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# G-inflation


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arXiv:1008.0603

# What’s “G”?

# Galileon field

# The Galileon field

$$\mathcal{L}_1 = \phi$$

$$\mathcal{L}_2 = (\nabla\phi)^2$$

$$\mathcal{L}_3 = (\nabla\phi)^2 \,\Box\phi$$

$$\mathcal{L}_4 = (\nabla\phi)^2 \Big[ 2(\Box\phi)^2 - 2(\nabla_\mu\nabla_\nu\phi)^2 - \frac{R}{2}(\nabla\phi)^2 \Big]$$

$$\mathcal{L}_5 = (\nabla\phi)^2 \left[ (\Box\phi)^3 + \cdots \right]$$

Field equations are 2nd order

Galilean shift symmetry <u>in flat space</u>

$$\partial_\mu\phi \rightarrow \partial_\mu\phi + b_\mu$$

$$\mathcal{L}_n \sim \partial^{2(n-1)}\phi^n$$

Nicolis et al. '09;
Deffayet et al. '09

# Our Lagrangian

$$\mathcal{L} = \frac{R}{2} + K(\phi, X) - F(\phi, X)\Box\phi$$

where $X := -\frac{1}{2}(\nabla\phi)^2$

**Field equations are 2nd order**

Deffayet, Pujolas, Sawicki, Vikman 1008.0048;
TK, Yamaguchi, Yokoyama 1008.0603

# Simple motivation

The Galileon field has been used to explain current cosmic acceleration

Antonio De Felice talk yesterday

Why don't we use the Galileon field to drive inflation in the early universe?

# Talk plan

I. Introduction

II. G-inflation

III. Primordial perturbations

IV. Pre-G-inflation?

V. Summary & Outlook


G-inflation

# Standard picture of inflation

One (or more) canonical scalar field(s) rolling slowly down a nearly flat potential

$$\mathcal{L} = X - V(\phi), \quad X = -\frac{1}{2}(\partial\phi)^2$$

$$3M_{\rm Pl}^2 H^2 \simeq V(\phi)$$

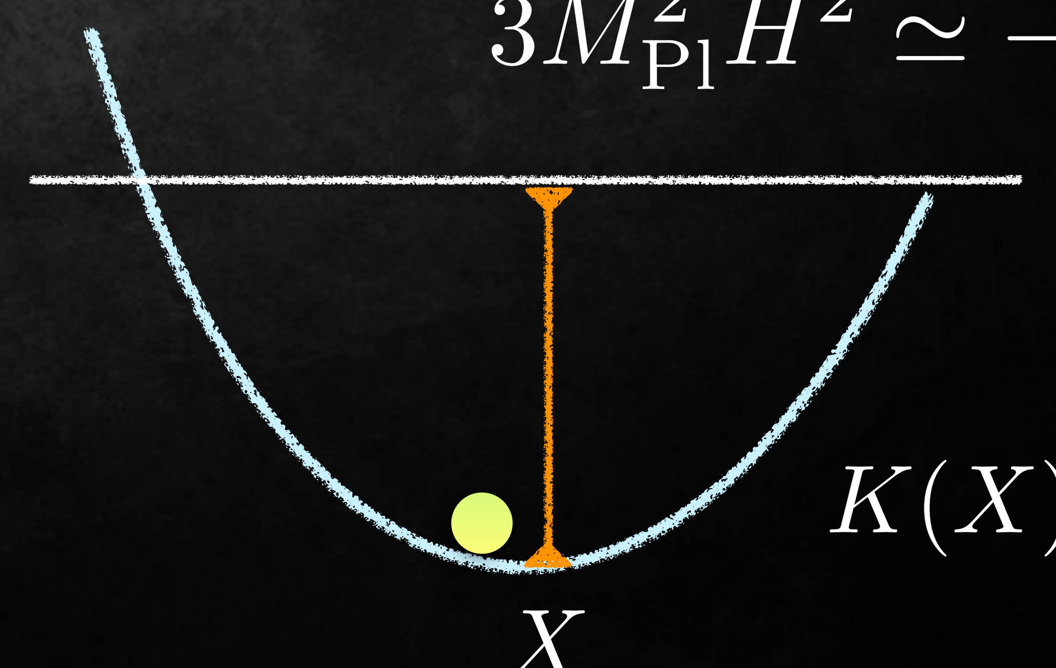
# Kinematically driven inflation

$$\mathcal{L} = K(\phi, X)$$

$K = K(X)$ →

$$\frac{d}{dt}\left(a^3 K_X \dot{\phi}\right) = 0$$

$$3M_{\rm Pl}^2 H^2 \simeq -K$$




"k-inflation"

Armendariz-Picon et al. '99;

"Ghost condensate"

Arkani-Hamed et al. '04

# G-inflation: background

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\Box\phi$$

$$\begin{aligned} 3H^2 &= \rho \\ -3H^2 - 2\dot{H} &= p \end{aligned}$$

+ Scalar field EOM is automatically satisfied

$$\begin{aligned} \rho &= 2XK_X - K + 3F_X H\dot{\phi}^3 - 2F_\phi X \\ p &= K - 2\left(F_\phi + F_X\ddot{\phi}\right)X \end{aligned}$$

# de Sitter G-inflation

$$K = K(X), \quad F = fX, \quad f = \mathrm{const}$$

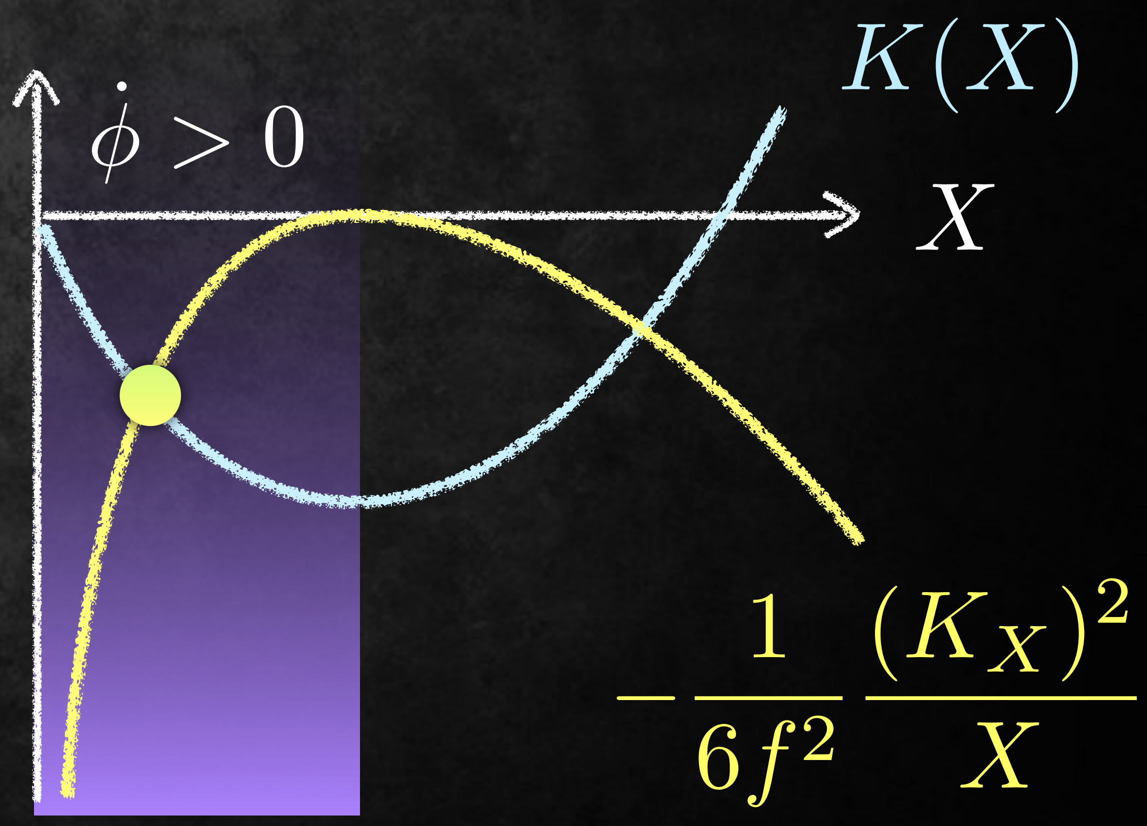
Look for exactly de Sitter solution:

$$H = \mathrm{const}$$
$$\dot{\phi} = \mathrm{const}$$

satisfying:

$$3H^2 = -K$$
$$K_X = -3fH\dot{\phi}$$

$$\longrightarrow \quad K = -\frac{1}{6f^2}\frac{(K_X)^2}{X}$$



# Example

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = \frac{X}{M^3}$$

$$\frac{H^2}{M_{\rm Pl}^2} \simeq \frac{1}{6}\frac{M^3}{M_{\rm Pl}^3}\frac{\mu}{M_{\rm Pl}}$$

$\mu \ll M_{\rm Pl}$ →

$$X \simeq \left(1 - \frac{\sqrt{3}\mu}{M_{\rm Pl}}\right)\mu M^3$$

# Quasi-dS G-inflation

$$K = K(X), \quad F = f(\phi)X$$

Quasi-de Sitter solution:

Required to get $n_s - 1 \neq 0$

$$H = H(t), \quad \dot{\phi} = \dot{\phi}(t)$$

Small rate of change

satisfying:

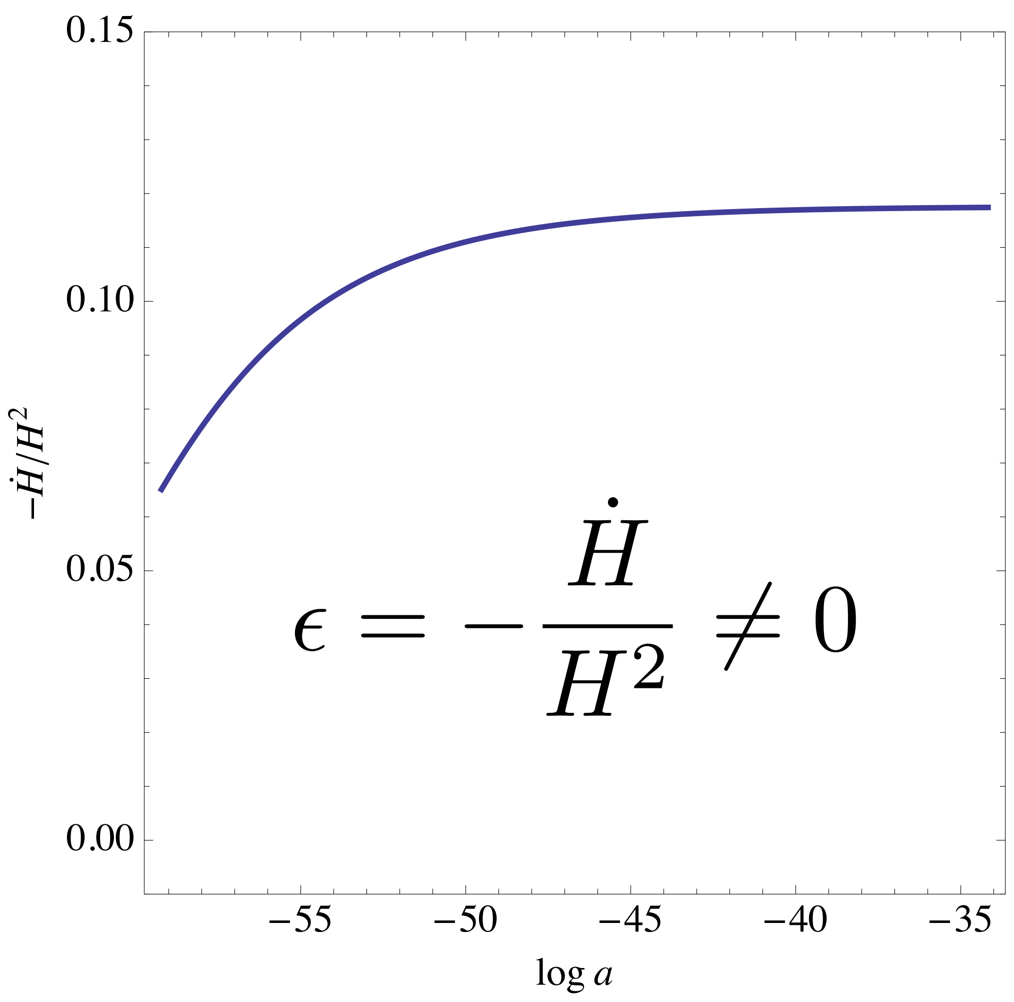
$$\epsilon = -\frac{\dot{H}}{H^2} \ll 1$$

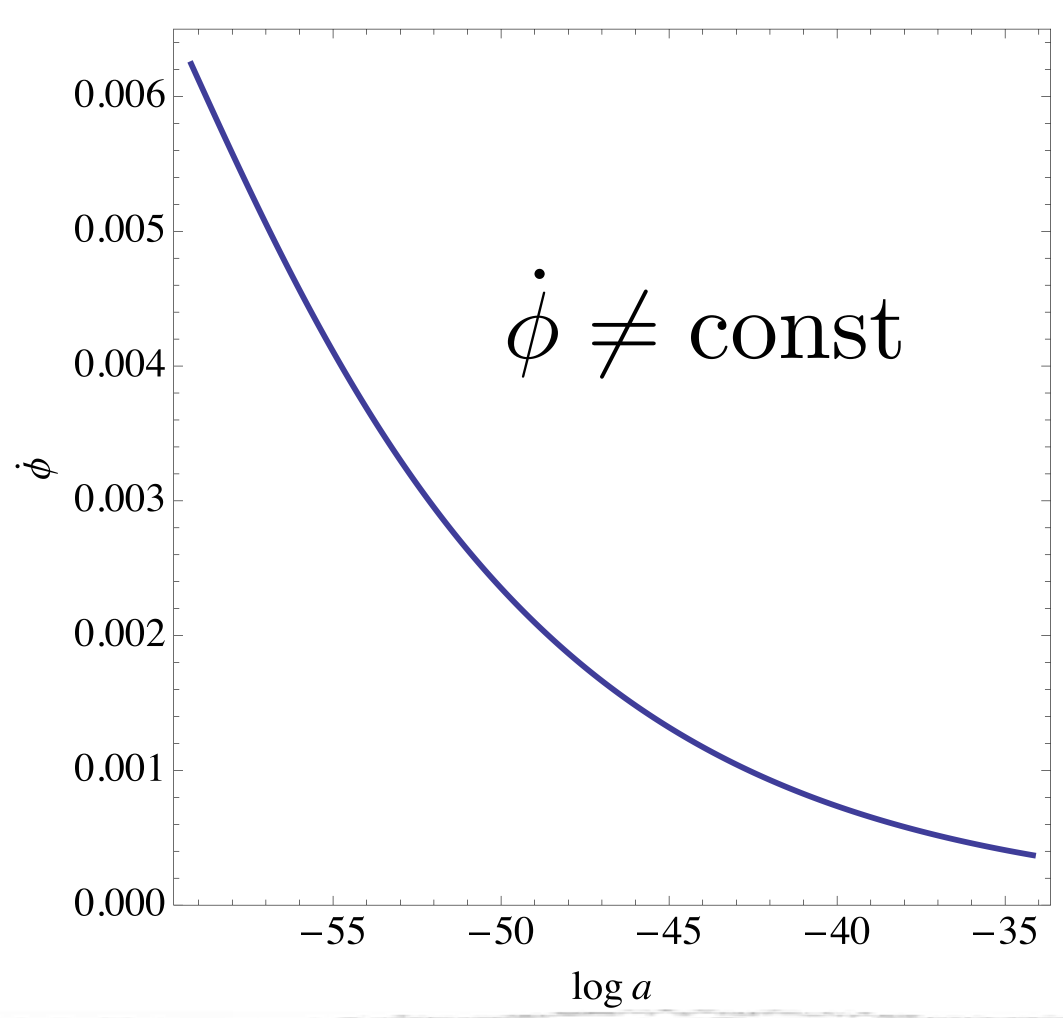
$$\begin{aligned} H^2 &\simeq -K(X) \\ K_X &\simeq -3f(\phi)H\dot{\phi} \end{aligned}$$

$$\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} \ll 1$$

# Numerical Example

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = \frac{e^{\alpha\phi}}{M^3}X$$



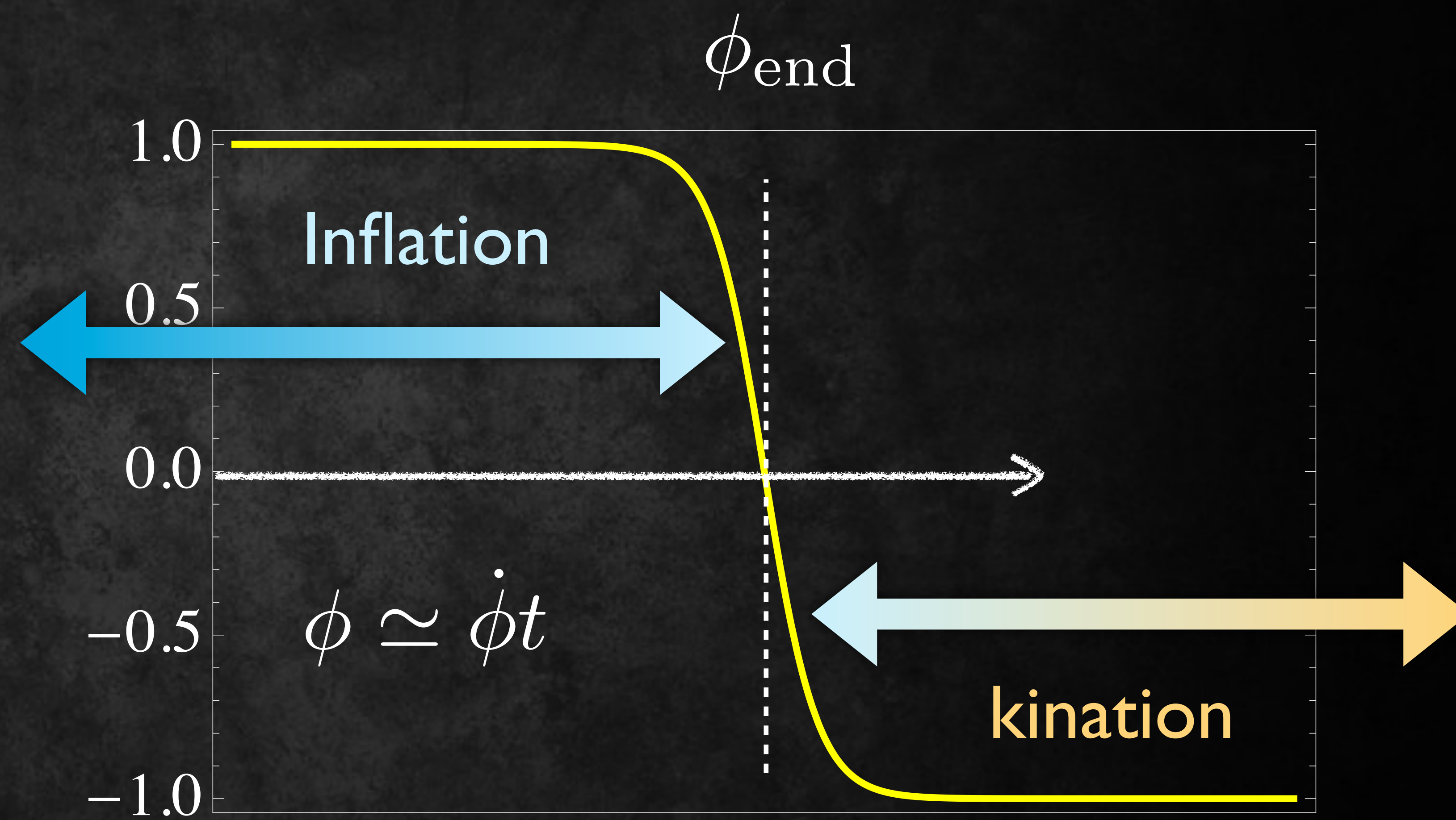
# Graceful exit & Reheating

<u>Basic idea</u>

$$K = -A(\phi)X + \cdots$$

Example:

$$A = \tanh\left[\lambda(\phi_{\rm end} - \phi)\right]$$



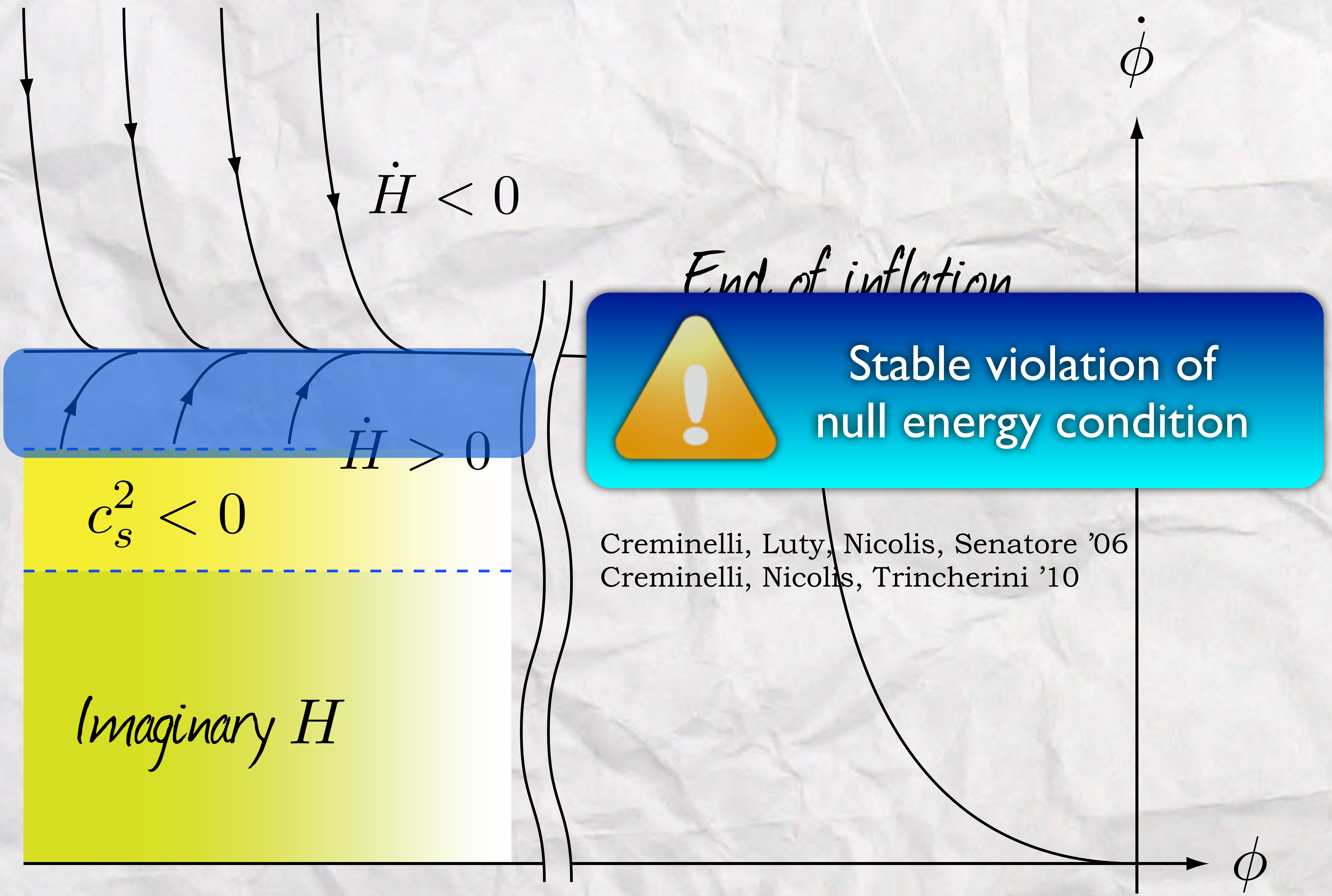

$$\rho \simeq p \simeq X \propto a^{-6}$$

*~ massless, canonical field (normal sign)*

Reheating through gravitational particle production

Ford '87

# Phase diagram



# Primordial perturbations

# Cosmological perturbations

$$ds^2 = -(1+2\alpha)dt^2 + 2a^2\beta_{,i}dtdx^i + a^2(1+2\mathcal{R})\delta_{ij}dx^i dx^j$$

$$\phi = \phi(t)$$

Unitary gauge: $\delta\phi = 0$

1. Expand the action to 2nd order
2. Eliminate $\alpha$ and $\beta$ using constraint eqs
3. Quadratic action for $\mathcal{R}$

$$\delta T_i{}^0 = -F_X \dot{\phi}^3 \alpha_{,i}$$

Uniform $\phi$ hypersurfaces
≠ comoving hypersurfaces

# Quadratic action

$$S^{(2)} = \frac{1}{2}\int d\tau d^3x\, z^2 \left[\mathcal{G}(\mathcal{R}')^2 - \mathcal{F}(\vec{\nabla}\mathcal{R})^2\right]$$

where

No ghost and gradient instabilities if

$$\mathcal{G} > 0, \quad c_s^2 = \mathcal{F}/\mathcal{G} > 0$$

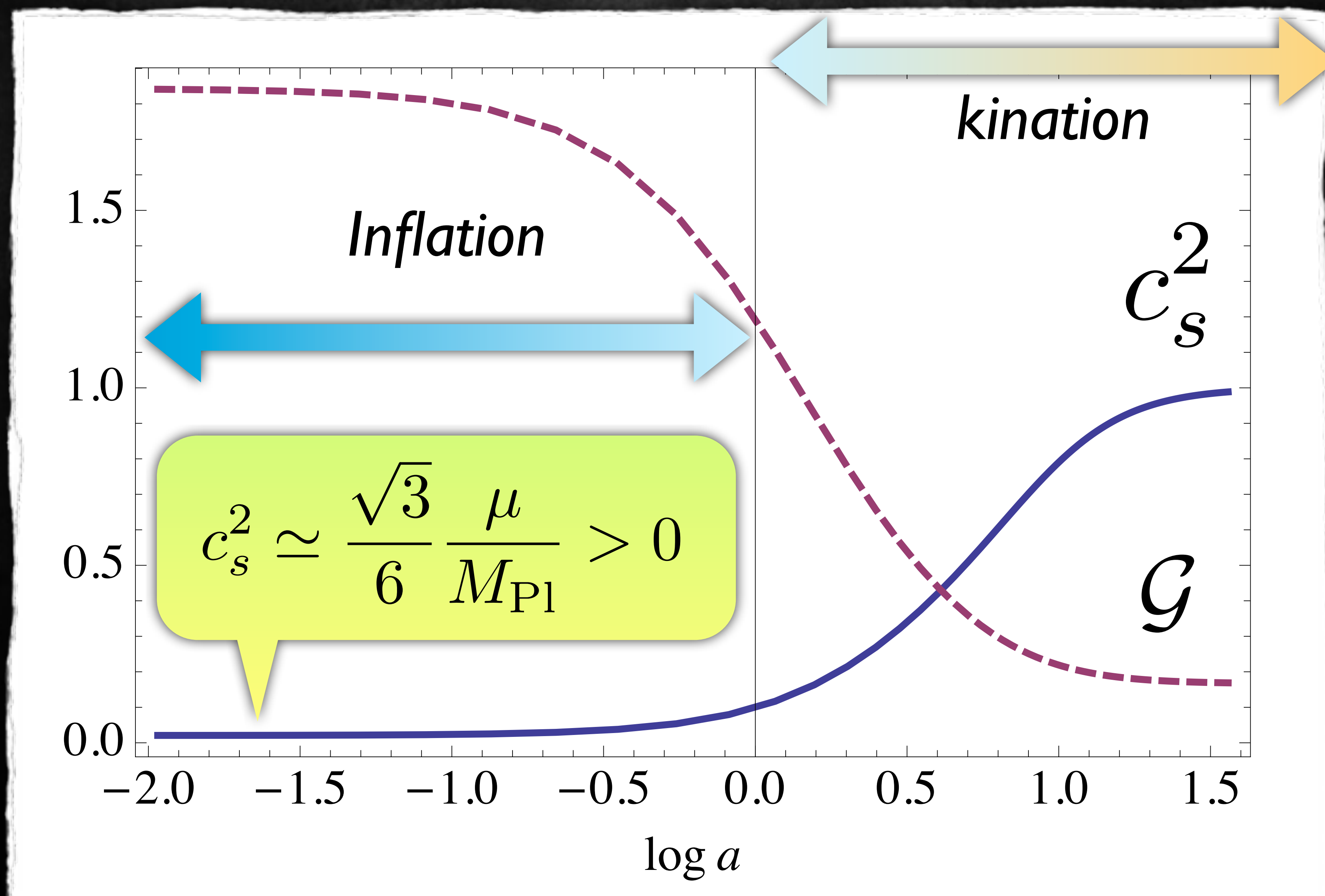
$z$

$$\mathcal{F} = K_X + 2F_X\left(\ddot{\phi} + 2H\dot{\phi}\right) - 2F_X^2X^2 + 2F_{XX}X\ddot{\phi} - 2\left(F_\phi - XF_{\phi X}\right)$$

$$\mathcal{G} = K_X + 2XK_{XX} + 6F_XH\dot{\phi} + 6F_X^2X^2 - 2\left(F_\phi + XF_{\phi X}\right) + 6F_{XX}HX\dot{\phi}$$

# Stable example

$$K = -A(\phi)X + \frac{X^2}{2M^3\mu}, \quad F = \frac{X}{M^3}$$



# Primordial spectrum

Consider G-inflation with:

$$K = K(X), \quad F = f(\phi)X$$

New variables:

$$\begin{aligned} dy &= c_s d\tau \\ \tilde{z} &= (\mathcal{F}\mathcal{G})^{1/4} z \\ u &= \tilde{z}\mathcal{R} \end{aligned}$$

$$\frac{d^2u}{dy^2} + \left(k^2 - \frac{\tilde{z}_{,yy}}{\tilde{z}}\right) u = 0$$

$$\frac{\tilde{z}_{,yy}}{\tilde{z}} \simeq \frac{1}{(-y)^2}\left[2 + 3\epsilon\mathcal{C}(X)\right]$$

$$\mathcal{C}(X) = \frac{K}{K_X}\frac{Q_X}{Q}$$

$$Q(X) = \frac{(K - XK_X)^2}{18Xc_s^2\sqrt{\mathcal{F}\mathcal{G}}}$$

$$\frac{\dot{c}_s}{Hc_s} = \mathcal{O}(\epsilon)$$

# Primordial spectrum

Normalized mode: $u = \frac{\sqrt{\pi}}{2}\sqrt{-y}H^{(1)}_{3/2+\epsilon\mathcal{C}}(-ky)$

$$\mathcal{P}_{\mathcal{R}} = \frac{Q}{4\pi^2}\bigg|_{c_s k = 1/(-\tau)}, \quad n_s - 1 = -2\epsilon\mathcal{C}$$

$\propto f_{,\phi}$

where $Q(X) = \frac{(K - XK_X)^2}{18M_{\rm Pl}^4 X c_s^2\sqrt{\mathcal{FG}}}$

$\mathcal{R}$ can be generated even from exact de Sitter

* Tensor mode dynamics: unchanged

# Tensor-to-scalar ratio

$$K = -X + \frac{X^2}{2M^3\mu}, \quad F = fX \quad \longrightarrow \quad H^2 \sim \frac{\mu M^3}{M_{\rm Pl}^2}$$

$$(f \simeq M^{-3})$$

$$r \simeq \frac{16\sqrt{6}}{3}\left(\frac{\sqrt{3}\mu}{M_{\rm Pl}}\right)^{3/2}$$

Standard consistency relation is violated

$$r \neq 16\epsilon$$

$\propto f_{,\phi}$

$$M = 0.00435 \times M_{\rm Pl}, \ \mu = 0.032 \times M_{\rm Pl}$$

$$\longrightarrow \quad \mathcal{P}_{\mathcal{R}} = 2.4 \times 10^{-9}, \ r = 0.17$$

*r* can be large!

Definition:

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}}, \quad \mathcal{P}_T = \frac{8}{M_{\rm Pl}^2}\left(\frac{H}{2\pi}\right)^2$$

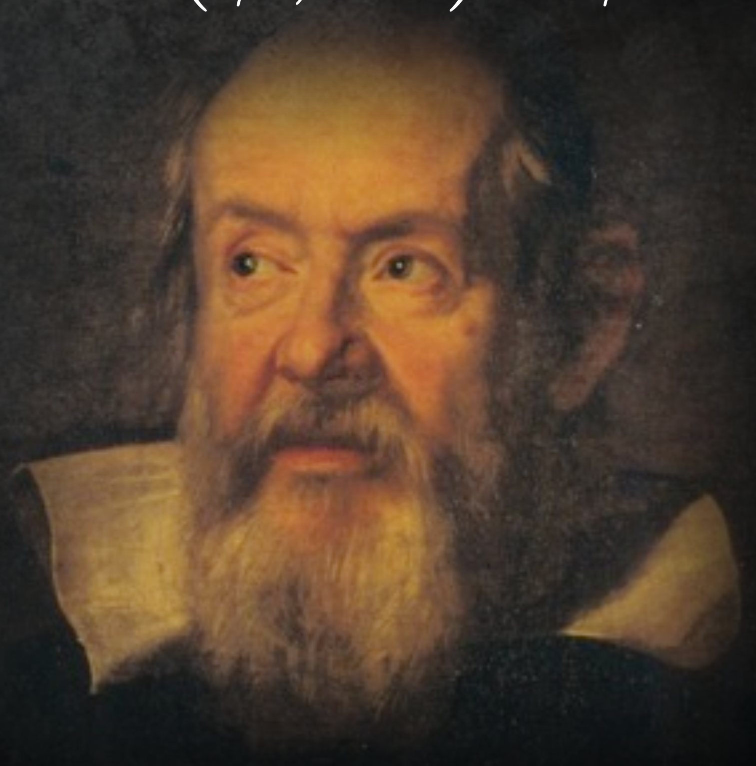
# Summary & Outlook

# Summary & Outlook

- G-inflation: A novel class of inflation models

$$\mathcal{L}_\phi = K(\phi, X) - F(\phi, X)\Box\phi$$

- $n_s - 1 \simeq 0$
- Large $r$
- ~~Consistency relation~~



- Non-Gaussianity?

TK, Yamaguchi, Yokoyama *in progress*


Thank you!