Baryon with Massive Strangeness in Holographic QCD

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Work in progress

Chiral Symmetry or Quark Mass

	Chiral sym.	Quark mass
D3-D7 D4-D6 etc.	unclear	easy
D4-D8/$\overline{\text{D8}}$ (Sakai-Sugimoto)	clear	technical

Real QCD
knows chiral sym.
but quark mass ≠ 0

→ **SS model + mass**

Bound-State Approach

in Holographic QCD

c.f.) Bound-state approach to strangeness in the Skyrme model

[Callan-Klebanov ’85]

Baryon with Massive Strangeness

$$0 < m_{u,d} \ll m_s$$

~~SU(3)~~$_f$

Hyperon = SU(2) baryon + K meson

Bound-state

	Skyrme	Sakai-Sugimoto
SU(3) rotation	80s	arXiv:0910.1179
Bound-state	Callan-Klebanov '85	today

Plan

1. Skyrme model [review]

2. Sakai-Sugimoto model

   Simplest case

   K meson

   Improvements

   Quark mass term

   Vector mesons

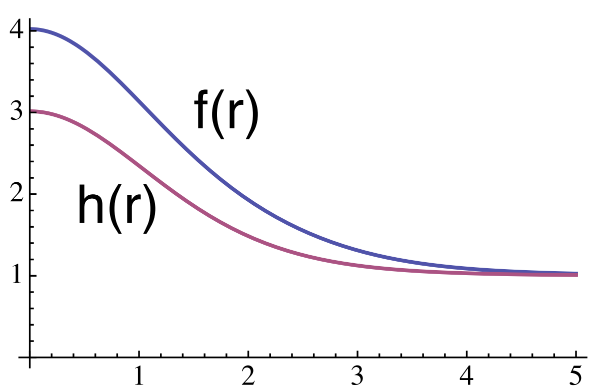
Skyrmion + Kaon

$$L \sim \mathrm{Tr}(U^{-1}\partial_\mu U)^2 + \mathrm{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2$$

ansatz: $U = \sqrt{U_\pi} U_K \sqrt{U_\pi}$

$$U_\pi = \begin{pmatrix} e^{iF(r)\hat{x}\cdot\tau} & 0 \\ 0 & 1 \end{pmatrix}, \; U_K \sim \exp\left[\begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix}\right]$$

$$L \sim \int dr\, r^2 \left[f(r)\dot{k}^\dagger \dot{k} - h(r)\partial_r k^\dagger \partial_r k - (m_K^2 + V(r))k^\dagger k \right]$$



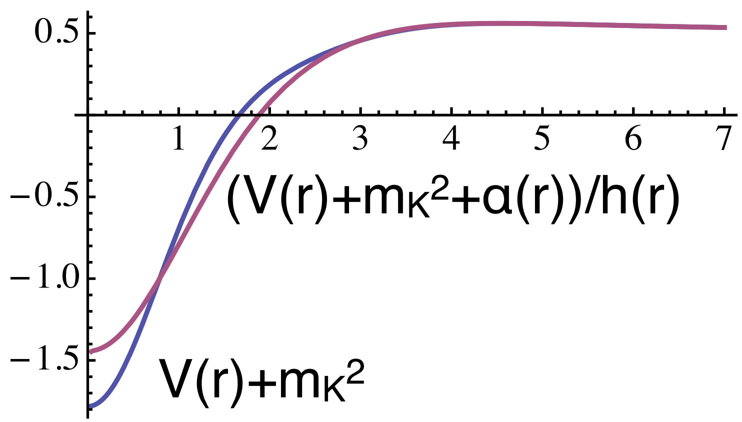
$$K(x^\mu) = k(r,t)Y(\Omega_2)$$

Non-canonical due to Skyme term

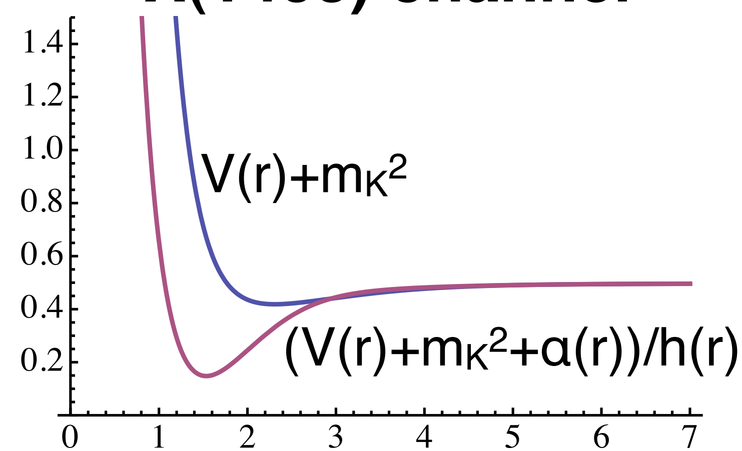
Bound-state Potential

$$-\left[\frac{1}{r^2}\partial_r(h(r)r^2\partial_r) - V(r) - m_K^2\right]k = \left[f(r)E_n^2 + 2\lambda(r)E_n\right]k$$

Λ, Σ, Σ* channel



Λ(1405) channel



	Λ	Σ	Σ*	Λ(1405)
mass (theory)	1048	1122	1303	1281
mass (exp)	1115	1190	1385	1405

Sakai-Sugimoto Model

5 dim U(N_f) YM-CS theory [Sakai-Sugimoto]

$$S = -\kappa \int d^4x dz \, \mathrm{Tr}\left[\frac{1}{2}h(z)\mathcal{F}_{\mu\nu}^2 + k(z)\mathcal{F}_{\mu z}^2\right] + \frac{N_c}{24\pi^2}\int_{\mathrm{M}_5}\omega_5$$

[μ=0,1,2,3]

Curved space: $h(z) = (1+z^2)^{-1/3}, \;\; k(z) = 1+z^2$

2 parameters: $\kappa = 0.00746, \;\; M_{\mathrm{KK}} = 948\,[\mathrm{MeV}] \to 1$

Baryon is soliton (N_f=2 [Hata-Sakai-Sugimoto-Yamato])

$$A_\alpha^{\mathrm{cl}} = -if(\xi)g\partial_\alpha g^{-1}, \;\; \widehat{A}_0^{\mathrm{cl}} = \frac{27\pi}{\lambda}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right], \;\; A_0^{\mathrm{cl}} = \widehat{A}_\alpha^{\mathrm{cl}} = 0$$

SU(2) U(1) [α=1,2,3,z]

Pion and Vector Mesons

Kaluza-Klein modes are mesons

vector mesons

$$\mathcal{A}_\mu(x,z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x)\psi_n(z)$$

↑ Higgsing

$$\mathcal{A}_z(x,z) = \sum_{n=1}^{\infty} \varphi^{(n)}(x)\phi_n(z) + \varphi^{(0)}(x)\phi_0(z)$$

pion

$$S_{\rm YM} = -\int d^4x \sum_{n\geq 1} {\rm Tr}\left[\frac{1}{2}(\partial_\mu B_\nu^{(n)} - \partial_\nu B_\mu^{(n)})^2 + \underline{\lambda_n (B_\mu^{(n)} - \partial_\mu \varphi^{(n)})^2}\right]$$

vector-meson mass

$$-\int d^4x \; {\rm Tr}(\partial_\mu \varphi^{(0)})^2 + ({\rm interactions})$$

Skyrme from Sakai-Sugimoto

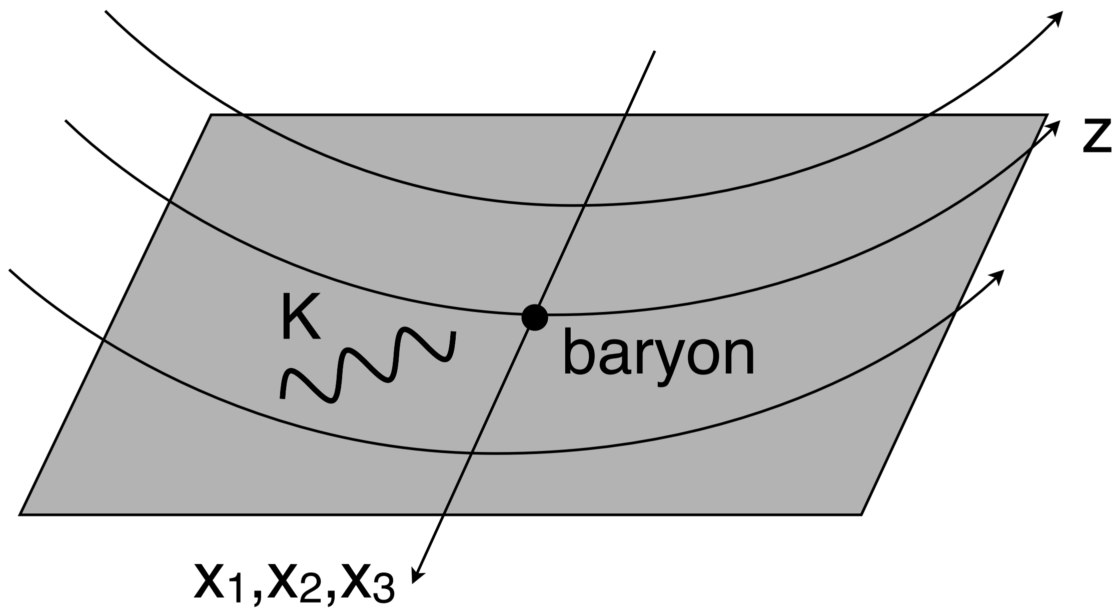
A_z=0 gauge

$$A_\mu(x,z) = U^{-1}(x)\partial_\mu U(x)\,\psi_+(z) + \sum_{n\geq 1} B_\mu^{(n)}(x)\psi_n(z)$$

$$S_{SU(N)} = -\kappa \int d^4x dz\, \mathrm{Tr}\left[k(z)F_{\mu z}^2 + \frac{1}{2}h(z)F_{\mu\nu}^2\right]$$

$$= -\int d^4x \left(\frac{f_\pi}{4}\mathrm{Tr}(U^{-1}\partial_\mu U)^2 + \frac{1}{32e^2}\mathrm{Tr}[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2\right)$$

Holographic Bound-state Approach



Consider reduction to 4 dim

K meson fluctuation: a_z

Yang-Mills part

$$S_{SU(3)} = -\kappa \int d^4x dz \,\mathrm{Tr}\left[\frac{1}{2}h(z)F_{\mu\nu}^2 + k(z)F_{\mu z}^2\right]$$

2-flavor baryon: A^{inst}

Kaon: a_z

$$A_0 = \begin{pmatrix} A_0^{\mathrm{inst}} & 0 \\ 0 & 0 \end{pmatrix} \quad A_i = \begin{pmatrix} A_i^{\mathrm{inst}} & 0 \\ 0 & 0 \end{pmatrix} \quad A_z = \begin{pmatrix} A_z^{\mathrm{inst}} & a_z \\ a_z^\dagger & 0 \end{pmatrix}$$

$$A_0^{\mathrm{inst}} = \frac{1}{2}\widehat{A}_0^{\mathrm{cl}} \qquad a_z \sim K(x^\mu)\frac{1}{k(z)}$$

$$D_M^{(\mathrm{inst})} a_N = \partial_M a_N + i A_M^{\mathrm{inst}} a_N$$

$$S_{\mathrm{YM}} = S_{\mathrm{inst}} + S_{\mathrm{fluc}}$$

$$\bar{D}_M^{(\mathrm{inst})} a_N^\dagger = \partial_M a_N^\dagger - i a_N^\dagger A_M^{\mathrm{inst}}$$

$$\begin{aligned} S_{\mathrm{fluc}} =& -2\kappa \int d^4x dz \, k(z) \bar{D}_\mu^{(\mathrm{inst})} a_z^\dagger D_\mu^{(\mathrm{inst})} a_z \\ =& -\frac{1}{\pi} \int dz \frac{1}{k(z)} \Big[\partial_\mu K^\dagger \partial^\mu K + K^\dagger A_\mu^{\mathrm{inst}} A_\mu^{\mathrm{inst}} K \\ & + i \left(\partial^\mu K^\dagger A_\mu^{\mathrm{inst}} K - K^\dagger A_\mu^{\mathrm{inst}} \partial^\mu K \right) \Big] \end{aligned}$$

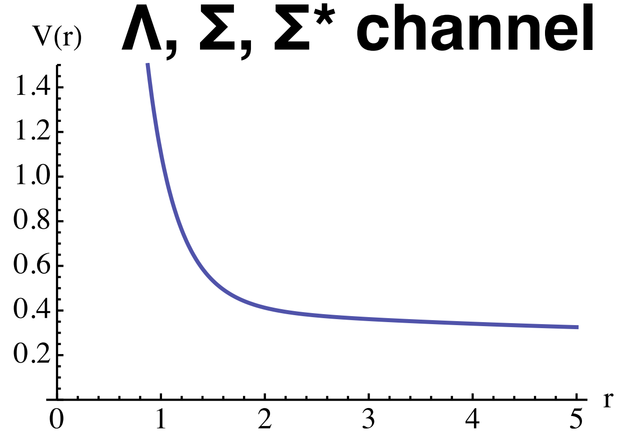
Canonical kinetic term

Simply add standard K mass: $m_K^2 K^\dagger K$

Bind ?

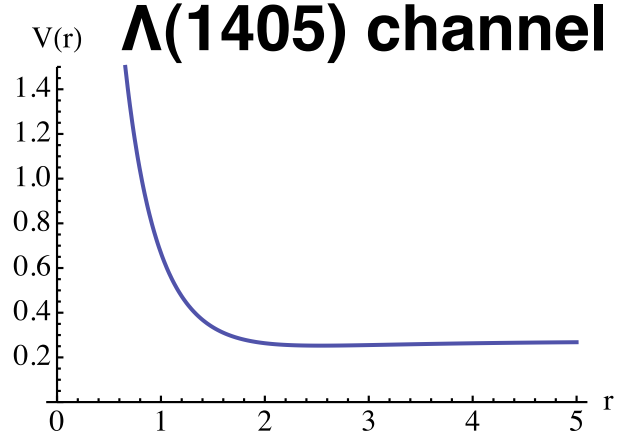
$$\left[- \frac{1}{r^2} \partial_r \left(r^2 \partial_r \right) + V(r) + m_K^2 \right] k_n(r) = \left[E_n^2 + 2\Psi_0 \, E_n \right] k_n(r)$$

Λ, Σ, Σ* channel



$$\Psi_0 \equiv \frac{1}{\pi} \int dz \, \frac{1}{k(z)} A_0^{\text{inst}} \propto \frac{1}{\lambda}$$

Λ(1405) channel



⟵ E ~ 490 MeV

c.f) m_K ~ 495 MeV

Thanks to (too) large A_0

Slightly bound in Λ(1405) channel (surprising)
but not in normal Λ, Σ, Σ* channel

Not similar to Skyrme case

Repulsive potential

Considering only a_z might be naive

Improvements

1. Quark/meson mass in holographic QCD
2. Vector mesons

1. Quark/meson Mass Term

Mass term by Wilson line [Hashimoto-Hirayama-Lin-Yee]

$$S_{\text{mass}} = c \int d^4x \, \text{PTr}\left[M_q \left(e^{-i\int_{-\infty}^{\infty} \mathcal{A}_z dz} - \mathbf{1}_3 \right) \right] + \text{c.c.}$$

Study Kaon mass in baryon background

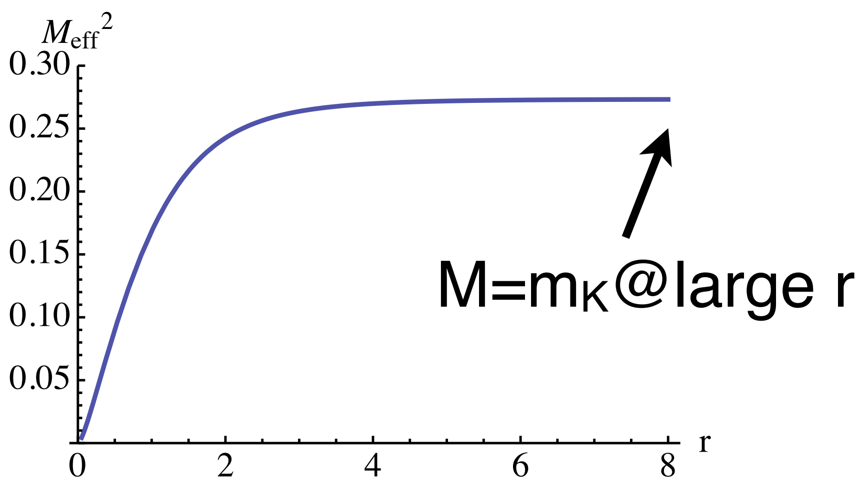
$$\text{P}\exp\left[-i\int_{-\infty}^{\infty} A_z dz \right], \quad A_z = \begin{pmatrix} A_z^{\text{inst}} & a_z \\ a_z^\dagger & 0 \end{pmatrix}$$

Inserting two a_z to the path-ordering

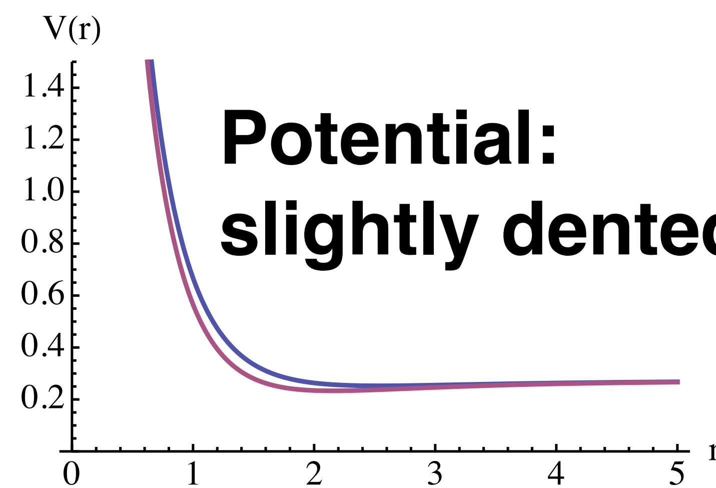
r-dependent Kaon Mass

$$S_{\text{kaon mass}} = -\int d^4x \int_{-\infty}^{\infty} dz_2 \int_{-\infty}^{z_2} dz_1\, a_z^\dagger(z_1) a_z(z_2)$$
$$\times \left[f_\pi^2 \left(m_K^2 - \frac{m_\pi^2}{2} \right) \cos(I_{(z_1,z_2)}) + \frac{f_\pi^2 m_\pi^2}{2} \cos(f(r) - I_{(z_1,z_2)}) \right]$$

$$\int_a^b dz\, A_z^{\text{inst}} = I_{(a,b)} \hat{x}^i \tau_i \qquad f(r) = I_{(-\infty,\infty)} = \pi \left(1 - \frac{r}{\sqrt{r^2+\rho^2}} \right)$$



$m_\pi = 0$ used



Potential: slightly dented

2. Vector Mesons: a_μ

K meson: a_z, Vector mesons(K*): a_μ

$$A_0 = \begin{pmatrix} A_0^{\rm inst} & a_0 \\ a_0^\dagger & 0 \end{pmatrix}, \quad A_\alpha = \begin{pmatrix} A_\alpha^{\rm inst} & a_\alpha \\ a_\alpha^\dagger & 0 \end{pmatrix}$$

c.f.) [Scoccola-Min-Nadeau-Rho,'89] Hidden Local Symmetry case

$$\begin{aligned} S_{\rm fluc} =& -2\kappa \int d^4x dz \, {\rm Tr}\Big[h(z)(\bar{D}_\mu^{\rm (inst)} a_\nu^\dagger D_\mu^{\rm (inst)} a_\nu - \bar{D}_\mu^{\rm (inst)} a_\nu^\dagger D_\nu^{\rm (inst)} a_\mu - i\, a^{\dagger\mu} F_{\mu\nu}^{\rm (inst)} a^\nu) \\ &+ k(z)\Big(\bar{D}_\mu^{\rm (inst)} a_z^\dagger D_\mu^{\rm (inst)} a_z - \bar{D}_\mu^{\rm (inst)} a_z^\dagger D_z^{\rm (inst)} a_\mu - \bar{D}_z^{\rm (inst)} a_\mu^\dagger D_\mu^{\rm (inst)} a_z \\ &+ \underline{\bar{D}_z^{\rm (inst)} a_\mu^\dagger D_z^{\rm (inst)} a_\mu + i(a_z^\dagger F_{\mu z}^{\rm (inst)} a_\mu - a_\mu^\dagger F_{\mu z}^{\rm (inst)} a_z)} \Big) \Big] \end{aligned}$$

- Without baryon, $(\partial_z a_\mu)^2$ gives vector-meson mass term

Approx. to Study Vector Mesons

Vector-meson mass term

$$\int dz\, k(z)\partial_z a_\mu^\dagger \partial_z a_\mu = \int dz\, h(z) \sum_{n,\,m} \lambda_n B_\mu^{(n)\dagger} B_\mu^{(m)} \psi_n \psi_m = \sum_n \lambda_n B_\mu^{(n)\dagger} B_\mu^{(n)}$$

Using this,

$$\begin{aligned} &\partial_z a_\mu^\dagger \partial_z a_\mu + i(a_z^\dagger F_{\mu z}^{(\mathrm{inst})} a_\mu - a_\mu^\dagger F_{\mu z}^{(\mathrm{inst})} a_z) \\ &= h(z)^4 \sum_{n,\,m} \lambda_n \underline{|B_\mu^{(n)} \psi_n - \frac{i}{\lambda_n} h(z)^{-4} F_{\mu z}^{(\mathrm{inst})} a_z|^2} \\ &\qquad - L_{\mathrm{KK}} h(z)^{-4} a_z^\dagger F_{\mu z}^{(\mathrm{inst})} F_{\mu z}^{(\mathrm{inst})} a_z \end{aligned}$$

Approx. $a_\mu = i L_{\mathrm{kk}} h(z)^{-4} F_{\mu z}^{(\mathrm{inst})} a_z$ $\quad L_{\mathrm{kk}} = \sum_n \frac{1}{\lambda_n}$

c.f.) [Scoccola-Min-Nadeau-Rho,'89]

L_{KK}: An Undetermined Parameter

$$L_{\mathrm{kk}} = \sum_n \frac{1}{\lambda_n}$$

n	1	2	3	4	5	6	7
λ_n	0.67	1.6	2.9	4.6	6.6	9.0	12
$\lambda_n^{1/2} M_{KK}$[MeV]	[776]	1187	1607	2021	2434	2845	3256

May converge $\displaystyle\sum_{n=1}^{7} \frac{1}{\lambda_n} = 3.0 \quad \sum_{n=1}^{30} \frac{1}{\lambda_n} = 3.5$

A trial choice: **L_{KK}=3**

Toward Noncanonical Kinetic Term

Approx. $a_\mu = i L_{\rm kk} h(z)^{-4} F^{(\rm inst)}_{\mu z} a_z$

$$L_{\rm kk}^2 h(z)^{-4} (|F^{(\rm inst)2}_{\mu z}| \partial_\mu a_z^\dagger \, \partial_\mu a_z - \partial_\mu a_z^\dagger F^{(\rm inst)}_{\nu z} F^{(\rm inst)}_{\mu z} \partial_\nu a_z)$$

$$\begin{aligned} S_{\rm fluc} = & -2\kappa \int d^4x dz \, {\rm Tr}\Big[h(z) (\bar{D}^{(\rm inst)}_\mu a_\nu^\dagger D^{(\rm inst)}_\mu a_\nu - \bar{D}^{(\rm inst)}_\mu a_\nu^\dagger D^{(\rm inst)}_\nu a_\mu - i \, a^{\dagger\mu} F^{(\rm inst)}_{\mu\nu} a^\nu) \\ & + k(z) \Big(\bar{D}^{(\rm inst)}_\mu a_z^\dagger D^{(\rm inst)}_\mu a_z - \bar{D}^{(\rm inst)}_\mu a_z^\dagger D^{(\rm inst)}_z a_\mu - \bar{D}^{(\rm inst)}_z a_\mu^\dagger D^{(\rm inst)}_\mu a_z \\ & + \bar{D}^{(\rm inst)}_z a_\mu^\dagger D^{(\rm inst)}_z a_\mu + i (a_z^\dagger F^{(\rm inst)}_{\mu z} a_\mu - a_\mu^\dagger F^{(\rm inst)}_{\mu z} a_z) \Big) \Big] \end{aligned}$$

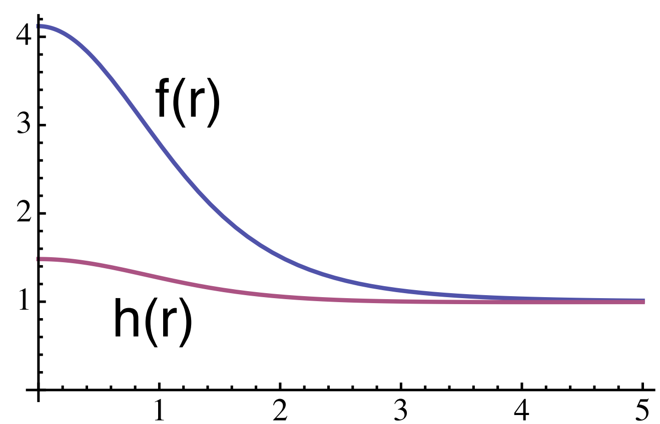
Recall that Skyrme-term comes from $F_{\mu\nu}F^{\mu\nu}$

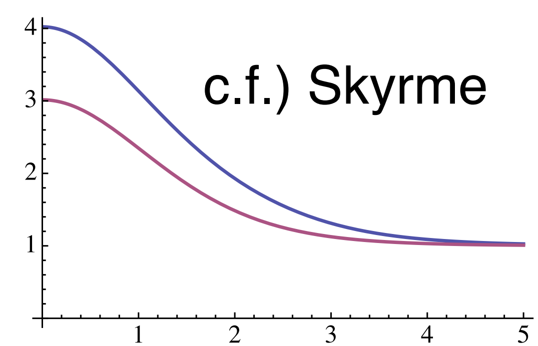
Noncanonical as Skyrme, But...

$$\mathcal{L}_{\rm kaon} = \left[f(r)\dot{k}^\dagger \dot{k} - h(r)\partial_r k^\dagger \partial_r k - (m_K^2 + V(r))k^\dagger k + \cdots \right]$$

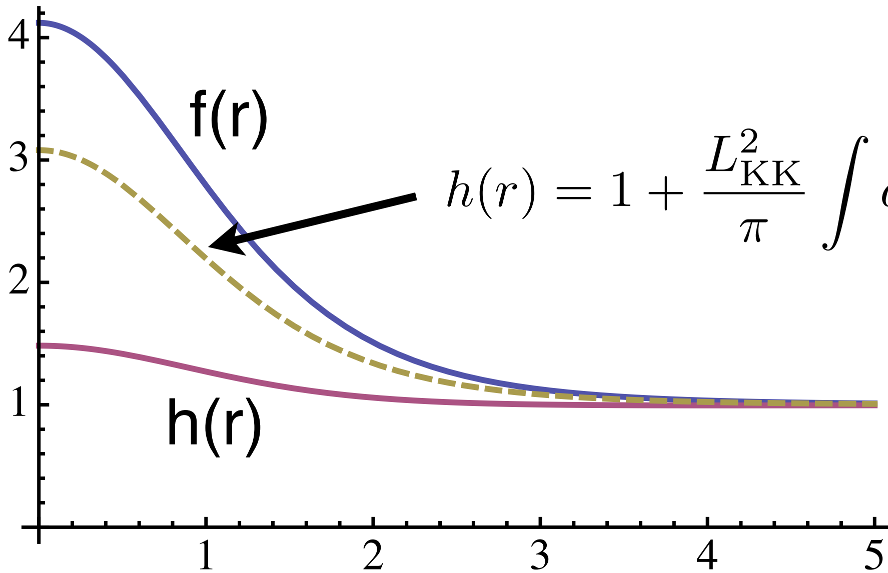
$$f(r) = 1 + \frac{L_{\rm KK}^2}{\pi}\int dz\, k(z)^{1/3} |F_{iz}^{({\rm inst})\,2}|$$

$$h(r) = 1 + \frac{L_{\rm KK}^2}{\pi}\int dz\, k(z)^{1/3} \left(\frac{2}{3}|F_{iz}^{({\rm inst})\,2}| - |F_{0z}^{({\rm inst})\,2}|\right)$$





F_{0z} Reduces h(r)



$$h(r) = 1 + \frac{L_{\rm KK}^2}{\pi}\int dz\, k(z)^{1/3}\left(\frac{2}{3}|F_{iz}^{(\rm inst)\,2}| - \cancel{|F_{0z}^{(\rm inst)\,2}|}\right)$$

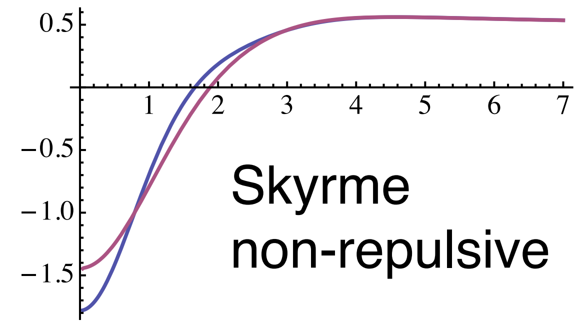
Skyrme vs SS: $U^{-1}\partial_\mu U \leftrightarrow F_{\mu z}^{(\rm inst)}$

$$\boxed{A_0^{\rm inst} \sim \frac{1}{\lambda}}$$ should be small, but **large**

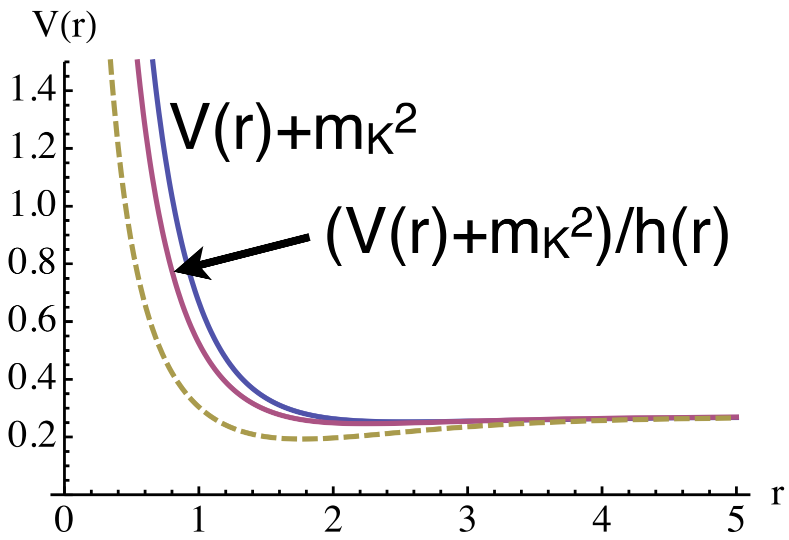
Still Slightly Bound

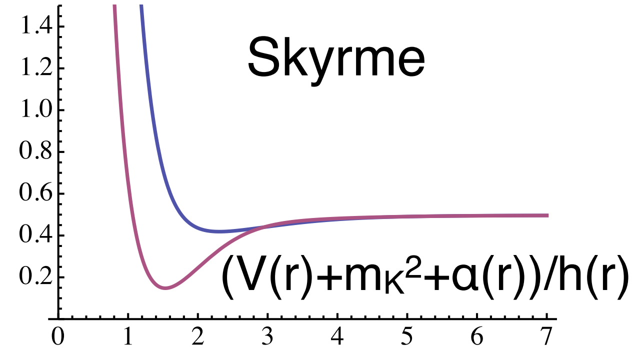
Λ, Σ, Σ^*: **unclear**

What potential?
Vector mesons matter



$\Lambda(1405)$: **weakly bound?**





Toward Holographic Bound-State Approach to Strangeness

Kaon only: E~490 MeV

r-depend kaon mass

Vector mesons may important

A_0 could be too large: **GOOD** or **BAD**

Not Strongly Bound Λ(1405)

Is Λ(1405) a N-$\bar{K}$ weak bound-state?