Recent topics on modulated reheating (preheating) and non-gaussianity

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Based on

Kohri, Lyth, Valenzuela-Toledo, arXiv:0904.0793 [hep-ph]

Kamada, Kohri and Shuichiro Yokoyama, arXiv:1008.1450 [astro-ph]

Abstract

In “Modulated” Reheating or Preheating” scenario, fluctuation of a light field σ contributes to curvature perturbation and produces sizable amount of f_{NL}

Modulated reheating scenario can be still consistent with Affleck-Dine baryogenesis. We can check the scenario in future experiments such as Planck, PolarBeaR and LiteBIRD

Introduction

- Inflation paradigm is attractive
  - Solving horizon problem
  - Solving flatness problem
  - Solving GUT monopole problem
  - Producing density (curvature) fluctuation
- Observation (WMAP)
  - Power spectrum of density fluctuation

$$\sqrt{\mathcal{P}_\zeta} = (4.9 \pm 0.2) \times 10^{-5}$$

  - Spectral index

$$n = \frac{d \ln P_\zeta}{d \ln k} + 1 = 0.96 \pm 0.03$$

  - Running of n(k)

$$-0.07 < \frac{dn}{d \ln k} < 0.02$$

Tensor to scalar ratio vs spectral index

In chaotic inflation $V = \frac{1}{2} m_\phi^2 \phi^2$

- Slow-roll parameters

$$\varepsilon = \frac{1}{2}(M_G \frac{V'}{V})^2 = 2(M_G/\phi)^2 \ll 1$$

$$\eta = M_G^2 \frac{V''}{V} = 2(M_G/\phi)^2 \ll 1$$

- Spectral index

$$n_s - 1 = 2\eta - 6\varepsilon < 0$$

- Tensor to scalar ratio

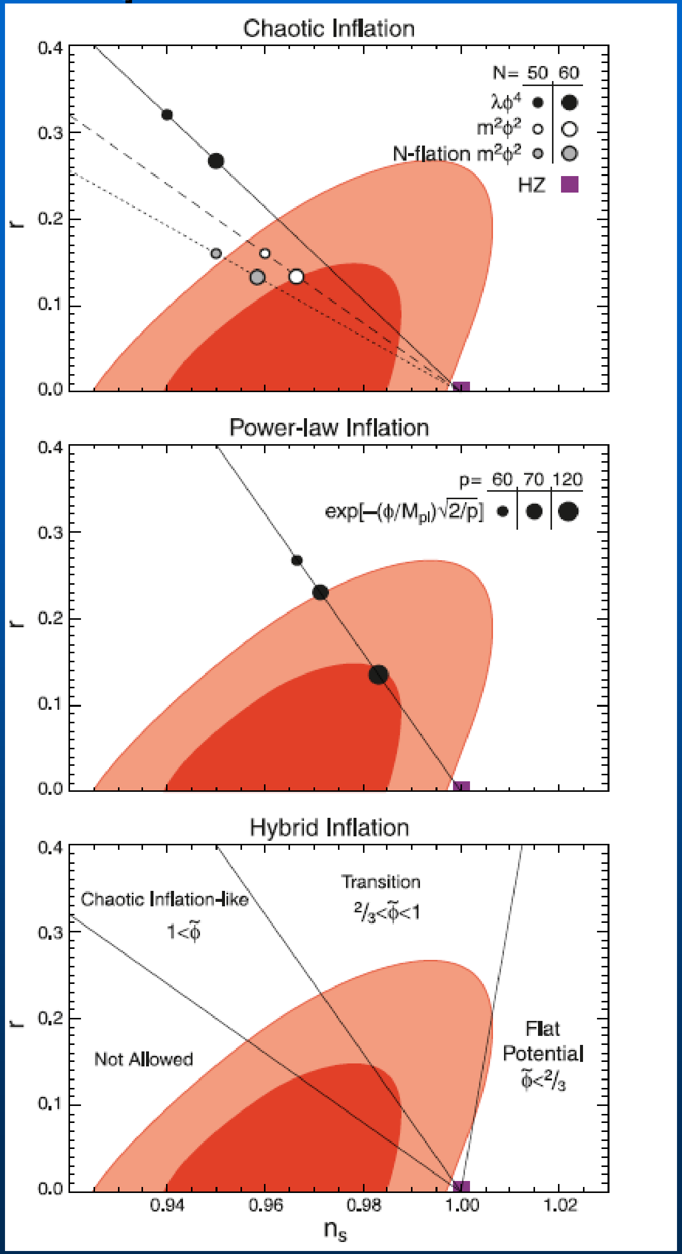
$$r \sim 16\varepsilon \sim 0.1$$

- E-folding number during inflation

$$N \sim \frac{\phi_*^2 - \phi_{end}^2}{M_G^2} \sim 50 - 60$$

- Field value at horizon exit

$$\phi_* \sim 7M_G \sim 2M_P$$



Constraints on non-gaussianity

- WMAP 7-year reported

$$f_{NL}^{local} = 32 \pm 21 \quad (68\% C.L.)$$

Komatsu et al, (2010)

Gaussian perturbation

- Correlators
  1. 2-point correlation function

$$\left\langle \zeta_{\vec{\mathbf{k}}} \zeta_{\vec{\mathbf{k}}'} \right\rangle = (2\pi)^3 \delta^3\left(\vec{\mathbf{k}} + \vec{\mathbf{k}}'\right) P_\zeta(k)$$

$$\zeta_{\vec{\mathbf{k}}} = \int \zeta(\vec{\mathbf{x}}) e^{-i\vec{k}\cdot\vec{\mathbf{x}}} d^3x$$

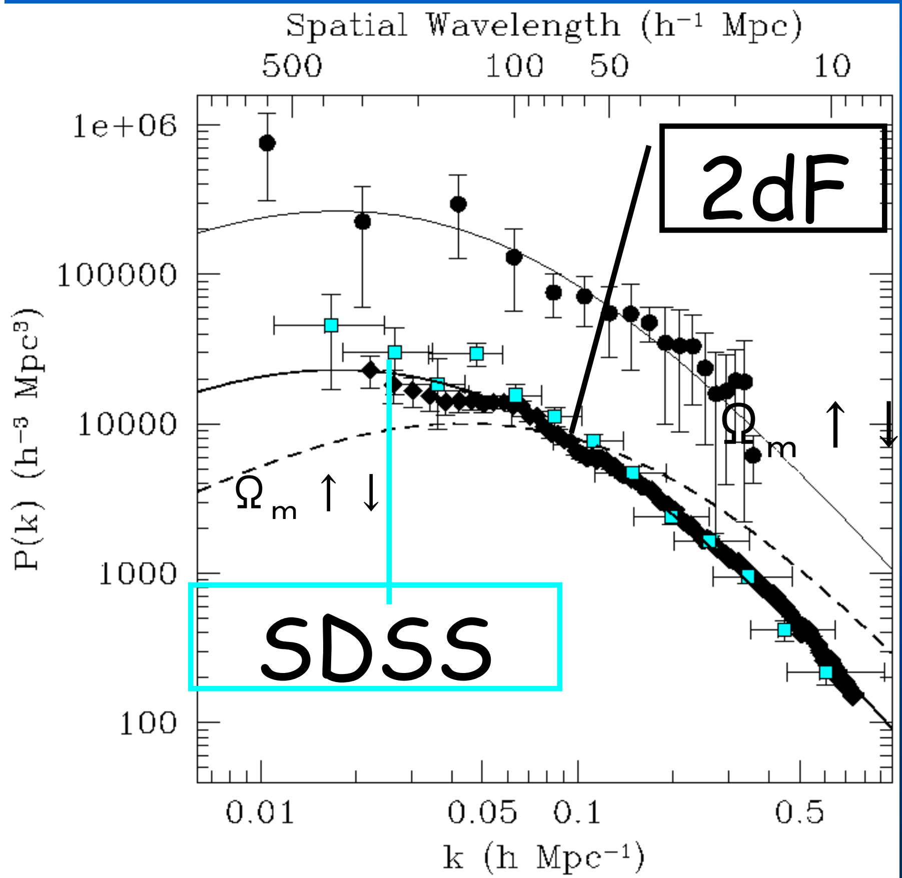
  2. Odd-number point correlation funcs.

$$\left\langle \zeta_{\vec{\mathbf{k}}} \right\rangle = 0 \qquad \left\langle \zeta_{\vec{\mathbf{k}}_1} \zeta_{\vec{\mathbf{k}}_2} \zeta_{\vec{\mathbf{k}}_3} \right\rangle = 0$$

$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2} P_\zeta \left(\sim \textit{scale} \text{ invariant} \propto k^0 \,? \right)$$

Power spectrum

Red galaxies, b = 1



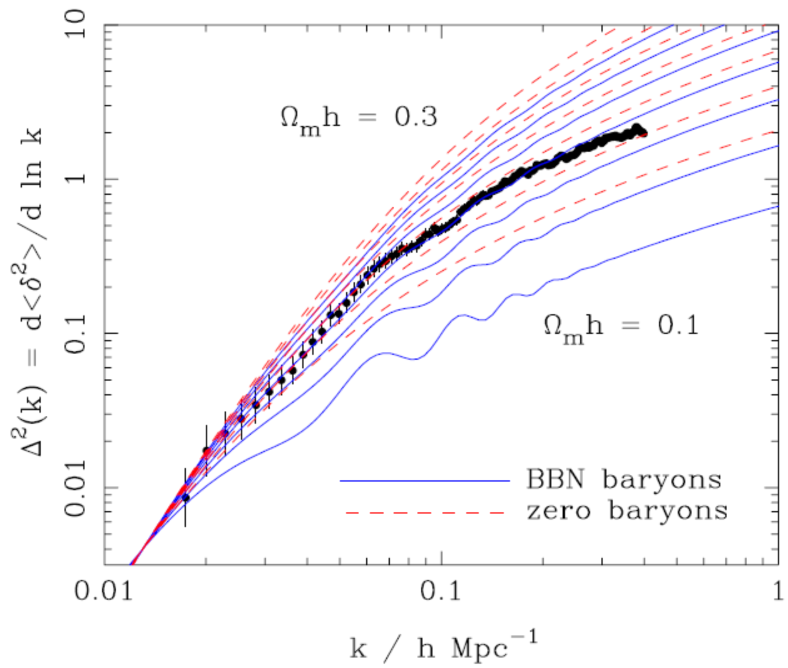
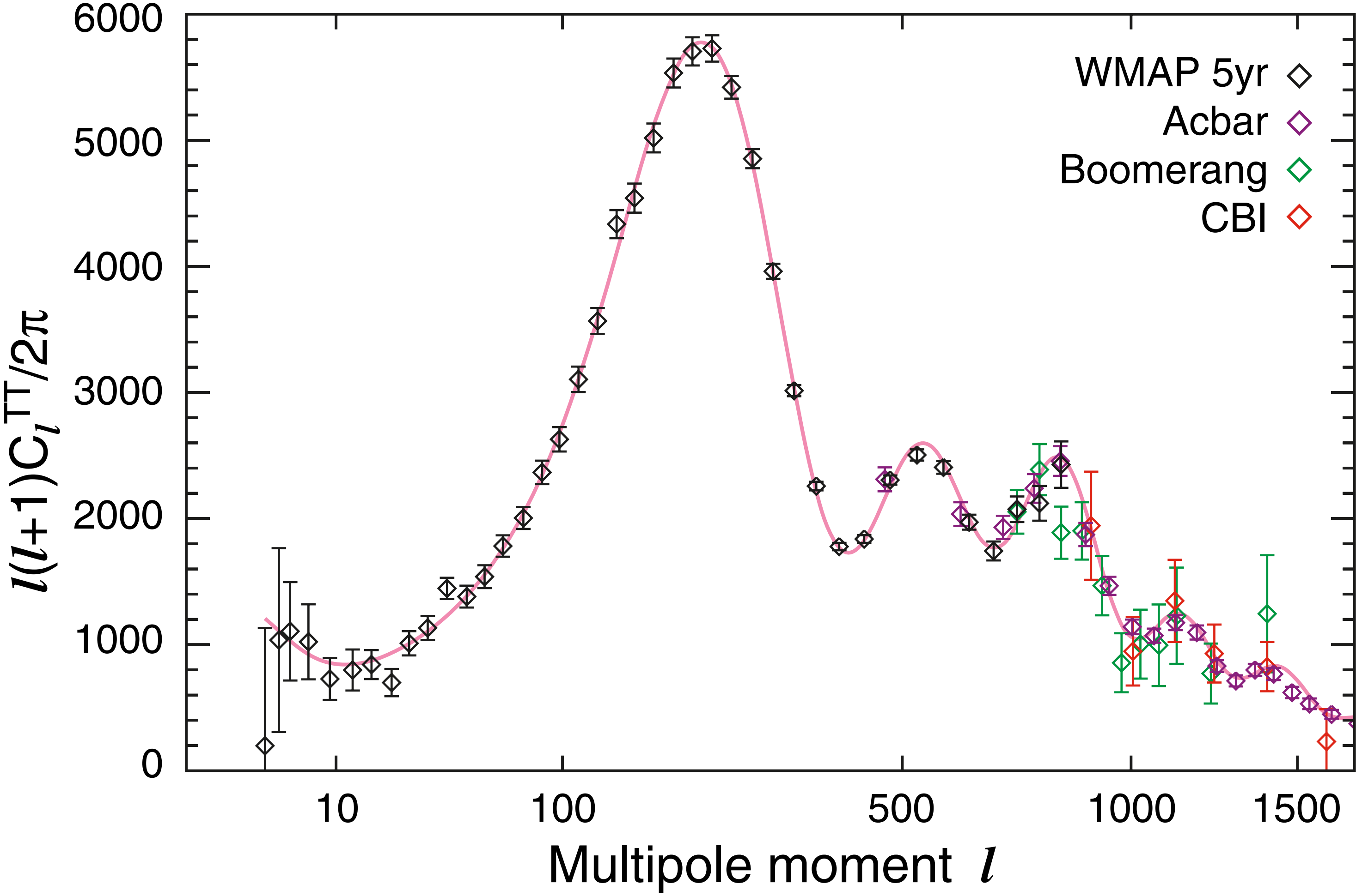


Figure 19.5: The galaxy power spectrum from the 2dFGRS, shown in dimensionless form, $\Delta^2(k) \propto k^3 P(k)$. The solid points with error bars show the power estimate. The window function correlates the results at different k values, and also distorts the large-scale shape of the power spectrum An approximate correction for the latter effect has been applied. The solid and dashed lines show various CDM models, all assuming $n = 1$. For the case with non-negligible baryon content, a big-bang nucleosynthesis value of $\Omega_b h^2 = 0.02$ is assumed, together with $h = 0.7$. A good fit is clearly obtained for $\Omega_m h \simeq 0.2$.

http://msowww.anu.edu.au/2dFGRS/ Olive and Peacock, Particle Data Group (09)

Cosmic Microwave Background (CMB) Anisotropy

$\times \mathcal{O}(10^{-13})$



Non-gaussian perturbation

- 3-point correlator and bispectrum B

$$\left\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \right\rangle = (2\pi)^3 \delta^3\left(\vec{\mathbf{k}}_1 + \vec{\mathbf{k}}_2 + \vec{\mathbf{k}}_3\right) B_\zeta\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\right)$$

- Nonlinearity parameter f_{NL}

$$B_\zeta\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\right) = \frac{6}{5} f_{NL}\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3\right)[P_\zeta(\mathbf{k}_1)P_\zeta(\mathbf{k}_2) + P_\zeta(\mathbf{k}_2)P_\zeta(\mathbf{k}_3) + P_\zeta(\mathbf{k}_3)P_\zeta(\mathbf{k}_1)]$$

$$\begin{aligned} \zeta(\vec{x}) &= \zeta_{gaussian} + \zeta_{nongaussian} \\ &= \zeta_{gaussian} + b(\zeta_{gaussian})^2 \end{aligned}$$

$$b = \frac{3}{5} f_{NL}$$

δ N formalism and curvature perturbation

Sasaki-Stewart (`96), Sasaki-Tanaka(`98),Lyth-Rodriguez (`04)

- Curvature perturbation ζ with choosing a gauge in which threads are comoving and the slice of uniform-energy density

$$a(\vec{x},t) = a(t)e^{\zeta(\vec{x},t)}$$

$$g_{ij} = a^2(\vec{x},t)\gamma_{ij}(\vec{x})$$

$$\begin{aligned}\delta N(x,t) &= \ln a(x,t)/a(x,0) - \ln a(t)/a(0)\\ &= \ln a(x,t)/a(t) - \ln a(x,0)/a(0)\\ &= \zeta(\vec{x},t) - 0\end{aligned}$$

$$\boxed{\zeta(\vec{x},t) = \delta N(x,t)}$$

[*Genelrally* a=a(t)e$^{\psi(x,t)}$ with $\psi(x,t)=\zeta(x,t)$ *for uniform − density* slice, $\psi(x,0)=0$ for *flat* slicing]

δ N formalism

- Curvature perturbation

$$\zeta = \delta N = \underbrace{\frac{\partial N}{\partial \phi}\delta\phi}_{\text{g part}} + \underbrace{\frac{1}{2}\frac{\partial^2 N}{\partial \phi^2}(\delta\phi)^2 + \ldots}_{\text{non-g part}}$$

- Power spectrum

$$\mathcal{P}_{\zeta_\phi} = (N_\phi)^2 \mathcal{P}_{\delta\phi}$$

- Nonlinear parameter

$$\frac{5}{6} f_{NL} = \left(\frac{\mathcal{P}_{\zeta_\phi}}{\mathcal{P}_{\zeta,obs}} \right)^2 \frac{N_{\phi\phi}}{(N_\phi)^2}$$

Practice in chaotic inflation

- Potential $$V = \frac{1}{2} m_\phi^2 \phi^2$$
- Slow roll parameters

$$\varepsilon = \frac{1}{2}(M_G \frac{V'}{V})^2 = 2(M_G / \phi)^2 \ll 1$$

$$\eta = M_G^2 \frac{V''}{V} = 2(M_G / \phi)^2 \ll 1$$

- Horizon exit

$$\phi_* \sim 7M_G \sim 2M_P$$

Practice in chaotic inflation model part II

- e-folding number N

$$N = \ln \frac{a(t_{end})}{a(t_*)} = \int_{t_*}^{t_{end}} \frac{da}{a} = \int_{t_*}^{t_{end}} H dt$$

- Friedman equation $H^2 = \frac{V}{3M_G^2}$

- Slow-roll $\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}$

$$Hdt = \frac{H}{\dot{\phi}} d\phi = -\frac{3H^2}{V'} d\phi = -\frac{1}{M_G^2}\frac{V}{V'} d\phi$$

Practice in chaotic inflation model part III

- e-folding number N

$$N \sim \frac{\phi_*^2 - \phi_{end}^2}{M_G^2} \sim 60 \qquad N_\phi \sim \frac{2\phi_*}{M_G^2} \qquad N_{\phi\phi} \sim \frac{2}{M_G^2}$$

- Spectrum $\mathcal{P}_\zeta = (N_\phi)^2 \mathcal{P}_{\delta\phi} \sim \left(\frac{2\phi_*}{M_G^2}\right)^2 \left(\frac{H}{2\pi}\right)^2 \sim \left(\frac{\phi_*}{\pi M_G}\right)^2 \left(\frac{m_\phi \phi_*}{2M_G^2}\right)^2$

$$\sim 3 \times 10^{-9}$$

$$(\rightarrow m_\phi \sim 10^{-5} M_G \sim 10^{13} GeV)$$

- Non-gaussianity

$$\frac{5}{6} f_{NL} = \left(\frac{\mathcal{P}_\zeta}{\mathcal{P}_{\zeta,obs}}\right)^2 \frac{N_{\phi\phi}}{(N_\phi)^2} \sim \frac{1}{2(\phi_*/\mathrm{M_G})^2}$$

$$\sim \eta - 2\varepsilon \sim O(0.01)$$

Large non-gaussianity?

$$f_{NL} \sim O(10)$$

Modulated Reheating

Dvali, Gruzinov, Zaldarriaga(04)
Kofman (04)
Zaldarriaga (04)

- If decay rate $\Gamma = 1/t_{dec} \sim T_R^2/M_{pl}$ depends on another field σ $\Gamma = \Gamma(\sigma)$

$$\frac{a_{dec}}{a_i} \propto t_{dec}^{2/3} \propto \Gamma^{-2/3}$$

$$\frac{a_f}{a_{dec}} \propto t_{dec}^{-1/2} \propto \Gamma^{1/2}$$

$$N = \ln\left(\frac{a_{dec}}{a_i}\right) + \ln\left(\frac{a_f}{a_{dec}}\right) = \left(-\frac{2}{3} + \frac{1}{2}\right)\ln(\Gamma) + \ldots$$

$$= -\frac{1}{6}\ln(\Gamma) + \ldots$$

Calculation in modulated reheating

$$N_\sigma = \frac{\partial N}{\partial \Gamma}\frac{\partial \Gamma}{\partial \sigma} \qquad N_{\sigma\sigma} = \frac{\partial N}{\partial \Gamma}\frac{\partial^2 \Gamma}{\partial \sigma^2} + \frac{\partial^2 N}{\partial \Gamma^2}\left(\frac{\partial \Gamma}{\partial \sigma}\right)^2$$

$$\mathcal{P}_{\zeta_\sigma} = (N_\sigma)^2 (H_* / 2\pi)^2$$

$$\frac{6}{5} f_{NL} = \left(\frac{\mathcal{P}_{\zeta_\sigma}}{\mathcal{P}_{\zeta,\mathrm{obs}}}\right)^2 \frac{N_{\sigma\sigma}}{(N_\sigma)^2}$$

Affleck-Dine baryogenesis with modulated reheating

Kamada, Kohri, Yokoyama (2010)

How does it depend on T_R?

- The baryon number often depends on the reheating temperature in cosmologies with SUSY or SUGRA

$$\frac{n_B}{s} \propto \left(T_R\right)^p, \quad p = 0,\ 1, \ldots$$

Is baryonic-Isocurvature perturbation induced?

- Baryonic-Isocurvature perturbation

$$S_B \equiv \frac{\delta(n_B/s)}{n_B/s} = p\frac{\delta T_R}{T_R}$$

- Adiabatic curvature perturbation

$$\sqrt{P_\zeta} = \delta N = -\frac{1}{6}\frac{\delta\Gamma}{\Gamma} = \frac{1}{3}\frac{\delta T_R}{T_R}$$

$$\Gamma \sim T_R^2 / M_{pl}$$

Is it observationally allowed?

- Theoretically

$$\frac{S_B}{\sqrt{P_\zeta}} = -3p$$

- Observationally

$$\left|\frac{S_B}{\sqrt{P_\zeta}}\right| \leq O(1)$$

$$\Rightarrow |p| < 1/3$$

Affleck-Dine baryogenesis

- Powerful mechanism in SUSY or SUGRA cosmology

Φ : AD fileld which carries baryon #

$$W = O(1)\frac{\Phi^{n+3}}{M_*^n} \qquad K = |\Phi|^2 + |I|^2 + \lambda |\Phi|^2 |I|^2 / M_{pl}{}^2$$

$$V = e^{K/M_{pl}} \left[\left(W_i + \frac{W K_i}{M_{pl}^2} \right) K^{i,\bar{j}} \left(W_{\bar{j}}^* + \frac{W^* K_{\bar{j}}}{M_{pl}^2} \right) - 3\frac{|W|^2}{M_{pl}^2} \right]$$

$$= V_{mass} \underbrace{- c_H H^2 |\Phi|^2 + \frac{|\Phi|^{2n+4}}{M_*^{2n}}}_{|\Phi| \sim \left(H M_*^n\right)^{\frac{1}{n+1}}} + \left(a_B m_{3/2} \frac{\Phi^{n+3}}{M_*^n} + h.c. \right)$$

"Mass" term before reheating

$$T = \left(H M_G T_R^2 \right)^{1/4}$$

i. Soft-mass term

$$m_{soft}^2 |\Phi|^2$$

ii. Thermal mass term from plasma

$$h^2 T^2 |\Phi|^2$$

iii. Thermal log from 2nd order and running of gauge coupling

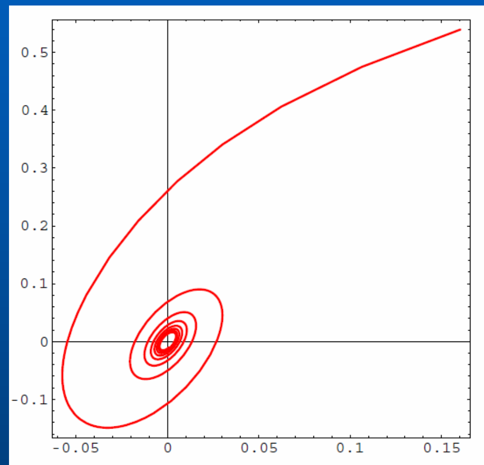
$$\alpha_g^2 T^4 Log\left(h^2 |\Phi|^2 / T^2 \right)$$

n_B

Baryon number generation

- Oscillation of Affleck-Dine Field Φ starts at

$$\sqrt{|V''|} \geq H$$

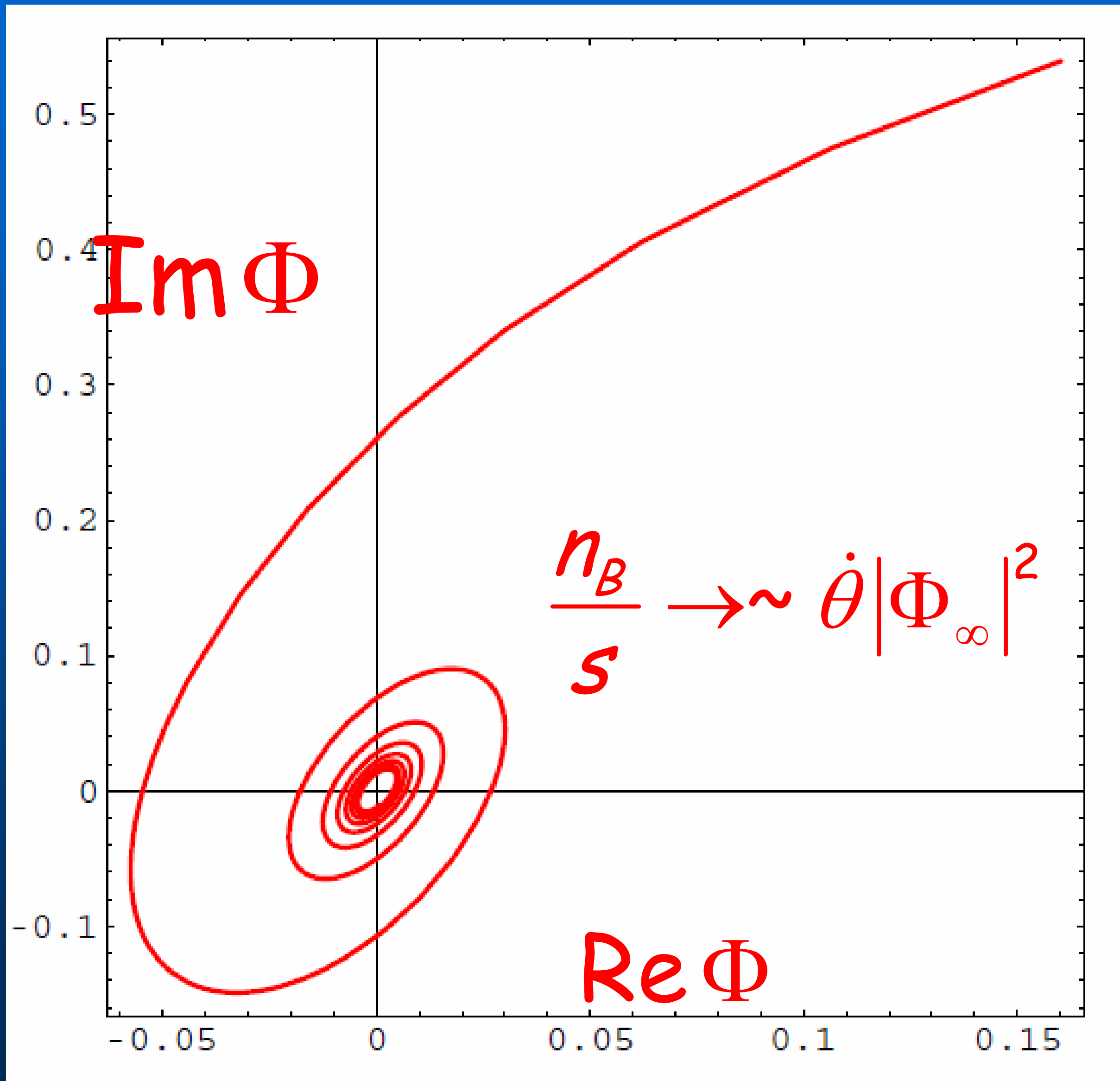


- Baryon number

$$n_B = iq\left(\frac{d\Phi^*}{dt}\Phi - \Phi^*\frac{d\Phi}{dt}\right) \sim \frac{\mathrm{Im}V_{osc}}{H_{osc}}$$

$$\sim m_{3/2}\left|\Phi_{osc}\right|^2$$

E.O.M.



M. Fujii, Master Thesis, Univ. of Tokyo, 2001

Baryon to entropy ratio

- Dilution before reheating

$$\frac{n_B}{s} \sim \frac{n_B(t_R)}{T_R^3} \sim \frac{n_B(t_{osc})\left(H_R / H_{osc}\right)^2}{T_R^3}$$

$$\sim \frac{m_{3/2} M_*^{2/(n+1)}}{M_G^2} T_R H_{osc}^{-2n/(n+1)}$$

T_R dependence on baryon#

i. Soft-mass term

$$\frac{n_B}{s} \propto T_R^{\,1} \qquad \text{with } H_{osc} \sim m_{soft}$$

ii. Thermal mass term

$$\frac{n_B}{s} \propto T_R^{\,0} \qquad \text{with n=3, } H_{osc} \propto T_R^{\,2/3},$$

$$M_* \sim 10^{16}\,\text{GeV}, T_R \sim 10^6\,\text{GeV}$$

iii. Thermal log term

$$\frac{n_B}{s} \propto T_R^{\,0} \qquad \text{with n=1, } H_{osc} \propto T_R^{\,1}$$

$$M_* \sim 10^{23}\,\text{GeV}, T_R \geq 10^7\,\text{GeV}$$

Isocurvature mode originated from the phase component

Kawasaki, Nakayama, F.Takahashi (09)

$$S_{B,phase} \sim \cot(n\theta_{\inf} + \alpha)\left(\frac{H_{\inf}}{M_*}\right)^{n/(n+1)}$$

- In case of thermal mass term

$$H_{\inf} \leq 10^9\,\mathrm{GeV}$$

with n=3, $M_* \sim 10^{16}\,\mathrm{GeV}$, $T_R \sim 10^6\,\mathrm{GeV}$

- In case of thermal log

$$H_{\inf} \leq 10^{13}\,\mathrm{GeV}$$

with n=1, $M_* \sim 10^{23}\,\mathrm{GeV}$, $T_R \geq 10^7\,\mathrm{GeV}$

Detectable non-gaussianity and tensor to scalar ratio

$$L_{int} = g\varphi\bar{\psi}\psi$$

$$\Gamma_\varphi \sim g^2 m_\varphi \qquad g(\sigma) \sim g\left[1+\lambda\left(\frac{\sigma}{M_{cut}}\right)^2\right]$$

- Non-gasussianity

$$f_{NL} \sim -10^2 \lambda^{-1}\left(\frac{\sigma}{0.1M_{cut}}\right)^{-2}$$

- Tensor to scalar ratio

$$r = 16\varepsilon \sim 0.1\left(H_{inf}/10^{13}\,GeV\right)^2$$

Modulated preheating

Kohri, Lyth, Valenzuela-Toledo (09)

Model of preheating

Kofman, Linde, Starobinsky ('94),('97)

See also Fujisaki, Kumekawa, Yamaguchi, Yoshimura (95)

- Motivated by particle physics (Chaotic Inflation, A-term Inflation, Inflection point Inflation.), we may adopt ½ $m^2\phi^2$ for inflaton potential

$$\mathcal{L} = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{2} g^2 \chi^2 \phi^2$$

Reduction of ρ_ϕ and resonant production of ρ_χ

- See massless preheating models.

$$\mathcal{L} = \frac{1}{4} \lambda \phi^4 + g^2 \chi^2 \phi^2$$

See Bond, Frolov, Huang, Kofman, arXiv:0903.3407

Modulated Preheating

See Podolsky, Felder, Kofman, Peloso (05)

- Coupling constant g can depend on another field

Ackerman et al (05)

$$g^2 = g^2\left(1 + \frac{\sigma^2}{M^2}\right)^2$$

$$(10^{-10} < g^2 < 10^{-6})$$

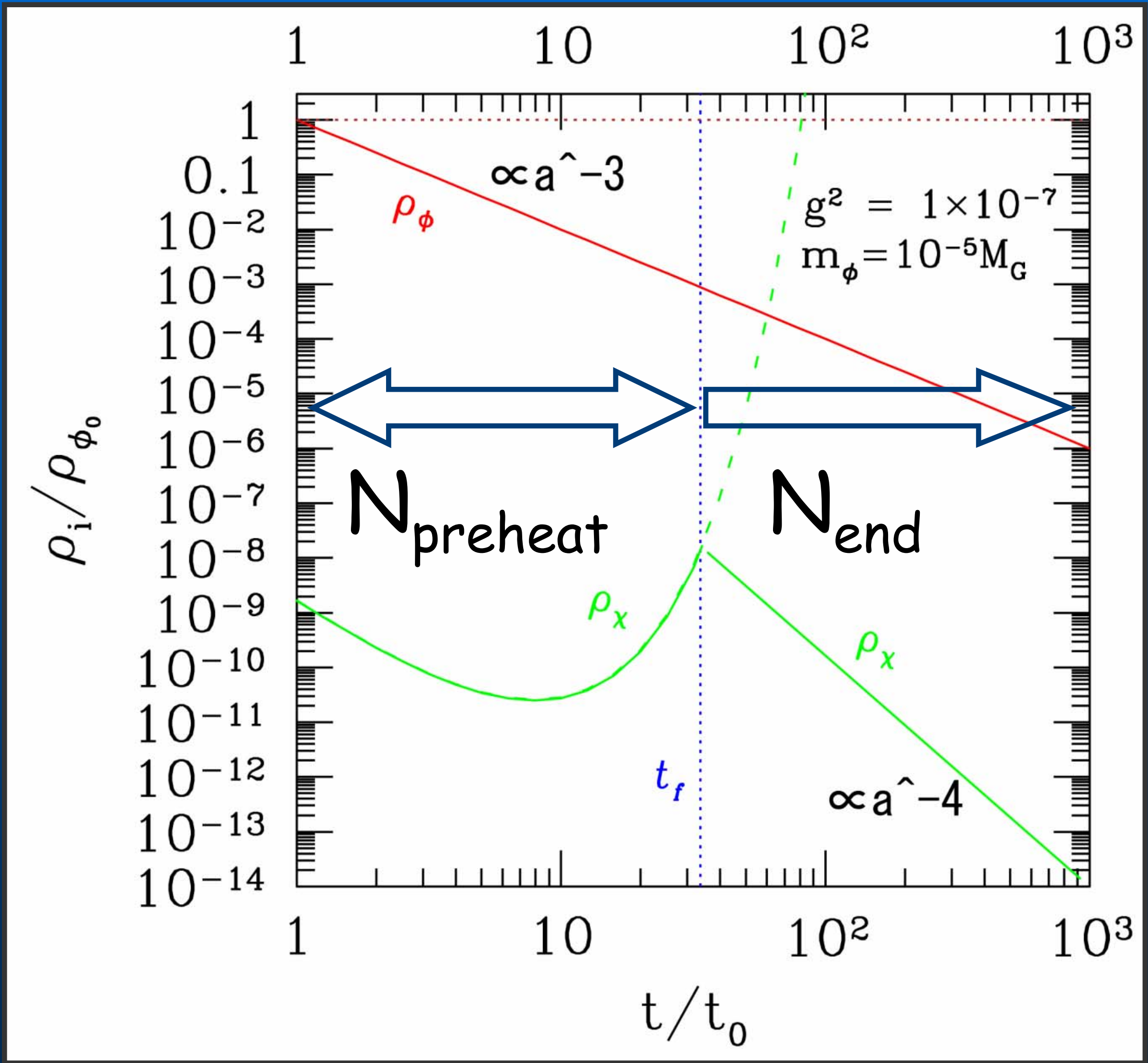
Broad resonance

- Then perturbation produced by σ is important

$$N_\sigma = g_\sigma \frac{\partial N}{\partial g} \qquad N_{\sigma\sigma} = g_{\sigma\sigma}\frac{\partial N}{\partial g} + (g_\sigma)^2 \frac{\partial^2 N}{\partial g^2}$$

Preheating $\rho_\chi \propto e^{2\mu m_\phi t}$

Kofman,Linde,Starobinsky(97)

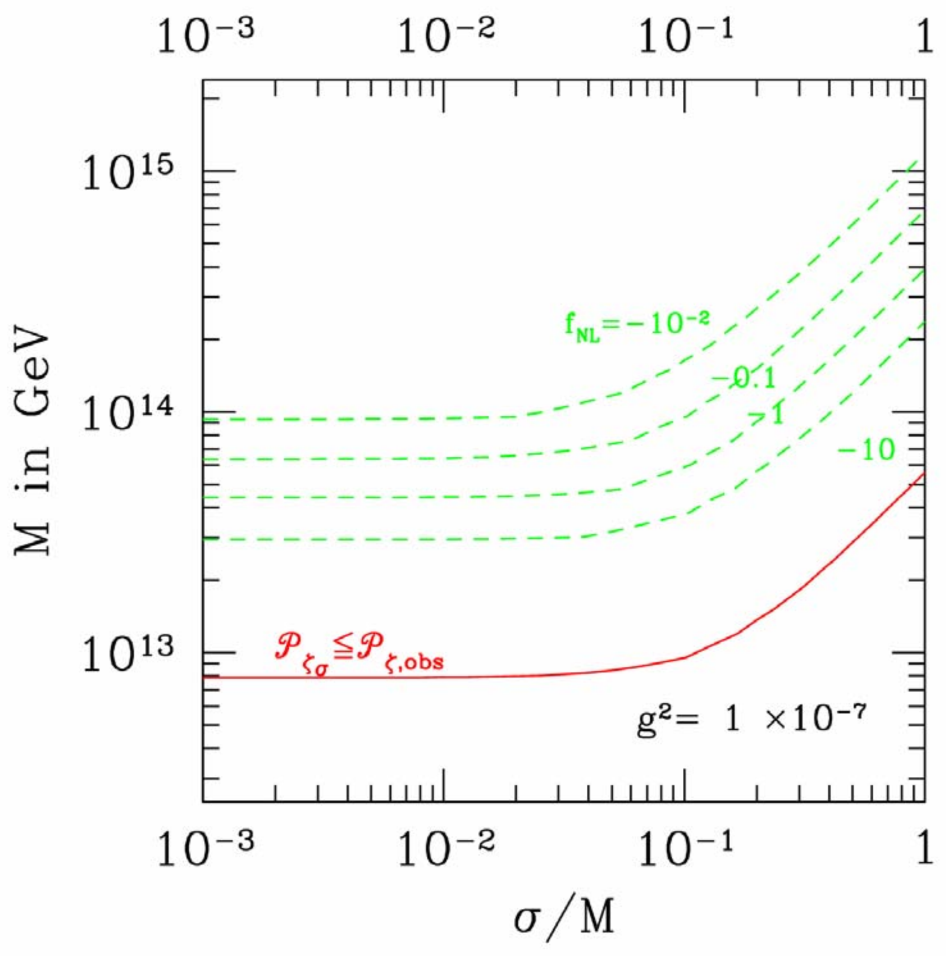


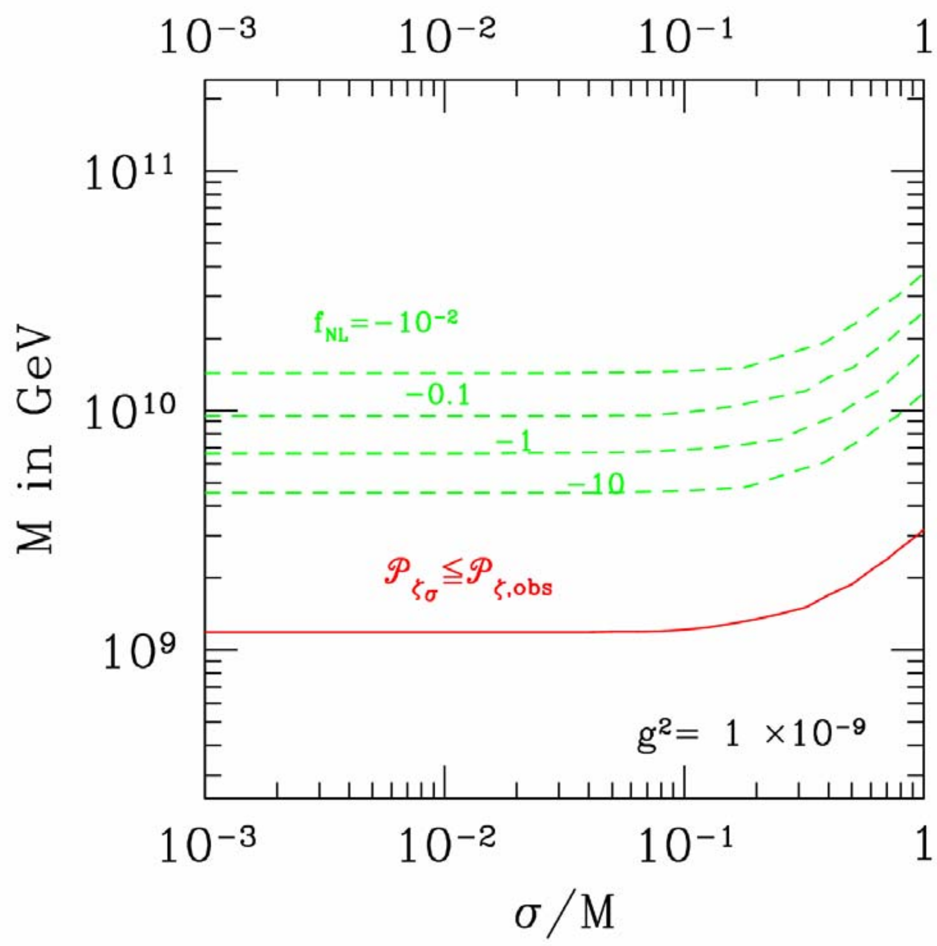
$$\Phi(t) \approx \frac{M_p}{m_\phi t}$$

$$t_f = gM_p / m_\phi^2 \quad (g\Phi(t_f) = m_\phi)$$

$$g\Phi(t_1)n_\chi(t_1) \equiv \frac{1}{2}m_\phi^2\Phi^2(t_1)$$

Non-gaussianity





Conclusion

- Modulated reheating/preheating scenarios are quite attractive because coupled massless σ quanta can contribute to curvature perturbation and f_{NL}

- This scenario will be able to be checked by Planck, PolarBeaR and LiteBIRD.

Angular spectrum of CMB anisotropy

- Temperature fluctuation

$$\frac{\delta T}{T}(\vec{n}) = \sum_{\ell,m} (-1)^{\ell} a_{\ell m} Y_{\ell m}(\vec{n})$$

- Angular spectrum

$$\left\langle a_{\ell m} a^{*}_{\ell' m'} \right\rangle = \delta_{\ell m} \delta_{\ell' m'} C_{\ell}$$

- Correlation function of temperature fluc.

$$\left\langle \frac{\delta T}{T}(\vec{e}_1) \frac{\delta T}{T}(\vec{e}_2) \right\rangle = \sum_{\ell} \frac{2\ell + 1}{4\pi} C_{\ell} P_{\ell}(\cos\theta)$$

- Curvature fluctuation

$$a_{\ell m} = \frac{4\pi}{(2\pi)^3} i^{\ell} \int \Theta_{\ell}(\vec{k}) Y^{*}_{\ell m} d^3\vec{k} \qquad \Theta_{\ell}(\vec{k}) = T_{\ell}(k)\ \zeta_{\vec{k}}$$

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How large/small is g²?

- For no radiative corrections to the potential

$$g^2 \ll 10^{-6} \qquad (V_{\text{1-loop}} \sim g^4 \phi^4 \log(\phi / Q))$$

But see Barnaby, Huang, Kofman, Pogosyan, **arXiv:0902.0615**

- Massless fluctuation of χ during inflation $g^2 < 10^{-10} \quad (gM_p < H_* \sim m_\phi)$

Mathew Equation

Mode expansion

$$\hat{\chi}(t,\mathbf{x}) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left(\hat{a}_k \chi_k(t)\, e^{-i\mathbf{kx}} + \hat{a}_k^{+} \chi_k^{*}(t)\, e^{i\mathbf{kx}} \right)$$

Equation of Motion

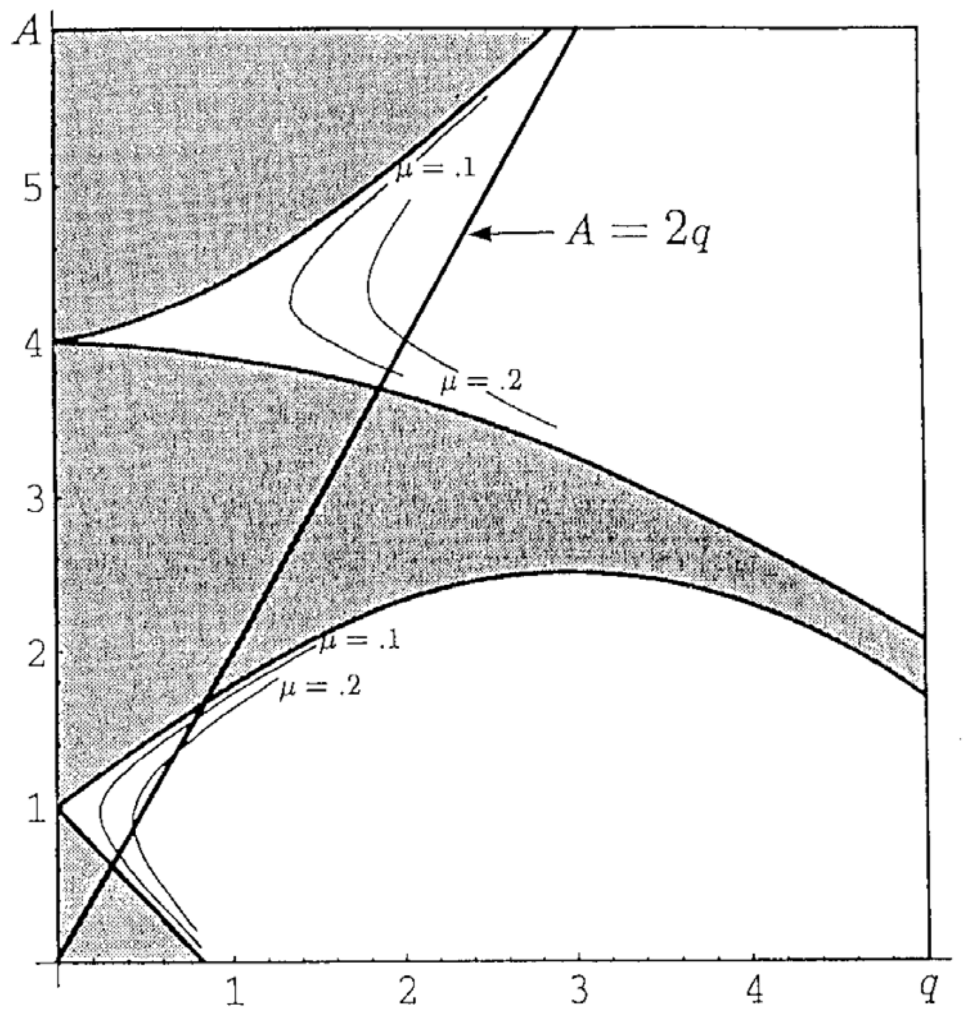
$$\ddot{\chi}_k + 3\frac{\dot{a}}{a}\dot{\chi}_k + \left(\frac{\mathbf{k^2}}{a^2} + g^2\phi^2 \right)\chi_k = 0$$

Kofman,Linde,Starobinsky(97)

Instability band

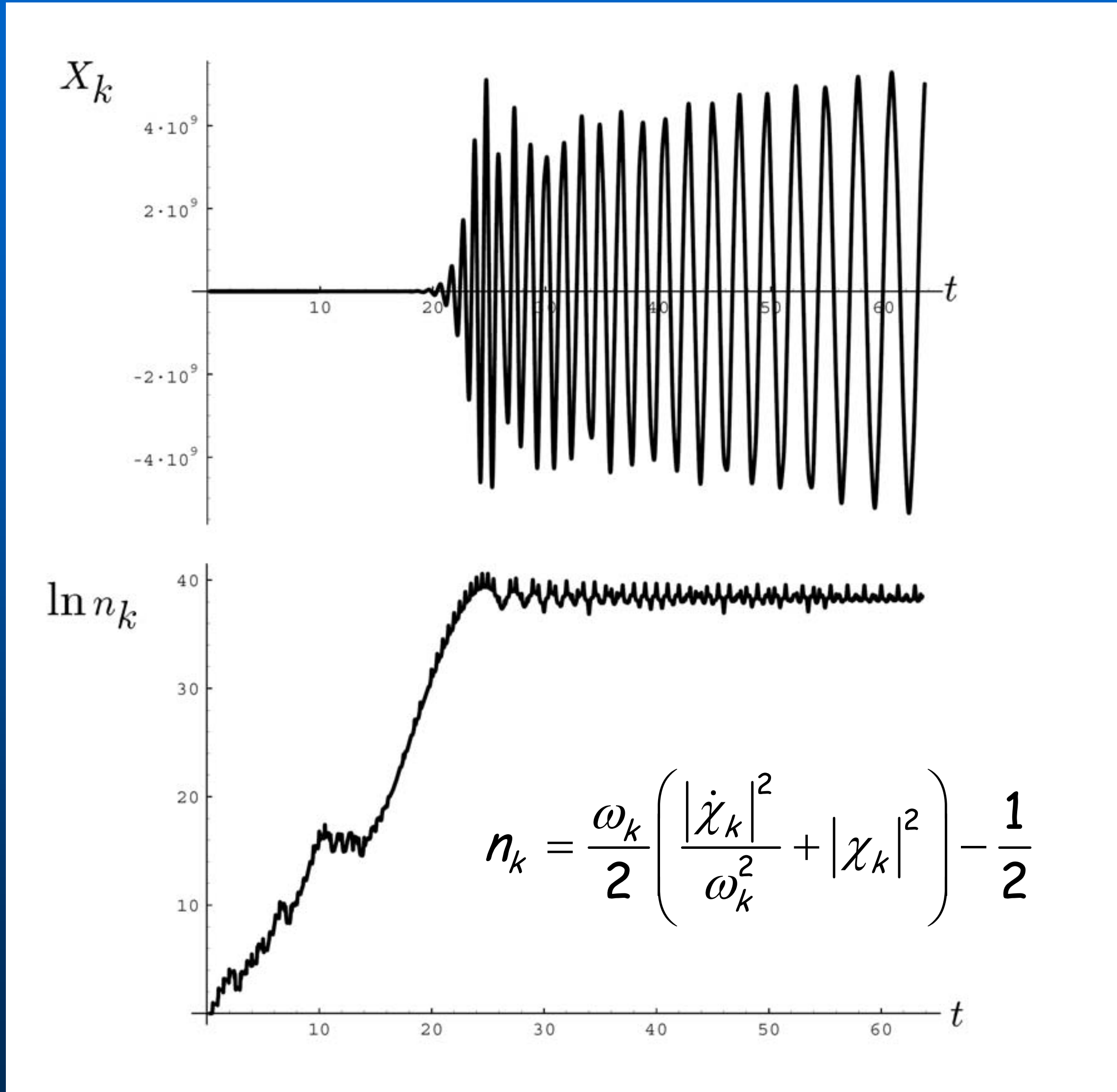
$$q \equiv g^2\Phi^2 / m_\phi^2$$

$$A = k^2 / m_\phi^2 + 2q$$



Evolution

Kofman,Linde,Starobinsky(97)



$$n_k = \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2}$$

End of preheating

Kofman,Linde,Starobinsky(97)

$$q \equiv g^2 \Phi^2 / m_\phi^2$$

$$\Phi(t) \approx \frac{M_p}{m_\phi t}$$

$$A = 2q$$

$$q(t_f) = O(1)$$

$$t_f = g M_p / m_\phi^2 \quad (g\Phi(t_f) = m_\phi)$$

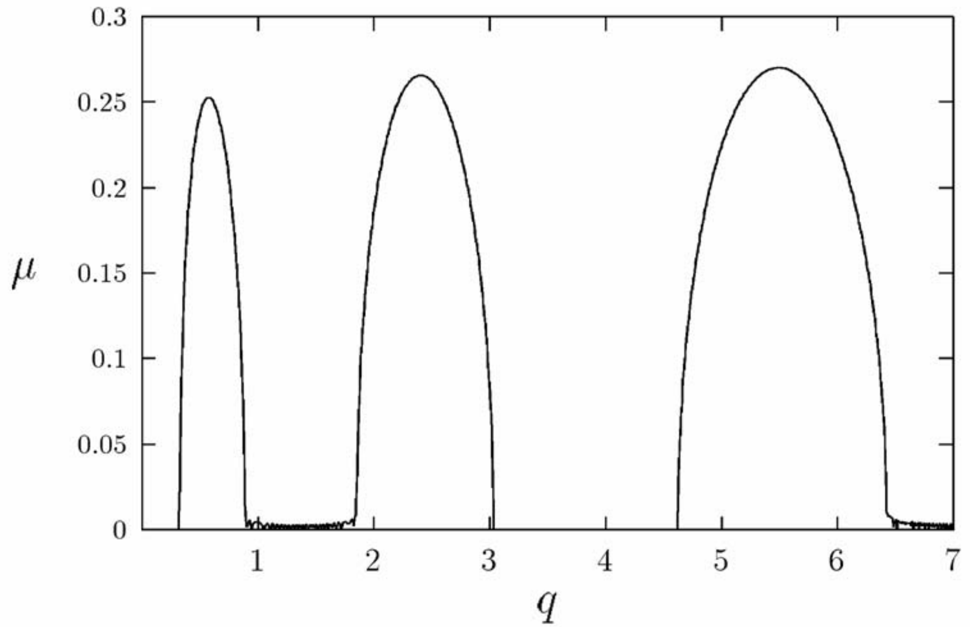


FIG. 7. The structure of the resonance bands for the Mathieu equation along the line $A = 2q$, which correspond to excitations with $k = 0$ in our model. The modes with small k are especially interesting because the momenta of the excitations are redshifted during the expansion of the universe. A small plateau at $10 \lesssim t \lesssim 15$ on Fig. 5 corresponds to the time where stochastic resonance ceases to exist, all modes are redshifted to small k, and the system spends some time in the interval with $1 \lesssim q \lesssim 2$, which is outside the instability zone. The last stage of the resonance shown in Fig. 6 corresponds to the resonance in the first instability band with $q < 1$.

Does narrow parametric resonance occur in expandin universe?

- Narrow resonance $\chi_k \propto e^{\mu m_\phi t}$ $\mu = \sqrt{(q/2)^2 - (2k/a/m_\phi - 1)^2}$

$$k/a \sim m_\phi \qquad \Delta(k/a) \sim q m_\phi \qquad (q \equiv g^2 M_P^2 / m_\phi^2 < 1)$$

$$g^2 < 10^{-10}$$

- Conditions for efficient resonance

$$\Delta t = \frac{q m_\phi}{d(k/a)/dt} \sim q/H \gg m_\phi^{-1} \quad (q \equiv g^2 M_P^2 / m_\phi^2 > 1)$$

$$g^2 > 10^{-10}$$

Big difference from Enqvist, Jokinen, Mazumdar, Vaihkonen (05)

We may need another mechanism

- Broad resonance

$$10^{-10} < g^2 < 10^{-6}$$

- Large fluctuation?
- Large non-gaussianity?

Modulated Preheating

See Podolsky, Felder,Kofman, Peloso (05)

- Coupling constant g can depend on another field

Ackerman et al (05)

$$g^2 = g^2\left(1+\frac{\sigma^2}{M^2}\right)^2$$

$$(10^{-10} < g^2 < 10^{-6})$$

Broad resonance

- Then perturbation produced by σ is important

$$N_\sigma = g_\sigma \frac{\partial N}{\partial g} \qquad N_{\sigma\sigma} = g_{\sigma\sigma}\frac{\partial N}{\partial g} + (g_\sigma)^2 \frac{\partial^2 N}{\partial g^2}$$

Is χ nonrelativistic or relativistic?

- χ's momentum at the resonance $k \sim \sqrt{g m_\phi M_p}$

$$(k/a) = \sqrt{g m_\phi M_p}\left(\frac{a_*}{a}\right) = \sqrt{g m_\phi M_p}\left(t m_\phi\right)^{-2/3}$$

- Mass of χ

$$\mathrm{m}_\chi(t) = g\,\Phi(t) = g\,M_p (a_*/a)^{3/2} = g\,M_p\left(m_\phi t\right)^{-1}$$

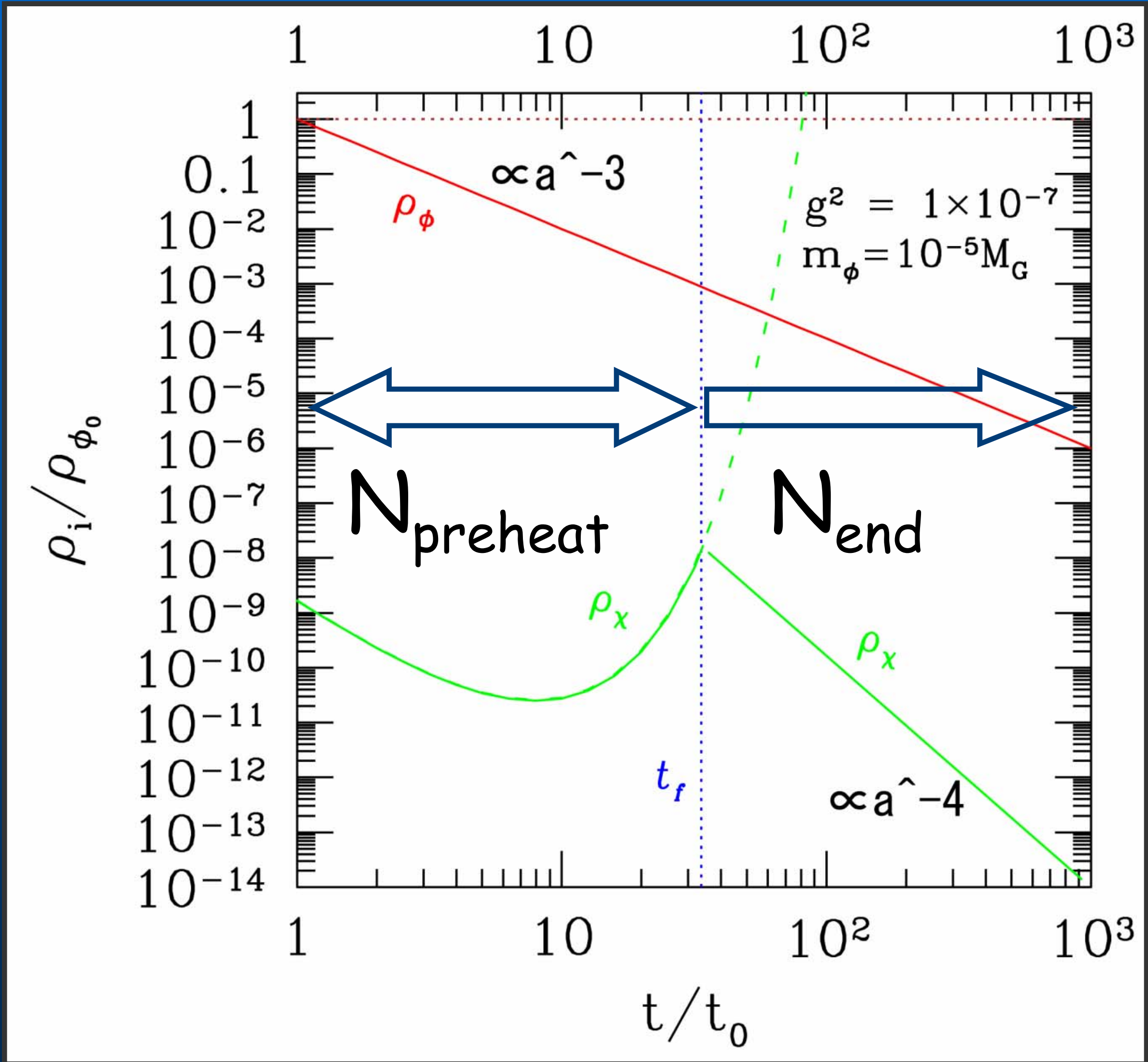
- After $t \gtrsim t_f$ $\quad t_f = g M_p / m_\phi^2 \quad (g\Phi(t_f) = m_\phi)$

$$\frac{k/a(t_f)}{m_\chi(t_f)} \gtrsim (g/10^{-5})^{-1/6}$$

Relativistic particle $\rho \propto a^{-4}$ for $t \gtrsim t_f$

Preheating $\rho_\chi \propto e^{2\mu m_\phi t}$

Kofman,Linde,Starobinsky(97)

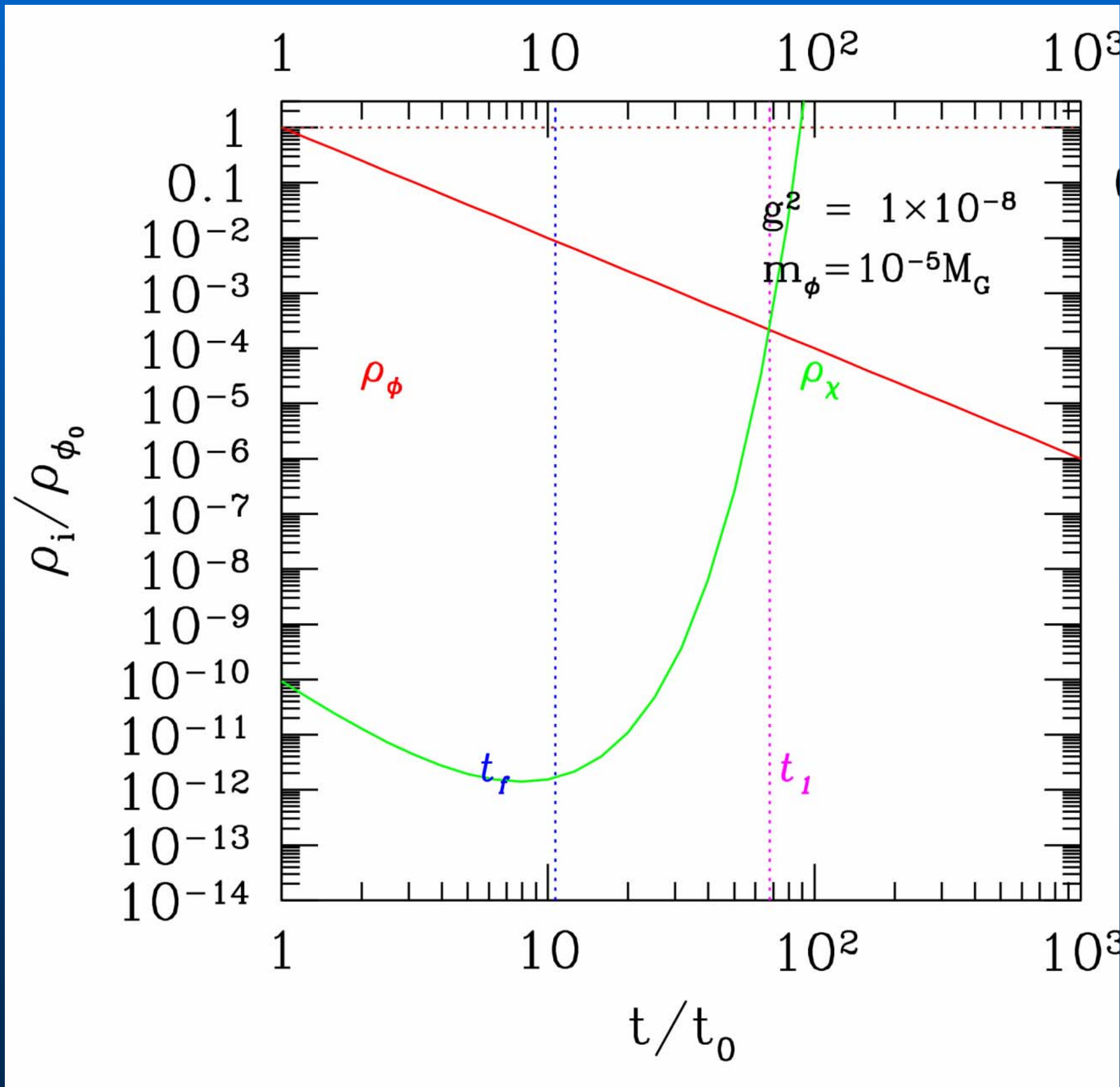


$$\Phi(t) \approx \frac{M_p}{m_\phi t}$$

$$t_f = gM_p / m_\phi^2 \quad (g\Phi(t_f) = m_\phi)$$

$$g\Phi(t_1) n_\chi(t_1) \equiv \frac{1}{2} m_\phi^2 \Phi^2(t_1)$$

$g^2=10^{-8}$



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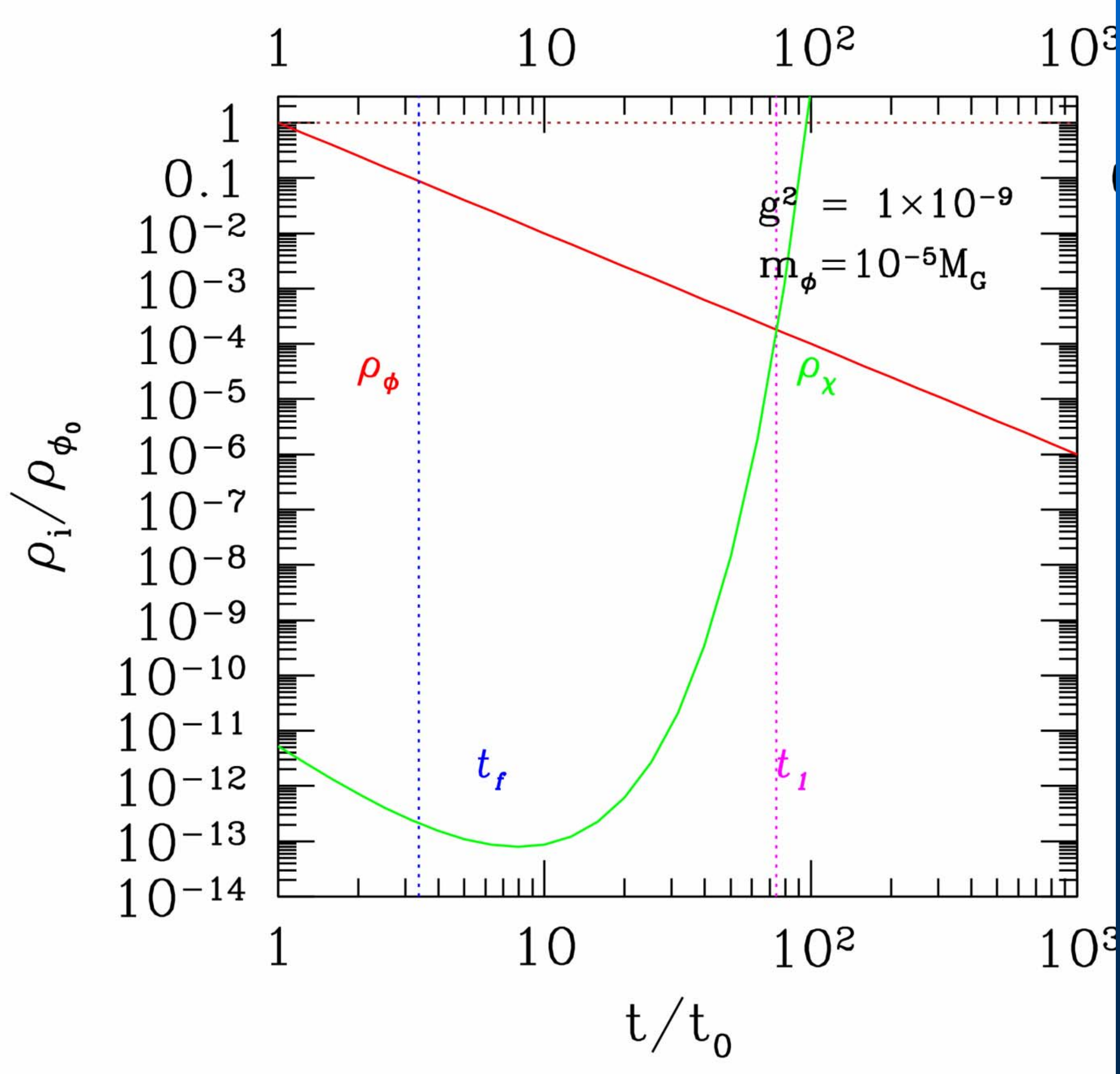
$$g\Phi(t_1) n_\chi(t_1) \equiv \frac{1}{2} m_\phi^2 \Phi^2(t_1)$$

$g^2=10^{-9}$



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Analytical estimate of 1st stage

- During preheating

  ϕ and χ are massive

$$\frac{a_f}{a_i} = (t_f / m_\varphi^{-1})^{2/3} = (gM_p / m_\varphi)^{2/3}$$

$$t_f = gM_p / m_\phi^2 \quad (g\Phi(t_f) = m_\phi)$$

$$N_{\mathrm{preheat}} = \ln\left(\frac{a_f}{a_i}\right)$$

$$\frac{\partial N_{\mathrm{preheat}}}{\partial g} = \frac{2}{3g} \qquad \frac{\partial^2 N_{\mathrm{preheat}}}{\partial g^2} = -\frac{2}{3g^2}$$

Analytical estimation of 2nd stage

- After preheating

  ϕ is massive, χ is almost massless

$$N_{\rm after} = \ln\left(\frac{a(t)}{a(t_{\rm f})}\right) = \frac{1}{3}\ln\left(\frac{\rho_\phi^{\rm end}}{\rho(t)}\right) = \frac{1}{3}\ln\left(\frac{\rho_{\rm tot}^{\rm end} - \rho_\chi^{\rm end}}{\rho(t)}\right)$$

$$\rho_\chi^{\rm end} = Ag^{-1}\,{\rm Exp}[Bg]$$

$$\frac{\partial N_{\rm after}}{\partial g} \sim -10^5\,\frac{\rho_\chi^{\rm end}}{\rho_{\rm tot}^{\rm end}} \qquad \frac{\partial^2 N_{\rm after}}{\partial g^2} \sim -10^{10}\,\frac{\rho_\chi^{\rm end}}{\rho_{\rm tot}^{\rm end}}$$

Spectrum and non-linear parameter

$$N_\sigma = g_\sigma \frac{\partial N}{\partial g} \qquad N_{\sigma\sigma} = g_{\sigma\sigma} \frac{\partial N}{\partial g} + (g_\sigma)^2 \frac{\partial^2 N}{\partial g^2}$$

$$\mathcal{P}_{\zeta_\sigma} = (N_\sigma)^2 (H_* / 2\pi)^2 \times (1 + R_{\text{1-loop}})$$

$$\frac{6}{5} f_{\text{NL}} = \left(\frac{\mathcal{P}_{\zeta_\sigma}}{\mathcal{P}_{\zeta,\text{obs}}} \right)^2 \frac{N_{\sigma\sigma}}{(N_\sigma)^2} \times (1 + R_{\text{1-loop}})$$

$$R_{\text{1-loop}} = \left[\frac{N_{\sigma\sigma}}{N_\sigma} \left(\frac{H_*}{2\pi} \right) \right]^2 \ln(kL)$$

Non-gaussianity