Relating Gauge theories via Gauge/Bethe Correspondence (part I)

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Outline

1. *Why are we here?*
2. *Bethe Ansatz for the XXX spin chain*
   - Parameters in a spin chain
   - XXX
   - Algebraic Bethe Ansatz
3. *Effective twisted superpotential in two dimensions*
   - Basics of $\mathcal{N} = (2, 2)$ field theories
   - Effective theory in the Coulomb branch
4. *The Gauge/Bethe Correspondence*

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Motivations

- Nekrasov and Shatashvili introduced a correspondence between supersymmetric gauge theories and integrable models.
- Today's talk will describe the simplest example of this correspondence (two dimensional $\mathcal{N} = (2, 2)$ gauge theory and lattice spin chain)
- Main interest: We can translate problems from one side to the other
- Fresh perspective on existing problems
- New questions
- Valuable tool for both sides: new insights.

Tomorrow's talk

- Summary: Gauge/Bethe correspondence
- The *tJ* model
- Supergroup symmetry
- Bethe Ansatz
- The Dictionary
- Quiver gauge theories
- Relation via brane cartoons
- Generalizations/Open questions
- Summary

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Parameters of a spin chain

- A spin chain-type system lives on a one-dimensional lattice of length L.
- To each point k we associate a representation Λ of the symmetry group K and call the corresponding Hilbert space $\mathscr{H}_k$.
- The dynamics is described by the Hamiltonian

$$\mathcal{H} = -\sum_{k=1}^{L-1} \left[\Pi_{k,k+1} - 1 \right] ,$$

where $\Pi_{k,k+1}$ is the permutation operator.

- Boundary conditions via an operator $\mathcal{K} \in \mathrm{End}(\mathscr{H})$.
- $\mathcal{K}$ depends on $r = \mathrm{rank}(K)$ twist parameters $\{ \hat{\vartheta}_a \}_{a=1}^{r}$.

Hilbert space

- $\mathcal{H}$ commutes with the total maximal torus $T \subset K$
- we can decompose the Hilbert space of states into a direct sum

$$\mathscr{H} = \bigotimes_{k=1}^{L} \mathscr{H}_k = \bigoplus_{a=1}^{r} \bigoplus_{N_a=0}^{L} \mathscr{H}_{N_a}^{(a)} .$$

- $\Psi \in \mathscr{H}_{N_a}^{(a)}$ is a magnon describing a state with N_a particles of *species* a.
- Ψ depends on the rapidities (quasi-momenta) $\{ \lambda_i^{(a)} \}_{i=1}^{N_a}$.
- For a general spin chain, each point $k = 1, \ldots, L$ can carry a different representation $\Lambda_k = [\Lambda_k^1, \ldots \Lambda_k^r]$ of the symmetry group, and one can turn on inhomogeneities $\nu_k^{(a)}$

Concrete example: XXX

- We want to study the XXX spin chain on a periodic line of length L
- At each point the spin carries the fundamental representation of $su(2)$
- The Hilbert space is $\mathscr{H}_k = \mathbb{C}^2$
- At each point k we have spin variables S_k^a, $a = 1, 2, 3$, which are Pauli matrices.
- The Hamiltonian (particle exchange) can be written as

$$\mathcal{H} = -\sum_{k=1}^{L-1} \Pi_{k,k+1} = \sum_a \sum_k \left(S_k^a S_{k+1}^a - \frac{1}{4} \right)$$

Algebraic Bethe Ansatz: Lax operator

- Introduce an auxiliary space $V = \mathbb{C}^2$
- Fundamental object: Lax operator $L_{k,a}(\lambda) \in \mathrm{End}(\mathscr{H}_k \otimes V)$

$$\begin{aligned} L_{k,a}(\lambda) &= \lambda \, \mathbb{I}_k \otimes \mathbb{I}_a + \imath \sum_a S_k^a \otimes \sigma^a \\ &= \begin{pmatrix} \lambda + \imath S_k^3 & \imath S_k^- \\ \imath S_k^+ & \lambda - \imath S_k^3 \end{pmatrix} \\ &= \left(\lambda - \frac{\imath}{2} \right) \mathbb{I}_{k,a} + \imath P_{k,a} \end{aligned}$$

where $P : a \otimes b \mapsto b \otimes a$

Algebraic Bethe Ansatz: Yang-Baxter equation

The properties of the Lax operator are related to the

fundamental commutation relation

$$R_{a_1,a_2}(\lambda - \mu) L_{k,a_1}(\lambda) L_{k,a_2}(\mu) = L_{k,a_2}(\mu) L_{k,a_1}(\lambda) R_{a_1,a_2}(\lambda - \mu)$$

Let's stop a moment to understand this equation

- This is an operator equivalence in $\mathscr{H}_k \otimes V \otimes V$ (indices a_1 and a_2)
- The commutation of L_{k,a_1} and L_{k,a_2} (same Hilbert space, different auxiliary space) requires an intertwiner
- The intertwiner R_{a_1,a_2} depends only on the auxiliary spaces
- The fundamental commutation relation is valid for any value of the spectral parameters λ and μ.

Algebraic Bethe Ansatz: Monodromy

- The Lax operator is essentially a connection along the chain, defining the transport between site k and $k+1$:

$$\Psi_{k+1} = L_k \Psi_k$$

  where $\Psi_k \in (\mathscr{H}_k)^{\otimes 2}$
- We have a closed chain: it is natural to calculate the monodromy

$$T_{L,a}(\lambda) = L_{L,a}(\lambda) L_{L-1,a}(\lambda) \cdots L_{1,a}(\lambda)$$

  note that we are using always the same auxiliary space. $T_{L,a}$ is an operator in $\mathscr{H} \otimes V$.
- It is convenient to write it making V explicit:

$$T_{L,a}(\lambda) = \begin{pmatrix} A(\lambda) & B(\lambda) \\ C(\lambda) & D(\lambda) \end{pmatrix}$$

Algebraic Bethe Ansatz: Monodromy

- The monodromy $T_{L,a}$ obeys the same commutation relation as before:

$$R_{a_1,a_2}(\lambda - \mu) T_{a_1}(\lambda) T_{a_2}(\mu) = T_{a_2}(\mu) T_{a_1}(\lambda) R_{a_1,a_2}(\lambda - \mu)$$

- It is a polynomial in λ of degree L. The first two terms are easy to write:

$$T_{L,a}(\lambda) = \lambda^L + \imath \lambda^{L-1} \sum_a (S^a \otimes \sigma^a) + \dots$$

- We know enough to introduce the transfer matrix

$$t(\lambda) = \operatorname{Tr}_a T_{L,a}(\lambda)$$

Algebraic Bethe Ansatz: Transfer matrix

- From the fundamental commutation relations we find that the transfer matrix for different values of the spectral parameter commute

$$[t(\lambda), t(\mu)] = 0$$

- $t(\lambda)$ is a polynomial in λ:

$$t(\lambda) = 2\lambda^L + \sum_{l=0}^{L-2} \lambda^l Q_l$$

- From the commutation relation we learn that the operators $Q_l \in \mathrm{End}(\mathscr{H})$ commute:

$$[Q_l, Q_m] = 0$$

Algebraic Bethe Ansatz: commuting Hamiltonians

- We have a family of $L - 1$ commuting integrals of motion.
- The original Hamiltonian is one of them:

$$\mathcal{H} = \frac{\imath}{2} \frac{d}{d\lambda} \log t(\lambda) \Big|_{\lambda = \imath/2} - \frac{L}{2}$$

- We have as many degrees of freedom as integrals of motion.
- The system is integrable!

Algebraic Bethe Ansatz: Spectrum

We can calculate the spectrum explicitly.

- Write the Fundamental commutation relations

$$[B(\lambda), B(\mu)] = 0$$
$$A(\lambda)B(\mu) = f(\lambda - \mu)B(\mu)A(\lambda) + g(\lambda - \mu)B(\lambda)A(\mu)$$
$$D(\lambda)B(\mu) = h(\lambda - \mu)B(\mu)D(\lambda) + k(\lambda - \mu)B(\lambda)D(\mu)$$

- We want a highest weight representation for this algebra
- Reference state Ω such that

$$C(\lambda)\Omega = 0$$

- Choosing $\Omega = \prod_{k=1}^{L} e_\uparrow$

$$T(\lambda)\Omega = \begin{pmatrix} (\lambda + \imath/2)^L & * \\ 0 & (\lambda - \imath/2)^L \end{pmatrix} \Omega$$

Algebraic Bethe Ansatz: Spectrum

- Algebraic Bethe Ansatz: all the eigenvectors have the form

$$\Phi(\{\lambda\}) = B(\lambda_1)\cdots B(\lambda_l)\Omega$$

- Now we require this to be an eigenvector:

$$t(\lambda)\Phi(\{\lambda\}) = \Lambda(\lambda,\{\lambda\})\Phi(\{\lambda\})$$

- From here we obtain consistency conditions on the parameters $\{\lambda\}$.
- These consistency conditions are the Bethe Ansatz equations:

$$\boxed{\left(\frac{\lambda_i - \imath/2}{\lambda_i + \imath/2}\right)^L = \prod_{\substack{i,j=1\\ i\neq j}}^{N} \frac{\lambda_i - \lambda_j - \imath}{\lambda_i - \lambda_j + \imath}}$$

Mid-talk Summary

- We study a system of interacting spins on a chain of length L
- Each point carries the fundamental representation of $su(2)$. This is the XXX model
- To solve this model we use the Algebraic Bethe Ansatz
  - We construct a Lax operator (connection along the chain)
  - From the connection we build the monodromy and the transfer matrix
  - The commutation relations imply the existence of L integrals of motion. The system is integrable.
  - We construct a highest-weight representation for the algebra (fundamental commutation relations)
  - Consistency conditions for the representation lead to the Bethe Ansatz equations
  - The Bethe Ansatz equations have a potential, the Yang-Yang function.

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Outline

Basics of $\mathcal{N}=(2,2)$ field theories: field content

- Field theories in $1+1$ dimensions with two (real) positive and two (real) negative chirality supercharges.
- Superspace is described by the two bosonic coordinates $x^0,\ x^1$, and the four fermionic coordinates $\theta^{\pm}, \bar{\theta}^{\pm}$.
- Differential operators

$$D_{\pm} = \frac{\partial}{\partial \theta^{\pm}} - i\bar{\theta}^{\pm}\partial_{\pm}, \quad \bar{D}_{\pm} = -\frac{\partial}{\partial \bar{\theta}^{\pm}} + i\bar{\theta}^{\pm}\partial_{\pm}.$$

- The θ-expansion of the vector superfield in Wess--Zumino gauge is given by

$$\begin{aligned} V = {} & \theta^- \bar{\theta}^- (A_0 - A_1) + \theta^+ \bar{\theta}^+ (A_0 + A_1) - \theta^- \bar{\theta}^+ \sigma \\ & - \theta^+ \bar{\theta}^- \overline{\sigma} + i \theta^- \theta^+ (\bar{\theta}^- \overline{\lambda}_- + \bar{\theta}^+ \overline{\lambda}_+) \\ & + i \bar{\theta}^+ \bar{\theta}^- (\theta^- \lambda_- + \theta^+ \lambda_+) + \theta^- \theta^+ \bar{\theta}^+ \bar{\theta}^- D, \end{aligned}$$

Basics of $\mathcal{N} = (2,2)$ field theories: field content

- Define the gauge covariant derivative

$$\mathcal{D}_\pm = e^{-V} D_\pm e^{V}, \qquad \overline{\mathcal{D}}_\pm = e^{V} \overline{D}_\pm e^{-V}.$$

- A chiral superfield satisfies $\overline{\mathcal{D}}_\pm \Phi = 0$. The θ-expansion of the chiral superfield is given by

$$\Phi = \varphi(y^\pm) + \theta^\alpha \psi_\alpha(y^\pm) + \theta^+ \theta^- F(y^\pm),$$

- A twisted chiral superfield satisfies $\overline{\mathcal{D}}_+ \Sigma = \mathcal{D}_- \Sigma = 0$.
- The super field strength $\Sigma = \frac{1}{2}\{\overline{\mathcal{D}}_+, \mathcal{D}_-\}$ is a twisted chiral superfield and its θ-expansion is given by

$$\Sigma = \sigma(\tilde{y}^\pm) + i\theta^+ \overline{\lambda}_+(\tilde{y}^\pm) - i\bar{\theta}^- \lambda_-(\tilde{y}^\pm) + \theta^+ \bar{\theta}^- [D(\tilde{y}^\pm) - iA_{01}(\tilde{y}^\pm)],$$

Basics of $\mathcal{N}=(2,2)$ field theories: couplings

- the D-term: $\int d^2x\, d^4\theta\ K$, where K is an arbitrary (real) differential function of the superfields,
- the F-term (plus its Hermitian conjugate) : $\int d^2x\, d\theta^- d\theta^+\ W|_{\bar\theta_\pm=0} + \text{h.c.}$, where the superpotential W is a holomorphic function of the chiral multiplets,
- the twisted F-term (plus its Hermitian conjugate): $\int d^2x\, d\bar\theta^- d\theta^+\ \widetilde{W}\Big|_{\bar\theta_+=\theta^-=0} + \text{h.c.}$, where the twisted superpotential $\widetilde{W}$ is a holomorphic function of the twisted superfields.

Non-renormalization

The F-term and the twisted F-term do not get renormalized.

Decoupling

In the effective action, the F-term and twisted F-term cannot mix.

Basics of $\mathcal{N} = (2,2)$ field theories: action

- Gauge group G and chiral matter multiplets X_k
  - Q if they are in the fundamental,
  - $\overline{Q}$ if they are in the anti-fundamental,
  - B if they are in the bifundamental,
  - Φ if they are in the adjoint representation
- The kinetic term of the Lagrangian is

$$L_{\text{kin}} = \int d^4\theta \left(\sum_k X_k^\dagger e^V X_k - \frac{1}{2e^2} \operatorname{Tr}(\Sigma^\dagger \Sigma) \right),$$

additional terms:

- Fayet-Iliopoulos (FI) and theta-term:
  $L_{\text{FI},\vartheta} = -\frac{\imath}{2}\tau \int d\bar{\theta}^- d\theta^+ \operatorname{Tr}\Sigma + \text{h.c.},$
- The complex mass: $L_{\text{mass}} = \sum_{k,l} \int d^2\theta \, m_k{}^l \widetilde{X}_l X^k + \text{h.c.},$
- The twisted masses: $L_{\text{tw}} = \int d^4\theta \, (X^\dagger e^{\theta^- \bar{\theta}^+ \widetilde{m}_X + \text{h.c.}} X),$

Effective theory in the Coulomb branch

- Main objective: describe the Coulomb branch of the theory.
- Consider the low energy effective theory obtained for slowly varying σ fields after integrating out the massive matter fields.
- In this way, we obtain an effective twisted superpotential $\widetilde{W}_{\text{eff}}(\Sigma)$
- the vacua of the theory are the solutions of the equation

$$\exp\left[2\pi \frac{\partial \widetilde{W}_{\text{eff}}(\sigma)}{\partial \sigma_i}\right] = 1 .$$

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Explicit example: $U(1)$ gauge theory

- $U(1)$ gauge theory with one chiral superfield Q of charge 1 and twisted mass $\tilde{m}_Q$.
- By supersymmetry, the effective action must have the form

$$S_{\text{eff}}(\Sigma) = -\int d^4\theta \, K_{\text{eff}}(\Sigma, \overline{\Sigma}) + \frac{1}{2}\int d^2\theta \, \widetilde{W}_{\text{eff}}(\Sigma) + \text{h.c.}\,.$$

- In the absence of an F-term, the action $S(\Sigma, Q)$ is quadratic in Q, and the effective action can be evaluated exactly via a one-loop calculation:

$$e^{\imath S_{\text{eff}}(\Sigma)} = \int \mathcal{D}Q \, e^{\imath S(\Sigma, Q)}\,.$$

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Explicit example: $U(1)$ gauge theory

- We want to calculate

$$e^{\imath S_{\text{eff}}(\Sigma)} = \int \mathcal{D}Q \, e^{\imath S(\Sigma, Q)} .$$

- The bosonic determinant equals

$$\int \frac{d^2 k}{(2\pi)^2} \log(k^2 + |\sigma - \tilde{m}_Q|^2 + D) ,$$

- expanding in powers of D,

$$\log(k^2 + |\sigma - \tilde{m}_Q|^2 + D) = \log(k^2 + |\sigma - \tilde{m}_Q|^2) + \frac{D}{k^2 + |\sigma - \tilde{m}_Q|^2} + \dots$$

- The zeroth order term is cancelled by the fermionic determinant

Explicit example: $U(1)$ gauge theory

- after integrating over the momenta we obtain the effective twisted superpotential

$$\widetilde{W}_{\text{eff}}(\Sigma) = \frac{1}{2\pi}(\Sigma - \widetilde{m}_Q)\left(\log(\Sigma - \widetilde{m}_Q) - 1\right) - \imath\tau\,\Sigma\,,$$

we also added a Fayet-Iliopoulos term.

- In the general case, an F-term is possible but, thanks to the decoupling theorem, it would not change the expression of the effective twisted superpotential.

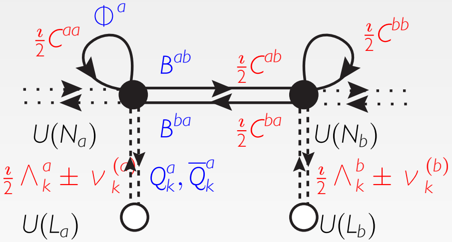
Quiver gauge theories

- We consider quiver gauge theories.
- The gauge group G and the flavor group F are direct products:

$$G = \prod_{a=1}^{r} U(N_a) , \qquad F = \prod_{a=1}^{r} U(L_a) .$$

- These theories can be represented via quiver diagrams.
  - Each factor $U(N_a)$ corresponds to a node
  - A bifundamental field in the representation $\overline{N_a} \otimes N_b$ is denoted by an arrow going from node a to node b
  - An adjoint field is an arrow starting and ending on the same node.
  - Each component $U(L_a)$ of the flavor group is represented by an extra node, joined by a dotted arrow to the relevant component of the gauge group

Quiver gauge theories



Quiver gauge theories: effective action

Contributions to the effective twisted superpotential:

- For each fundamental field Q_k with twisted mass $\widetilde{m}_k^{\mathrm{f}}$:

$$\widetilde{W}_{\mathrm{eff}}^{\mathrm{f}} = \frac{1}{2\pi} \sum_{i=1}^{N} \left(\sigma_i - \widetilde{m}_k^{\mathrm{f}} \right) \left(\log(\sigma_i - \widetilde{m}_k^{\mathrm{f}}) - 1 \right) .$$

- For each anti-fundamental field $\overline{Q}_k$ with twisted mass $\widetilde{m}_k^{\bar{\mathrm{f}}}$:

$$\widetilde{W}_{\mathrm{eff}}^{\bar{\mathrm{f}}} = \frac{1}{2\pi} \sum_{i=1}^{N} \left(-\sigma_i - \widetilde{m}_k^{\bar{\mathrm{f}}} \right) \left(\log(-\sigma_i - \widetilde{m}_k^{\bar{\mathrm{f}}}) - 1 \right) .$$

Quiver gauge theories: effective action

- For each adjoint field Φ with twisted mass $\widetilde{m}^{\text{adj}}$:

$$\widetilde{W}_{\text{eff}}^{\text{adj}} = \frac{1}{2\pi} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} \left(\sigma_i - \sigma_j - \widetilde{m}^{\text{adj}}\right) \left(\log(\sigma_i - \sigma_j - \widetilde{m}^{\text{adj}}) - 1\right).$$

- For each bifundamental field B^{12} in the representation $\overline{N_1} \otimes N_2$ and twisted mass $\widetilde{m}^{\text{b}}$:

$$\widetilde{W}_{\text{eff}}^{\text{b}} = \frac{1}{2\pi} \sum_{i=1}^{N_1} \sum_{p=1}^{N_2} \left(-\sigma_i^{(1)} + \sigma_p^{(2)} - \widetilde{m}^{\text{b}}\right) \left(\log(-\sigma_i^{(1)} + \sigma_p^{(2)} - \widetilde{m}^{\text{b}}) - \right.$$

where the $\sigma_i^{(a)}$ are the scalar components of the vector multiplet for the group $U(N_a)$.

Special example

- Consider a $U(N)$ gauge theory with flavor $U(L)$.
- Using the rules above, the effective twisted superpotential reads:

$$\begin{aligned}\widetilde{W}_{\mathrm{eff}}(\sigma) = {} & \frac{L}{2\pi}\sum_{i=1}^{N}\left(\sigma_i - \tilde{m}^{\mathrm{f}}\right)\left(\log(\sigma_i - \tilde{m}^{\mathrm{f}}) - 1\right) \\ & + \frac{L}{2\pi}\sum_{i=1}^{N}\left(-\sigma_i - \tilde{m}^{\bar{\mathrm{f}}}\right)\left(\log(-\sigma_i - \tilde{m}^{\bar{\mathrm{f}}}) - 1\right) \\ & + \frac{1}{2\pi}\sum_{\substack{i,j=1 \\ i\neq j}}^{N}\left(\sigma_i - \sigma_j - \tilde{m}^{\mathrm{adj}}\right)\left(\log(\sigma_i - \sigma_j - \tilde{m}^{\mathrm{adj}}) - 1\right)\end{aligned}$$

Special example

- The vacua of the theory are obtained from

$$\exp\left[2\pi\frac{\partial\widetilde{W}_{\text{eff}}(\sigma)}{\partial\sigma_i}\right]=1\,.$$

- Explicitly

$$\boxed{\left(\frac{\sigma_i-\widetilde{m}^{\text{f}}}{\sigma_i+\widetilde{m}^{\bar{\text{f}}}}\right)^L=\prod_{\substack{i,j=1\\ i\neq j}}^{N}\frac{\sigma_i-\sigma_j-\widetilde{m}^{\text{adj}}}{\sigma_i-\sigma_j+\widetilde{m}^{\text{adj}}}}$$

This is precisely the same equation we found before for the XXX spin chain

$$\widetilde{m}^{\text{f}}=\frac{\imath}{2}\qquad\widetilde{m}^{\bar{\text{f}}}=\frac{\imath}{2}\qquad\widetilde{m}^{\text{adj}}=\imath$$

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The dictionary

gauge theory			integrable model
number of nodes in the quiver	r	r	rank of the symmetry group
gauge group at a-th node	$U(N_a)$	N_a	number of particles of species a
effective twisted superpotential	$\widetilde{W}_{\text{eff}}(\sigma)$	$Y(\lambda)$	Yang-Yang function
equation for the vacua	$e^{2\pi\, d\widetilde{W}_{\text{eff}}} = 1$	$e^{2\pi \imath\, dY} = 1$	Bethe ansatz equation
flavor group at node a	$U(L_a)$	L_a	effective length for the species a
lowest component of the twisted chiral superfield	$\sigma_i^{(a)}$	$\lambda_i^{(a)}$	rapidity
twisted mass of the fundamental field	$\widetilde{m}_k^{f(a)}$	$\frac{\imath}{2}\Lambda_k^a + \nu_k^{(a)}$	highest weight of the representation and inhomogeneity
twisted mass of the adjoint	$\widetilde{m}^{\text{adj}(a)}$	$\frac{\imath}{2}C^{aa}$	diagonal of the Cartan matrix
twisted mass of the bifundamental field	$\widetilde{m}^{b(ab)}$	$\frac{\imath}{2}C^{ab}$	non-diagonal of the Cartan matrix
FI-term for $U(N_a)$	τ_a	$\hat{\vartheta}^a$	boundary twist parameter

To be continued . . .

Thank you for your attention