ has SL(2, **Z**) symmetry in Type IIB string theory,
so, we can think of τ as compex structure of torus.

★Therefore, we can think of the background as T^2-fibration over 6 dimensional manifold whose torus shape is different from place to place.

☆ A sketch of an internal manifold



What is happening when a fiber is singular?

At a singular point z_i,
complex structure behaves as $\tau(z) \sim \frac{1}{2\pi i} \ln(z - z_i)$

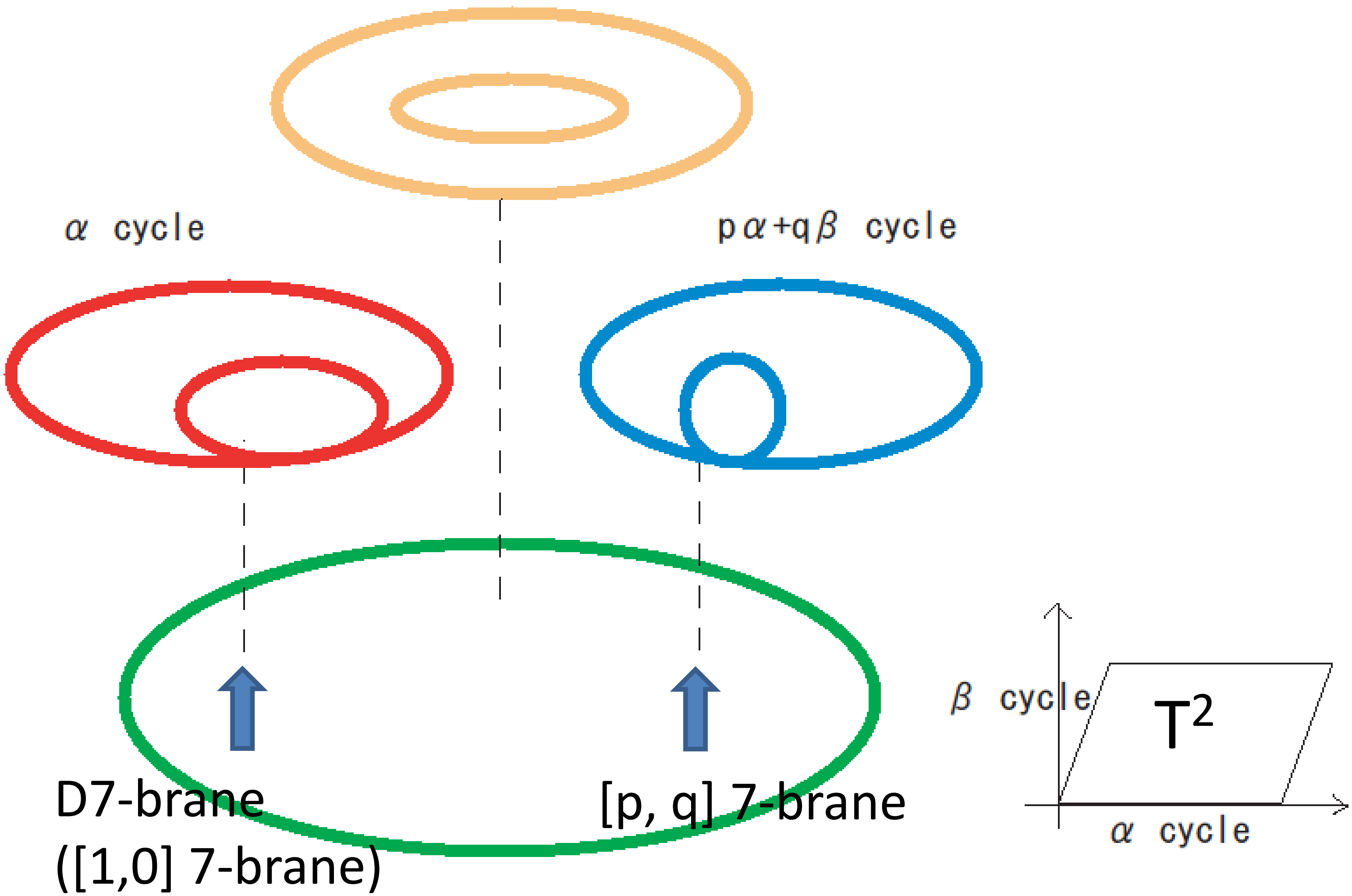
So, when circling around $z = z_i$, τ undergoes monodromy.

$$\tau \to \tau + 1 \quad (\text{i.e. } C_0 \to C_0 + 1)$$

C_0 is a magnetic charge of D7-brane,
So, we deduce that there is a D7-brane at $z = z_i$!

$z = z_i$ is a (complex) codimension 1 subspace of 6-dim mfd.
So, the dimension of the branes should be 1+3+(6-2)=1+7.
This is consistent with the dimension of D7-branes.

0	1	2	3	4	5	6	7	8	9	10	11
$\mathbb{R}^4$				S				$z,\ \bar{z}$		torus	
$\mathbb{R}^4$				B						torus	
7-brane								X	X	X	X

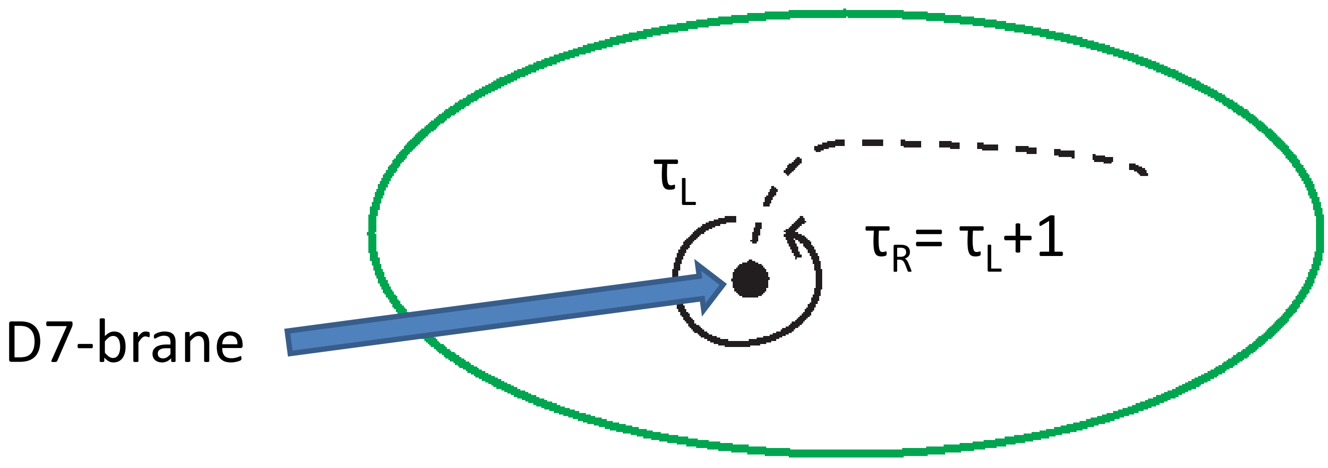


☆ A [p, q] 7-brane is a 7-brane on which a (p, q) string ends.
A (p, q) string is a bound state of p fundamental strings and q D1-strings.

Gaberdiel, Zwiebach '97

i.e. $\begin{pmatrix} p & r \\ q & s \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$ $\quad g_{p,q} = \begin{pmatrix} p & r \\ q & s \end{pmatrix} \in SL(2, \mathbf{Z})$

(i) Monodromy around a D7-brane ([1, 0] 7-brane)



$$\tau = \frac{\int_\beta \Omega}{\int_\alpha \Omega} \quad \Rightarrow \quad \begin{aligned} \alpha_R &= \alpha_L \\ \beta_R &= \alpha_L + \beta_L \end{aligned}$$

$$\begin{pmatrix} 1 & \tau_L \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}}_{M_{1,0}} = \begin{pmatrix} 1 & \tau_L + 1 \end{pmatrix} = \begin{pmatrix} 1 & \tau_R \end{pmatrix}$$

Monodromy matrix of 7-brane

$$\begin{pmatrix} \alpha_L & \beta_L \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \alpha_L & \alpha_L + \beta_L \end{pmatrix} = \begin{pmatrix} \alpha_R & \beta_R \end{pmatrix}$$

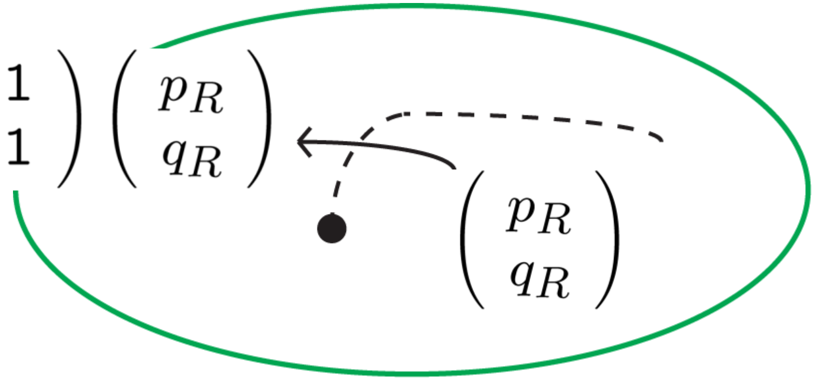
◊ Transformation of (p, q) string

(p, q) string → pα+qβ cycle

$$\begin{aligned} \begin{pmatrix} \alpha_R & \beta_R \end{pmatrix} \begin{pmatrix} p_L \\ q_L \end{pmatrix} &= \begin{pmatrix} \alpha_L & \beta_L \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p_R \\ q_R \end{pmatrix} \\ &= \begin{pmatrix} \alpha_L & \beta_L \end{pmatrix} \begin{pmatrix} p_L \\ q_L \end{pmatrix} \end{aligned}$$

⇒ $$\begin{pmatrix} p_L \\ q_L \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p_R \\ q_R \end{pmatrix}$$

$$\begin{pmatrix} p_L \\ q_L \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p_R \\ q_R \end{pmatrix}$$



Ex. D7-brane & Fundamental string

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \to \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

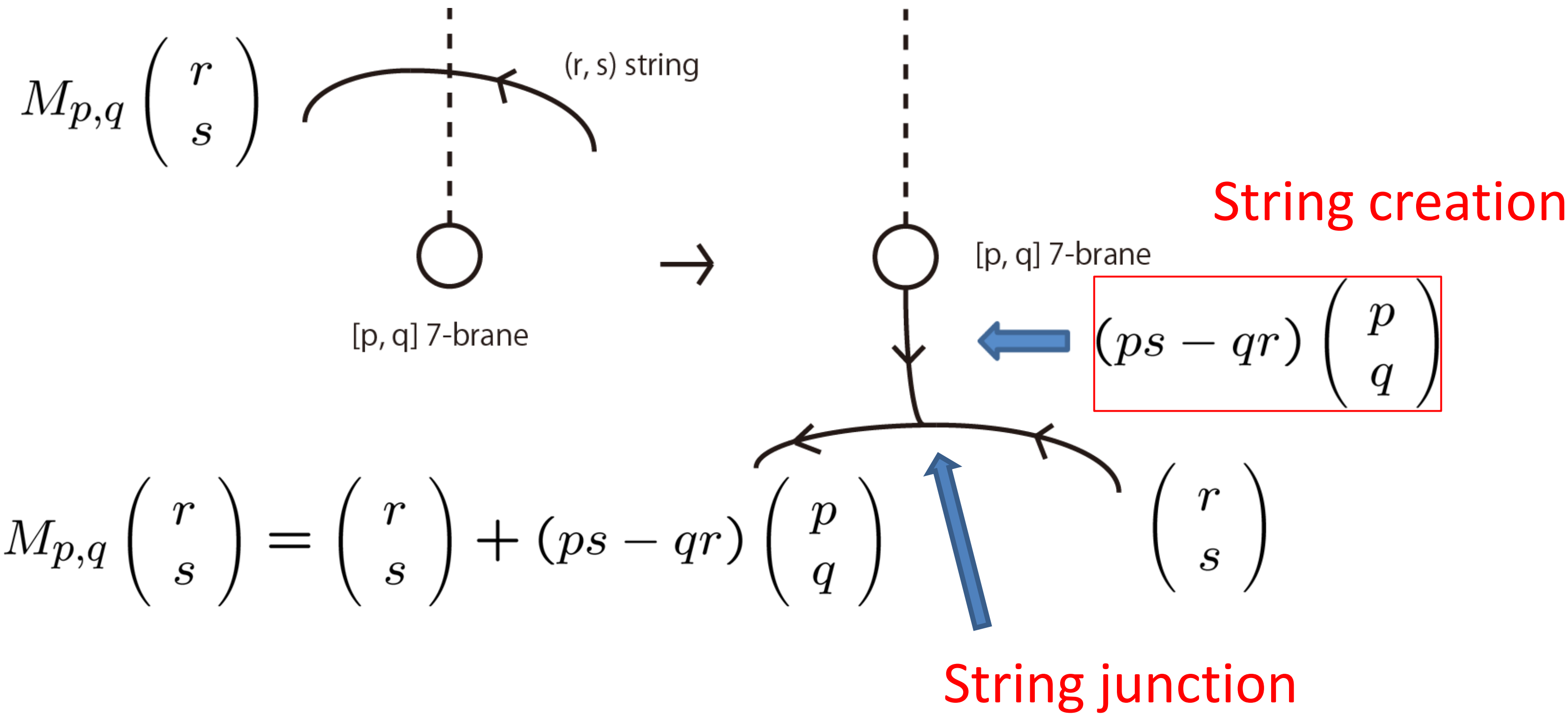
(ii) Monodromy around a [p, q] 7-brane

$$\begin{pmatrix} p \\ q \end{pmatrix} = M_{p,q} \begin{pmatrix} p \\ q \end{pmatrix} \Rightarrow g_{p,q} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = M_{p,q} g_{p,q} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow M_{1,0} = g_{p,q}^{-1} M_{p,q} g_{p,q}$$

$$M_{p,q} = g_{p,q} M_{1,0} g_{p,q}^{-1} = \begin{pmatrix} 1 - pq & p^2 \\ -q^2 & 1 + pq \end{pmatrix}$$

☆ A [p,q] 7-brane has a branch cut which changes the charge of a (r, s) string.



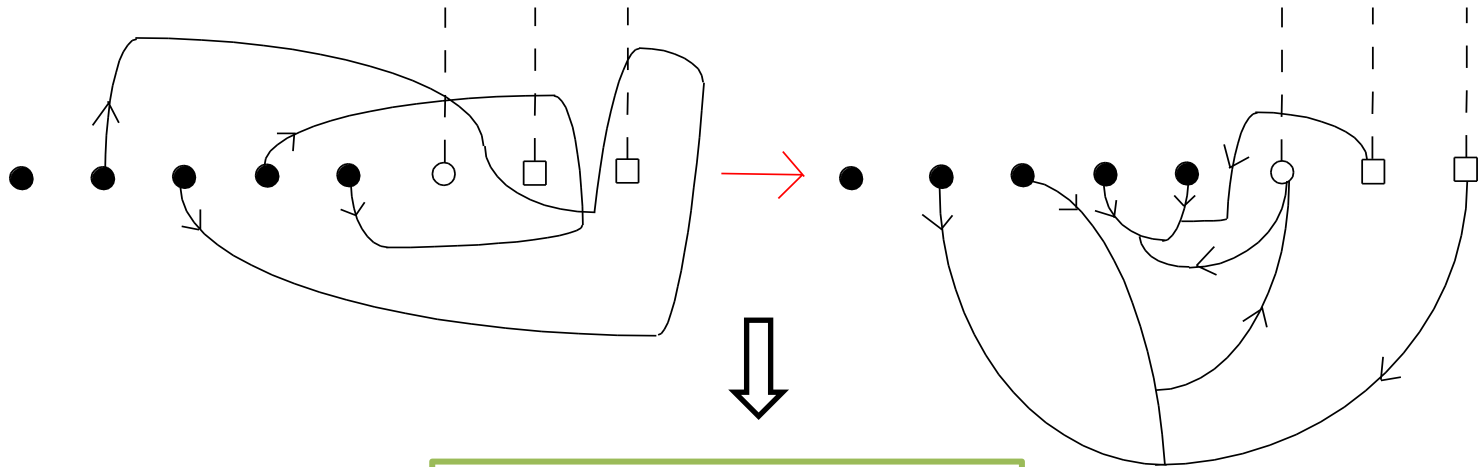
$$M_{p,q}\begin{pmatrix} r \\ s \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix} + (ps - qr)\begin{pmatrix} p \\ q \end{pmatrix}$$

☆ By using string junctions, we can generate $E_{6,7,8}$ algebras !

Ex. $E_6 \rightarrow A^5BC^2$ Gaberdiel, Zwiebach '97

$$E_6 \rightarrow SU(5) \times SU(2) \times U(1)$$

$$78 \rightarrow (24,1)_0 + (1,1)_0 + (1,3)_0 + (10,2)_{-3} + (\overline{10},2)_3 + (\bar{5},1)_{-6} + (5,1)_6$$



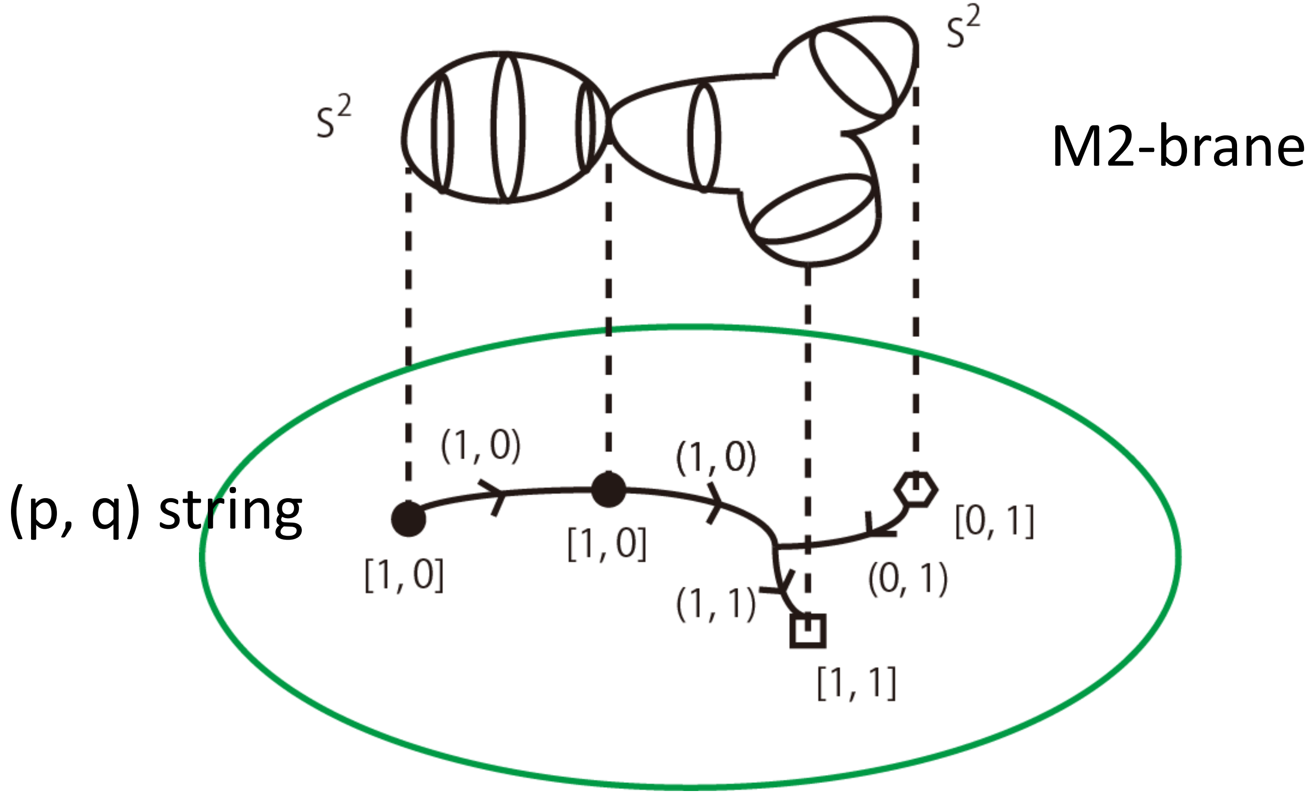
$\Lambda^2 5 \times \Lambda^2 5 \rightarrow \Lambda^4 5 \sim \bar{5}$

E_6 algebra

$$(10,2)_{-3} \times (10,2)_{-3} \rightarrow (\bar{5},1)_{-6}$$

● [1, 0]-brane (A-brane)
○ [1, -1]-brane (B-brane)
□ [1, 1]-brane (C-brane)

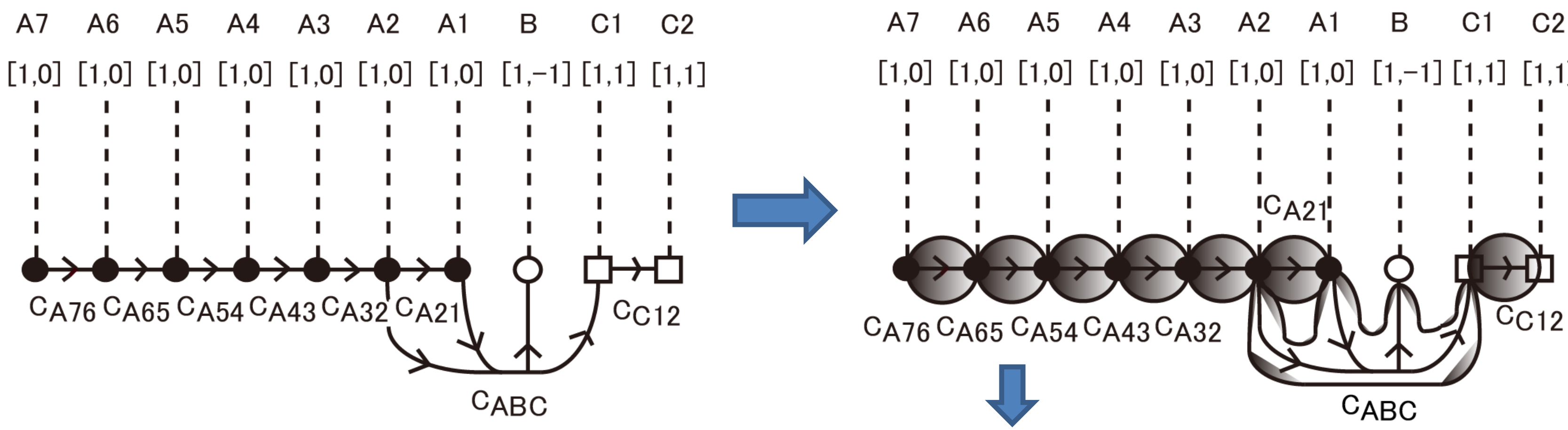
◊ In M-theory, a (p, q) string can be thought as a M2-brane.



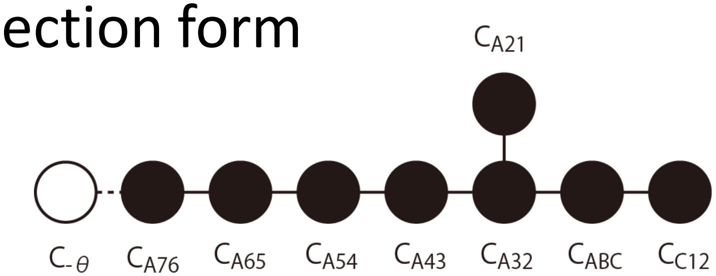
In order to obtain a gauge symmetry, i.e. massless gauge fields, the length of a (p, q) string, i.e. the size of a 2-cycle where an M2-brane wraps should vanish.

◇ Therefore, a gauge symmetry is realized as a collection of vanishing 2-cycles where each 2-cycle represents a simple root of the corresponding Lie algebra.

Ex. E_8 (A^7BC^2)



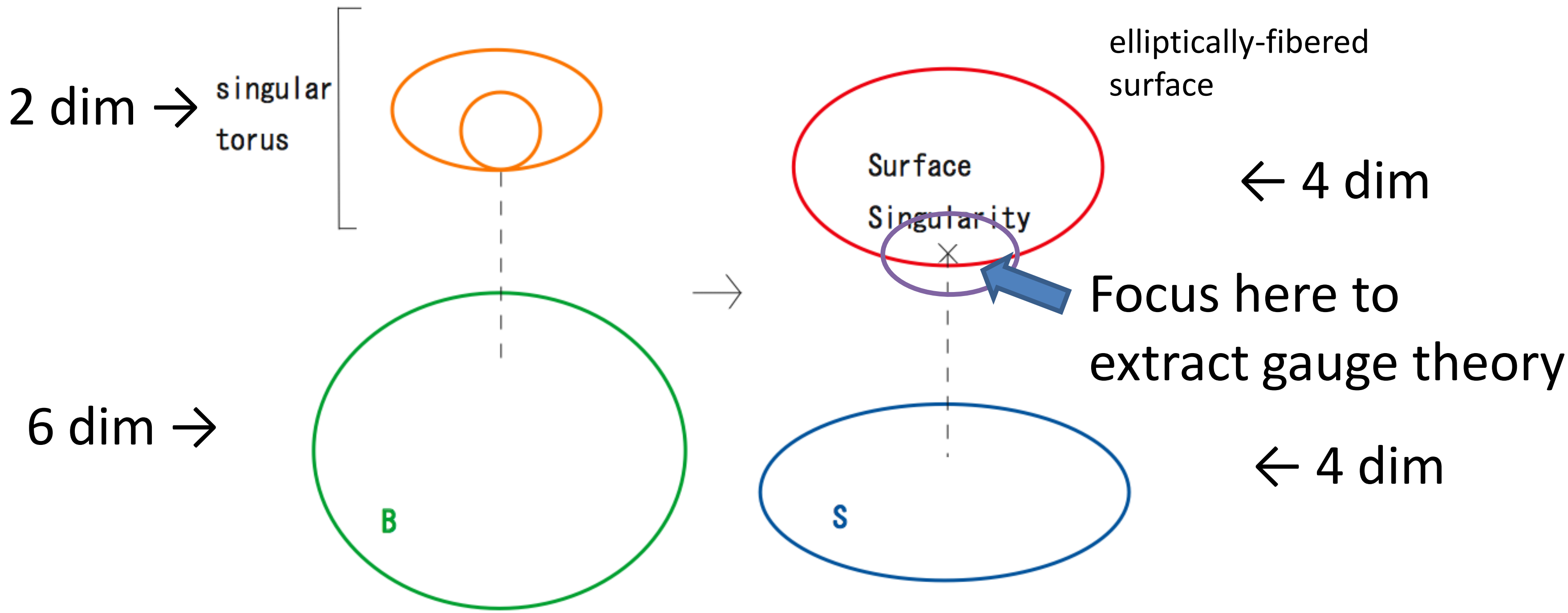
intersection form



The vanishing of those kinds of 2-cycles is called ADE singularity

$A_n \rightarrow SU(n+1)$, $D_n \rightarrow SO(2n)$, E_6, E_7, E_8

Gauge Symmetry in F-theory
→ Surface Singularities



Ex. SU(5)

5 D7-branes ↔ A_4 singularity

[4 vanishing 2-cycles]

Elliptically-fibered Calabi-Yau 4-fold equation

$$y^2 = x^3 + fx + g$$

f, g are sections of K_B^{-4}, K_B^{-6}

The location of the 7-brane ↔ 1-cycle of torus shrinks

$$\Delta = 4f^3 + 27g^2 = 0$$

◊ Tate form Bershadsky, et al '96

$$y^2 + A_1 xy + A_3 y = x^3 + A_2 x^2 + A_4 x + A_6$$

a_i is a section of K_B^{-i}

Ex. A_4 (SU(5)) singularity at z=0

$$A_1 = a_5,\ A_2 = za_4,\ A_3 = z^2 a_3,\ A_4 = z^3 a_2,\ A_5 = z^5 a_0$$

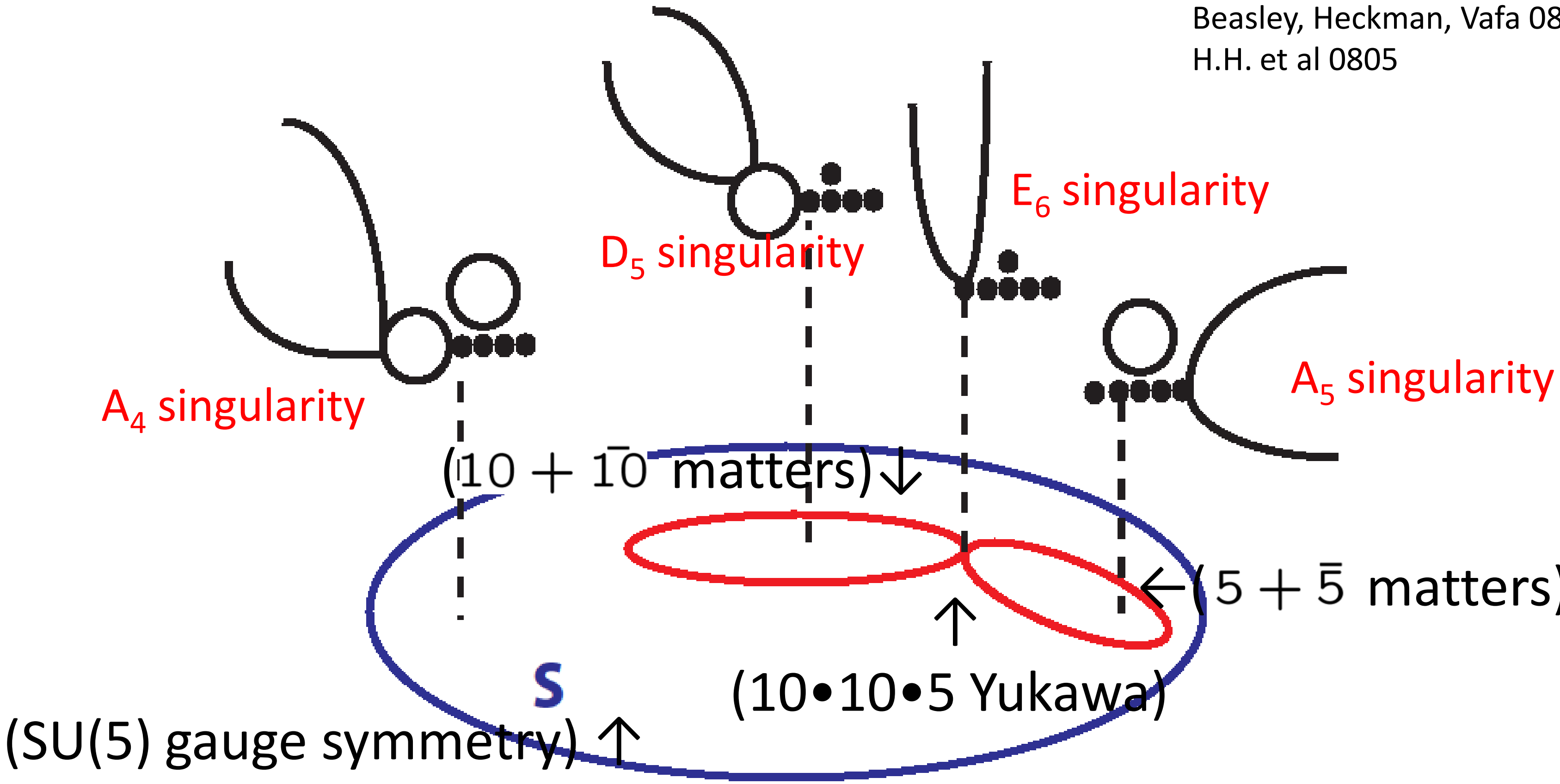
⇒ $$\Delta = z^5 (d_0(s_1, s_2) + d_1(s_1, s_2) z + d_2(s_1, s_2) z^2 + \mathcal{O}(z^3))$$

↑ 5 7-branes are located at z=0

$d_0(s_1, s_2) = 0$ ⇒ Intersection of other 7-brane

Ex. SU(5) GUT : Up-type Yukawa couplings

Donagi, Wijnholt 0802
Beasley, Heckman, Vafa 0802
H.H. et al 0805



SU(6) $\supset$ SU(5): $35 \to 24 + 1 + 5 + \bar{5}$

SO(10) $\supset$ SU(5): $45 \to 24 + 1 + 10 + \overline{10}$

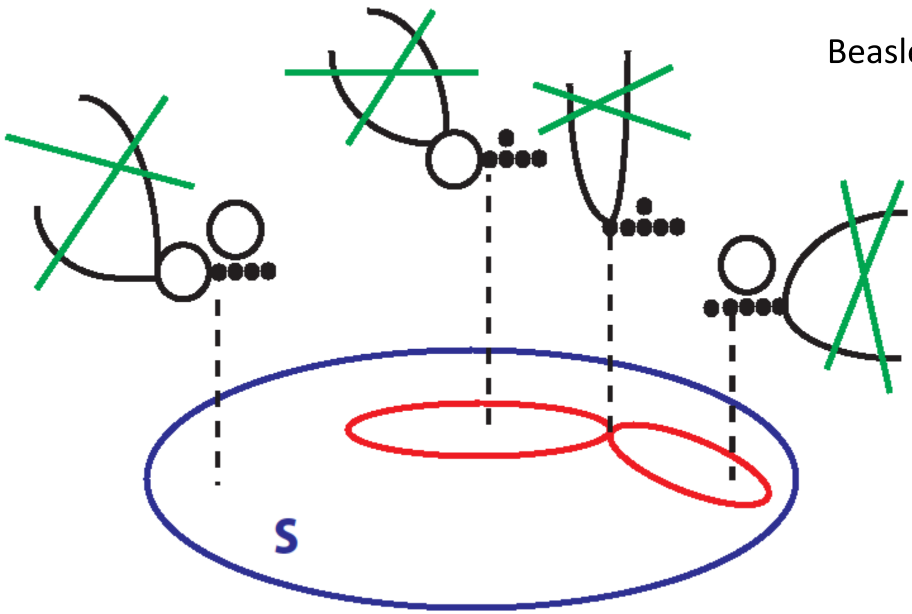
Chiral matters ← turning on **G**-flux

The number of massless matter can be counted by Hodge number with geometric data on matter curves and **G**-flux.

Therefore, it is enough to concentrate on the 2-cycles which vanishes somewhere on the surface **S** in order to extract the Information of matters and Yukawa couplings in GUT models.

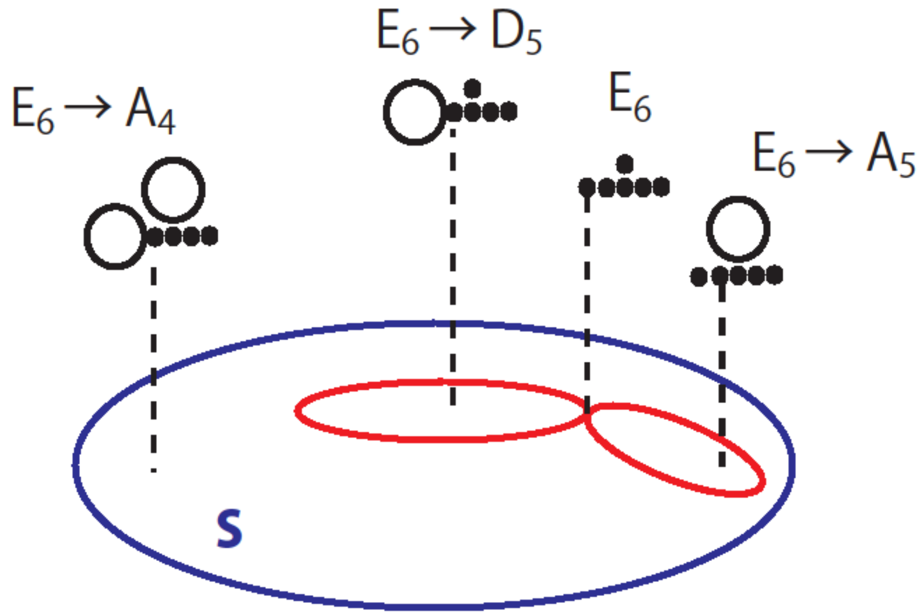
→ "(Semi-)Local Model Building"

Beasley, Heckman, Vafa 0802, 0806



After focusing on relevant “small” 2-cycles, those 2-cycles just form ADE dynkin diagrams.

Ex. SU(5) up-type Yukawa case



From the effective field theory on 7-brane point of view, this singularity deformation can be captured by gauge symmetry breaking by the **VEV** of adjoint-valued fields which corresponds to normal modes of 7-branes.

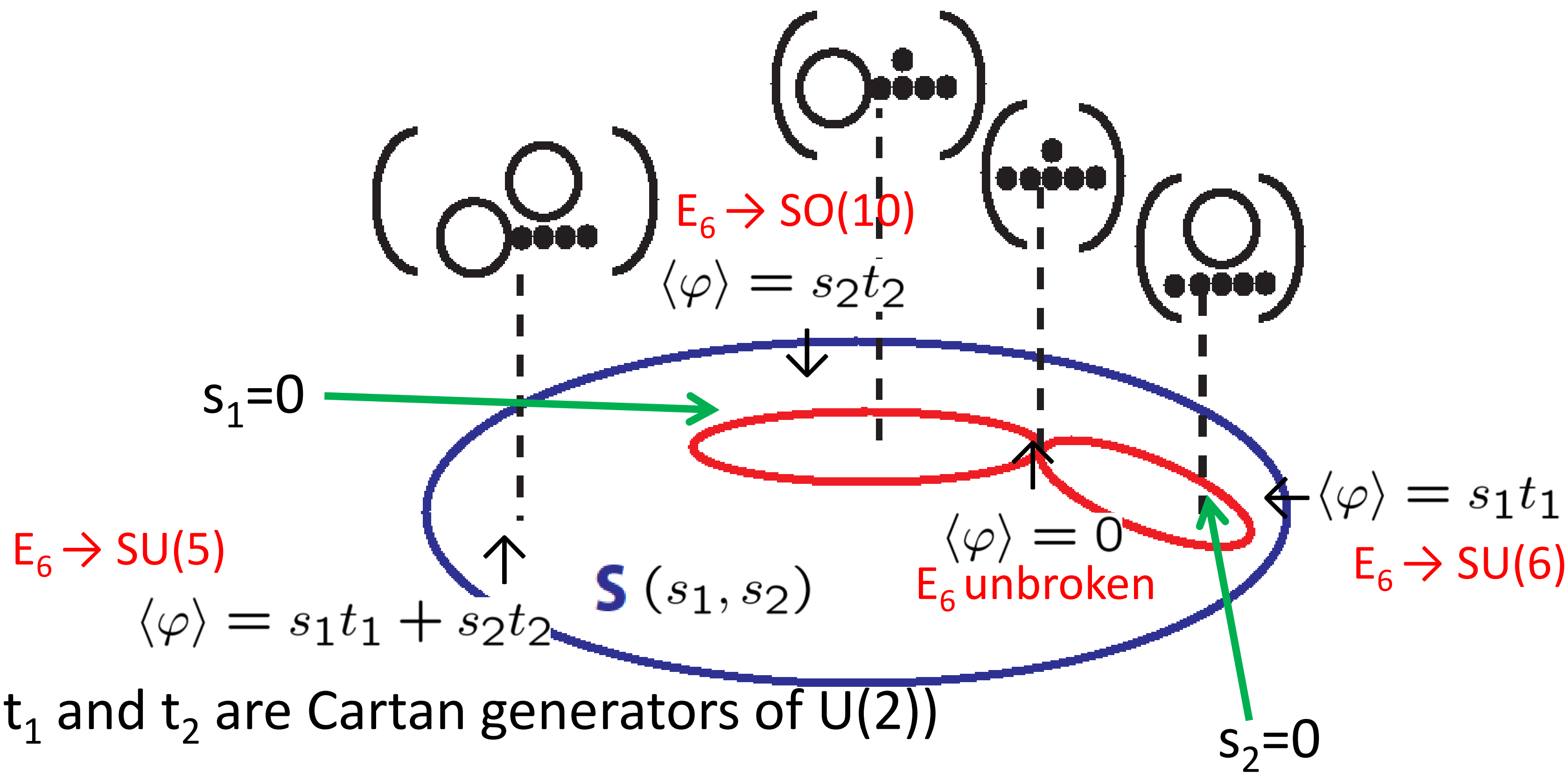
The **VEV** of the adjoint Higgs fields varies over the surface **S**

○ 8D gauge theory with varying VEV of adjoint Higgs fields

→ "Higgs bundle"

H.H. et al 0901

Ex. E_6 Higgs bundle ($E_6 \supset SU(5) \times \langle U(2) \rangle$)



(t_1 and t_2 are Cartan generators of U(2))

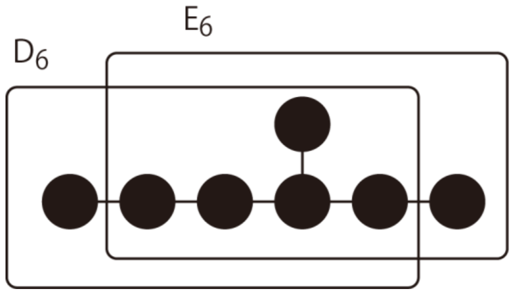
Matter wave function localizes here like the domain wall fermions.

◊ Necessary Ingredients for SU(5) GUT

(i) Up-type Yukawa Couplings → E_6 Singularity
(ii) Down-type Yukawa Couplings → D_6 Singularity

} E_7

We need at least E_7 Higgs bundle.



○ Maximal one is an E_8 Higgs bundle.

In case of E_8 Higgs bundle model, the adjoint Higgs fields take values at $SU(5)_{broken}$ and the space which their eigenvalues sweep is called "spectral surface".

$E_8 \supset SU(5)_{GUT} \times \langle SU(5)_{broken} \rangle$

$$\det(\xi I_{5\times 5} - \langle\varphi\rangle) = a_0\xi^5 + a_2\xi^3 + a_3\xi^2 + a_4\xi + a_5 = 0$$

4. Dimension-4 Proton Decay Problem in F-theory

◊ As an ordinary SU(5) theory, this SU(5) GUT local model also suffers from the dimension-4 proton decay problem. Therefore, we need to distinguish $\bar{5}_M \leftrightarrow \bar{5}_H$.

☆ Proposed Solutions so far

(i) to consider a global compactifications with $\mathbf{Z}_2$ symmetry

Tatar, Tsuchiya, Watari 0905
H.H. et al 0910

(ii) rank5- GUT Scenario with U(1) Flux

Tatar, Watari 0602

※ (iii) factorized spectral surface scenario

Tatar, Tsuchiya, Watari 0905
Marsano, Saulina, Schafer-Nameki 0906
... etc

((iii)* spontaneous R-parity violating scenario)

Tatar, Watari 0602
Tatar, Tsuchiya, Watari 0905
Blumenhagen et al 0908

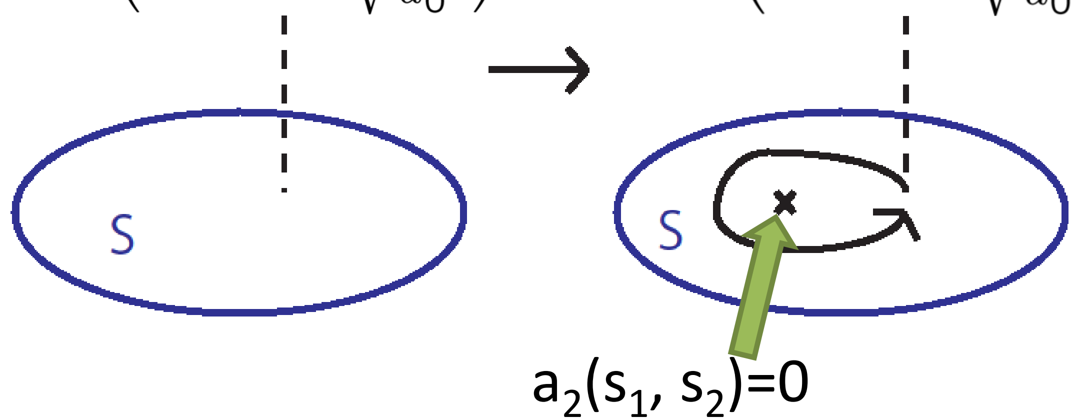
Today's Talk target

◊ Set up: $E_8 \supset SU(5)_{GUT} \times \langle SU(5)_{broken} \rangle$
Adjoint Higgs fields take values at $SU(5)_{broken}$ and we naively expect $SU(5)_{GUT} \times U(1)^4$ gauge symmetry on the surface **S**. However, those $U(1)^4$ generators are acted by the Weyl group $S_5 \subset SU(5)_{broken}$ and no U(1) symmetry is left in general.

Ex. Monodromy of Rank 2 Spectral Surface $\langle SU(2) \rangle$

$$a_0(\det(\xi I_{2\times 2} - \langle\varphi\rangle)) = a_0\xi^2 + a_2 = 0 \;\to\; \xi = \pm\sqrt{\frac{a_2}{a_0}}$$

$$\langle\varphi(s_1, s_2)\rangle = \begin{pmatrix} \sqrt{\frac{a_2}{a_0}} & o \\ 0 & -\sqrt{\frac{a_2}{a_0}} \end{pmatrix} \qquad - \begin{pmatrix} \sqrt{\frac{a_2}{a_0}} & o \\ 0 & -\sqrt{\frac{a_2}{a_0}} \end{pmatrix}$$



Therefore, it is important to reduce the monodromy for unbroken U(1) symmetry.

☆ Factorized spectral surface scenario: Marsano, Saulina, Schafer-Nameki 0906

Use an U(1) symmetry generated by reducing monodromy

Ex. (4 + 1) Factorization (S(U(4) × U(1)))

$$a_0\xi^5 + a_2\xi^3 + a_3\xi^2 + a_4\xi + a_5 \Rightarrow (c_0\xi^4 + c_1\xi^3 + c_2\xi^2 + c_3\xi + c_4)(d_0\xi + d_1)$$

Weyl group		Weyl group
S_5	⇒	S_4

Then, there seems to be one U(1) symmetry which might be useful for prohibiting dimension-4 proton decay operators.

○ We consider global E_8 Higgs bundle with (4+1) factorization.

Adj $E_8 \to SU(5)_{GUT} \times SU(5)_{broken}$

$$248 \to (24,1) + (1,24) + \underline{(10,5)} + \underline{(\bar{5},10)} + (\bar{10},\bar{5}) + (5,\bar{10})$$

Ex. [(4+1) factorization] $SU(5)_{broken} \to S(U(4) \times U(1))$

$\underline{(10, \xi_i)}$ (i=1, ..., 5) ⇐ $(10_M, \xi_a)$ (a=1,...,4)
$(10_{other}, \xi_5)$

$\underline{(\bar{5}, \xi_i + \xi_j)}$ ([i < j]=1, ..., 5) ⇐ $(\bar{5}_M, \xi_a + \xi_5)$ (a=1,...,4)
$(\bar{5}_H, \xi_a + \xi_b)$ ([a < b]=1,...,4)

(ξ_i: fundamental weights of $SU(5)_{broken}$)

Matter	10_M	10_{other}	$\bar{5}_M$	$\bar{5}_H$
U(1) charge	1	-4	-3	2

$$W_{(4+1)} \sim 10_M 10_M 5_H + \bar{5}_M 10_M \bar{5}_H + 10_{\text{other}} \bar{5}_H \bar{5}_H$$

No $\bar{5}_M 10_M \bar{5}_M$ terms!

Two Caveats

H.H. et al 1004

◊ Factorized spectral surface scenario seems work well at first sight. However, there are two caveats.

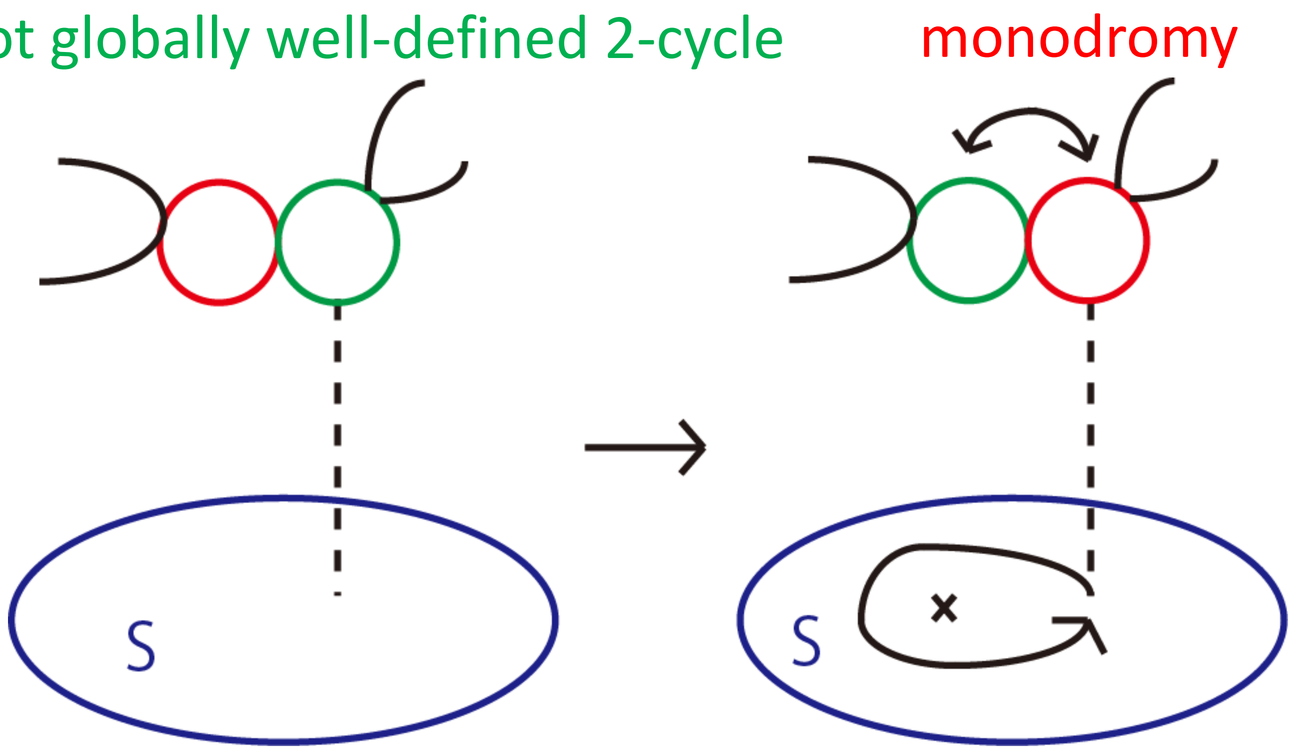
(i) Higgs bundle itself is an approximate description which captures the low energy physics of F-theory. Since the prohibition of proton decay operators requires very high accuracy, we need to know whether an approximation does not hurt an U(1) symmetry.

(ii) When $a_0 \sim 0$, the two roots of spectral surface equation run off to the infinity. This indicates that global E_8 Higgs bundle description fails around $a_0 \sim 0$. We have to make sure that dimension-4 proton decay operators are forbidden around the region.

☆ In order to see whether there is indeed an unbroken U(1) symmetry with high precision, we have to go back to the origin where an unbroken U(1) symmetry comes.

※ Remember that a gauge field comes from an M2-brane which wraps a "globally well-defined" 2-cycle.

× Not globally well-defined 2-cycle



☆ Since Higgs bundle configuration only cut off the relevant 2-cycles for GUT model building, it may misses some monodromy contribution which, by nature, is a global effect.

Therefore, in order to ensure an unbroken U(1) symmetry, we have to look for a monodromy invariant 2-cycle by considering a global compactification structure.

◊ The Method for the Analysis

(i) consider a global defining equation of CY_4
(For simplicity, we use K3-fibered CY_4)

(ii) identify the 2-cycles of a K3 surface from the defining equation of CY_4

(iii) Trace the movement of fibered 2-cycle when we we go around a monodromy point of a base surface S

(i) Set Up

☆ $E_8 \rightarrow E_6 \times \langle SU(3) \rangle$ Example

H.H. et al 1004

Rank 3 Spectral Surface $a_0\xi^3 + a_2\xi + a_3 = 0$

Elliptic-fibered CY_4 with E_6 singularity at z=0

$$y^2 = x^3 + (a_2 z^3 + f_0 z_4)x + (\tfrac{1}{4}a_3^2 z^4 + a_0 z_5 + g_0 z_6 + a_0'' z^7)$$

($a_{0,2,3}, f_0, g_0, a_0''$ are sections over **S**)

The dependence of spectral surface parameters on the defining equation of CY_4 can be read from Heterotic-F theory Duality.

Berglund, Mayr '98
Donagi, Wijnholt 0802
H.H. et al 0805

(ii) Identifying 2-cycles

We are now considering an elliptic-fibered K3 surface.
A K3 surface has 22 2-cycles in general. Two of them are now an elliptic-fiber and a zero section, which is not relevant in the present discussion. So, we need to indentify 20 2-cycles.

◊ Intersection form of an elliptically- fibered K3 surface

$$\begin{pmatrix} -C(E_8) & 0 & 0 & 0 & 0 \\ 0 & -C(E_8) & 0 & 0 & 0 \\ 0 & 0 & H & 0 & 0 \\ 0 & 0 & 0 & H & 0 \\ 0 & 0 & 0 & 0 & H \end{pmatrix} \qquad H = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

an ellipric-fiber and a zero section

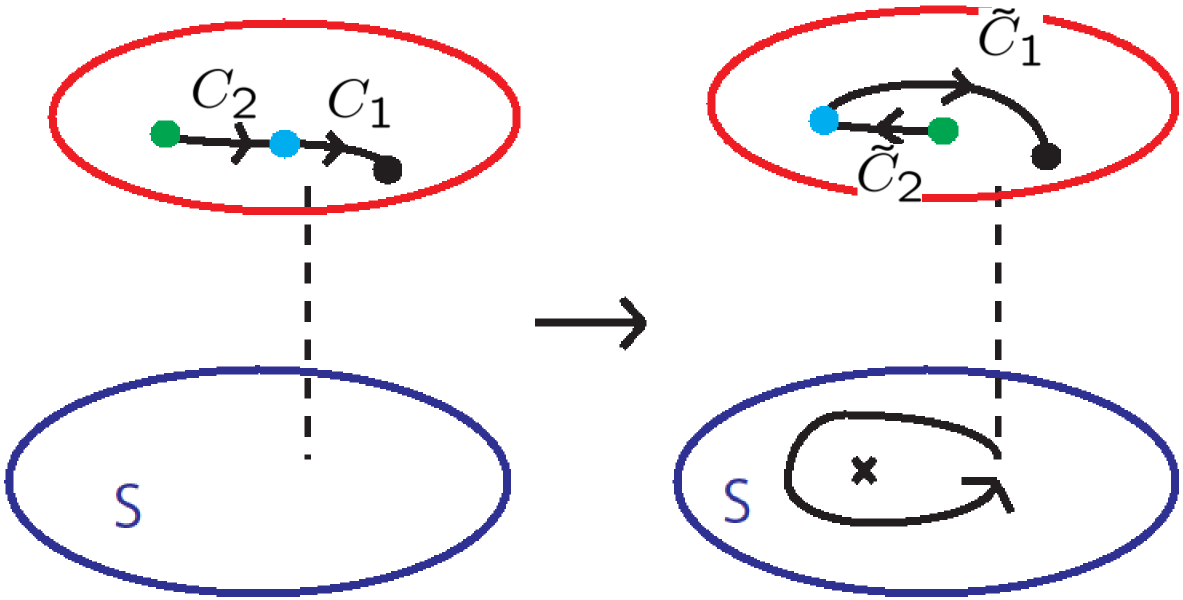
2-cycles which correspond to simple roots of $E_8 \times E_8$

other 2-cycles

relevant for U(1) gauge fields (20 2-cycles)

It is important to note that all those 20 2-cycles are NOT globally well-defined 2-cycles when we fiber a K3 surface over an surface **S** because of the monodromy.

◊ An easy example



$$\tilde{C}_1 = C_1 + C_2, \quad \tilde{C}_2 = -C_2$$

Monodromy of 2-cycles ⟷ The Change of the locations of 7-branes

Remember that the location of [p, q] 7-branes are where a 1-cycle of a torus degenerates and it can be computed from the discriminant.

$$y^2 = x^3 + \underbrace{(a_2 z^3 + f_0 z_4)}_{F} x + \underbrace{(\tfrac{1}{4} a_3^2 z^4 + a_0 z_5 + g_0 z_6 + a_0'' z^7)}_{G}$$

$$\Delta = 4F^3 + 27G^2 = z^8 \times (\text{degree 6 polynomial})$$

8 7-branes at z=0 ↑ (E_6 singularity)

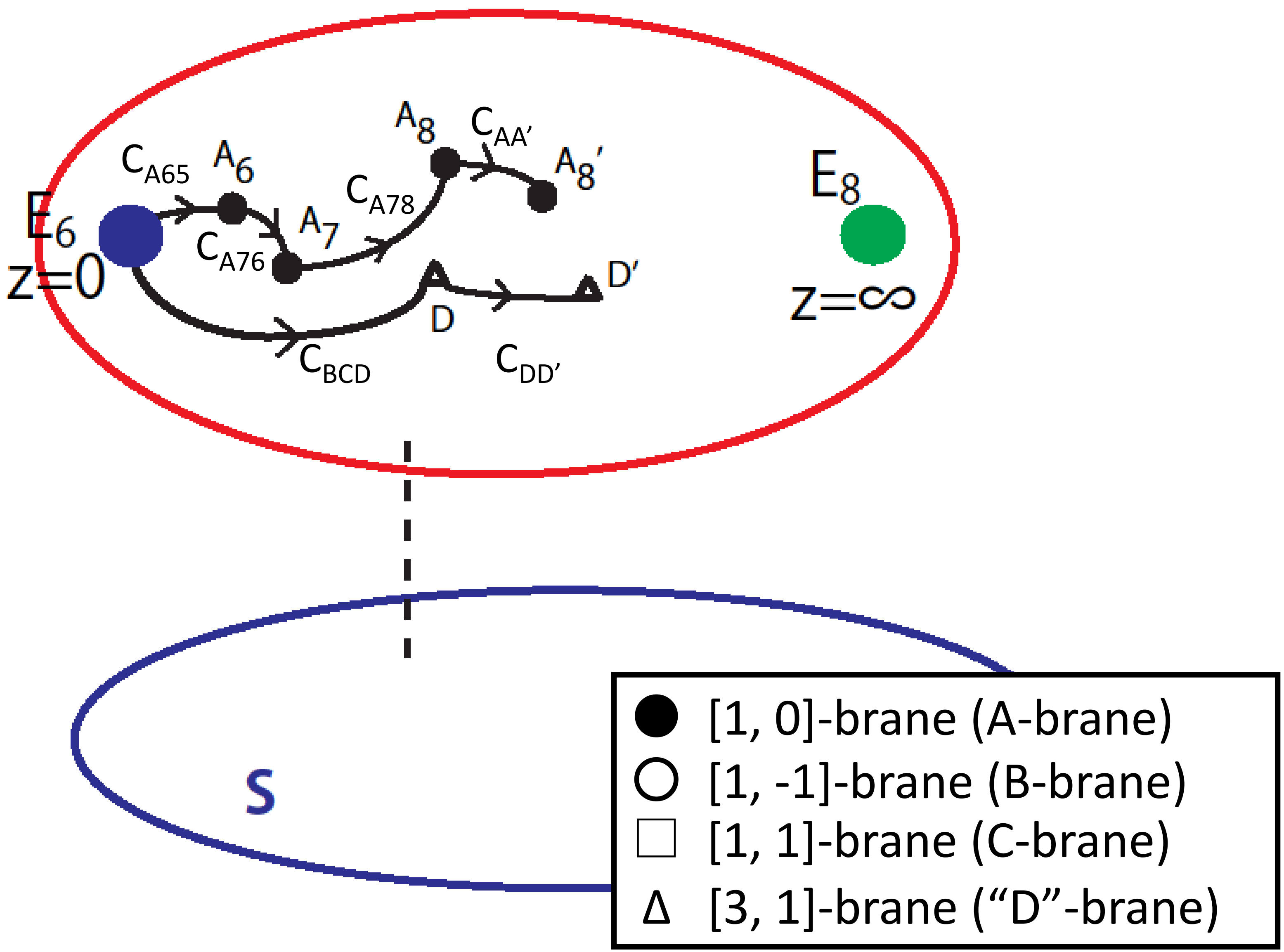
↑ 6 other 7-branes at $z=z_i$ (i=1,..,6)

↑

This degree 6 polynomial governs the movement of 7-branes and it also induces the monodromy of (p, q) strings which end on the 6 7-branes.

24-8-6=10 7-branes are concentrated at z=∞ and it generate Hidden E_8 singularity.

A schematic picture of our model



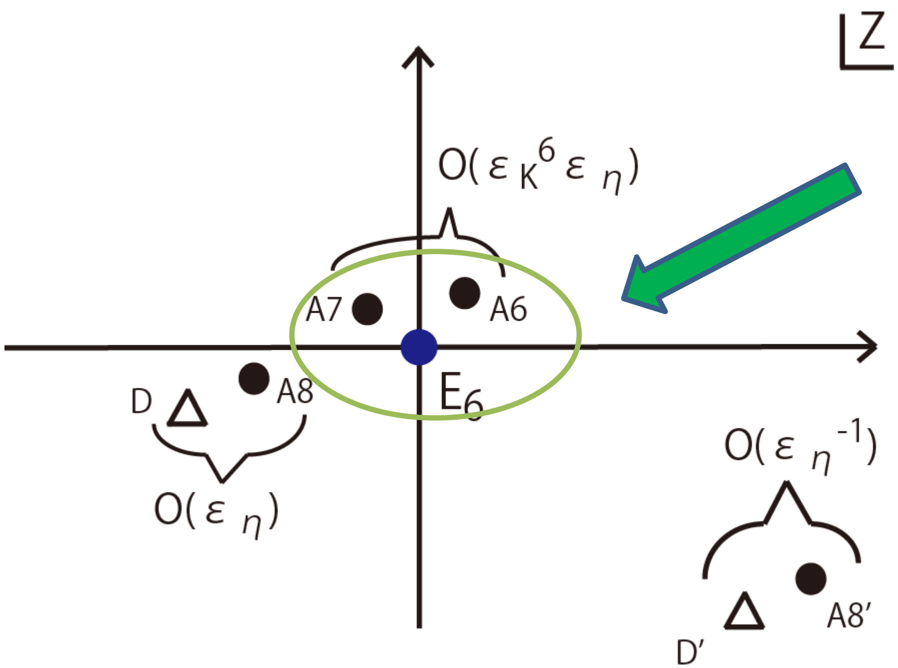
(iii) Monodromy of 2-cycles

☆ 8D Gauge Theory Region

First, we concentrate on the region which (semi-) local model building is concentrated on. This is achieved by the following scaling. $a_0 = a_{0,*}\epsilon_\eta, \quad a_2 = a_{2,*}\epsilon_K^2\epsilon_\eta, \quad a_3 = a_{3,*}\epsilon_K^3\epsilon_\eta$

with $|a_{r,*}| \sim \mathcal{O}(1), \quad 0 \neq |\epsilon_K| << 1, \quad 0 \neq |\epsilon_\eta| < 1$

The locations of 6 7-branes



This part consists of E_8 7-branes and is what we capture by Higgs bundle

○ Monodromy in the 8D gauge theory region

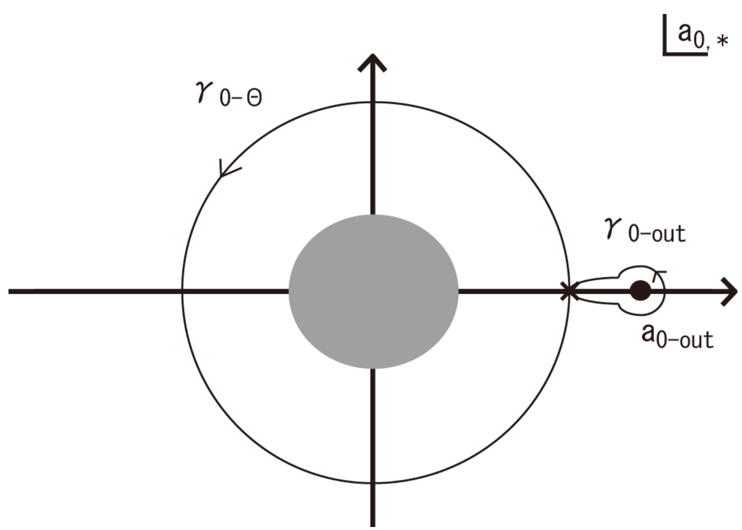
In Higgs bundle description, we only take the leading behavior of discriminant locus.

8D gauge theory region

(degree 6 polynomial) → $\frac{27}{16}a_3^4 + (\frac{27}{2}a_0 a_3^2 + 4a_2^3)z + 27a_0^2 z^2$

Monodromy locus $\Delta' \sim 4a_2^3(27a_0 a_3^2 + 4a_2^3) = 0$

a_2, a_3=constant slice



$$\rho(\gamma_{0-out}) = W_{C_{A76}}$$
$$\rho(\gamma_{0-\theta}) \sim W_{C_{-\theta}}$$

⇒ $S_3 \subset SU(3)$

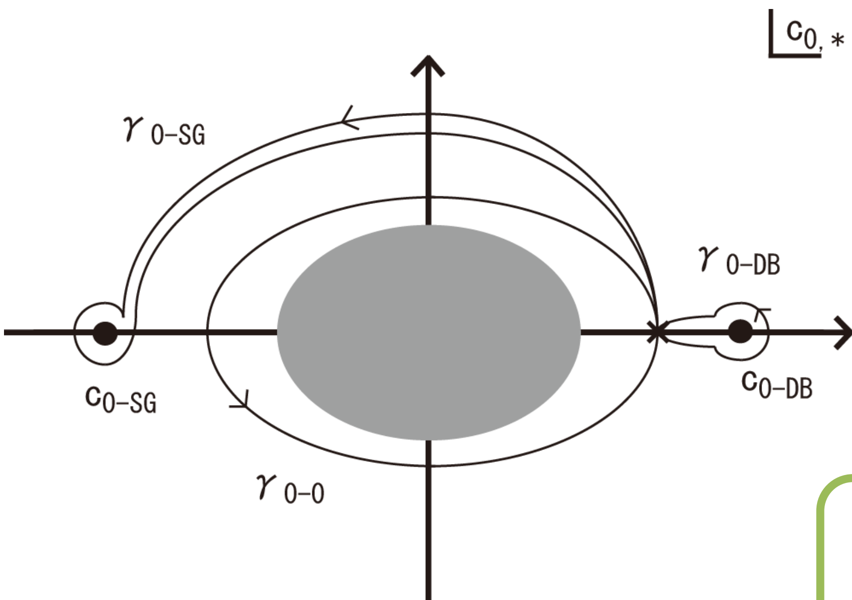
○ Monodromy in the 8D gauge theory region with (2+1) Factorization

$$a_0\xi^3 + a_2\xi + a_3 = (c_0\xi^2 + c_1\xi + c_2)(d_0\xi + d_1)$$

$$a_1 = c_0 d_1 + c_1 d_0 = 0$$
$$\to d_0 = c_0,\ d_1 = -c_1$$

$$\begin{cases} a_0 = c_0^2 \\ a_2 = c_0 c_2 - c_1^2 \\ a_3 = -c_1 c_2 \end{cases}$$

$$\Delta' \sim c_0^6 (4c_0c_2 - c_1^2)(c_0c_2 + c_1^2)^2$$



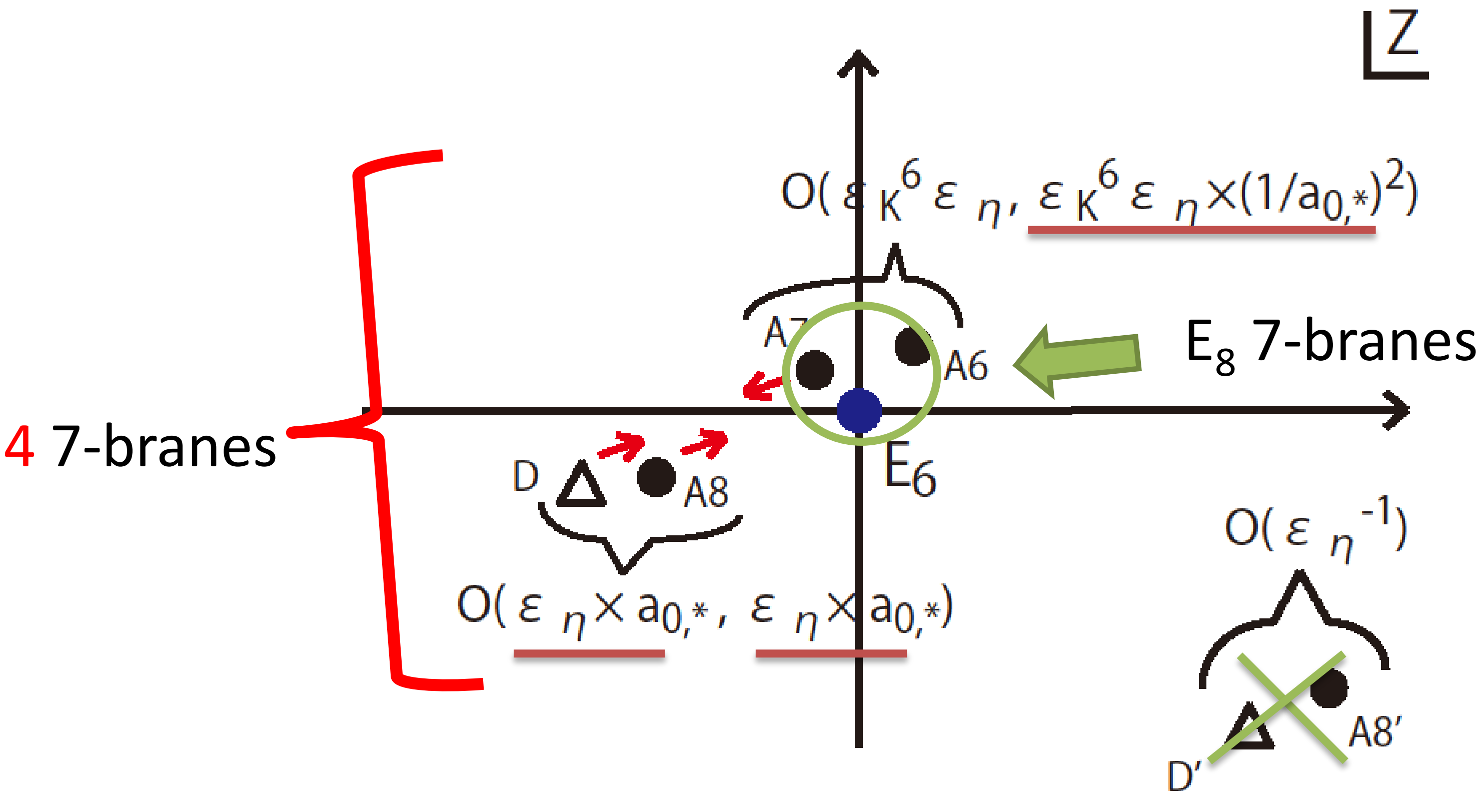
$$\rho(\gamma_{0-SG}) = W_{C_{A76}+C_{-\theta}}$$
$$\rho(\gamma_{0-DB}) = \rho(\gamma_{0-out})^2 = id$$
$$\rho(\gamma_{0-0}) = \rho(\gamma_{0-\theta})^2 \sim id$$

→ $Z_2 \subset SU(3)$

Factorization really reduces monodromy "approximately"

○ "Full" Monodromy without factorization

We explore into the region $a_{0,*} \sim |\varepsilon_K|^2$ .To make the analysis easier, we stay within the region $a_0'' \sim O(\varepsilon_\eta)$.



We include the movement of 4 7-branes, two of which are missed in 8D gauge theory description.

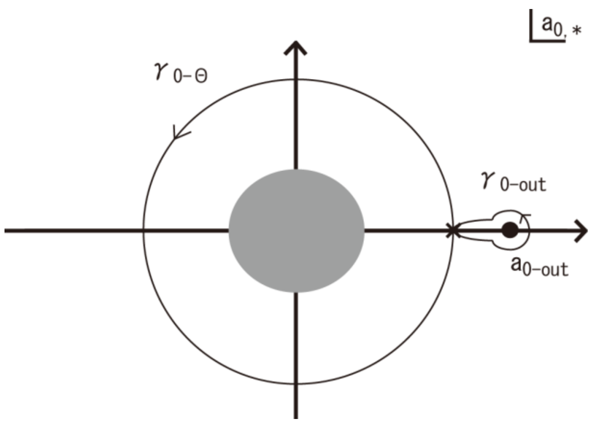
$$\Delta = 4F^3 + 27G^2 = z^8 \times (\text{degree 6 polynomial})$$

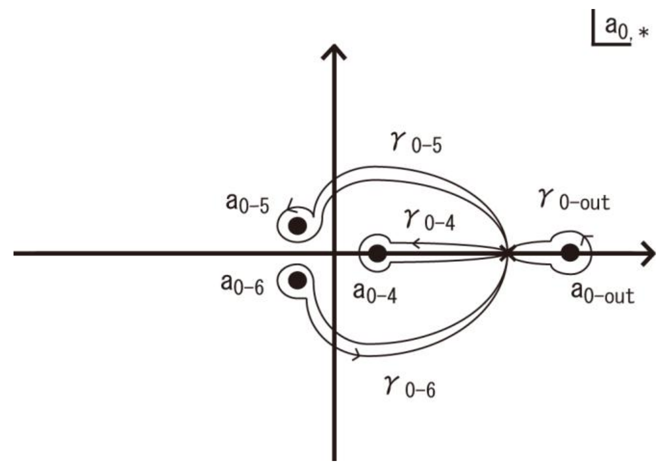
$a_{0,*} \sim |\varepsilon_K|^2$

degree 4 polynomial (4 7-branes)

“Full” Modnoromy locus

(↓8D gauge theory region)





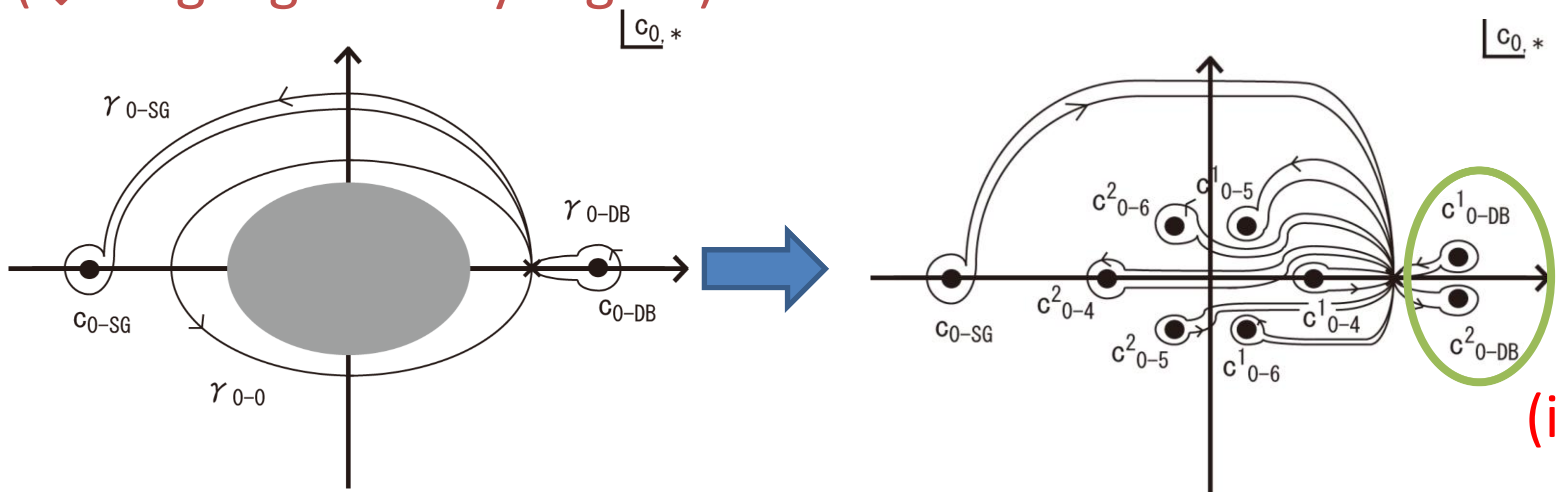
Instead of $\Upsilon_{0\text{-}\Theta}$, we have
New three other loops $\Upsilon_{0\text{-}4,5,6}$!

The monodromy of these loops mix the 2-cycles within E_8 with the ones without E_8!

○ "Full" Modnoromy with factorization

"Full" monodromy locus

(↓8D gauge theory region)



(i) $c_{0\text{-}DB}$ splits into two $c_{0\text{-}DB}{}^{1,2}$

$$\rho(\gamma_{0-SG}) = W_{C_{A76}+C_{-\theta}}$$
$$\rho(\gamma_{0-DB}) = \rho(\gamma_{0-out})^2 = id$$
$$\rho(\gamma_{0-0}) = \rho(\gamma_{0-\theta})^2 \sim id$$

(↑8D gauge theory region)

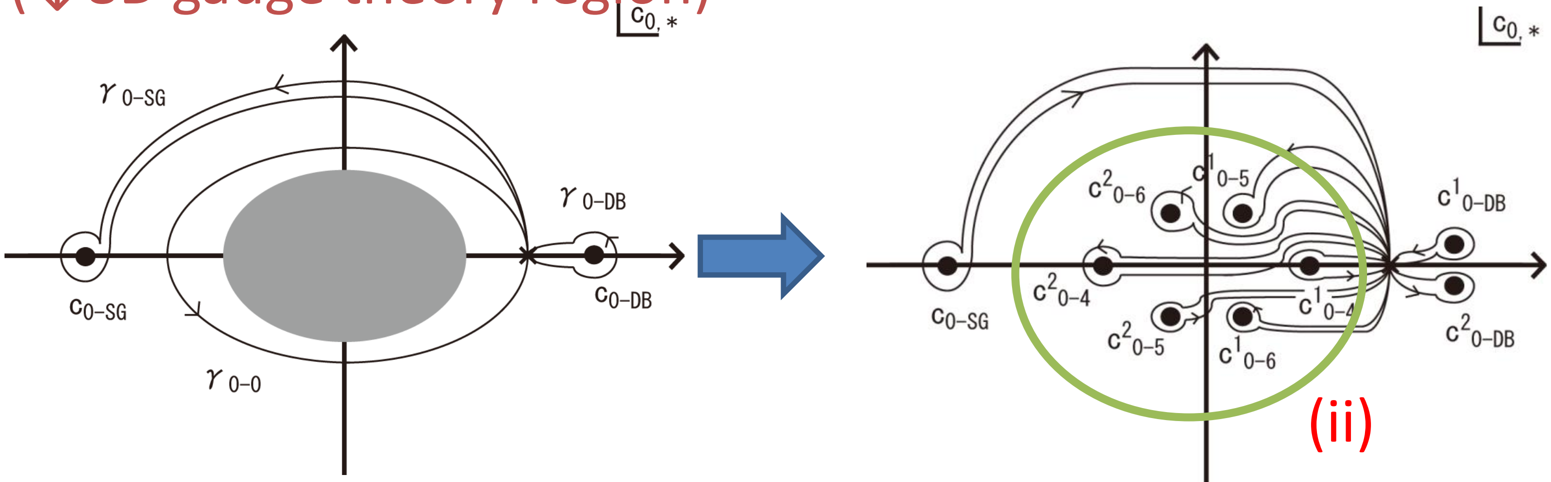
⇒

$$\rho(\gamma_{0-SG}) = W_{C_{A76}+C_{-\theta}}$$
$$\rho(\gamma^{1,2}_{0-DB}) = \rho(\gamma_{0-out}) = W_{C_{A76}}$$

$$\left.\begin{array}{l}\rho(\gamma_{0-SG}) \\ \rho(\gamma^{1,2}_{0-DB})\end{array}\right\} \quad S_3 \,! \subset SU(3)$$

“Full” monodromy locus

(↓8D gauge theory region)



(ii) New 6 $c_{0\text{-}4,5,6}^{1,2}$ points

$$
\begin{aligned}
\rho(\gamma_{0-0}) &= \rho(\gamma_{0-\theta})^2 \sim id \\
\rho(\gamma_{0-SG}) &= W_{C_{A76}+C_{-\theta}} \\
\rho(\gamma_{0-DB}) &= \rho(\gamma_{0-out})^2 = id
\end{aligned}
$$

$\Rightarrow$ $\rho(\gamma^{1,2}_{0-4,5,6})$

→mix the 2-cycles within E_8 with the ones without E_8!

(↑8D gauge theory region)

Even after factorization, monodromy is NOT reduced.
i.e. NO UNBROKEN U(1) SYMMETRY!

5. Conclusion

We revisited the dimension-4 proton decay problem in F-theory and found that a supposed unbroken U(1) symmetry in a simple factorization limit is indeed broken.

To avoid the problem, we need to tune more parameters of the internal space of compacitification.

Discussion

Although we show that dimension-4 proton decay operators are likely to be generated in a factorized spectral surface scenario, it is interesting to compute the coefficients of those operators. There may be a chance to suppressed the dangerous operators