Numerical study of Q-ball formation in gravity mediation

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TH, Takahashi, Yamaguchi, in preparation
Affleck-Dine mechanism

- Affleck-Dine field parametrising “flat directions”
  - In MSSM, there are a large number of fields like quarks, leptons, gauges, Higgs, and their superpartners.

\[
H_u = \begin{pmatrix} 0 \\ \Phi \end{pmatrix}, \quad L = \begin{pmatrix} \Phi \\ 0 \end{pmatrix}, \quad F = D = 0 \quad \therefore \quad V(\Phi) = 0
\]

- Global U(1) symmetry conserves baryon/lepton number

\[
n_{B,L} = i \beta (\Phi^* \dot{\Phi} - \Phi \dot{\Phi}^*)
\]

- Dynamical generation of baryon/lepton number
  - Soft SUSY terms and non-renormalisable terms lift the potential, driving $\Phi$ toward the origin
  - A-term like $\Phi^n + \Phi^{*n}$ kicks $\Phi$ to an angular direction

Affleck, Dine, NPB (1985)
Dine, Randall, Thomas (1996)
Scalar field with global U(1) charge: \( \mathcal{L} = |\partial_\mu \Phi| - V(\Phi) \)

If \( \frac{V(\Phi)}{|\Phi|^2} \) has a minimum for \( \Phi \neq 0 \), non-topological soliton with a given charge, \textit{Q-ball}, exists

In cosmological context,

- dark matter candidate
- baryon/lepton number inside Q-balls protected from spharelon process
- decay rate, evaporation rate, etc.

\textit{crucially depends on charge}

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Field equation and potential of Affleck-Dine field in gravity mediation

\[ \ddot{\Phi} + 3H \dot{\Phi} - \frac{1}{a^2} \nabla^2 \Phi = -V'(\Phi) \]

\[ V(\Phi) = m^2 |\Phi|^2 \left[ 1 + K \log \left( \frac{|\Phi|^2}{M_*^2} \right) \right] - cH^2 |\Phi|^2 + (\text{N.R.}) \]

1-loop correction from gauginos

\[ K = -0.1 \sim -0.01 \]

Enqvist, McDonald, PLB(1998)

\[ \frac{V(\Phi)}{|\Phi|^2} \text{ has a minimum at } \Phi \neq 0 \]

Hence this system has Q-ball solution
Numerical setup

Initial condition
(situation after starting to rotate in the phase space)

\[
\Phi_{in} = M_*, \\
\dot{\Phi}_{in} = i m M_* \epsilon
\]

adding small fluctuations as seed of Q-balls

\[
\left| \frac{\delta \Phi}{\Phi} \right|_{in} = O(10^{-7})
\]

Kasuya, Kawasaki, PRD (2000)

6th-order symplectic integrator by Yoshida (time) + finite difference (space)
(supported by Aphrodite code)

Regarding a region where \(|q(t, x)| > q_c\) as a Q-ball with \(q_c = q(t_{form})/5\)
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Result: 2D

\[ \epsilon = 1 \]

\[ N = 512^2 \]
Result: 3D

$N = 128^3$

charge density

Filaments

Enqvist, et al., PRD(2001)
Multamaki, Vilja, PLB(2002)
Result: 3D

$N = 128^3$

charge density

Large Q-balls at intersections

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Result: 3D

\[ N = 128^3 \]

charge density

Torn to small pieces
Result: 3D

\[ N = 128^3 \]

charge density

Relaxation

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Results: charge distributions

\[ f_{NQ} = aQ^b e^{-cQ^2} \]

\[ a = 71.2, b = 1.29, c = 1.86 \times 10^3 \]

\[ Q_{\text{peak}} = 1.9 \times 10^{-2} |\Phi_{in}|^2 m^{-2} \]

\[ \hat{Q} \equiv \frac{Q}{|\Phi_{in}|^2 m^{-2}} \]

~60% larger than existing result:

\[ Q_{\text{KK}}^{\text{max}} = 1.2 \times 10^{-2} |\Phi_{in}|^2 m^{-2} \]

Kasuya, Kawasaki, PRD (2000)
Result: relations

\[ \hat{Q} = 0.17 \hat{E}^{0.92} \]

\[ \tau = 5000 \]

\[ \epsilon = 1 \]

c.f gravity mediation: \( Q \propto E, \quad R \sim |K|^{-1/2} m^{-1} \)
Result: $2^{\text{nd}}$ stage formation in $\epsilon = 0.01$ case

Recall: $\Phi_{in} = M_*$, $\dot{\Phi}_{in} = imM_*\epsilon$

1$^{\text{st}}$ generation Q-ball: POSITIVE, EXCITED
2$^{\text{nd}}$ generation Q-ball: POSITIVE = NEGATIVE, mildly excited

Excited Q-balls release their excessive energy, producing negative Q-balls
Result: relations

\[
\begin{align*}
\Phi_{in} &= M_* \\
\dot{\Phi}_{in} &= i m M_* \epsilon \left\{ \frac{E}{mQ} \sim \frac{|\Phi|^2}{|\Phi|\dot{\Phi}} \sim \frac{1}{\epsilon} \ll 1 \rightarrow \frac{E}{mQ}\right|_{f_{in}} \sim O(10)
\end{align*}
\]
Result: charge distributions

For small $\varepsilon$, # of +/- Q-balls eventually become the same.

Peak charge of 1$\text{st}$-gen Q-balls scales as

$$Q \sim |\Phi \dot{\Phi}| \propto \varepsilon$$

The scaling becomes no longer valid for 2$\text{nd}$-gen Q-balls

cf. this scaling is broken down also in gauge mediation.

Kasuya, Kawasaki, PRD (2001)
Q-balls could be a promising source of GWs

- First numerical simulation: Kusenko et al.
  - Kusenko, Mazumdar, PRL (2008)
  - Kusenko, Mazumdar, Multamaki, PRD (2009)
- Analytical estimation with 'thermal-log term' contributions
  - Chiba, Kamada, Yamaguchi, PRD (2010)

Gravitational wave energy and spectrum

\[ \rho_{GW} = \frac{1}{32\pi G} \langle \dot{h}_{ij} \dot{h}_{ij} \rangle \]
\[ \Omega_{GW} = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \log k} \]

The basic structure of the spectrum is reflected by the existence of the filamentary structure before Q-ball formation epoch.
Energy spectrum at Q-ball formation epoch

\[ N = 200^3 \]
\[ \varepsilon = 1 \]
\[ Lm = 1 \]

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\[ \Omega_{GW} \left( |\Phi_{in}| / M_{pl} \right)^{-4} \]

GWs produced by the growth of filamentary structure
GWs produced by fragmentation

\[ Q \text{-ball size} \]
\[ k_Q \sim \sqrt{|K|} m \sim 0.3m \]
Box size effect

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Prediction of peak position seems robust

$\frac{k_{\text{peak}}}{m} \approx 0.06 \sim 0.07$

Peak appears at the scale that is several times larger than Q-ball size. Filaments may be crucial.

$N = 200^3$
$\varepsilon = 1$

$L = 2.0$

$L = 0.5$
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\[ \varepsilon = 0.01 : \text{early time} \quad mt \leq 1000 \]

\[ m t = 1000 \quad N = 200^3 \quad \varepsilon = 0.01 \quad L m = 1 \]
There are no significant change after the end of fragmentation process.
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Time evolution of amplitude for $\varepsilon = 0.01$

Even during the 2$^{nd}$-formation epoch, we cannot observe extra production of GWs. 2$^{nd}$-formation process is not so violent process.

$N = 200^3$
$\varepsilon = 0.01$

$k_{\text{phys}} = 0.06 \sim 1.2$
Difference between $\varepsilon = 1$ and $\varepsilon = 0.01$

In the case with elliptic orbit IC, the final GW amplitude is suppressed a bit.
Summary

- 3D simulations for Q-ball formation in gravity mediation.

- Charge distribution of Q-balls
  - 'circular' case: the peak charge is slightly larger than existing results
  - 'elliptic' case: eventually the same numbers of +/- Q-balls appear, and peak charge, scaling, ...

- Power spectrum of GWs from Q-balls
  - *early epoch*: large scale GWs may be associated with filamentary structure. But it remains unclear now.
  - formation(fragmentation) epoch: small scale significantly grows.
  - relaxation epoch: no more grows even during 2\textsuperscript{nd}-formation process, though it's crucial for the final shape of charge distribution.

- Filamentary structure plays a crucial role for both charge distributions and the peak amplitude of GWs.

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TH, Kawasaki, Takahashi, JCAP 06(2010)008
TH, Takahashi, Yamaguchi, in preparation