

KK DM

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- Our universe may be **supersymmetric** and **higher dimensional** at some fundamental level.

- Low energy physics in the measurable range depends on M_{susy} and M_{comp} .

- In this talk, I am going to focus on the case when $M_{\text{comp}} \ll M_{\text{susy}}$ so that we may see extra dimensions before observing SUSY particles.



A Hint?

Pamela & Fermi

A. Ibarra's talk

- TeV scale DM, which mainly **annihilates / decays into leptons** ($2\mu, 4\mu, 4e \dots$), can explain Pamela & Fermi 'anomalies' (with a largish BF maybe due to local dense clumps or other physical properties of DM).
- KK DM is naturally in TeV scales fitting the relic abundance and **preferably annihilates into leptons**. Obvious advantages over conventional DM candidates.

Today's focus



KK DM ($B_1, h_1, n_1, G_1, \dots$)
in UED and Split-UED.

Today's focus



KK DM ($B_1, h_1, n_1, G_1, \dots$)
in UED and Split-UED.

cf) In MSSM $\tilde{B}, \tilde{h}^0, \tilde{\nu}, \tilde{G}$

Contents

- A brief review of **UED model building**
- **KK-parity**
- An extension of UED, **split-UED**, very very brief sketch of phenomenology.
- Very very very brief review of flavor hierarchy problem in UED.



Let's suppose we are living
in higher dimensions

A simple model extra dimension

S^1



A simple model extra dimension

S^1



a trivial sol. to Einstein's eq.

A simple model extra dimension

S^1



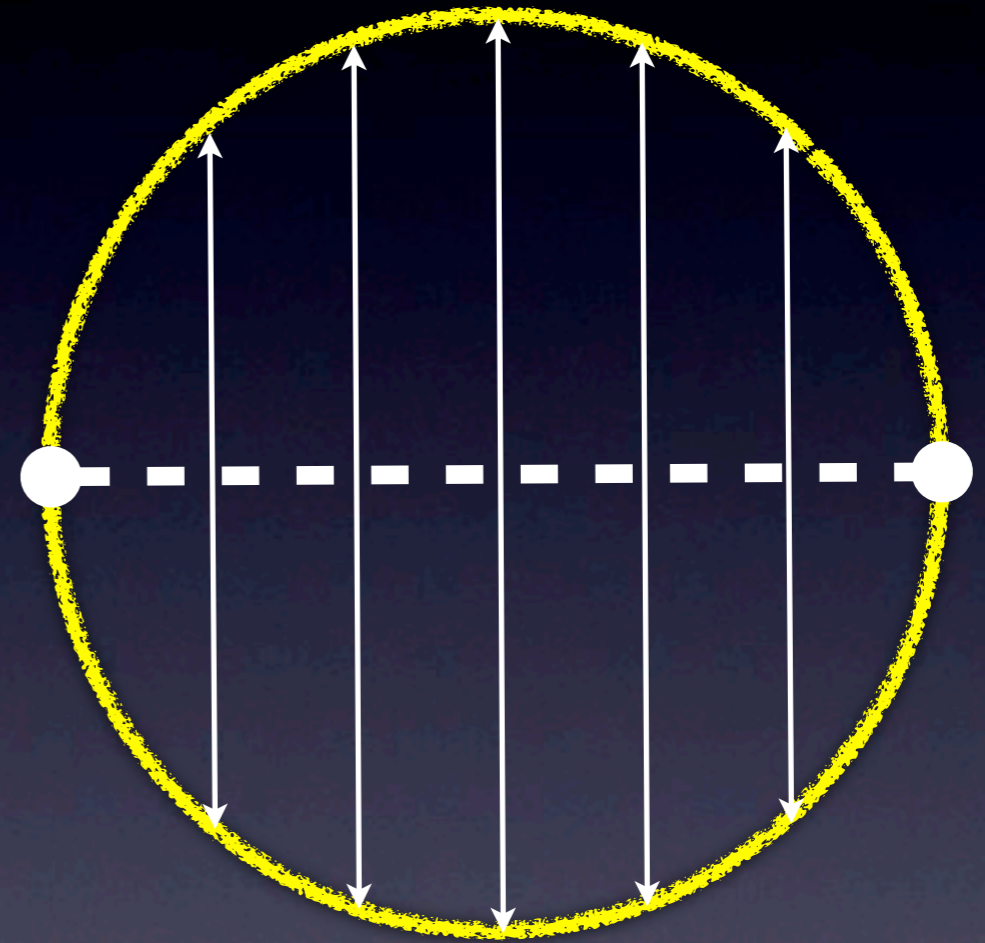
a trivial sol. to Einstein's eq.

Bad! Field theory with fermion fields
becomes **vectorlike** (L-R symmetric)
unlike the real world. :-)

“Orbifold”

$$S^1 / Z_2$$

Identify the opposite points:
Two “fixed points” appear.

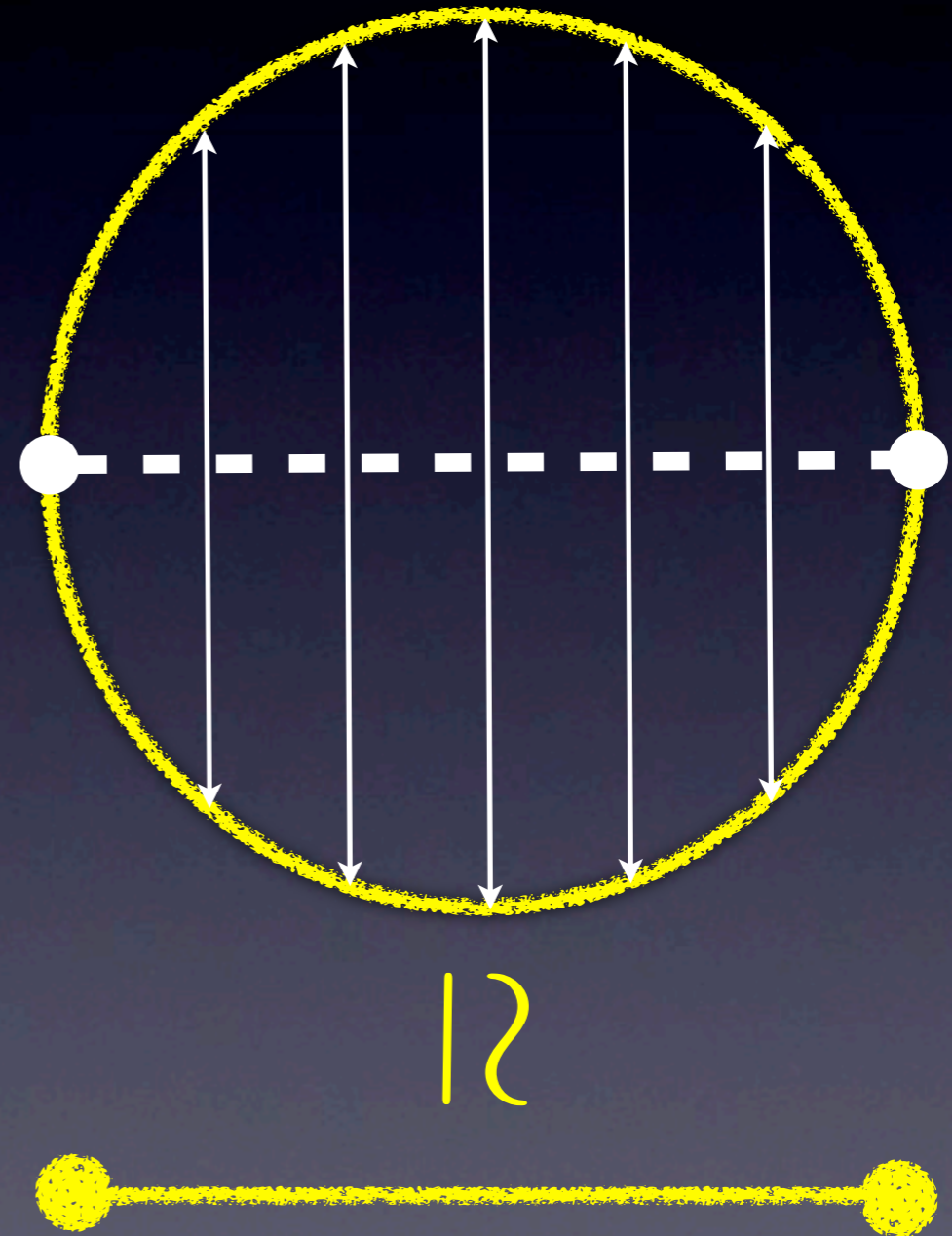


“Orbifold”

$$S^1 / \mathbb{Z}_2$$

Identify the opposite points:
Two “fixed points” appear.

Physical domain=Half Circle
 \cong an interval with “two
boundaries”.

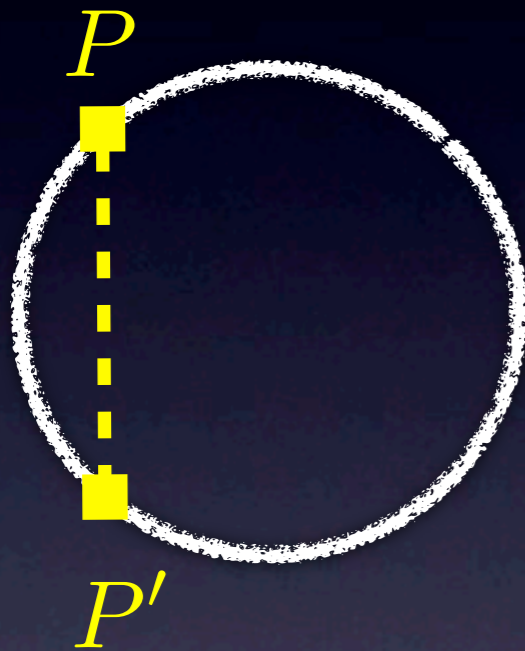


Fermion on orbifold



$$\Psi(P) \rightarrow \Psi(P')$$

Fermion on orbifold



$$\Psi(P) \rightarrow \Psi(P')$$

5D Lorentz symmetry dictates:

$$\Psi(P') = \pm \gamma_5 \Psi(P)$$

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or

$$\Psi_L(P) + \Psi_R(P) \rightarrow \pm(-\Psi_L(P) + \Psi_R(P))$$

Fermion on orbifold



$$\Psi(P) \rightarrow \Psi(P')$$

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L-chiral state has the opposite Z_2 parity to R-chiral state and vice versa.

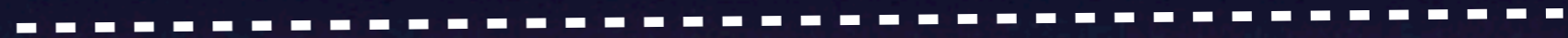
$$L \neq R$$

\Rightarrow Theory becomes chiral! :-)

Fermion on interval



periodic



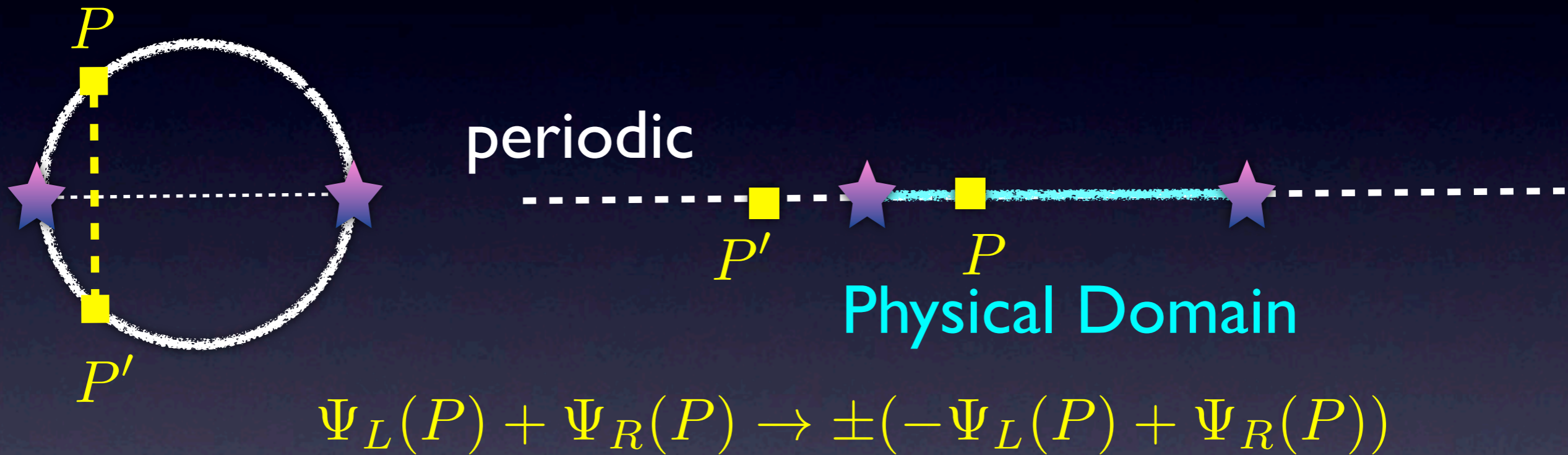
Fermion on interval



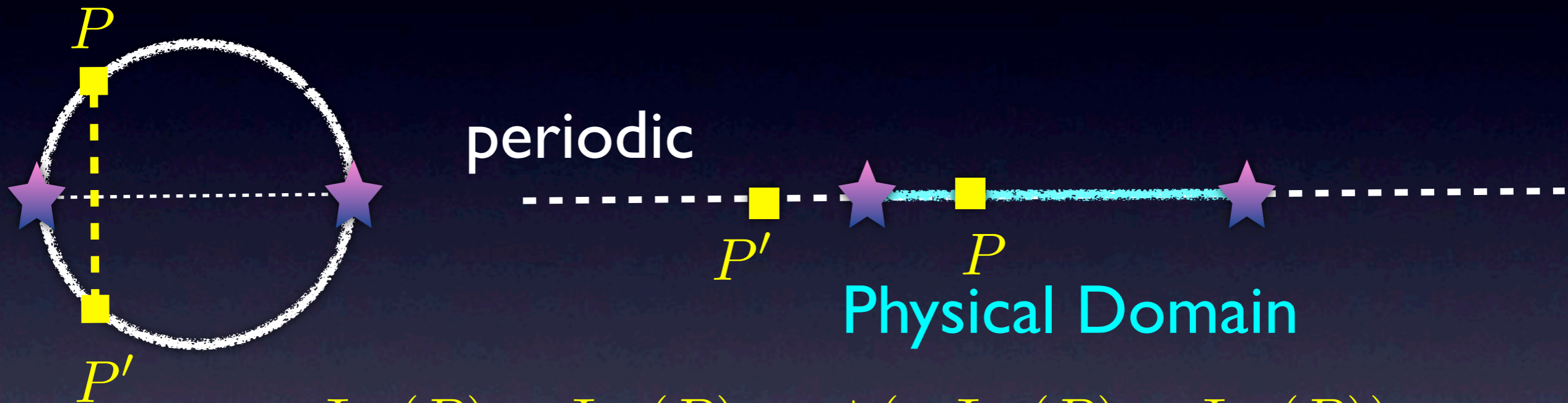
Fermion on interval



Fermion on interval



Fermion on interval



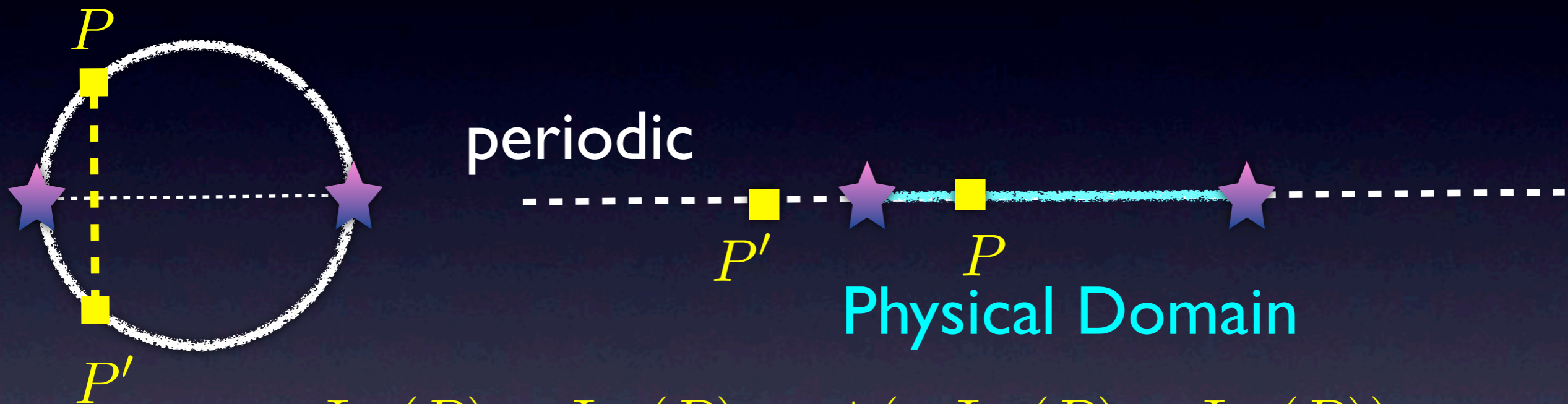
$$\Psi_L(P) + \Psi_R(P) \rightarrow \pm(-\Psi_L(P) + \Psi_R(P))$$

Either L-state or R-state is Odd at the fixed point.

\Rightarrow Satisfying Dirichlet BC at the end points.

(No massless zero mode)

Fermion on interval



$$\Psi_L(P) + \Psi_R(P) \rightarrow \pm(-\Psi_L(P) + \Psi_R(P))$$

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(No massless zero mode)

Lesson: By imposing proper BC (or orbifold transformation rule), one can project out a half of (unnecessary) chiral state.

UED

Appelquist, Cheng, Dobrescu, 2001

Model spacetime: $M^4 \times S^1/Z_2$

gauge symmetry: $G = SU(3)_c \times SU(2)_W \times U(1)_Y$

matter fields (5D): $\Psi(x^\mu, y) = (Q, U^c, D^c, L, E^c)$

$$\Psi(x^\mu, y) = \sum_n \Psi_L^n(x) f_L^n(y) + \Psi_R^n f_R^n(y) \quad \begin{array}{c} \text{Orbifold projection} \\ \downarrow \\ S^1/Z_2 \end{array}$$

$$\Psi^0(x^\mu, y) = (Q_L, U_L^c, D_L^c, L_L, E_L^c)$$

$$V_\mu^0 = (G_\mu^a, W_\mu^\alpha, B_\mu)$$

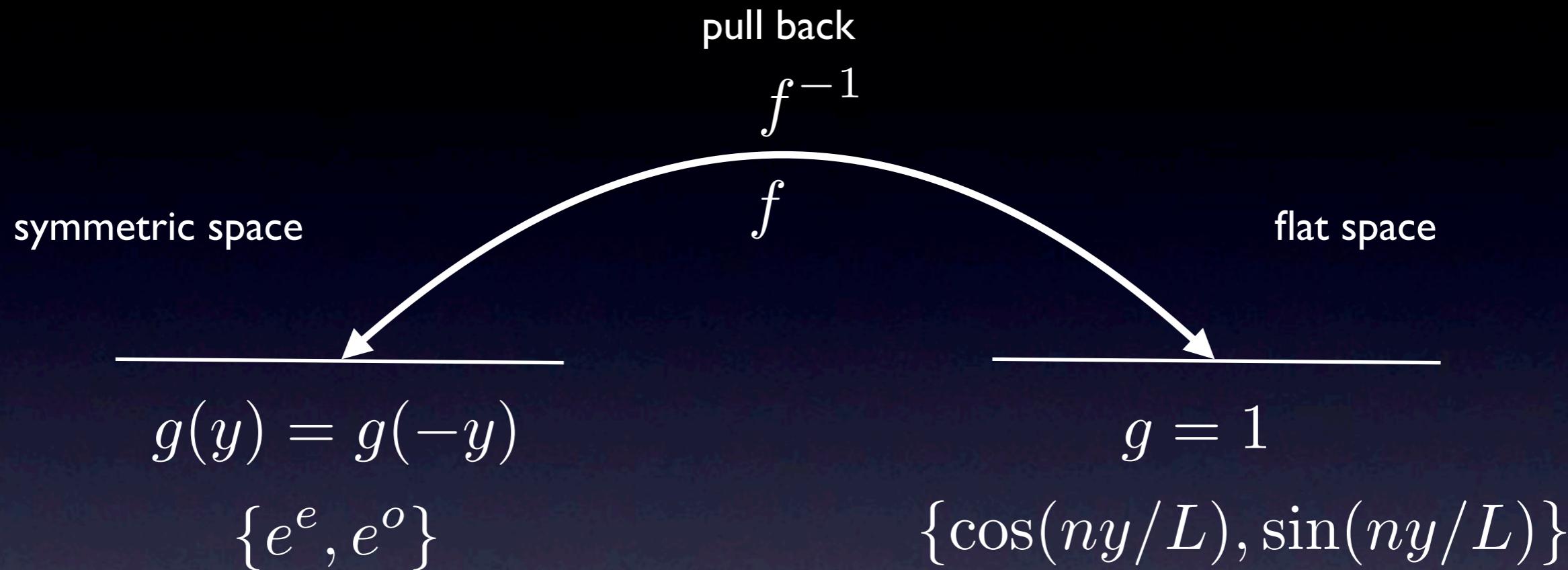
zero-modes = particles in the SM
+ KK excitations

KK-parity



- **Z_2 reflection** about the middle point of extra dimension.
- A remnant symmetry of 5D translational invariance, which is broken by end points (or fixed points in orbifold language).
- It is often claimed that KK-parity requires flat geometry. But ..

Indeed, KK-parity can be defined on any **symmetric space**.



$$P_{KK} : \begin{cases} e^e \rightarrow +e^e, & \text{even} \\ e^o \rightarrow -e^o, & \text{odd} \end{cases} \quad : \quad \begin{cases} \cos \rightarrow +\cos, & \text{even} \\ \sin \rightarrow -\sin, & \text{odd} \end{cases}$$

reflection
about the middle point

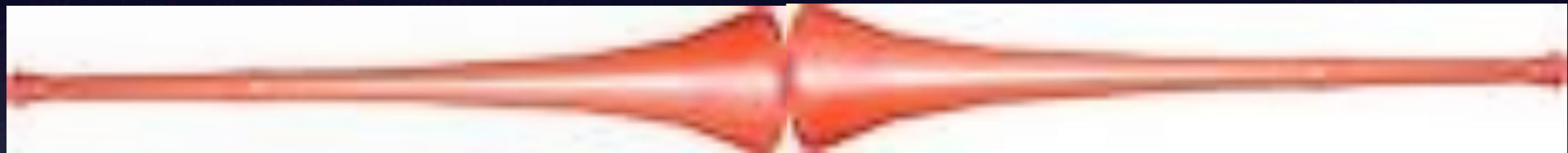
for a massive field

$$\{\cosh(ky), \sinh(ky)\}$$

Examples

Examples

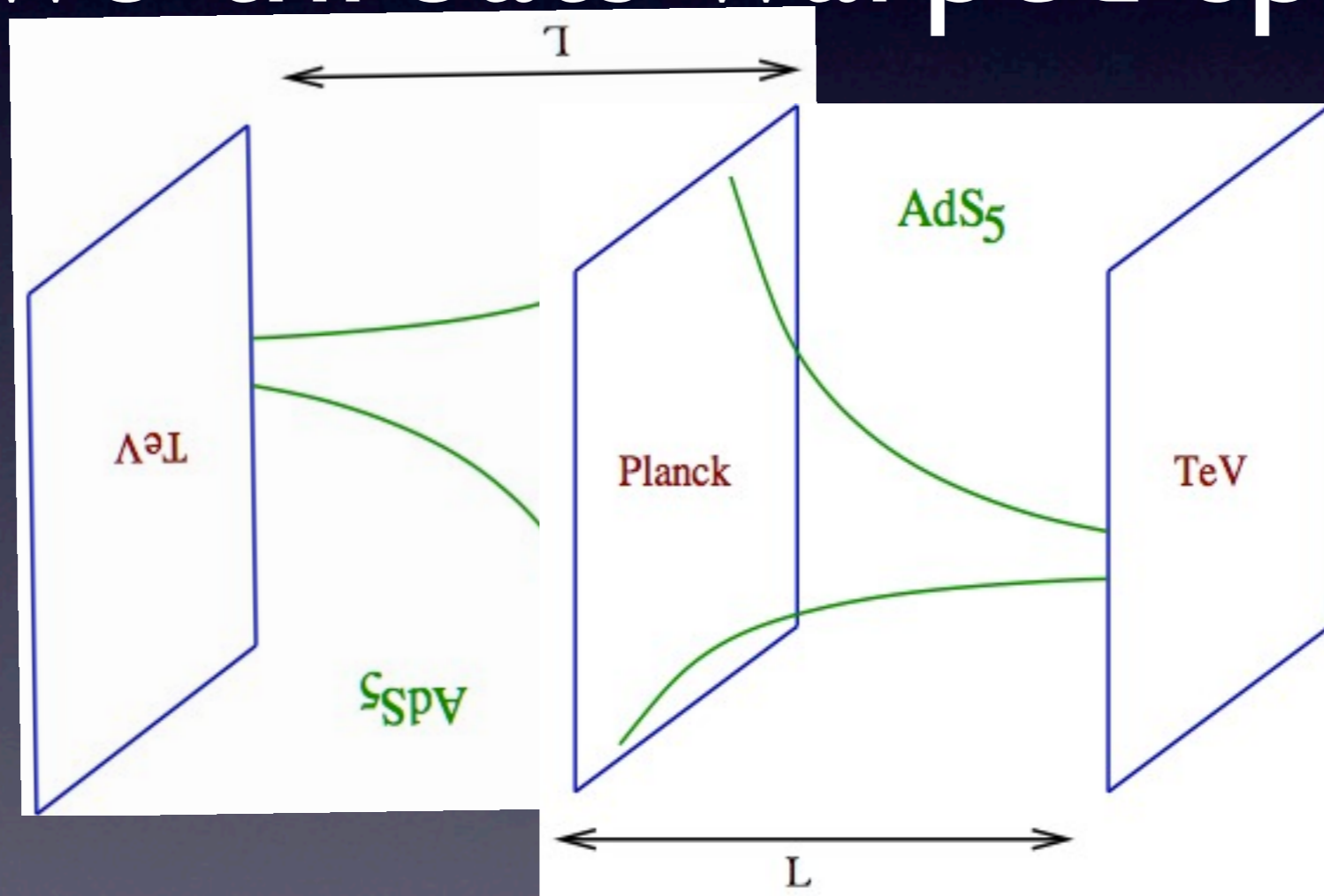
Vuvuzela space



Two Vuvuzelas glued together, FIFA2010

Examples

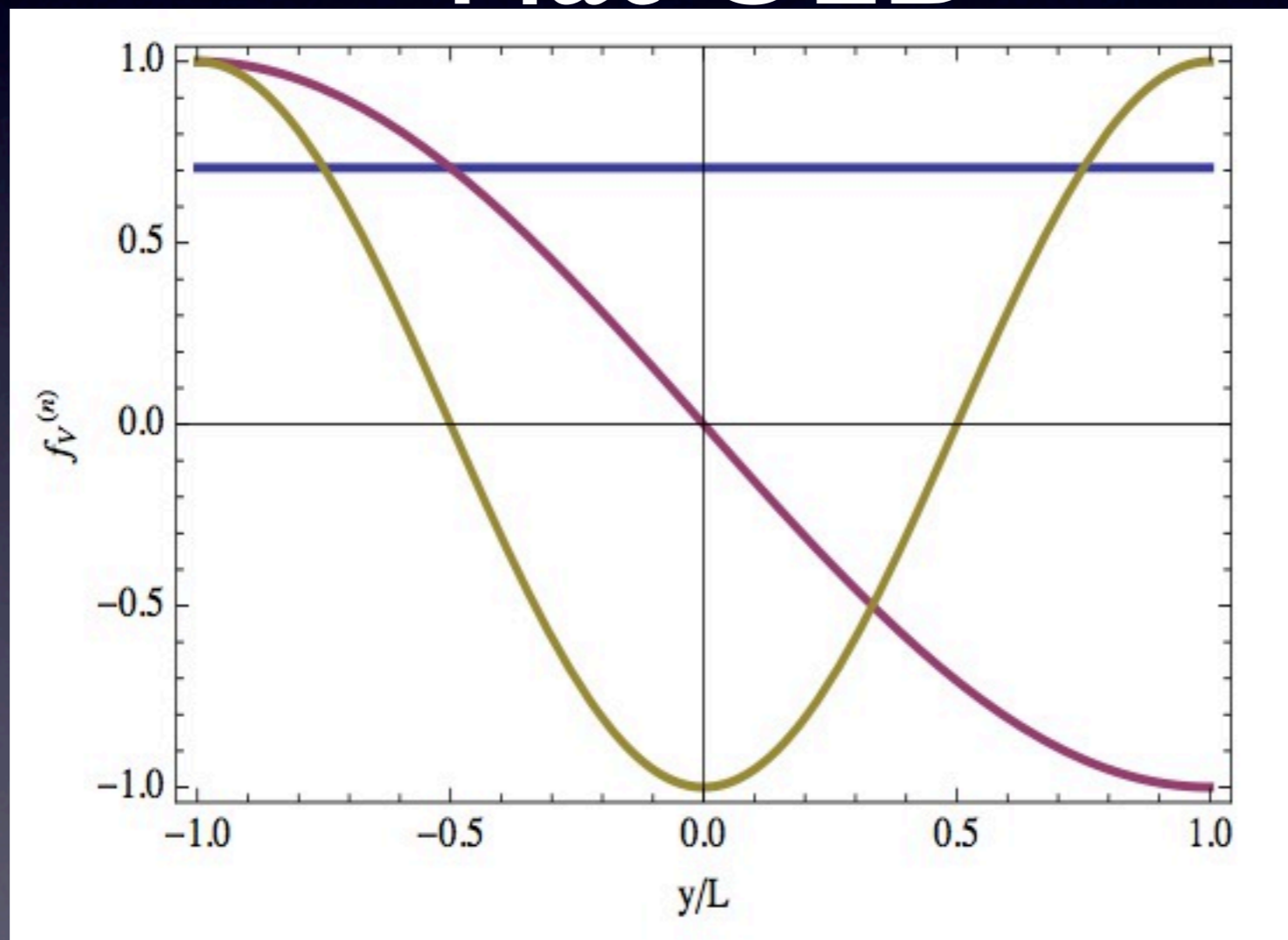
Two throats warped space



Agashe, Falkowski, Low, Servant (2008)

Examples

Flat UED



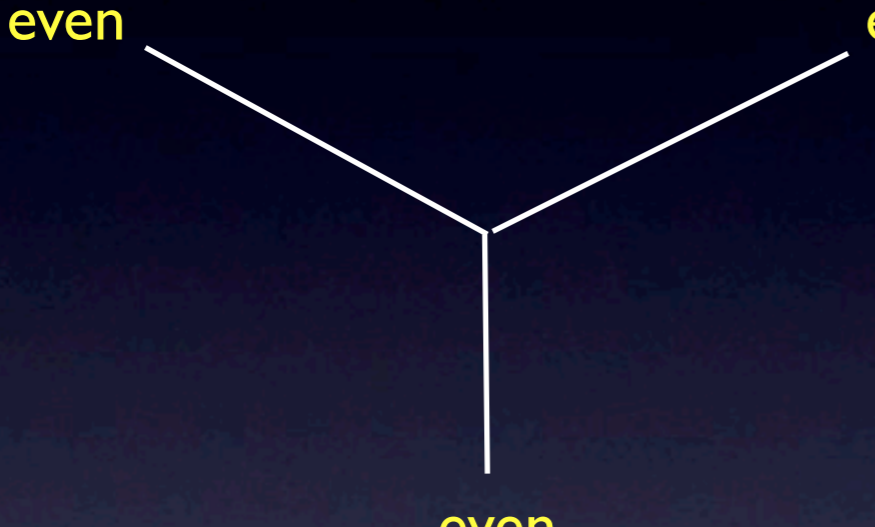
$n=0, 2$: even

$n=1$: odd

$$P_{kk} = (-1)^n$$

Interaction allowed/forbidden by KK-parity

even even




even

$$g \propto \int_{-L}^L dy \psi_{\text{even}} \psi_{\text{even}} \psi_{\text{even}} \neq 0$$

Allowed

odd even



even

$$g \propto \int_{-L}^L dy \psi_{\text{odd}} \psi_{\text{even}} \psi_{\text{even}} = 0$$

Forbidden

The lightest KK-odd particle cannot decay.
If it is neutral, it can be a DM candidate.

Underlying Math:

An odd function cannot be decomposed into finite number of even functions

The lightest KK-odd particle cannot decay.
If it is neutral, it can be a DM candidate.

UED :-)

UED is found attractive because

- The lightest KK odd particle (LKP, often the 1st KK partner of $U(1)_Y$ gauge boson) is a **perfect DM candidate**.
- Minimal UED has 2 new parameters (R, Λ), which allows for easy scanning of parameters space.
- Constraints from electroweak precision test **allow for new physics at a few hundred GeV**, which leads to a reach LHC phenomenology.
- An interesting **“bench mark” model** from the perspective of model discrimination at the LHC.

UED :-)

Troubles with UED

$$\frac{g_5^2 \Lambda}{24\pi^3} = \frac{g_4^2 R \Lambda}{24\pi^2} \sim 1$$
$$R \Lambda \sim \frac{24\pi^2}{g_4^2} \lesssim \frac{240}{g_4^2}$$

- Hierarchy problem not addressed. Indeed worse than the SM.
- Have flat profiles in bulk, so no flavor hierarchies understood.

What's the use of a model which does not address any problem?
-anonymous

Let's extend UED so that
UED becomes more
interesting and useful.

Two Extensions of UED

without introducing additional field contents

- Brane localized terms (Dim=5, 6) Carena, Tait, Wagner (2002)
- Bulk mass for fermion (Dim=4) SCP, Shu (2009) “split UED”

Bulk Mass

- $M_{\text{gauge}}=0$: gauge symmetry.
- $M_{\text{fermi}}\neq 0$: Dirac mass term in 5D is compatible with Lorentz symmetry and gauge symmetry.
- Q. Is M_5 compatible with KK-parity as well?

Quiz) KK-parity forbids 5D fermion mass.
Yes or No?

Quiz) KK-parity forbids 5D fermion mass.
Yes or No?



and

The word "NO" is rendered in large, bold, 3D red letters. The letters are thick and have a slight shadow underneath, giving them a three-dimensional appearance. They are set against a plain white background.

KK-parity {

- forbids KK-even mass
- allows KK-odd mass

because Dirac Bilinear is odd under the reflection

$$y \rightarrow -y$$

$$\Psi(x^\mu, y) \rightarrow \pm \gamma_5 \Psi(x^\mu, y)$$

$$\begin{aligned} \bar{\Psi} \Psi &\rightarrow (\gamma_5 \Psi)^\dagger \gamma^0 (\gamma_5 \Psi) \\ &= \Psi^\dagger \gamma_5 \gamma^0 \gamma_5 \Psi \\ &= -\Psi^\dagger \gamma^0 \Psi \\ &= -\bar{\Psi} \Psi \end{aligned}$$

$$m_5(y) \rightarrow m_5(-y) = -m_5(y)$$

$m_5 \bar{\Psi} \Psi$ is invariant

$$(\partial_y \pm m) f_{R/L}^{(0)} = 0$$

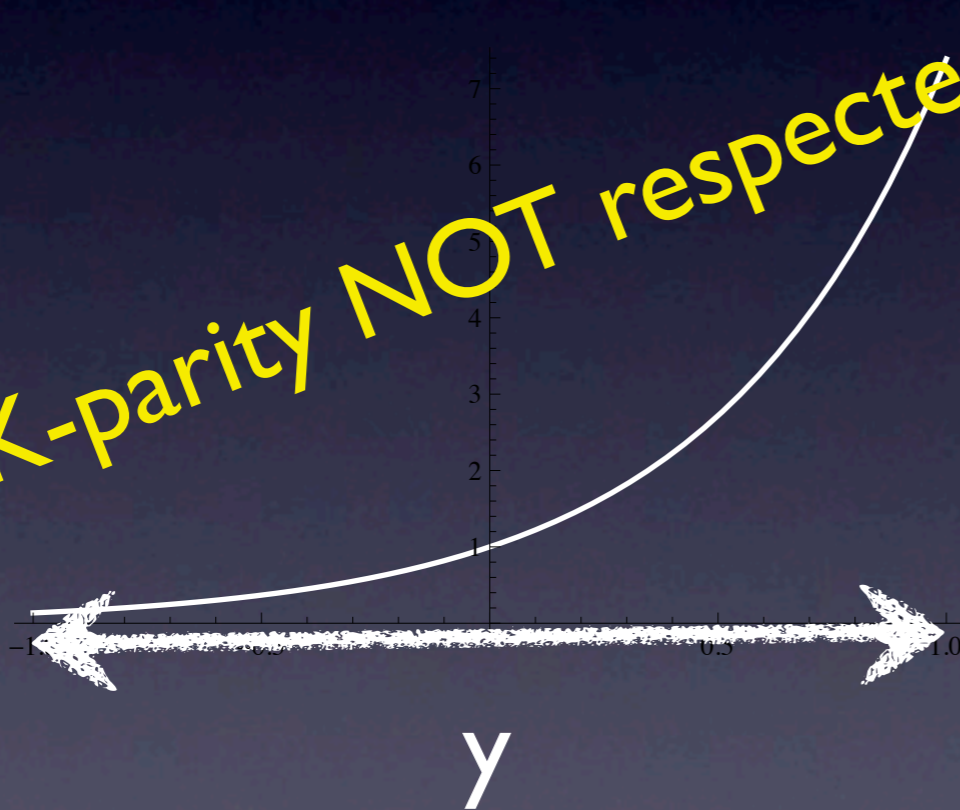
with “even mass”

$$m(-y) = m(y)$$

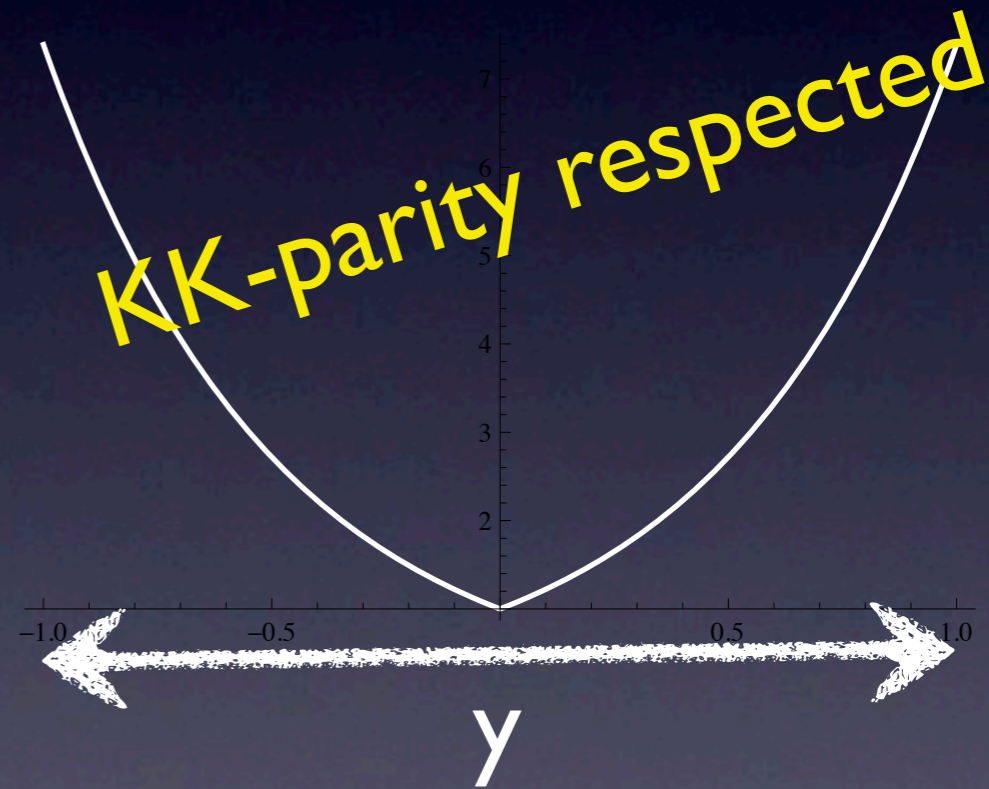
with “odd mass”

$$m(-y) = -m(y)$$

KK-parity NOT respected



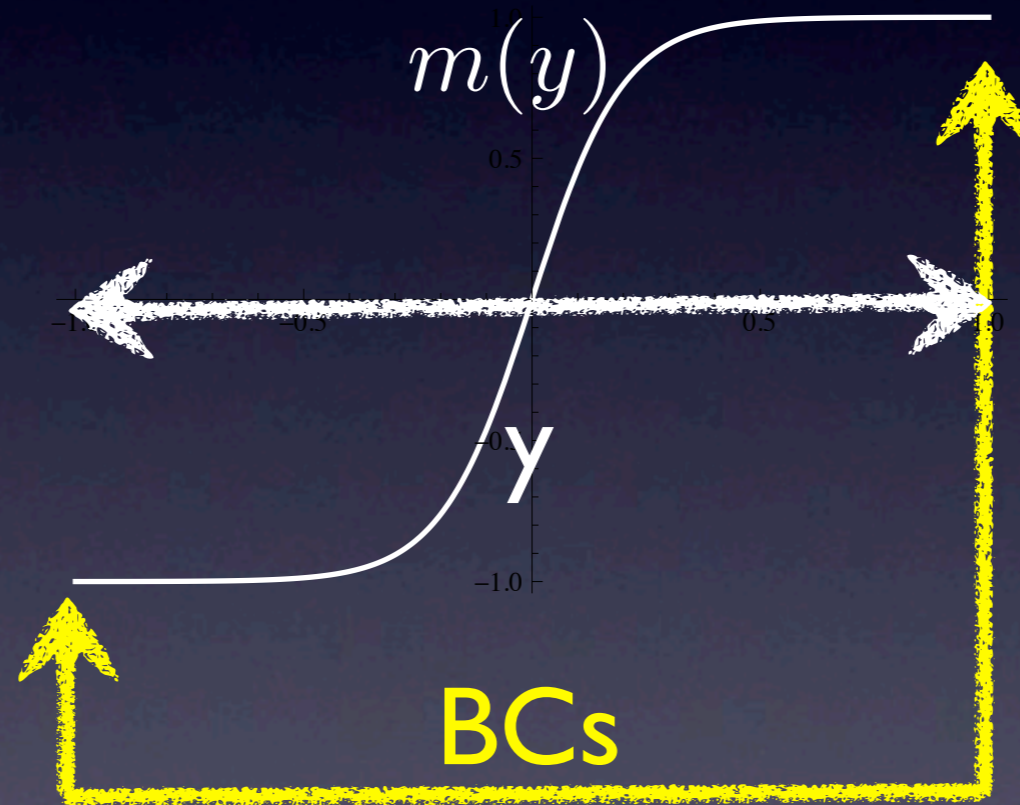
KK-parity respected



Odd mass on orbifold

Georgi, Grant, Hailu (2001)

$$M_5(y) \rightarrow M_5(-y) = -M_5(y)$$



The lowest energy configuration
interpolating boundary values: +M, -M

$$M \tanh \mu y \rightarrow M \theta(y)$$

Split UED

SCP, Shu, 2009

$$\Delta S = - \int d^5 x \mu \theta(y) \bar{\psi} \psi$$

- With the odd bulk mass, chiral zero mode remains massless but the profile of zero mode is exponentially localized.

$$f_{R/L}^{(0)} = \sqrt{\frac{\pm \mu}{1 - e^{\mp 2\mu L}}} e^{\mp \mu y}$$

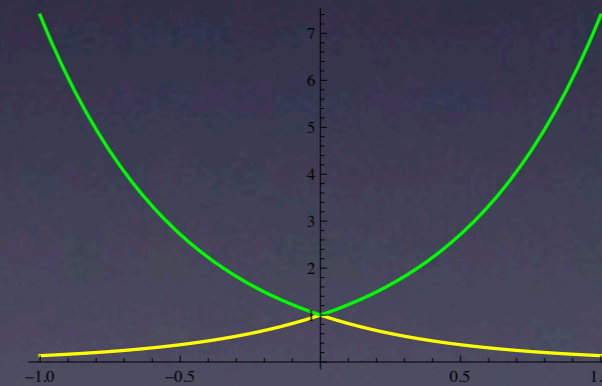
- KK-masses are deformed

$$m_n = \sqrt{\mu^2 + k_n^2}$$

$$n = 2, 4, \dots : k_n = \frac{n\pi}{L}$$

$$n = (1,)3, \dots : k_n = \mp \mu \tan k_n L$$

for DL/DR or RH/LH zero mode



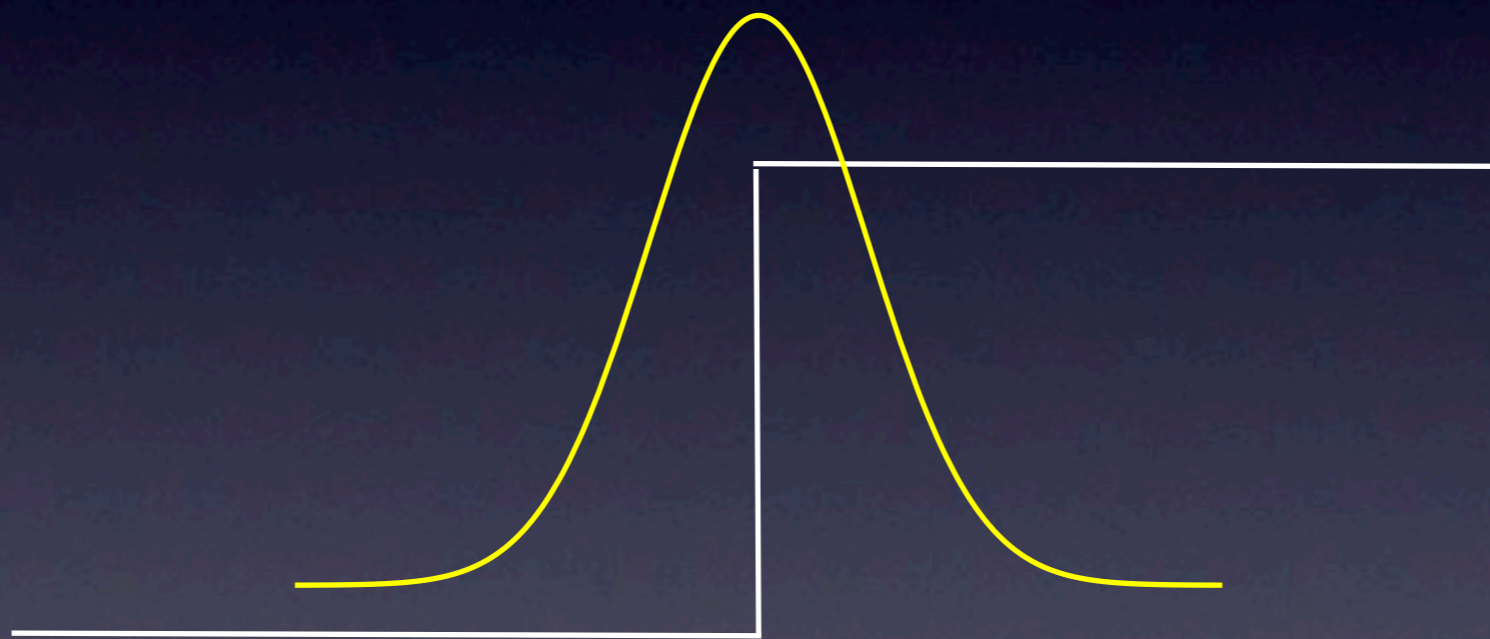
- There can exist a new ultralight mode.

reminder

Domain wall fermion

$$[-\infty, +\infty]$$

- A 'trapped fermion' exists in the presence of domain wall in infinite extra dimension. It is chiral (=massless).



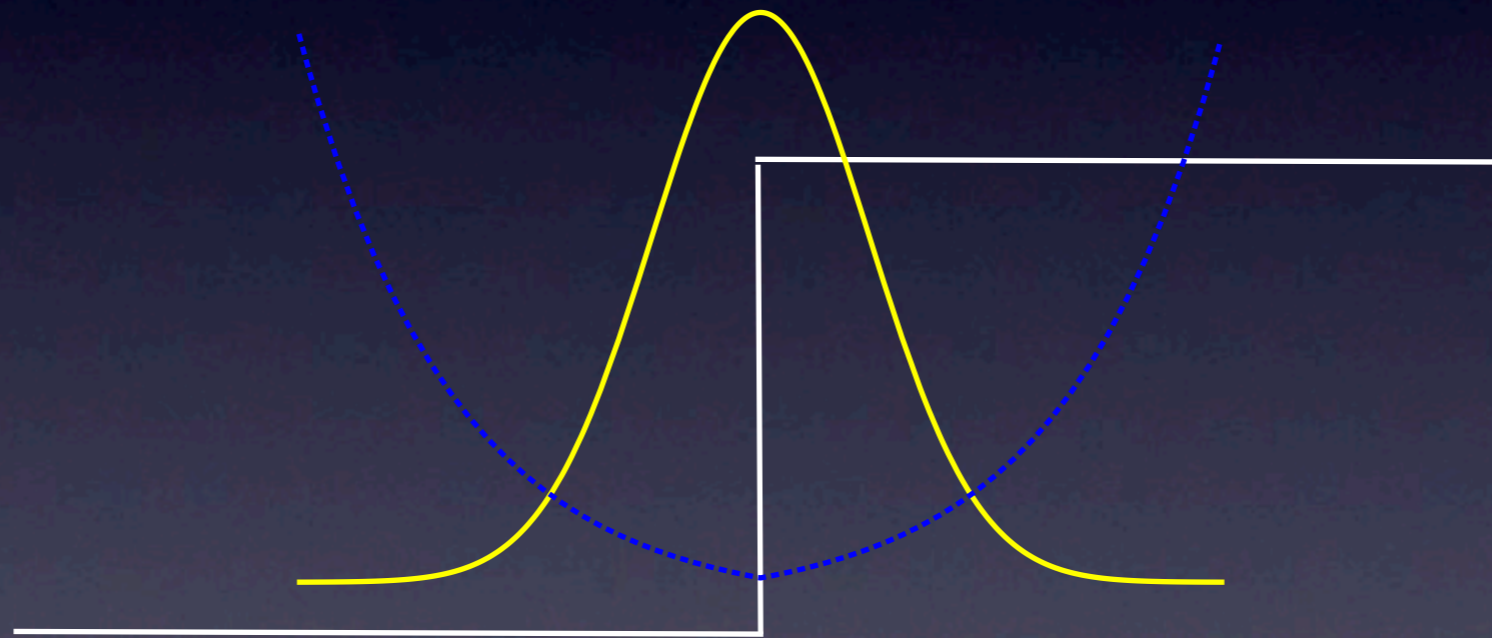
- The other chiral state is exponentially diverging (**non-normalizable**), which is not physical mode.

Only domain wall fermion is physical.

Domain wall fermion

$[-L, +L]$

- A 'trapped fermion' still exists in the presence of domain wall in finite extra dimension. It is chiral (=massless).



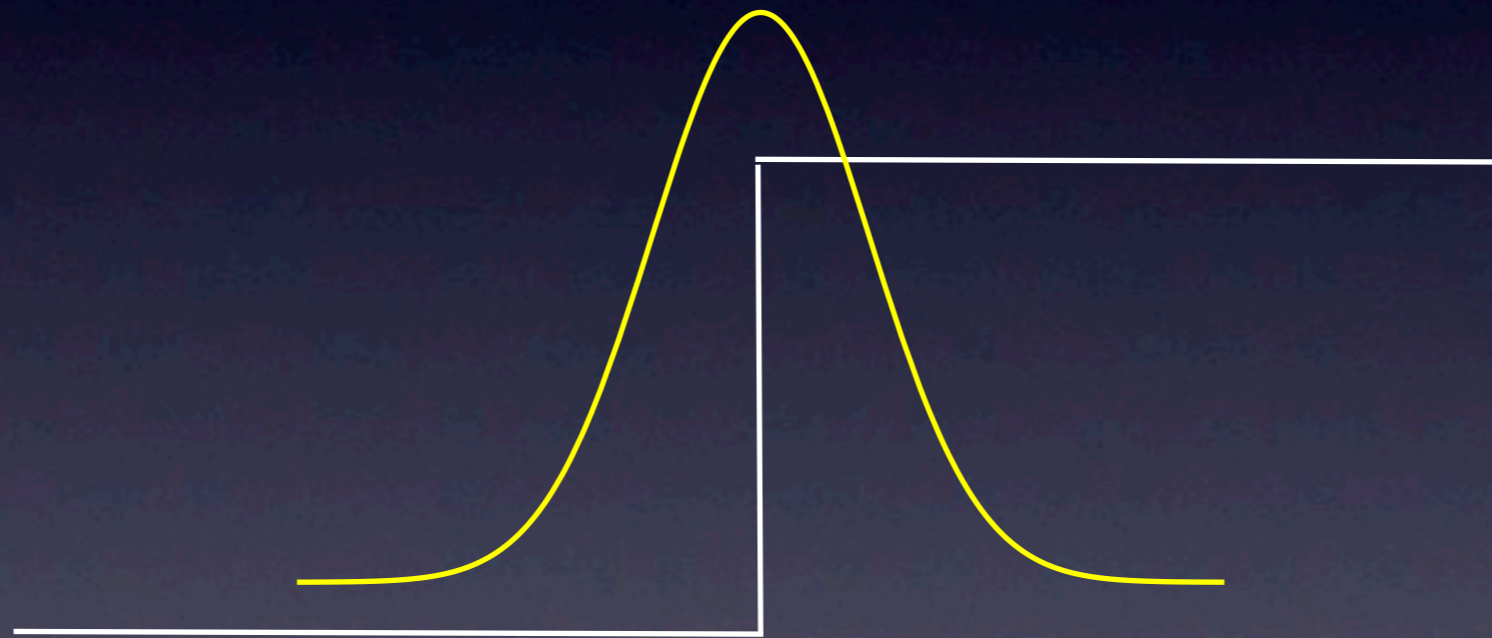
- The other chiral state is exponentially growing but **normalizable** since the extra dimension is finite. **This mode is also physical.**

Both [can be] physical

Domain wall fermion

$$[-L, +L] + BC-i$$

(i) Dirichlet BC for growing mode. \Rightarrow Domain wall fermion is physical zero mode

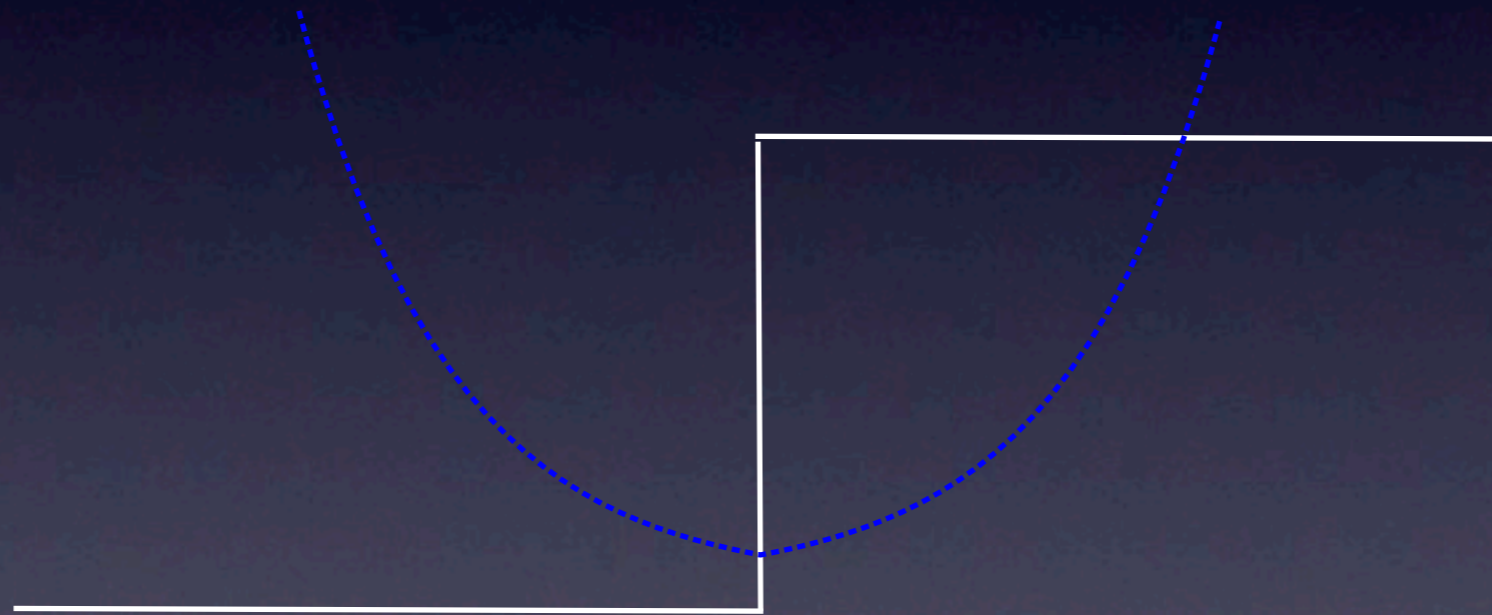


This case is totally OK as the domain wall fermion is a natural chiral zero mode

Domain wall fermion

$[-L, +L] + \text{BC-ii}$

(ii) Dirichlet BC for Domain wall mode. \Rightarrow Growing mode is physical zero mode

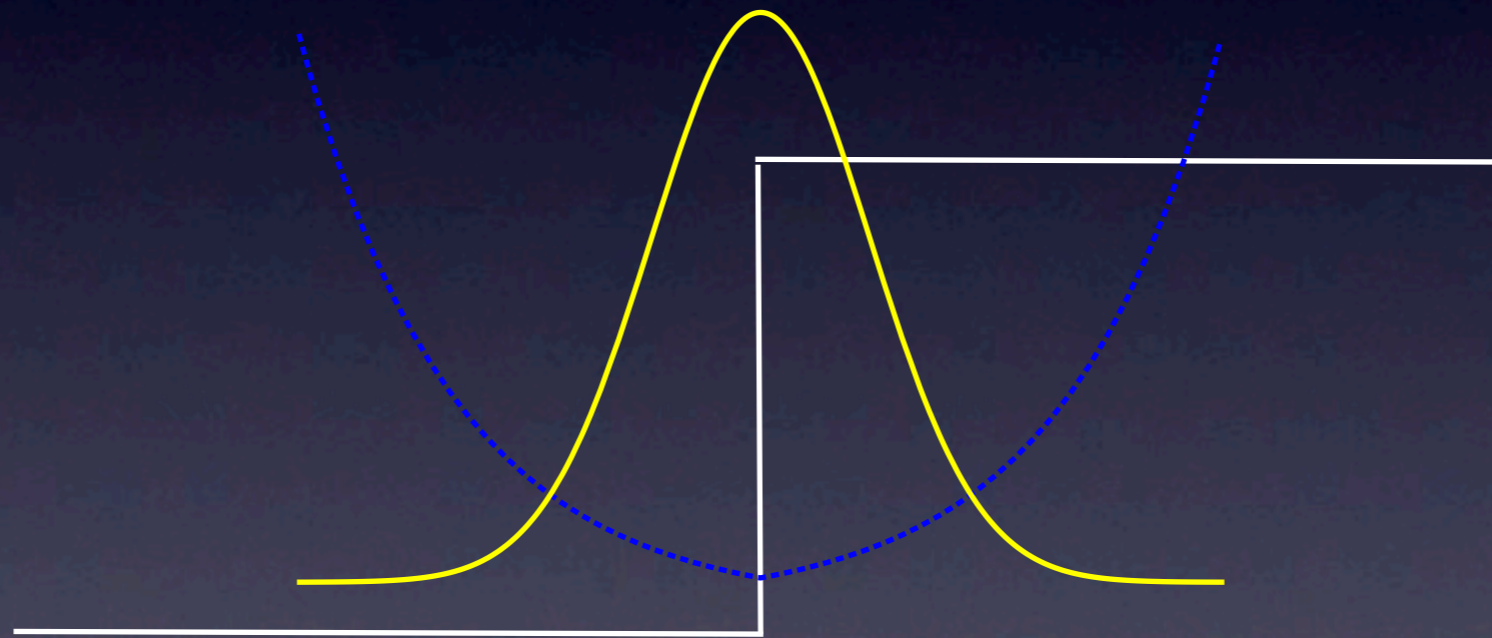


$$m_1 = 2|\mu|e^{-\mu L}$$

Domain wall fermion

$[-L, +L] + \text{BC-ii}$

(ii) Dirichlet BC for Domain wall mode. \Rightarrow Growing mode is physical zero mode

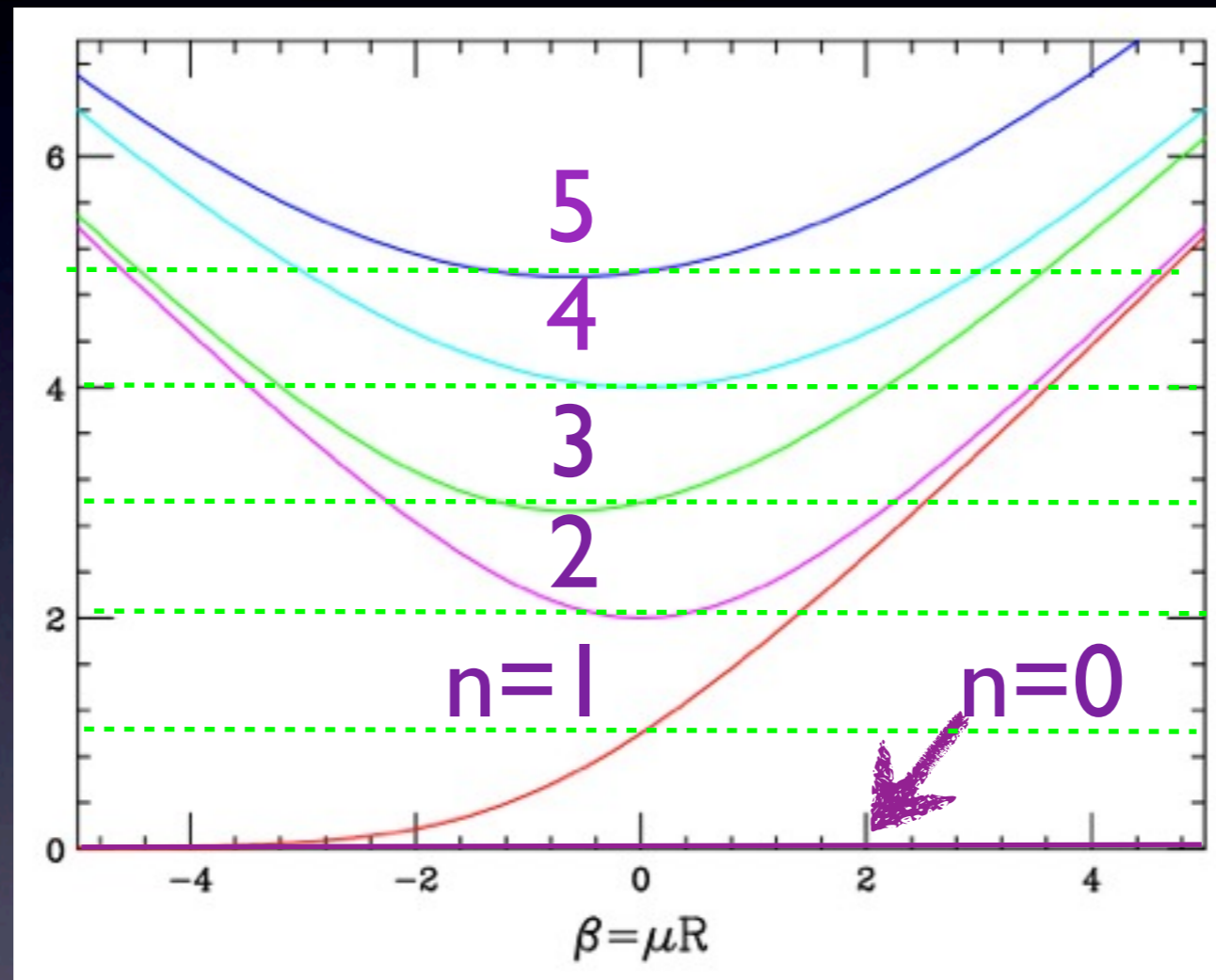


In this case, 1st excited KK mode, which approximately satisfies the Dirichlet BC, behaves like the domain wall fermion and is very light.

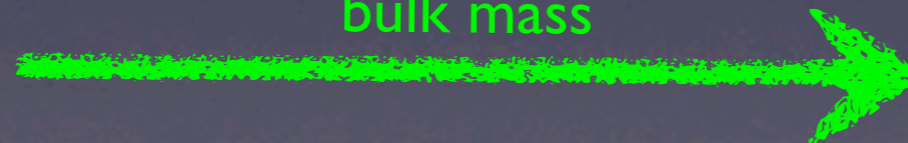
$$m_1 = 2|\mu|e^{-\mu L}$$

sign(μ)	zero mode chirality	location	ultralight KK mode
$\mu > 0$	RH	middle	no
$\mu < 0$	RH	end points	yes
$\mu > 0$	LH	end points	yes
$\mu < 0$	LH	middle	no

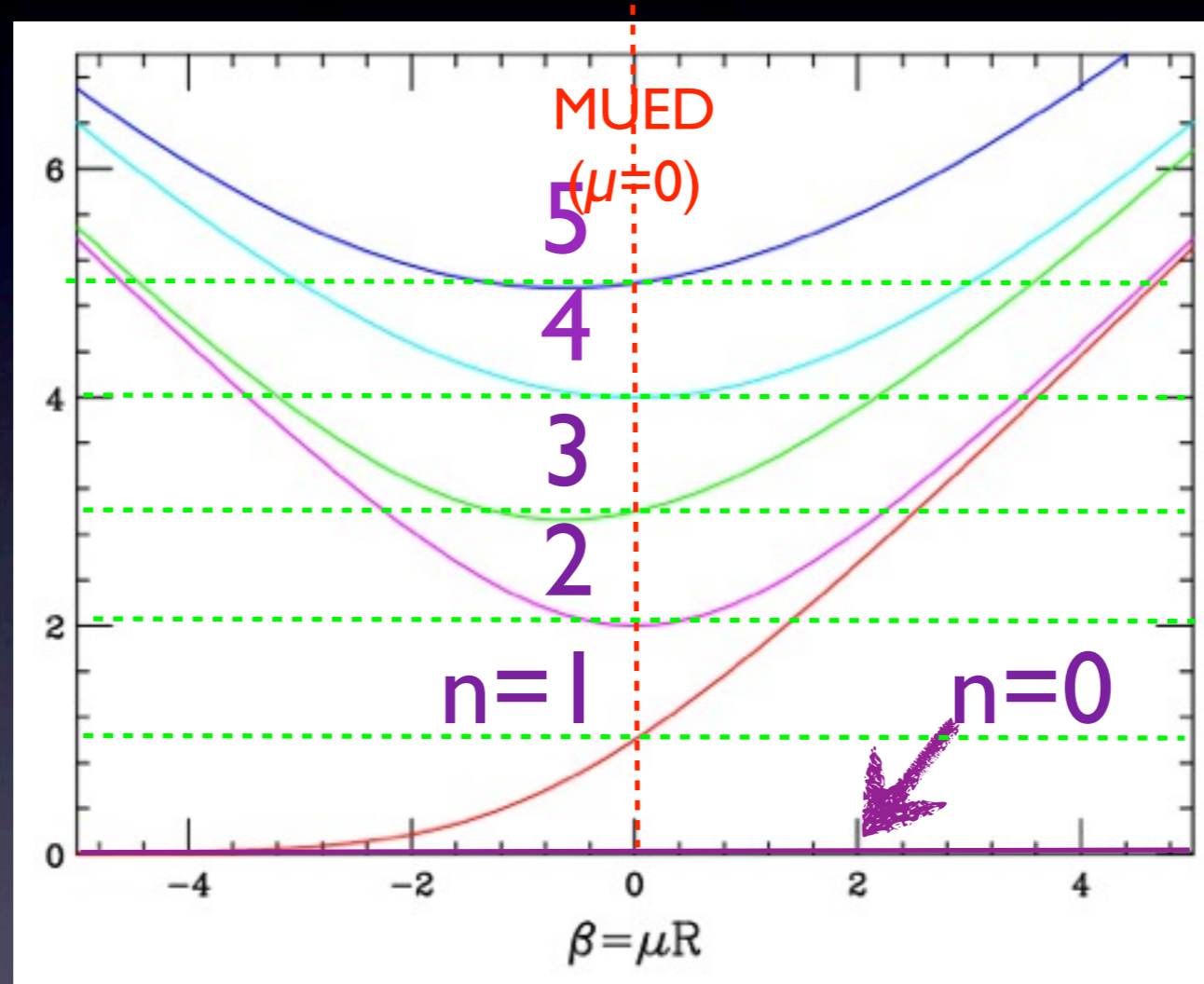
Full KK spectra -run with bulk mass-



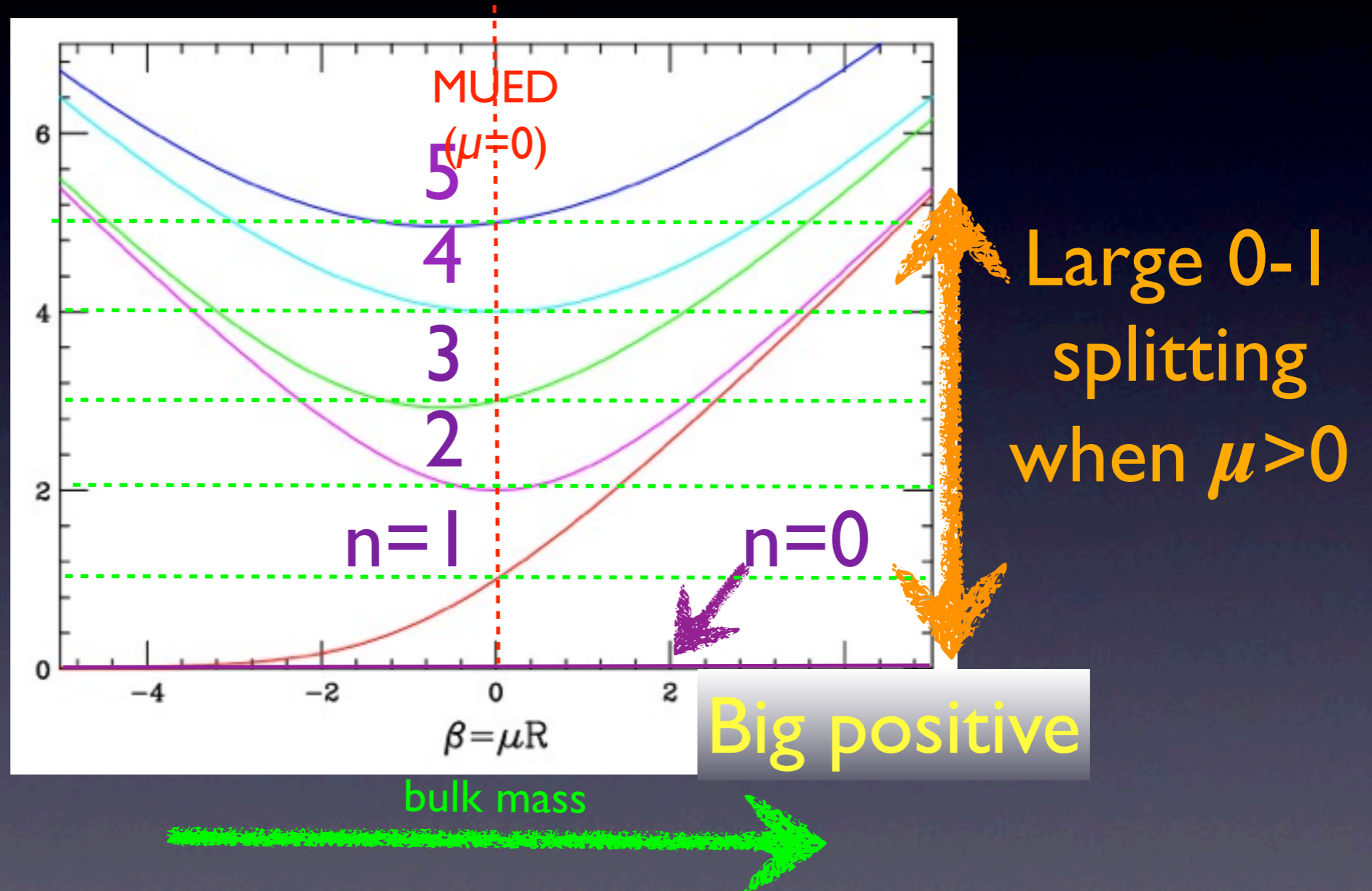
bulk mass



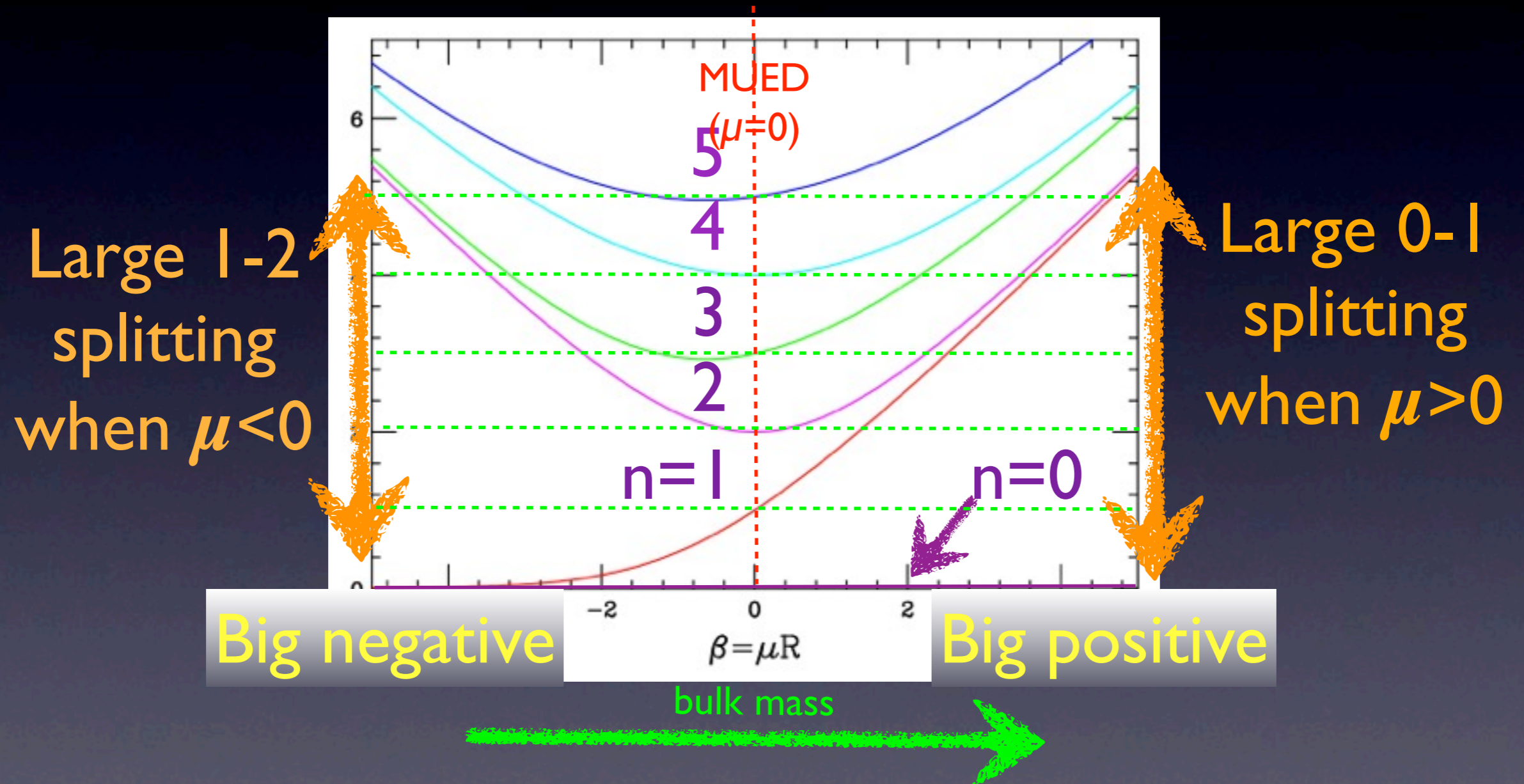
Full KK spectra -run with bulk mass-



Full KK spectra -run with bulk mass-



Full KK spectra -run with bulk mass-



Finally

Flavor hierarchy in split-UED + Localized Higgs

Csaki, Hubisz, Heinonen, SCP, Shu (arXiv:1007.0025)

- If Higgs is flat, we don't worry about flavor problem.
[Doing nothing and safe]
- However, we may be more ambitious and wish to address Yukawa hierarchy problem as fermions are not flat.
- Using Arkani-hamed-Schmaltz mechanism, Yukawa hierarchy can be realized by wave function overlaps with the Higgs vev profile.

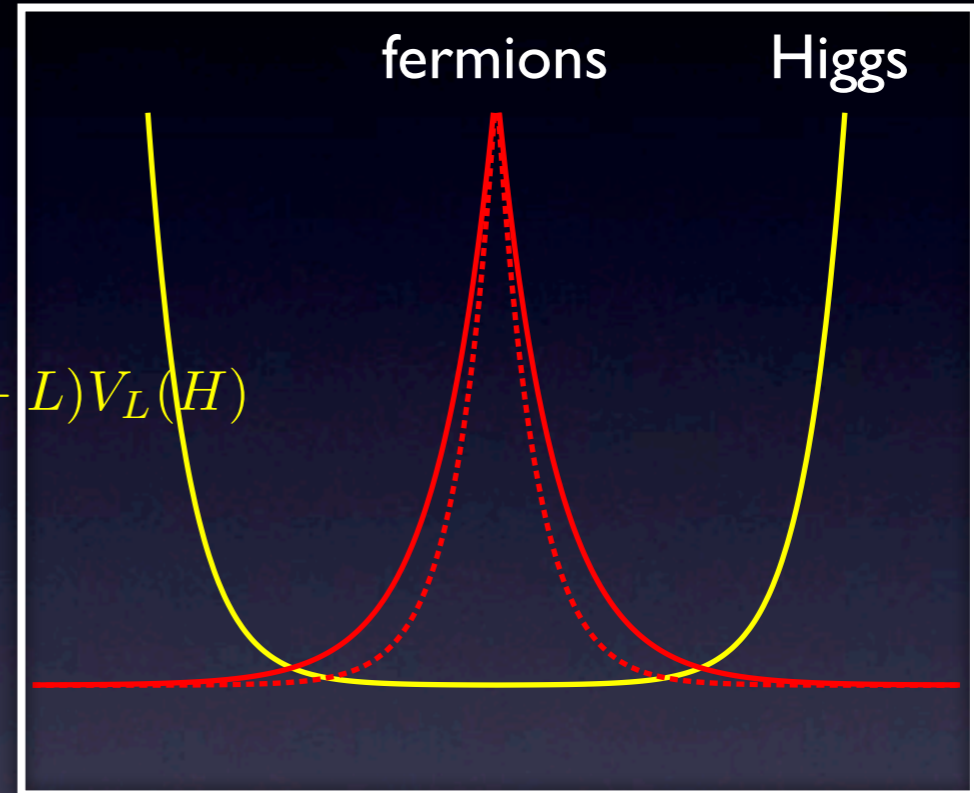
Higgs sector

Csaki, Hubisz, Heinonen, SCP, Shu (arXiv:1007.0025)

Our model

$$S = \int d^5x |D_M H|^2 - m_H^2 |H|^2 - \delta(y+L)V_{-L}(H) - \delta(y-L)V_L(H)$$

$$V_L(H) = V_{-L}(H) = \lambda(|H|^2 - v^2)^2$$



- The VEV profile is indeed localized toward end points at its lowest energy configuration. $v(y) = A \cosh(m_H y)$
- Higgs at boundaries and fermions in the middle is realized: a perfect situation for generating Yukawa hierarchy.

Flavor constraint

- Flavor changing gluon exchange is allowed after EWSB.
- Flavor bound is severe as we can expect (**No RS-GIM** like mechanism works in original UED)

$$C_K^4 \approx \left[\frac{L \cdot 500 \text{GeV}}{1000 \text{TeV}} \right]^2$$

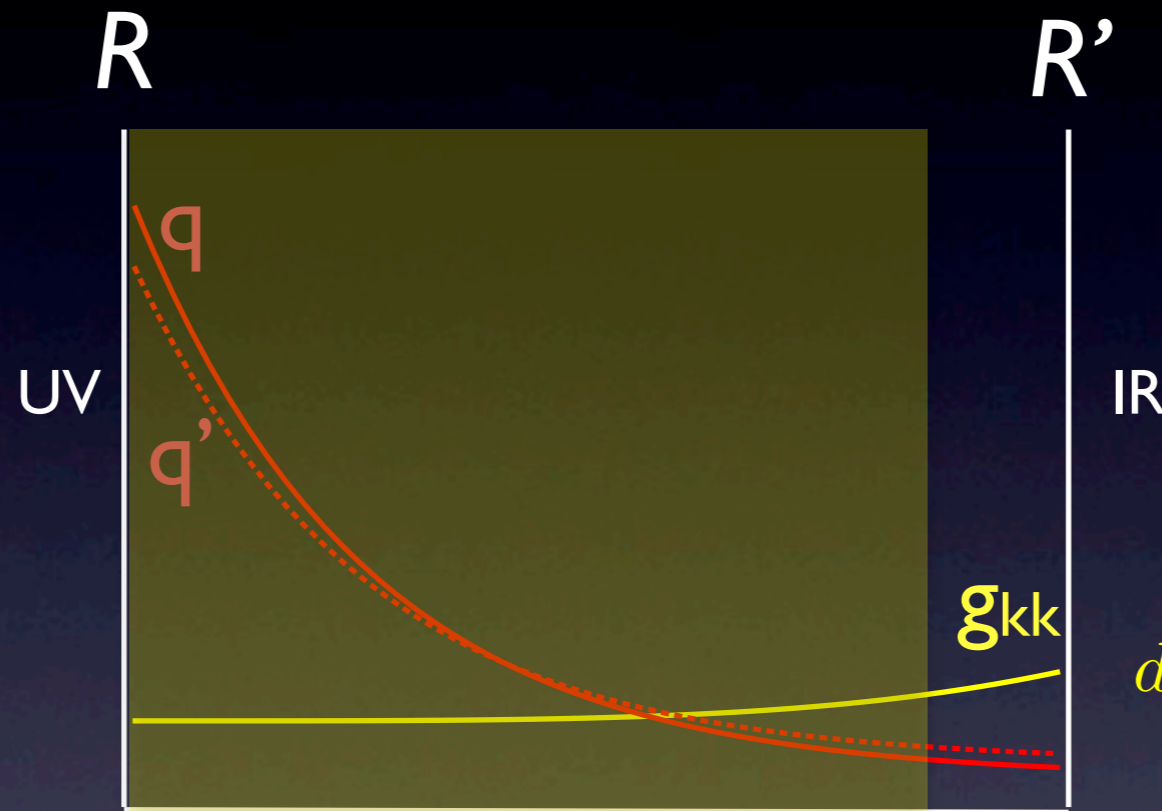
$$\begin{aligned} \text{Re}[C_K^4] &\leq (10^4 \text{TeV})^{-2} \\ \text{Im}[C_K^4] &\leq (10^5 \text{TeV})^{-2} \end{aligned} \quad \longrightarrow \quad L^{-1} \geq 500 \text{TeV}$$

Flavor problem is understood but hard to get tested at the LHC

Not the
end of the
story!

RS-GIM

Arkani-Hamed, Schmaltz
Grossman, Neubert
Gherghetta, Pomarol



$$R'/R \sim 10^{16}$$

$$ds^2 = \left(\frac{R}{z}\right)^2 (dx^2 - dz^2)$$

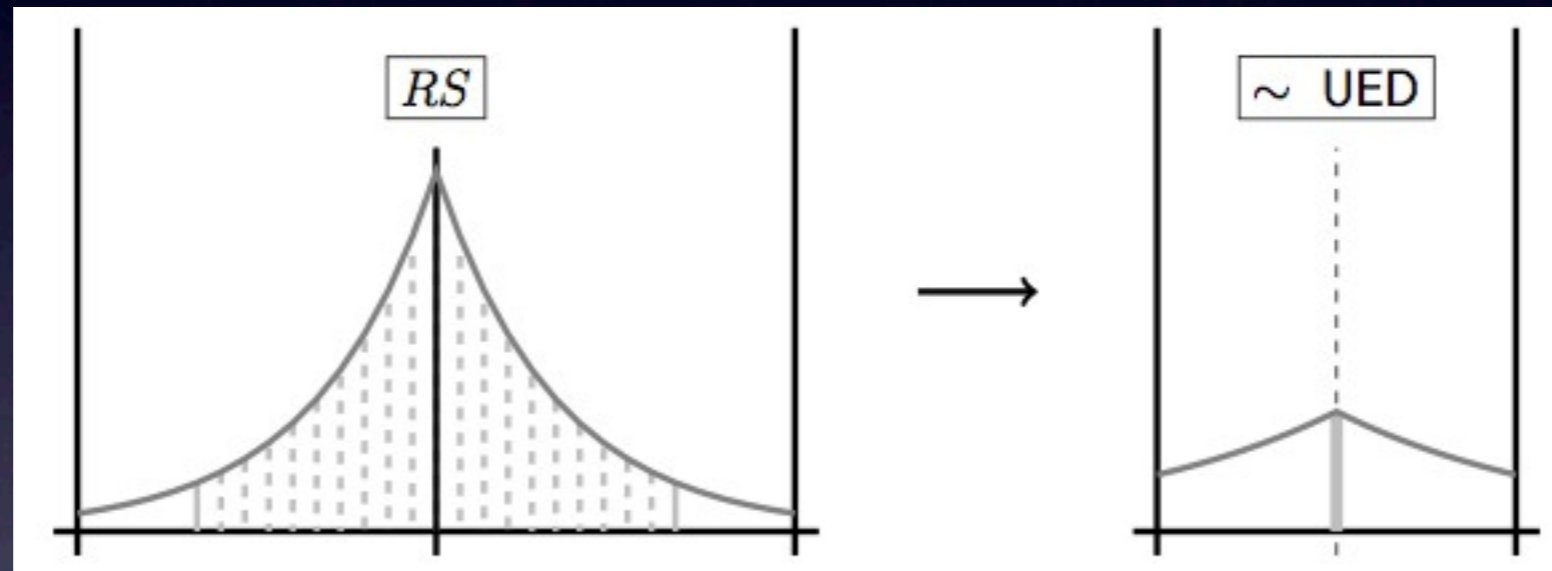
- In RS, the gauge boson KK-modes are basically flat throughout most of the bulk of the XD, varying mainly in the region of IR brane.
- Integrating out the region in the vicinity of the UV brane creates (after canonical normalization of the zero mode) flavor universal BLKT.

$$S_{\text{fermion}} = \int d^5x \left\{ \frac{i}{2} \bar{\Psi} \Gamma^\mu \overleftrightarrow{\partial}_\mu \Psi \right\} \kappa_f L \delta(y).$$

- The remaining non-universal pieces arise only near the IR brane, where the fermion wave functions are exponentially suppressed ~ **RS-GIM!**

RS to UED

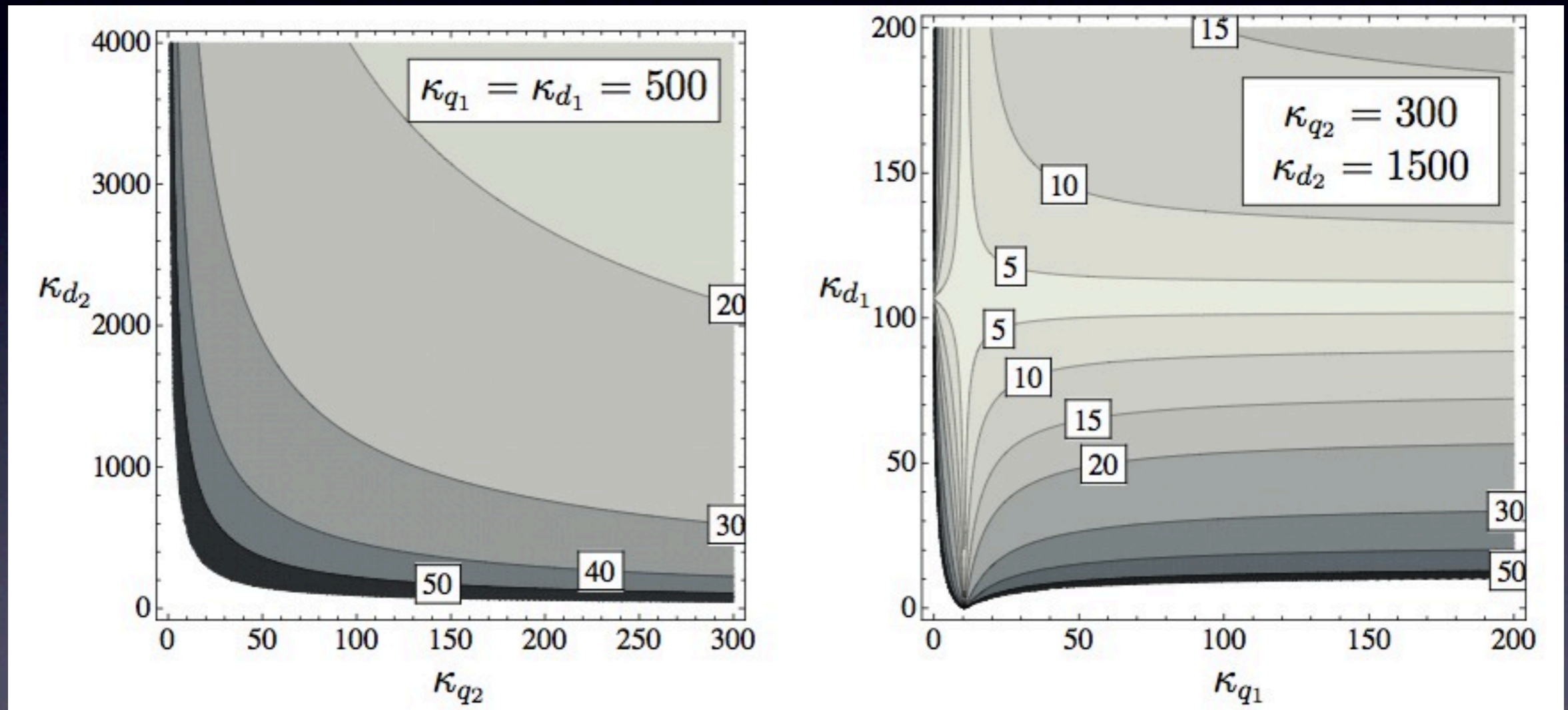
Interpret UED as an effective description of RS with two throats



- It is possible to reduce a warped geometry to an approximately flat XD by integrating out a large slice of the warped XD.

- The remaining warping is minimal and it is clear that this model will describe exactly the same physics as the complete warped XD, encapsulating RS-GIM mechanism.
- KK-parity is conserved and flavor hierarchy understood.

Allowed range



- Large BLKT allows rather low KK-scale (~ 5 TeV) :-)

Summary & Discussion

- **KK-parity** does not require flat geometry. KK-parity does allow 5D fermion mass.
- **Split-UED** allows 5D masses in a way of keeping KK-parity. Phenomenology becomes richer.
- Flavor hierarchy may be due to the non-flat profiles but theory is stringently constrained lack of RS-GIM like mechanism.
 - ✦ Regard UED as an effective theory of two-throats RS model and introduce BLKTs accordingly. (this talk)
 - ✦ Introduce flavor symmetry to make bulk masses degenerate (then the flavor structure is just as good as the SM; MFV) . There may be more ..