Numerical study of Q-ball formation in gravity mediation

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COLLABORATION WITH

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TH, Kawasaki, Takahashi, JCAP 06(2010)008 [arXiv:1003.1779]
TH, Takahashi, Yamaguchi, in preparation

Affleck-Dine mechanism

- Affleck-Dine field parametrising "flat directions"

  Affleck, Dine, NPB (1985)

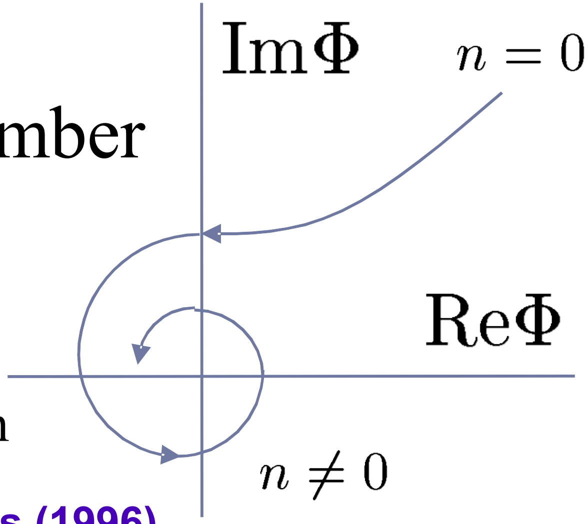
  - In MSSM, there are a large number of fields like quarks, leptons, gauges, Higgs, and their superpertnars.

$$H_u = \begin{pmatrix} 0 \\ \Phi \end{pmatrix} \quad L = \begin{pmatrix} \Phi \\ 0 \end{pmatrix} \qquad F = D = 0 \quad \therefore V(\Phi) = 0$$

- Global U(1) symmetry conserves baryon/lepton number

$$n_{B,L} = i\beta(\Phi^* \dot{\Phi} - \Phi \dot{\Phi}^*)$$

- Dynamical generation of baryon/lepton number
  - Soft SUSY terms and non-renormalisable terms lift the potential, driving Φ toward the origin
  - A-term like $\Phi^n + \Phi^{*n}$ kicks Φ to angular direction



Dine, Randall, Thomas (1996)

Q-ball

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Scalar field with global U(1) charge : $\mathcal{L} = |\partial_\mu \Phi| - V(\Phi)$

If $\dfrac{V(\Phi)}{|\Phi|^2}$ *has a minimum for* $\Phi \neq 0$

Coleman, NPB (1985)

⇒ *non-topological soliton with a given charge, Q-ball, exists*

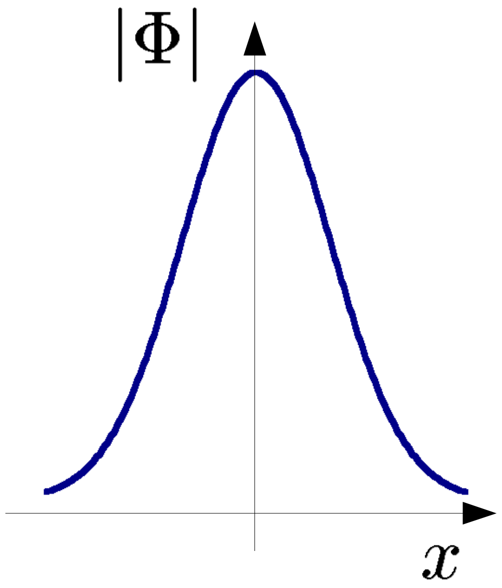
In cosmological context,

- ✔ dark matter candidate
- ✔ baryon/lepton number inside Q-balls protected from spharelon process

↑

decay rate, evaporation rate, etc....

crucially depends on charge



Equations

Field equation and potential of Affleck-Dine field in gravity mediation

$$\ddot{\Phi} + 3H\dot{\Phi} - \frac{1}{a^2}\nabla^2\Phi = -V'(\Phi)$$

$$V(\Phi) = m^2|\Phi|^2\left[1 + K\log\left(\frac{|\Phi|^2}{M_*^2}\right)\right] - cH^2|\Phi|^2 + (\mathrm{N.R.})$$

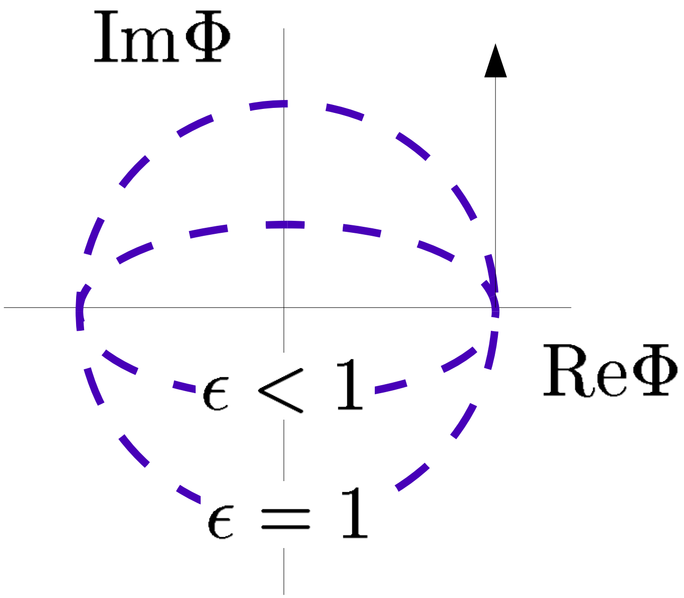
1-loop correction from gauginos $K = -0.1 \sim -0.01$

Enqvist, McDonald , PLB(1998)

$V(\Phi)/|\Phi|^2$ has a minimum at $\Phi \neq 0$

Hence this system has Q-ball solution

Numerical setup



Initial condition
(situation after starting to rotate in the phase space)

$$\Phi_{in} = M_*$$

$$\dot{\Phi}_{in} = imM_*\epsilon$$

adding small fluctuations as seed of Q-balls

$$\left|\frac{\delta\Phi}{\Phi}\right|_{in} = O(10^{-7})$$

Kasuya, Kawasaki, PRD (2000)

6th-order symplectic integrator by Yoshida (time)+ finite difference (space)
(supported by Aphrodite code)

Regarding a region where $|q(t,x)| > q_c$ as a Q-ball with $q_c = q(t_{form})/5$

Result : 2D

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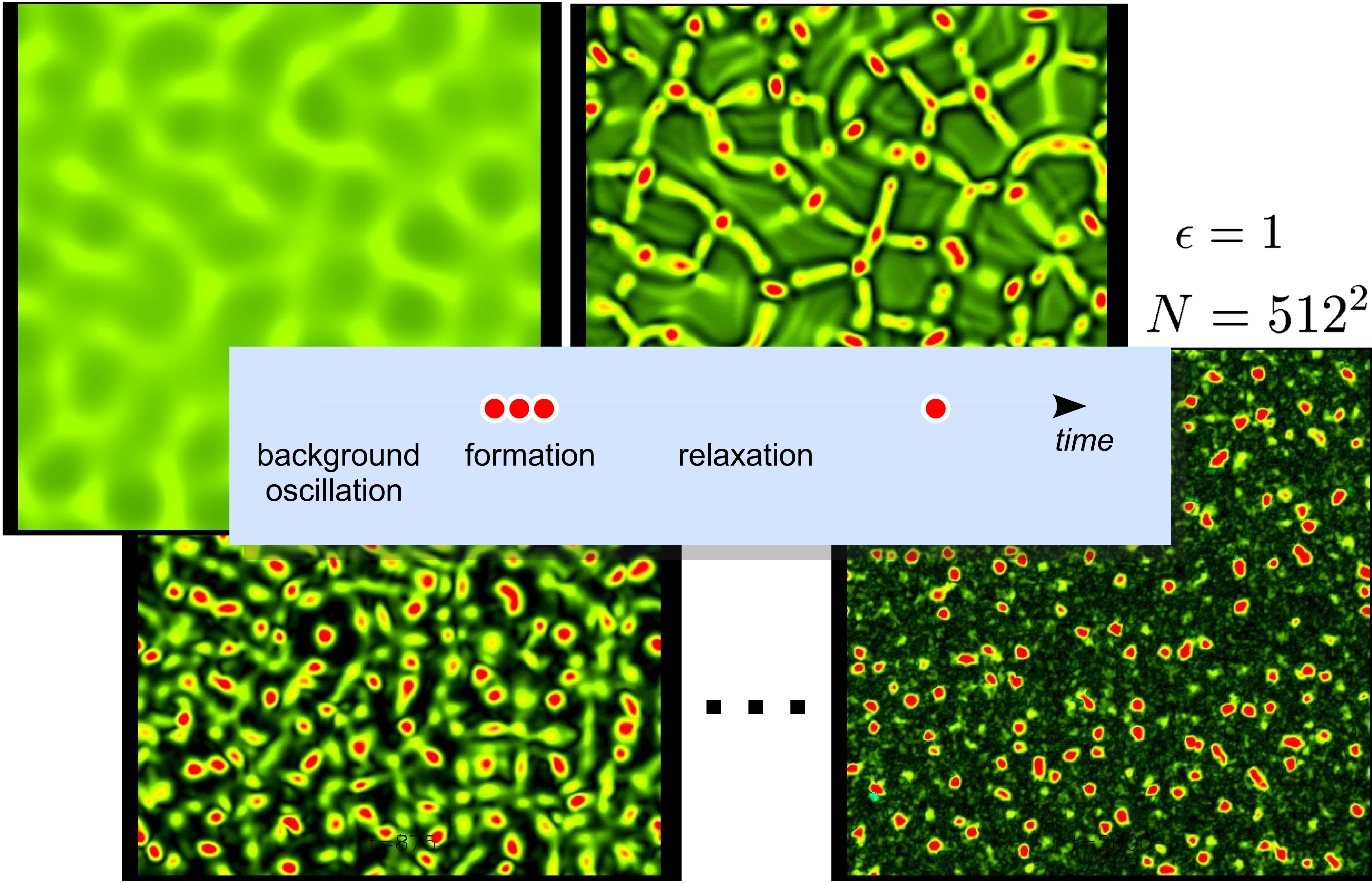

$$\epsilon = 1$$

$$N = 512^2$$

Result : 3D

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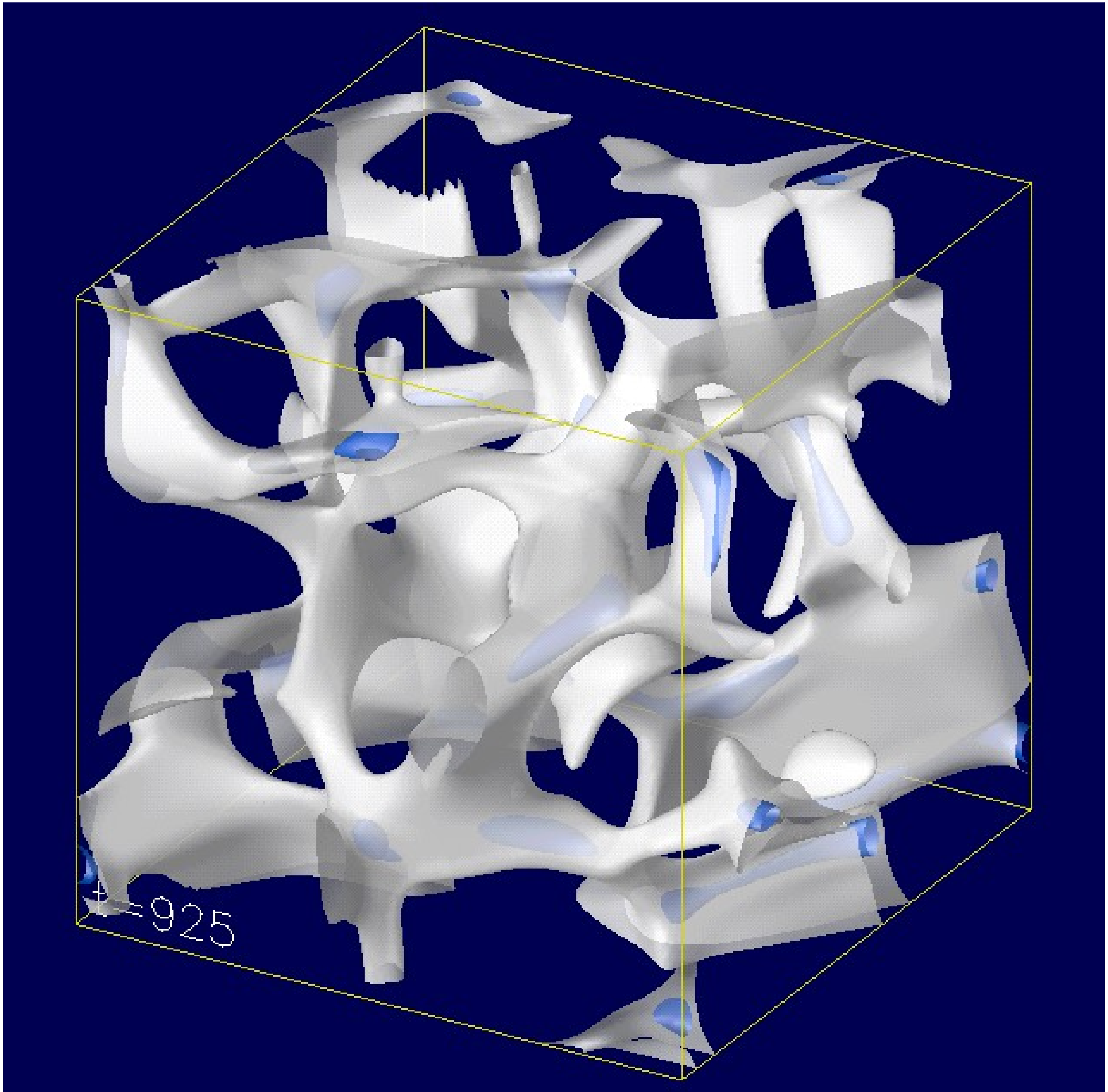
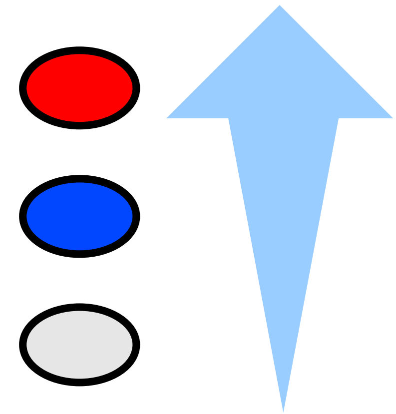


$N = 128^3$

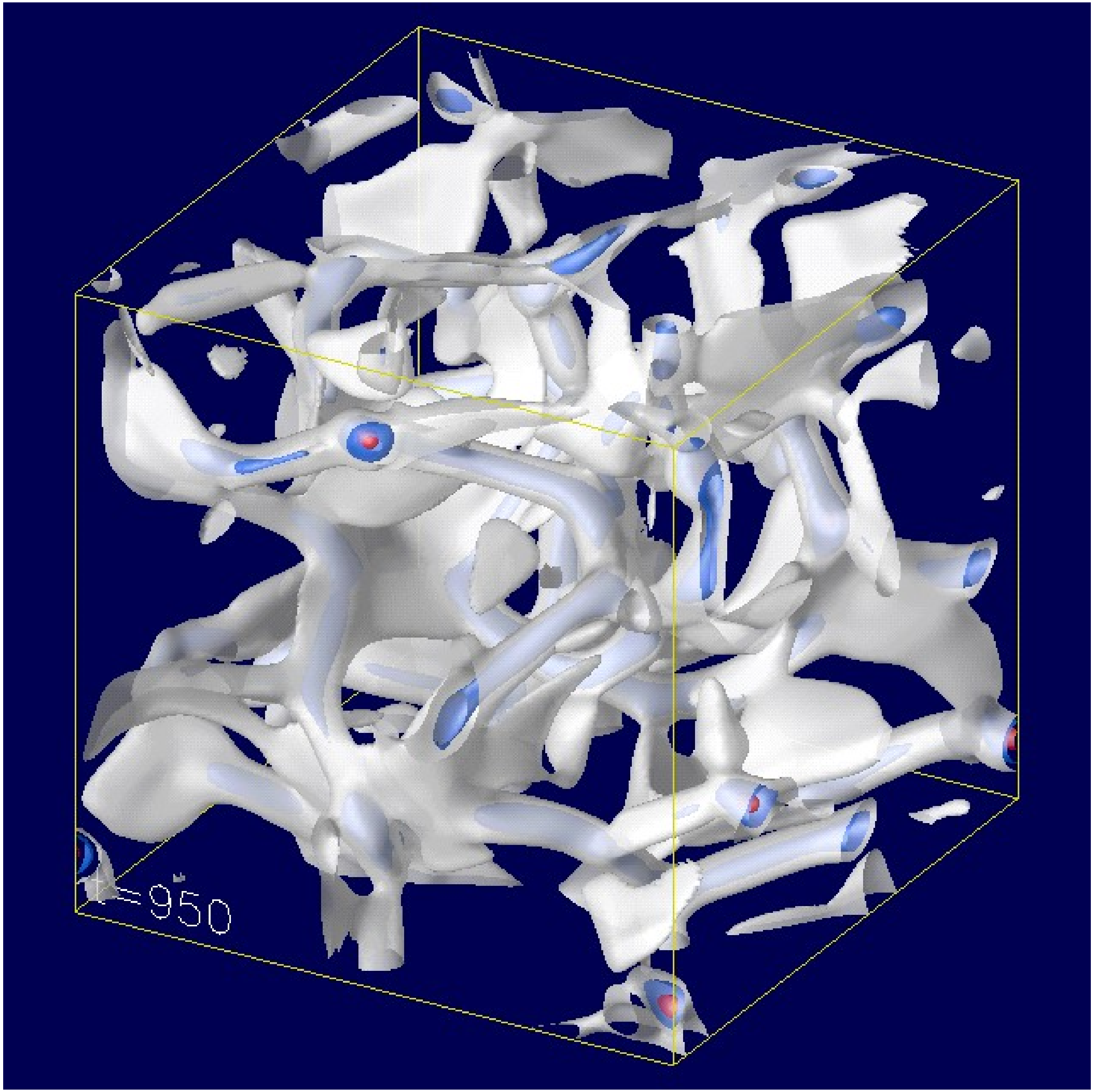


charge density

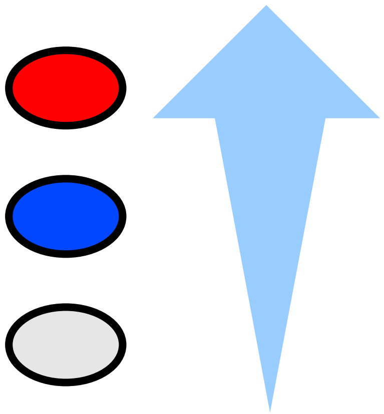
Filaments

Enqvist, et al. , PRD(2001)
Multamaki, Vilja, PLB(2002)

Result : 3D



$$N = 128^3$$



charge density

Large Q-balls at intersections

Result : 3D

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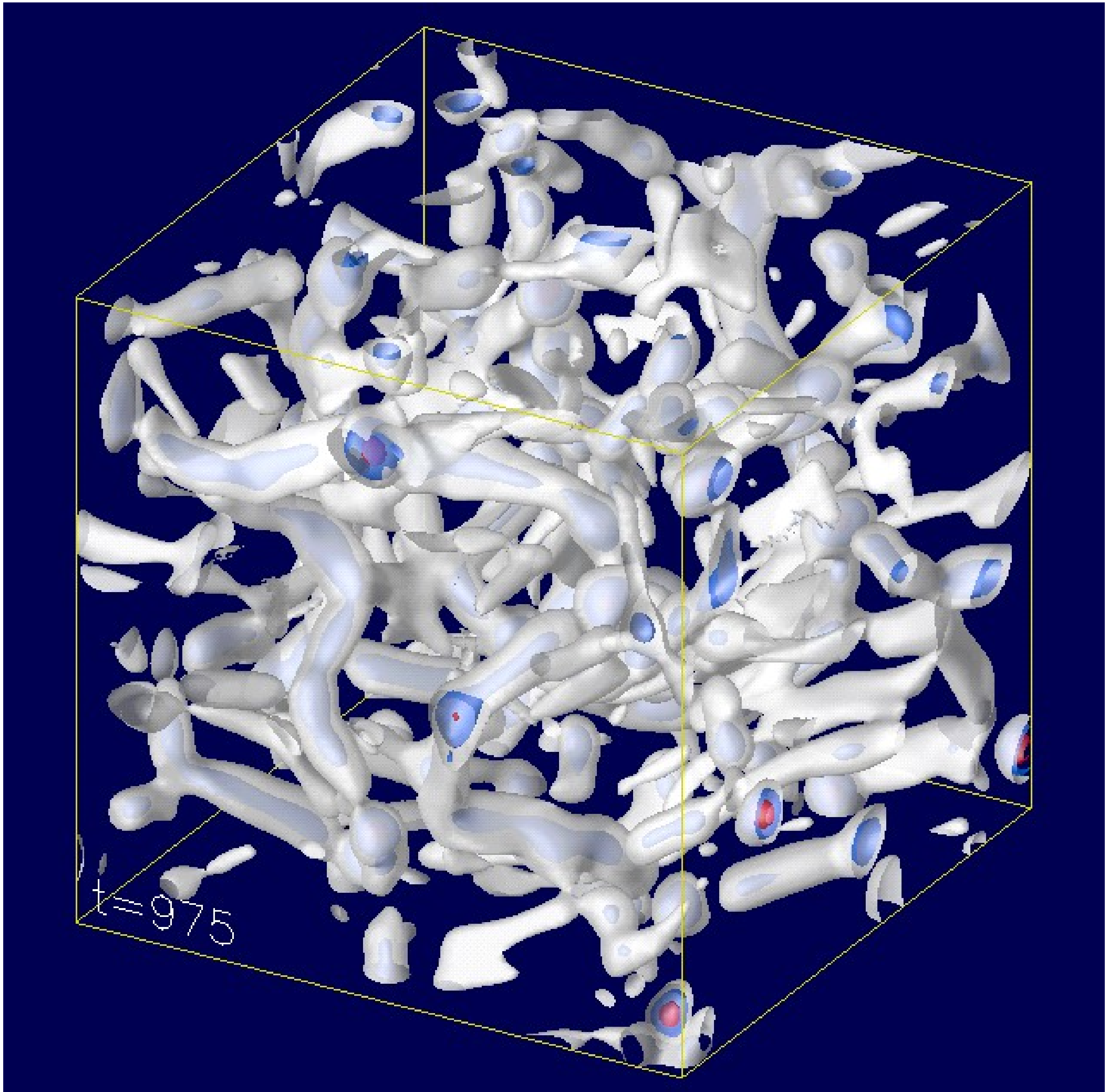
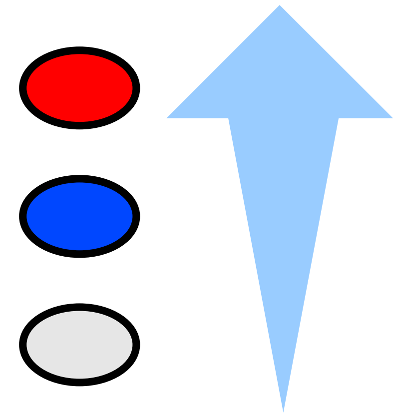


$$N = 128^3$$



charge density

Torn to
small pieces

Result : 3D

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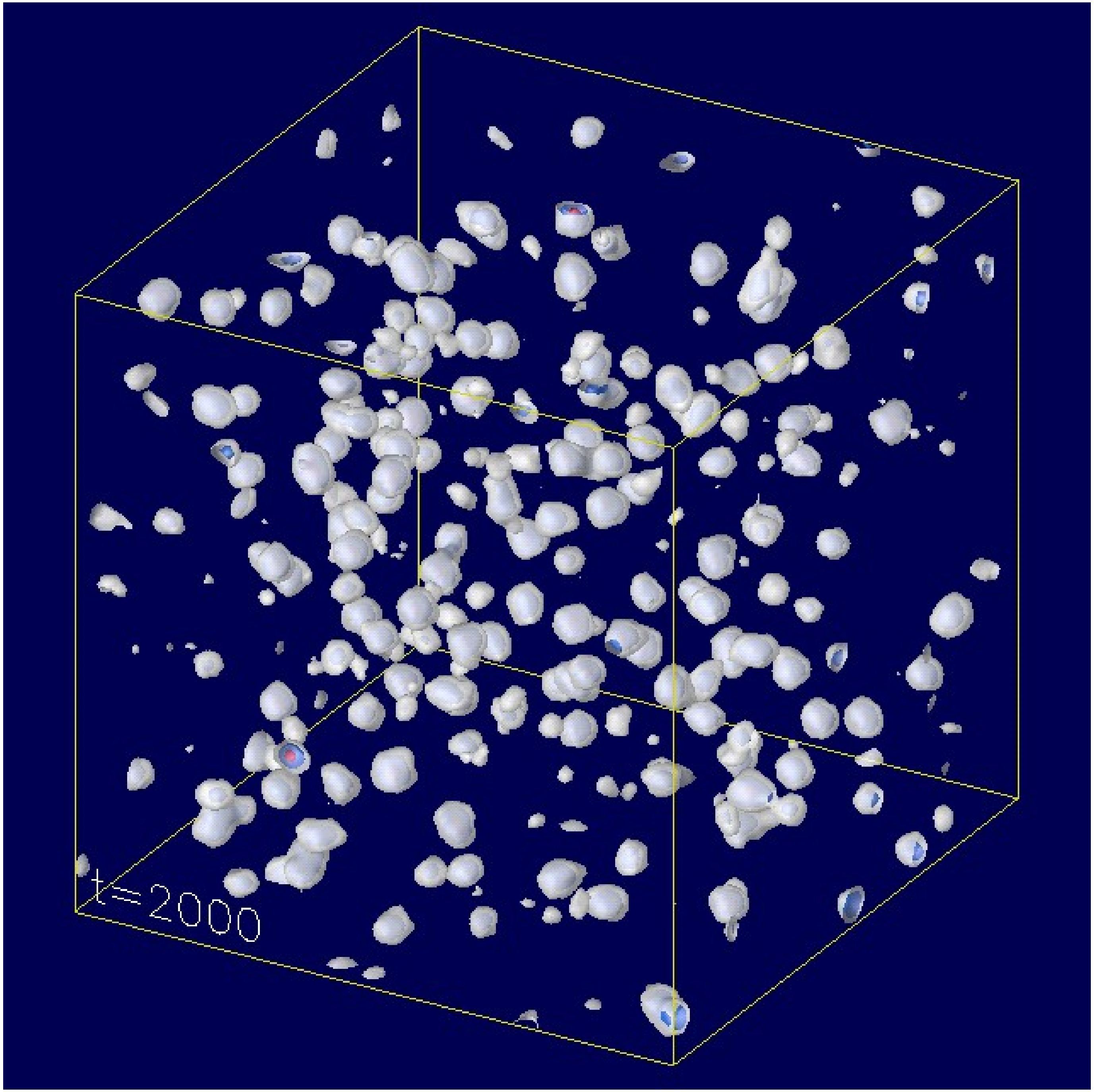
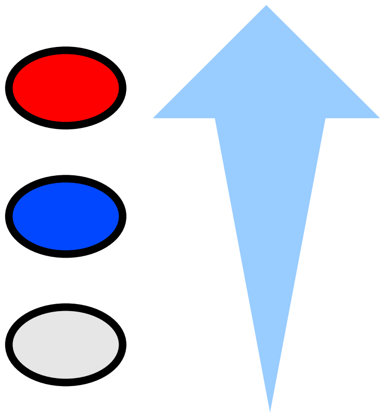


$$N = 128^3$$



charge density

Relaxation

Results : charge distributions

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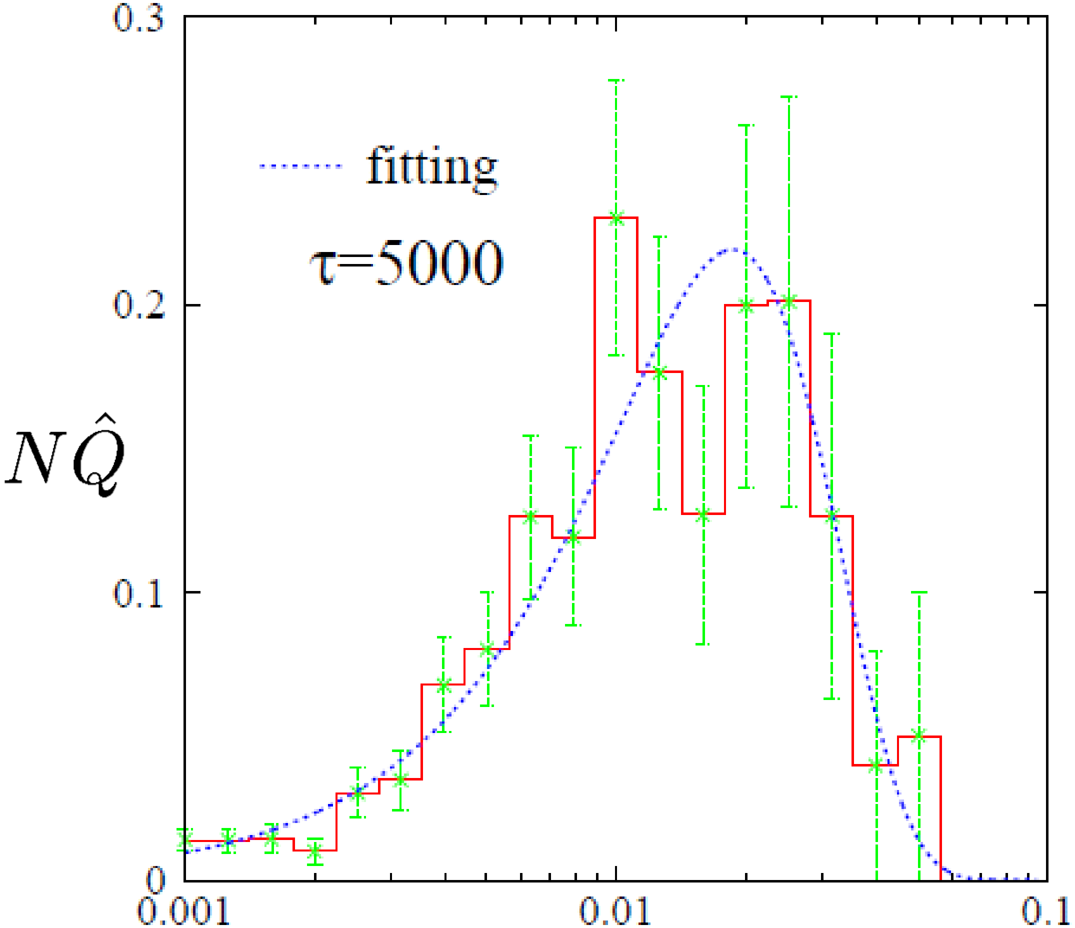

$$\hat{Q} \equiv \frac{Q}{|\Phi_{in}|^2 m^{-2}}$$

$$f_{NQ} = aQ^b e^{-cQ^2}$$

$$a = 71.2, b = 1.29, c = 1.86 \times 10^3$$

$$Q_{\rm peak} = 1.9 \times 10^{-2} |\Phi_{in}|^2 m^{-2}$$

~60% larger than existing result :

$$Q_{\rm max}^{\rm KK} = 1.2 \times 10^{-2} |\Phi_{in}|^2 m^{-2}$$

Kasuya, Kawasaki, PRD (2000)

Result : relations

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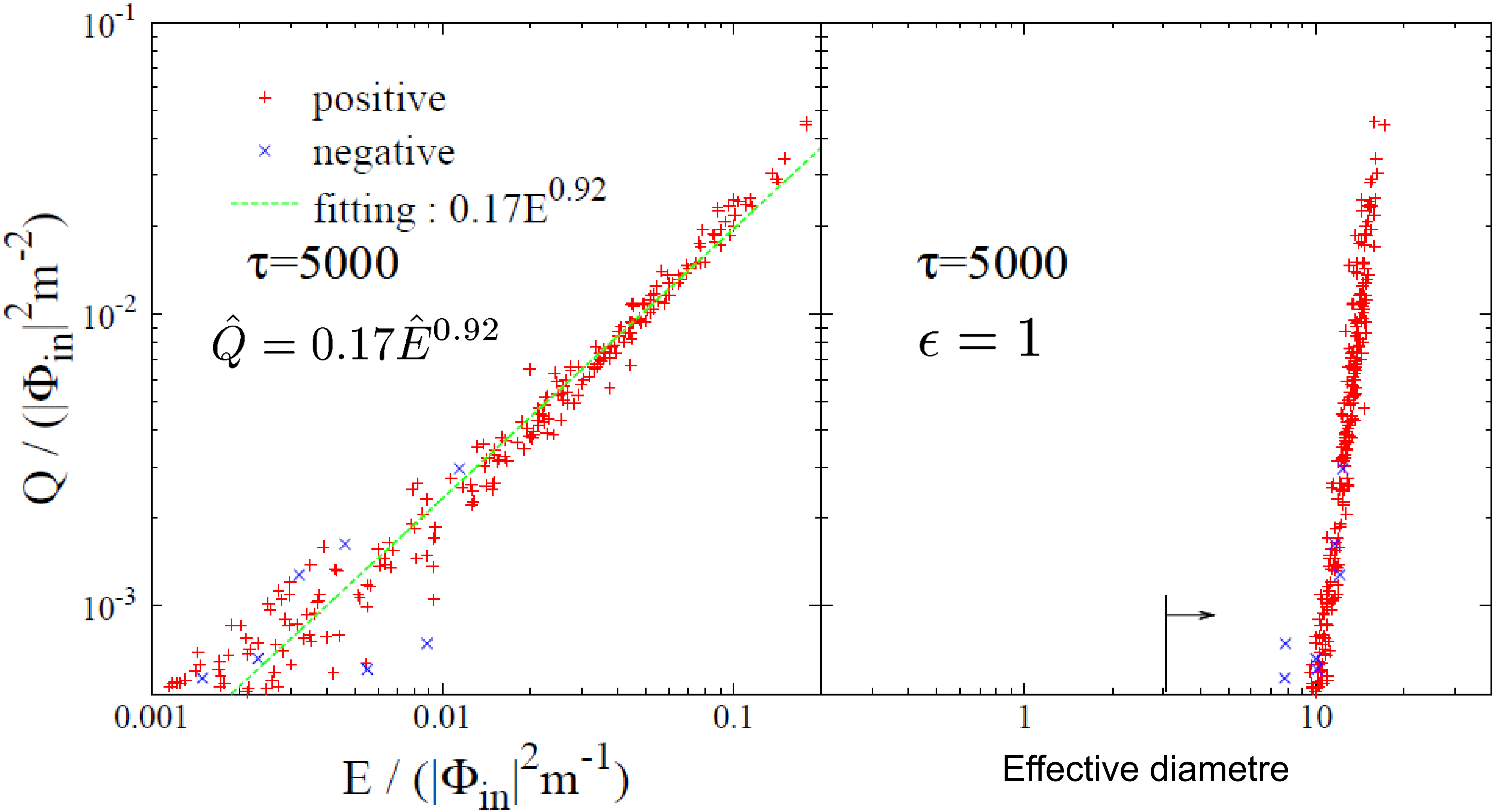

c.f gravity mediation : $Q \propto E, \quad R \sim |K|^{-1/2} m^{-1}$

Result : 2nd stage formation in $\epsilon = 0.01$ case

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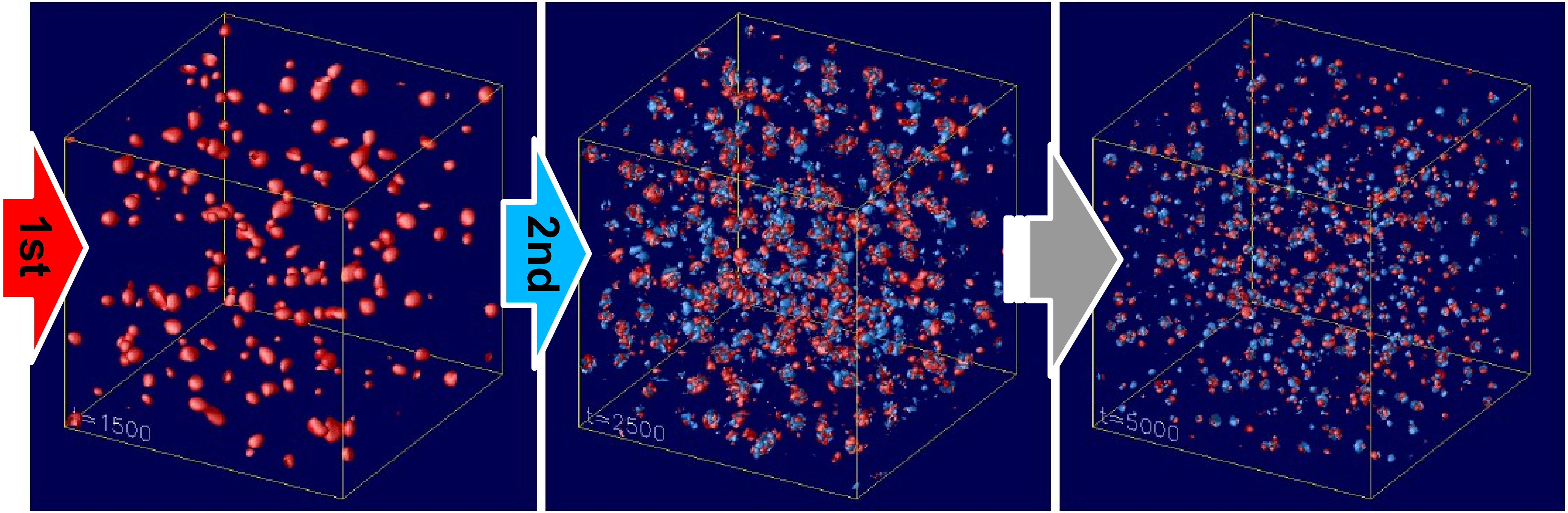
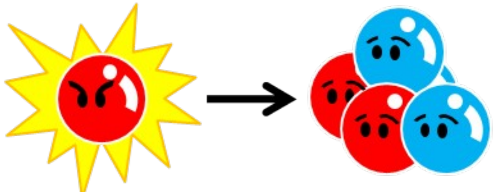
Recall : $\Phi_{in} = M_*$ $\dot{\Phi}_{in} = imM_*\epsilon$



(a) $\tau = 1500$ (b) $\tau = 2500$ (c) $\tau = 5000$



- 1st generation Q-ball : POSITIVE, EXCITED
- 2nd generation Q-ball : POSITIVE=NEGATIVE, mildly excited

Excited Q-balls release their excessive energy , producing negative Q-balls

Result : relations

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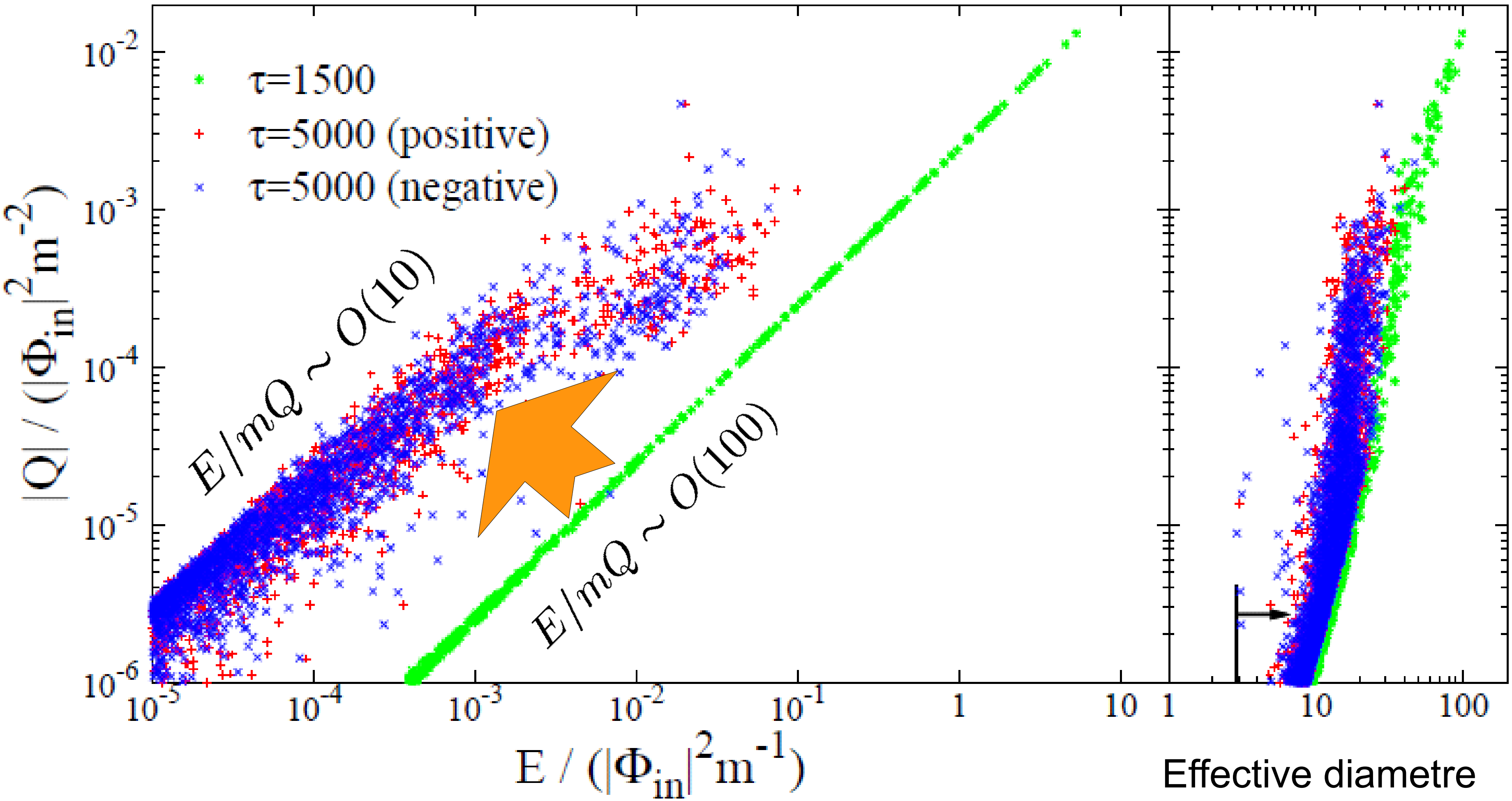
$$\left.\begin{aligned}\Phi_{in} &= M_* \\ \dot{\Phi}_{in} &= imM_*\epsilon\end{aligned}\right\} \quad \frac{E}{mQ} \sim \frac{|\Phi|^2}{|\Phi\dot{\Phi}|} \sim \frac{1}{\epsilon} \ll 1 \longrightarrow \left.\frac{E}{mQ}\right|_{fin} \sim O(10)$$



Result : charge distributions

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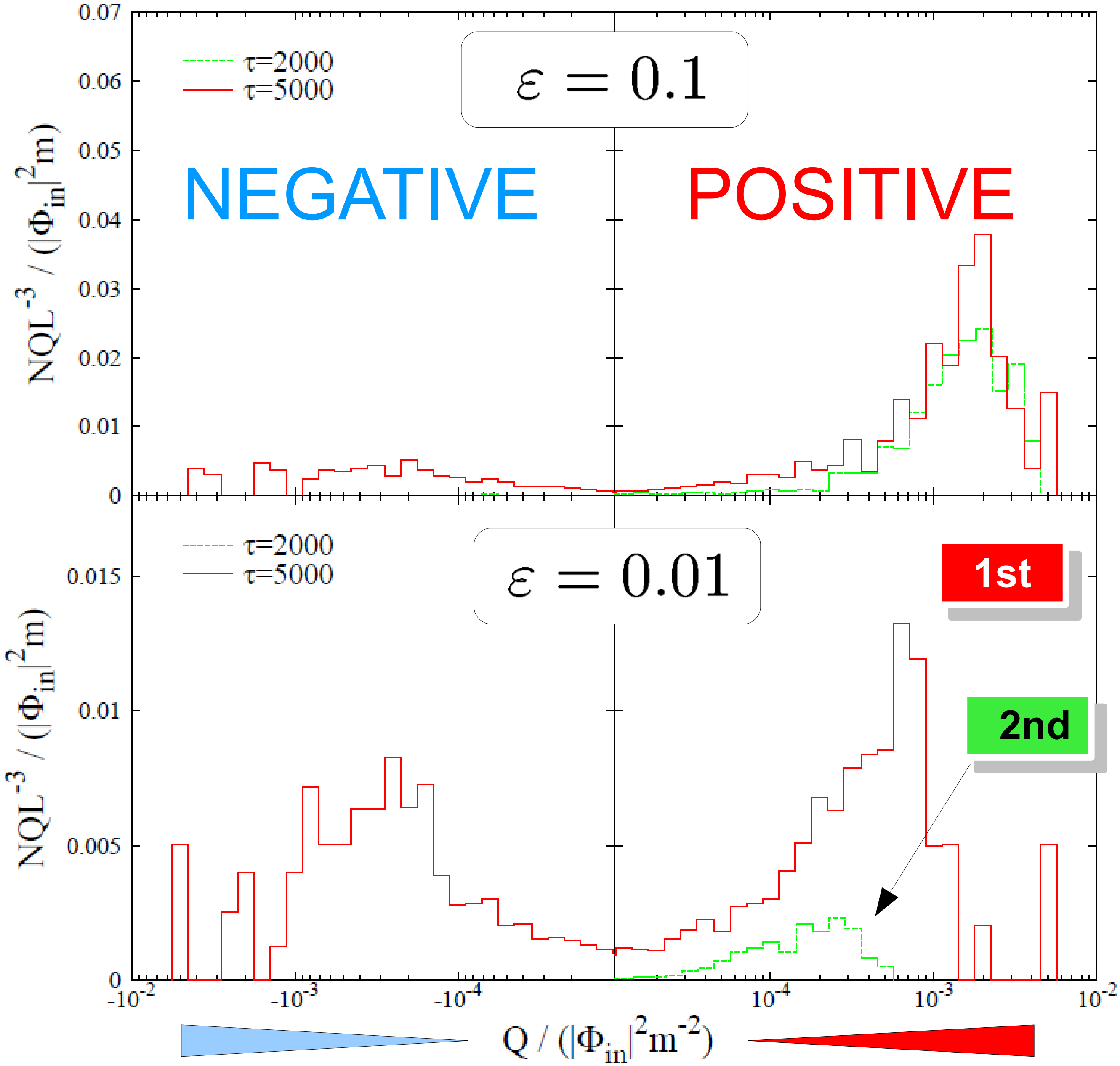

- For small ε , # of +/- Q-balls eventually become the same.
- Peak charge of 1st-gen Q-balls scales as

$$Q \sim |\Phi\dot{\Phi}| \propto \varepsilon$$

- The scaling becomes no longer valid for 2nd-gen Q-balls

cf. this scaling is broken down also in gauge mediation.

Kasuya, Kawasaki, PRD (2001)

Gravitational waves

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- Q-balls could be a promising source of GWs
  - First numerical simulation : Kusenko et al.

Kusenko, Mazumdar, PRL (2008)
Kusenko, Mazumdar, Multamaki, PRD (2009)

  - Analytical estimation with 'thermal-log term' contributions

Chiba, Kamada, Yamaguchi, PRD (2010)

- Gravitational wave energy and spectrum

$$\rho_{\rm GW} = \frac{1}{32\pi G}\langle \dot{h}_{ij}\dot{h}_{ij}\rangle \qquad \Omega_{\rm GW} = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\log k}$$

- The basic structure of the spectrum is reflected by the existence of the filamentary structure before Q-ball formation epoch.

Energy spectrum at Q-ball formation epoch

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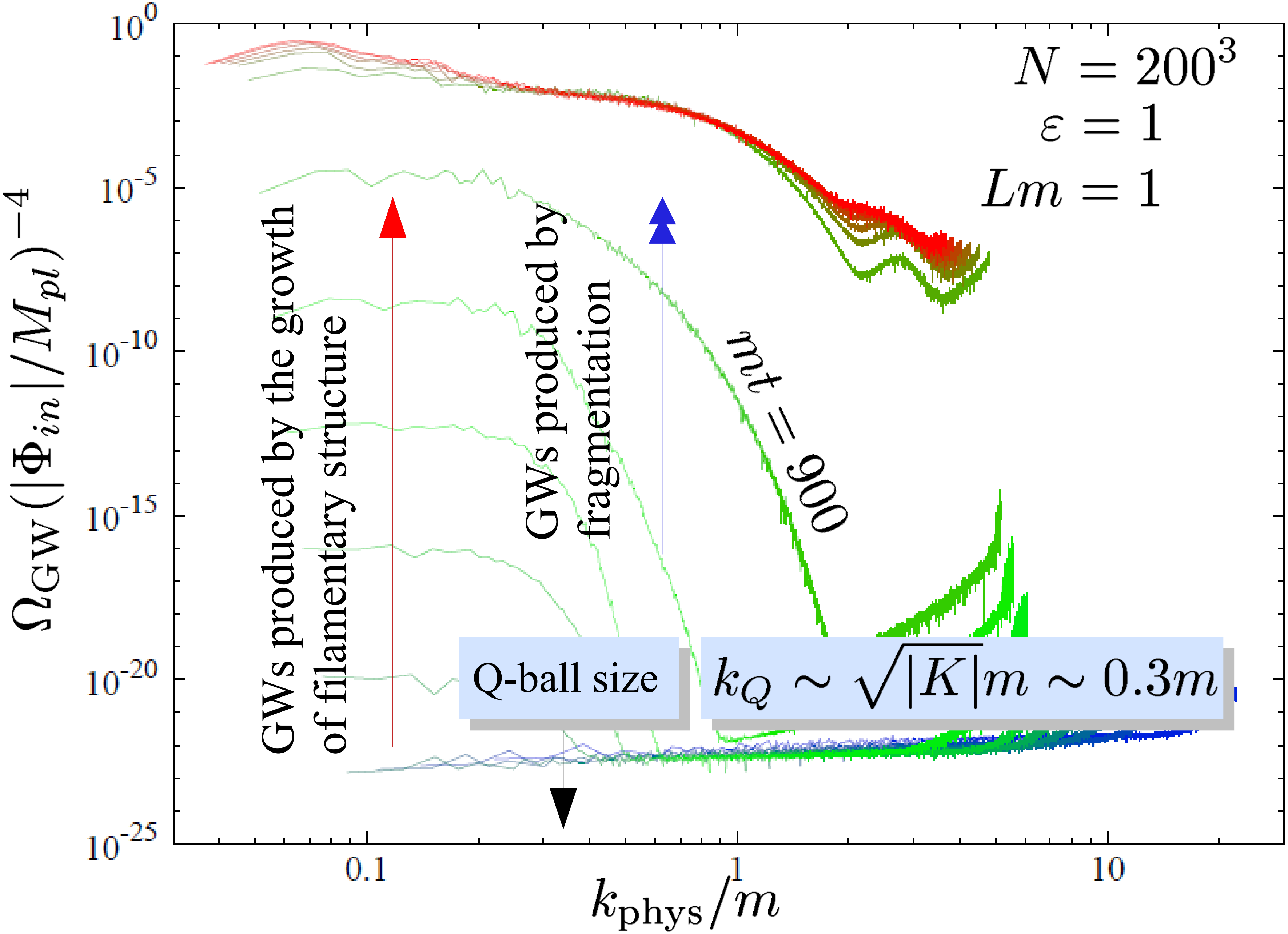
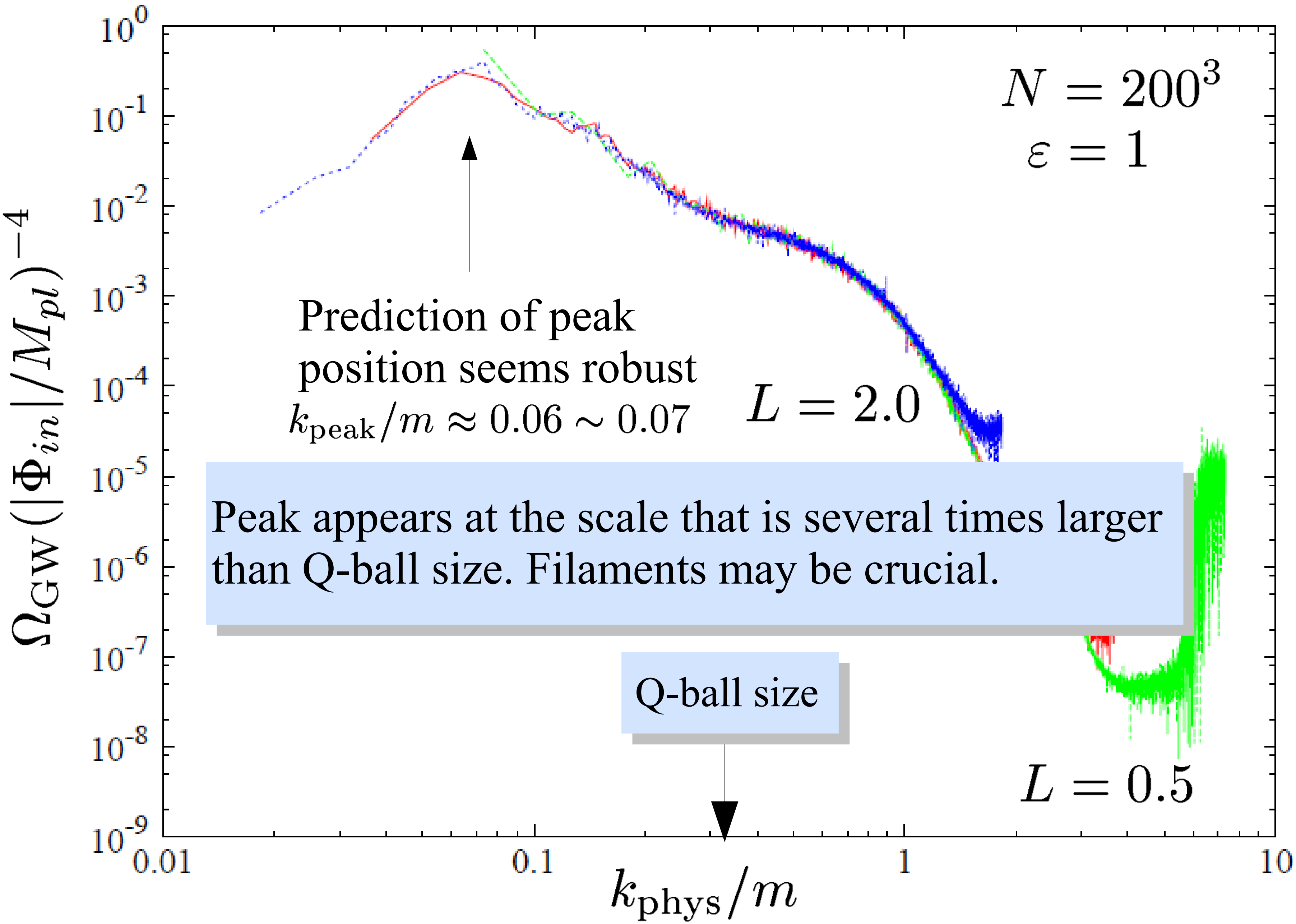
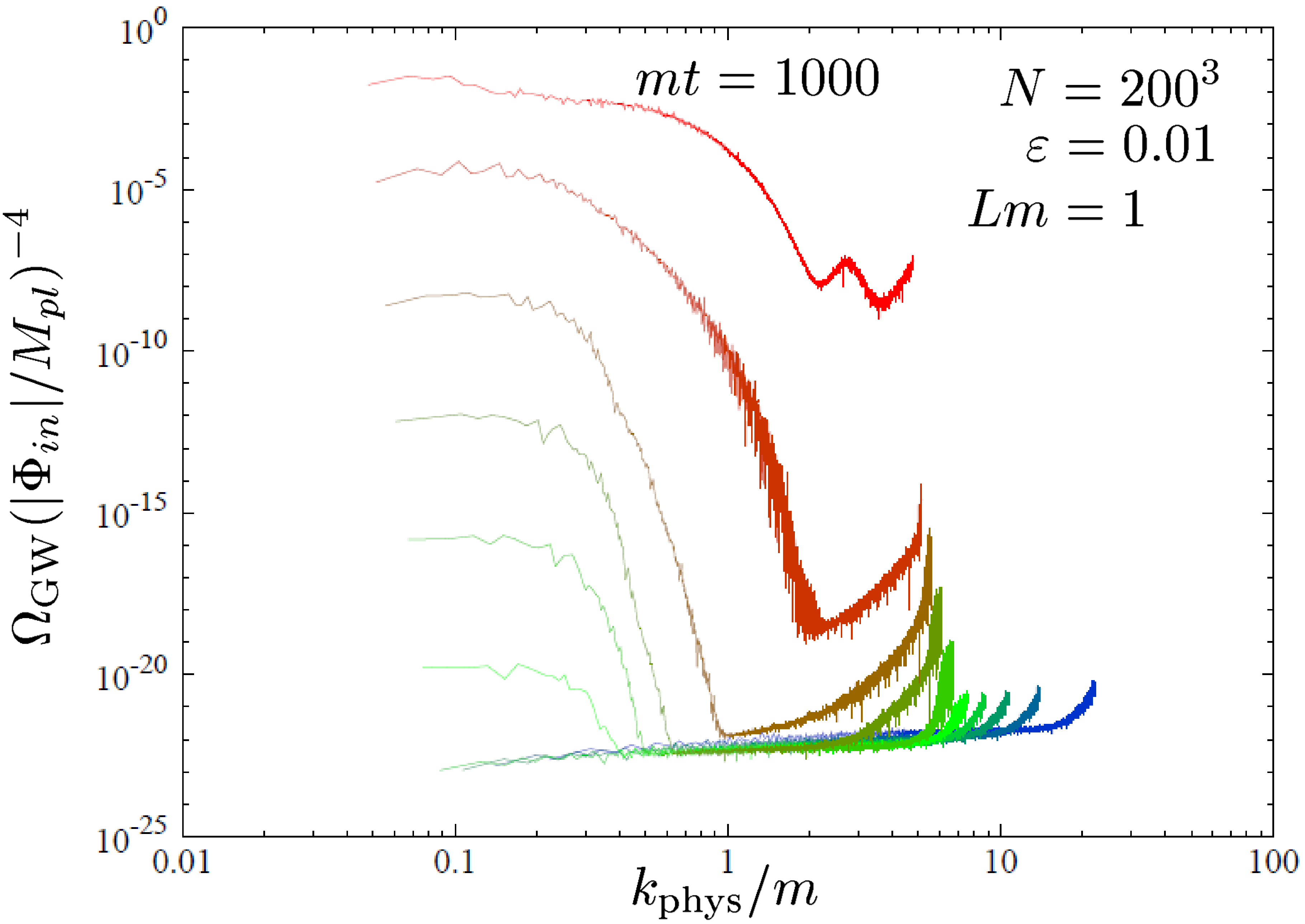


Box size effect



Peak appears at the scale that is several times larger than Q-ball size. Filaments may be crucial.

$\varepsilon = 0.01$: early time $mt \leq 1000$



$\varepsilon = 0.01$: late time $1000 < mt$

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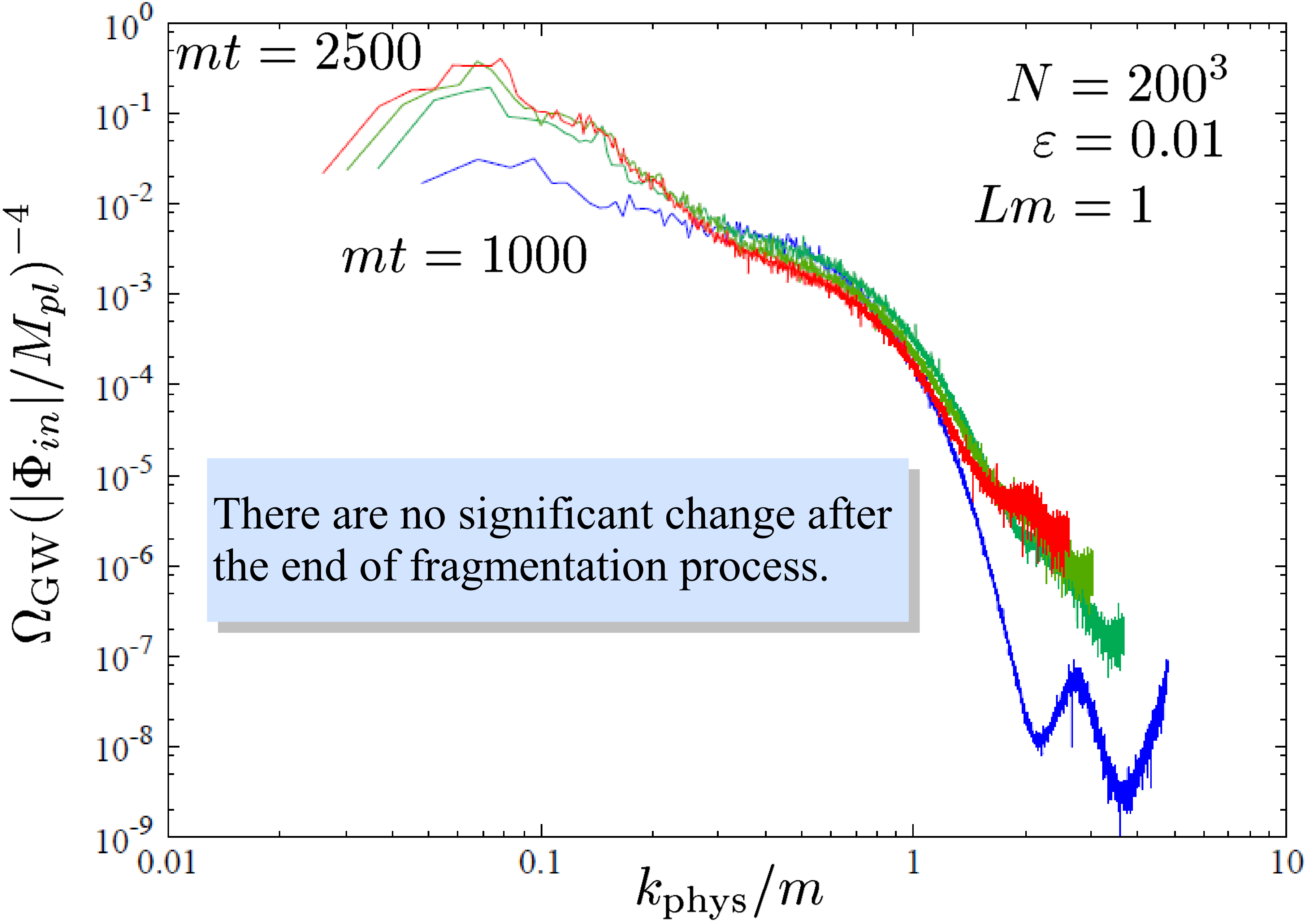
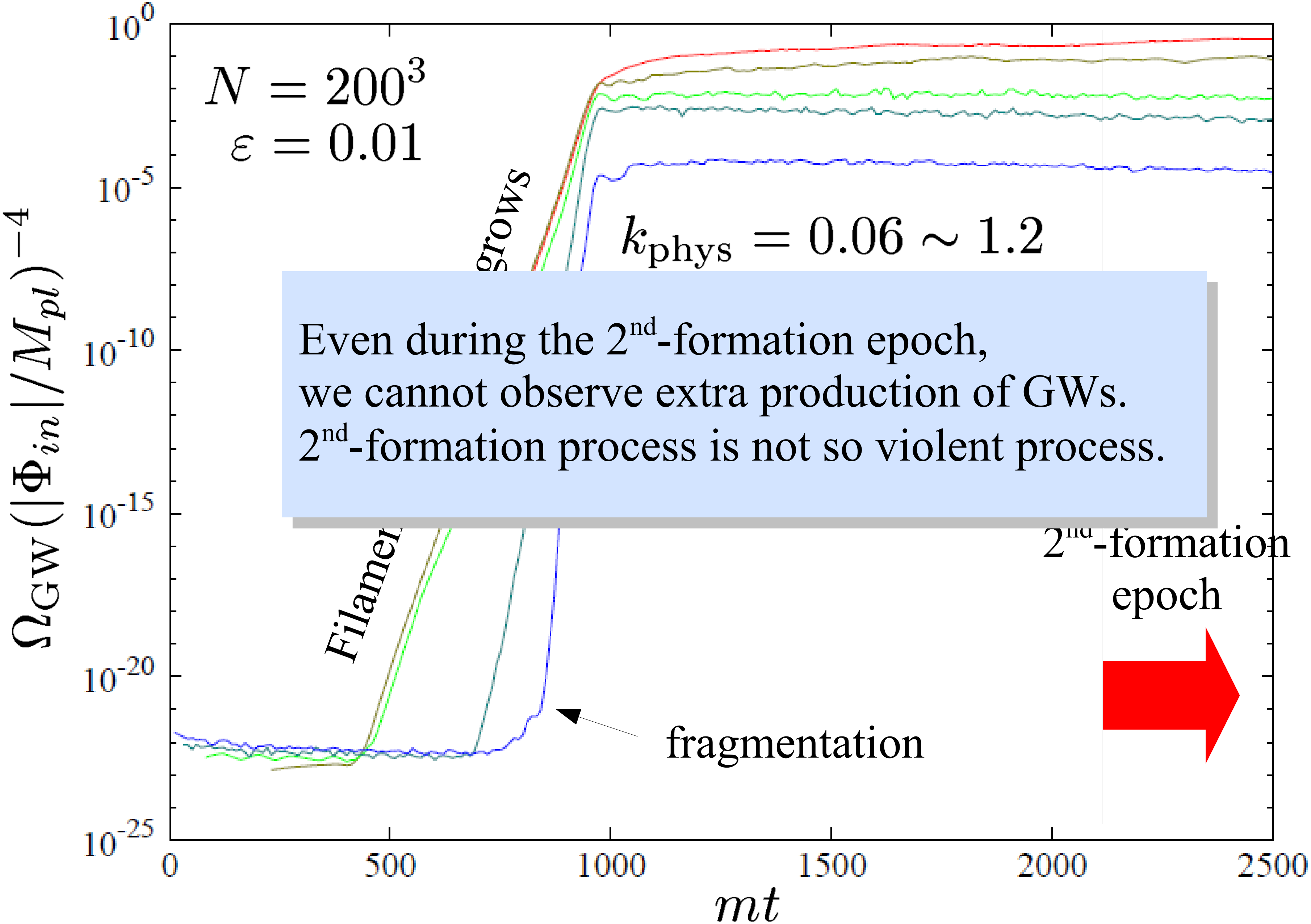


There are no significant change after the end of fragmentation process.

Time evolution of amplitude for $\varepsilon = 0.01$



Even during the 2nd-formation epoch,
we cannot observe extra production of GWs.
2nd-formation process is not so violent process.

Difference between $\varepsilon = 1$ and $\varepsilon = 0.01$

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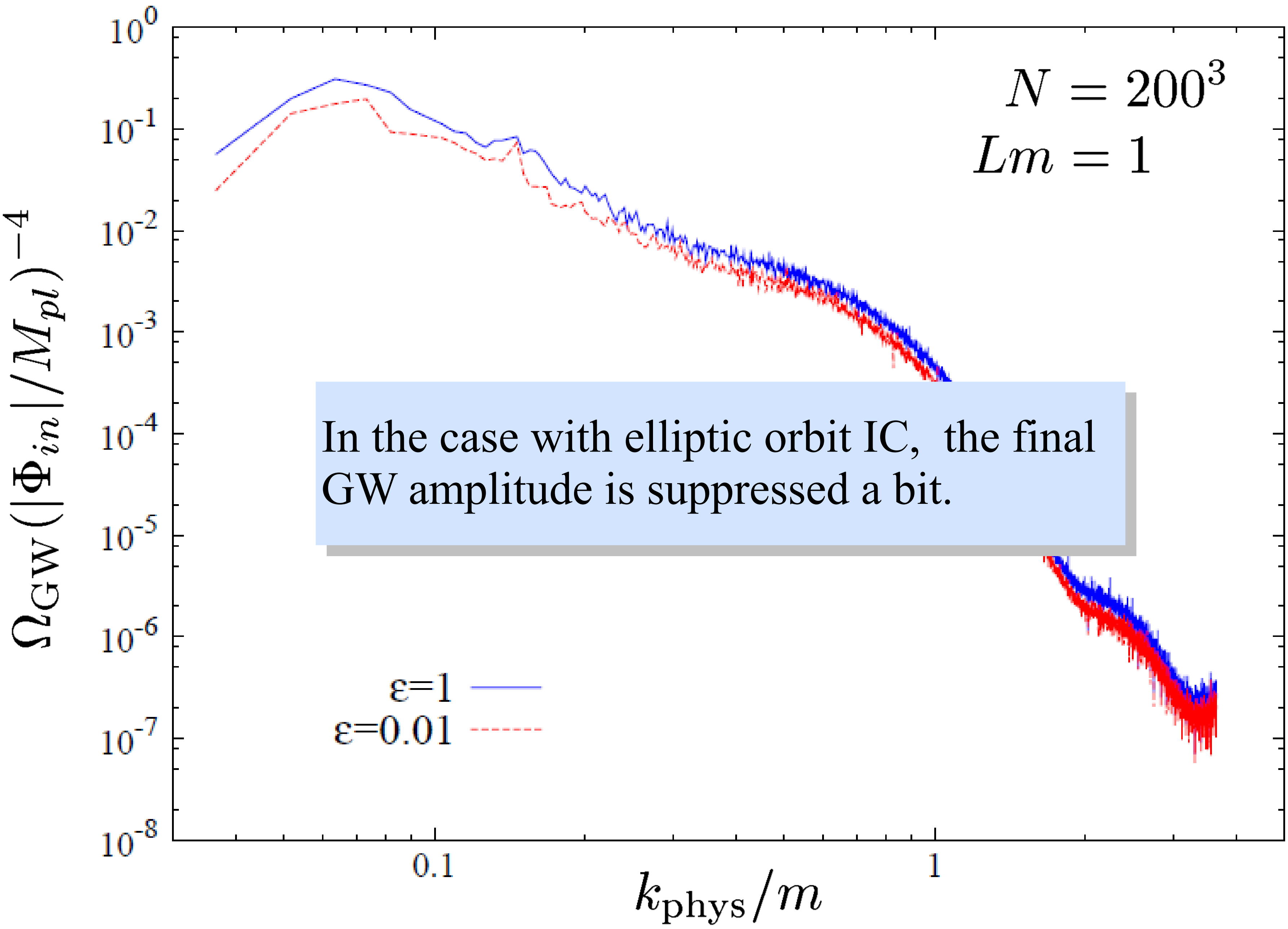

In the case with elliptic orbit IC, the final GW amplitude is suppressed a bit.

Summary

- 3D simulations for Q-ball formation in gravity mediation.
- Charge distribution of Q-balls **TH, Kawasaki, Takahashi, JCAP 06(2010)008**
  - 'circular' case : the peak charge is slightly larger than existing results
  - 'elliptic' case : eventually the same numbers of +/- Q-balls appear, and peak charge, scaling , ...
- Power spectrum of GWs from Q-balls **TH, Takahashi, Yamaguchi, in preparation**
  - *early epoch* : large scale GWs may be associated with filamentary structure. But it remains unclear now.
  - *formation(fragmentation) epoch* : small scale significantly grows.
  - *relaxation epoch* : no more grows even during 2nd-formation process, though it's crucial for the final shape of charge distribution.
- Filamentary structure plays a crucial role for both charge distributions and the peak amplitude of GWs.