4-14 August
SI 2010

Inflationary Cosmology I

Overview of Cosmological Perturbation Theory

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Progress in Cosmology (1)

1st stage: 1916 ~ 1980

- 1916~ General Relativity/Friedmann Universe
- 1929 Hubble's law: $V=H_0 R$ ···cosmological redshift
- 1946~ Big-Bang theory/Nuclear astrophysics
- 1960~ High redshift objects/Quasars
- 1965 Discovery of relic radiation from Big-Bang

  Cosmic Microwave Background: $T_0=2.7K$
- 1970~ BBNucleosynthesis vs Observed Abundance

  → Existence of Dark Matter

Friedmann equation

$$\boxed{H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2}}$$

$a = a(t)$ ··· cosmic scale factor (∝size of the universe)

$H \equiv \dfrac{da/dt}{a}$ ··· expansion rate (Hubble parameter)

ρ ··· mass density (=energy density/c^2)

$$K = \begin{cases} +1 & \cdots \text{ closed universe (3-sphere)} \\ 0 & \cdots \text{ flat universe (Euclid 3-space)} \\ -1 & \cdots \text{ open universe (3-hyperboloid)} \end{cases}$$

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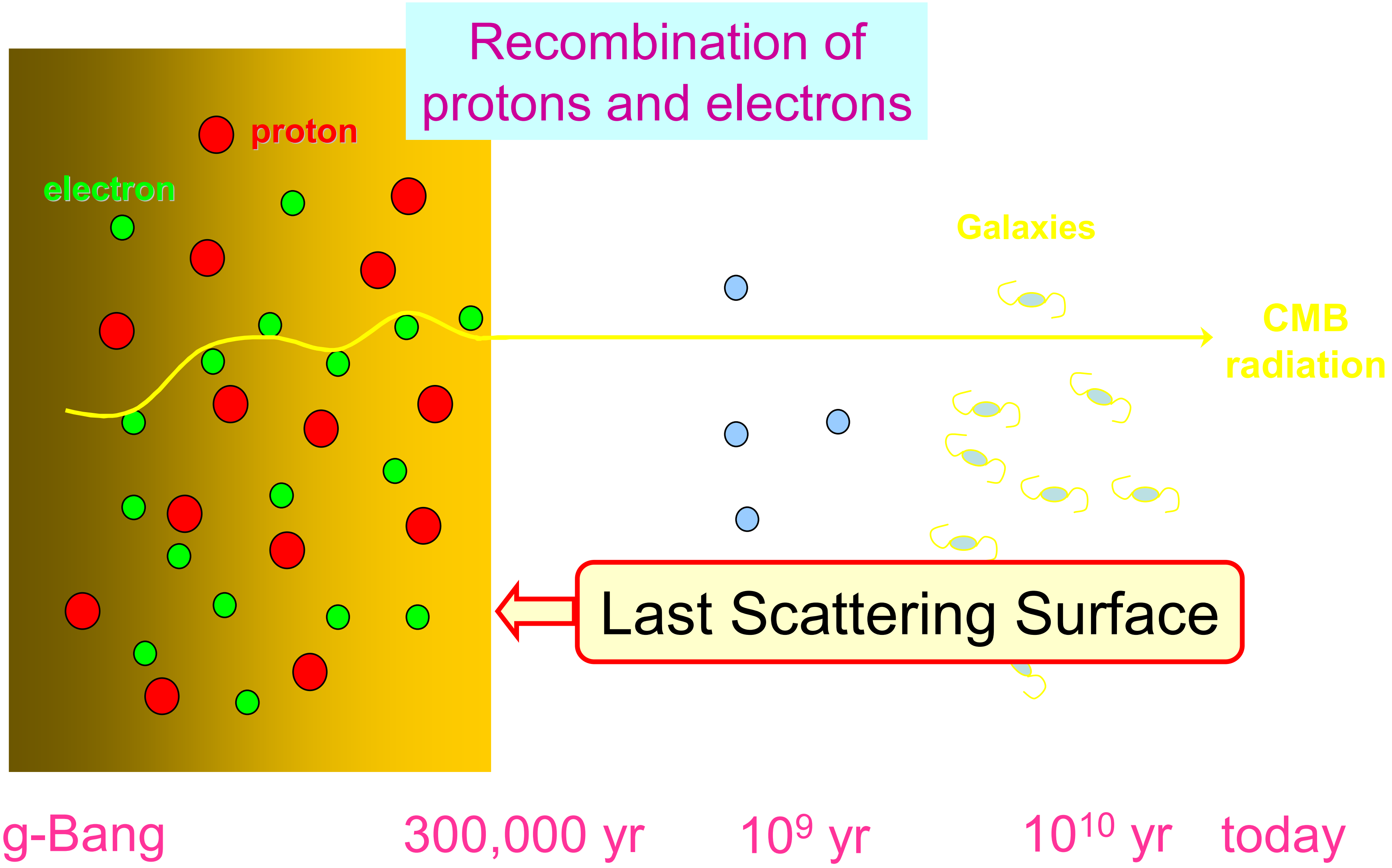
Friedmann (Big-Bang) Universe and CMB



Recombination of protons and electrons

Last Scattering Surface

When we observe sky with visible light…

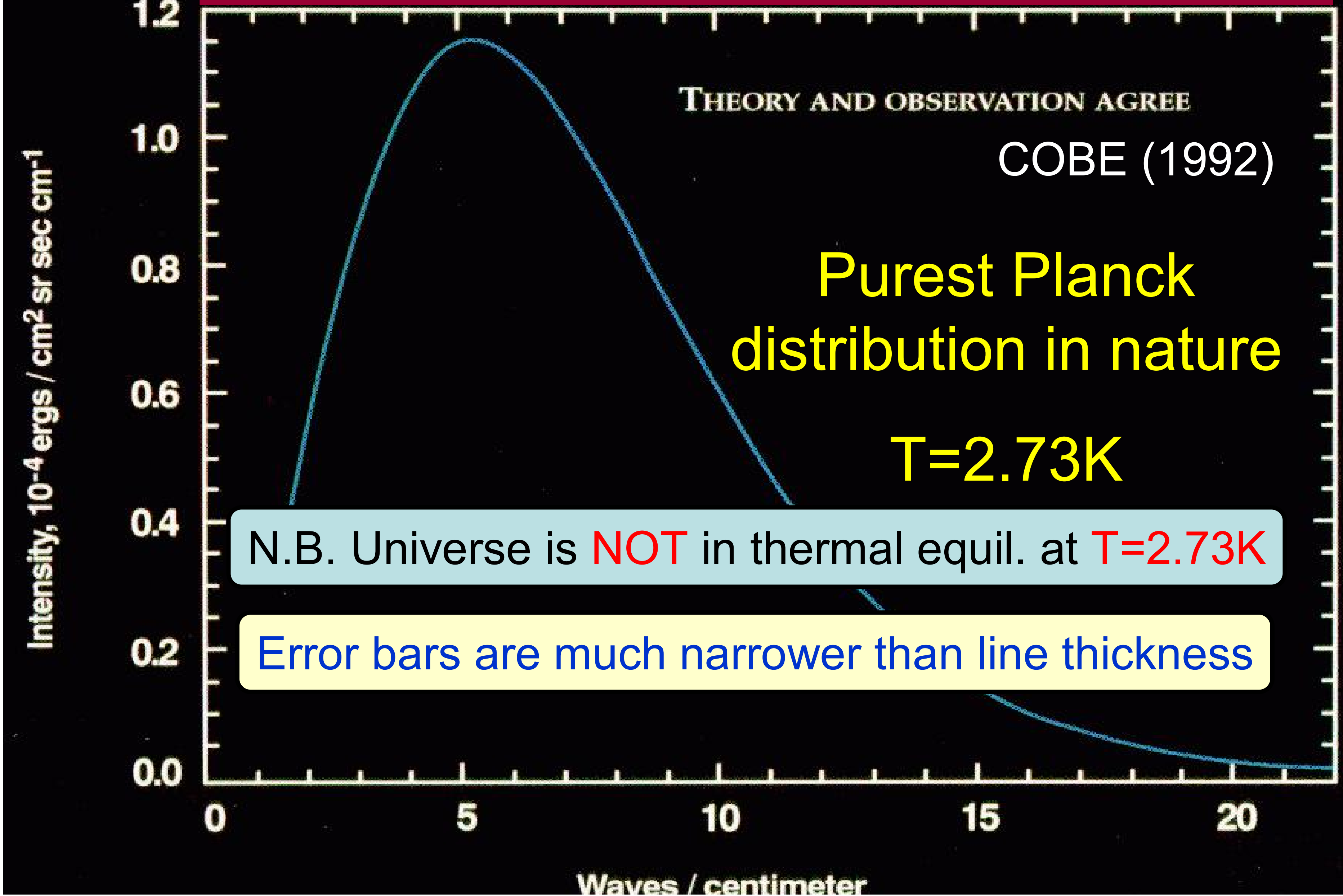
We can only see the cloud: last scattering surface for visible frequencies

⇕

With microwave frequencies, we can see the universe at 300,000 years old.

Photons from LSS get redshifted by ~1000

CMB Frequency Spectrum



COBE (1992)

Purest Planck distribution in nature

T=2.73K

N.B. Universe is NOT in thermal equil. at T=2.73K

Error bars are much narrower than line thickness

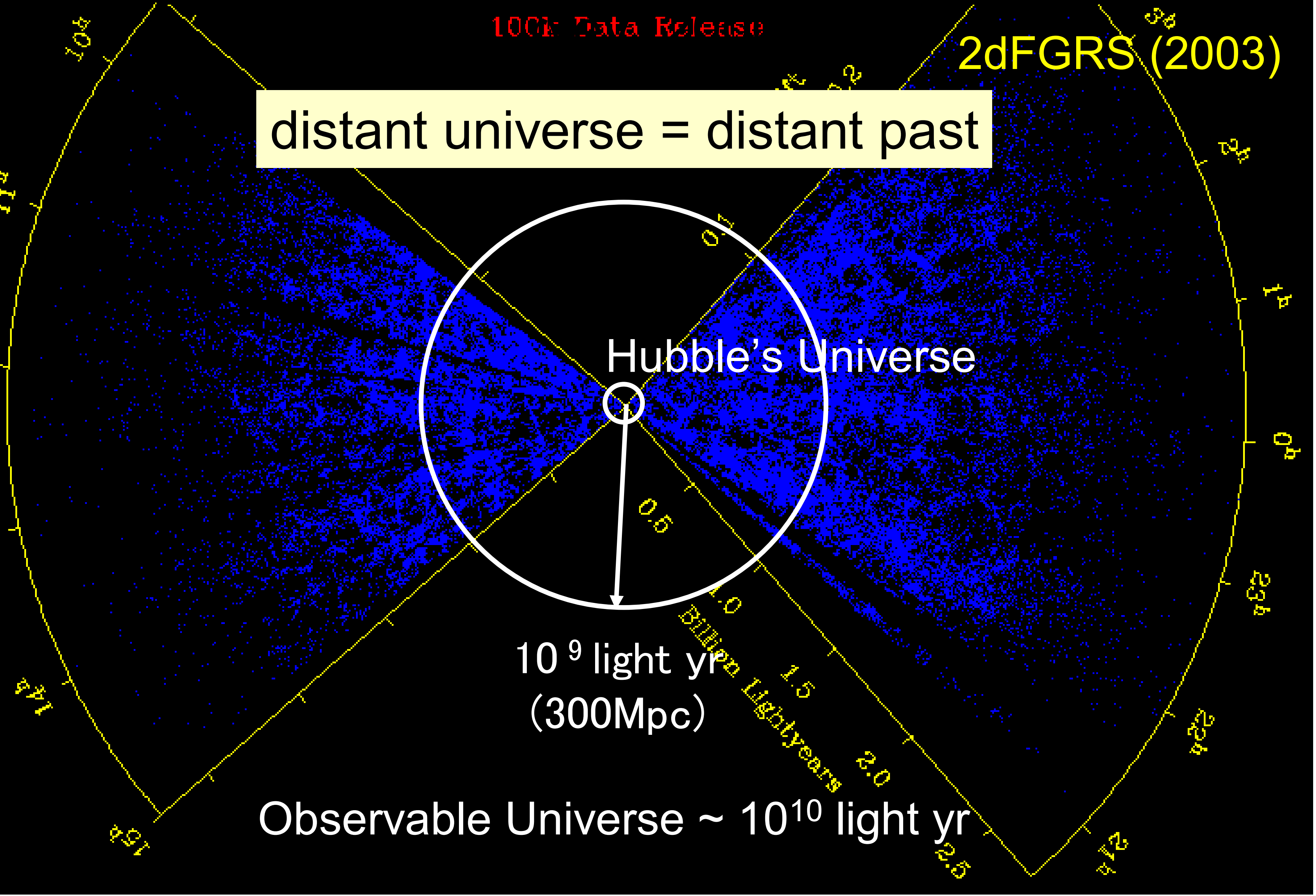
Establishment of homogeneous & isotropic Big-Bang Universe Model

Progress in Cosmology (2)

2nd stage: 1980 ~ 2003

- 1980~ Revelation of Large Scale Structure
  - Cosmological Perturbation Theory
  - Particle Cosmology/Inflationary Universe
- 1992 Detection of CMB anisotropy
  - Evidence for Inflationary Universe
- 2003 Accurate CMB angular spectrum
  - Confirmation of Flatness of the Universe
  - Strong evidence for Dark Energy

Observed Large Scale Structure



2dFGRS (2003)

distant universe = distant past

Hubble’s Universe

10^9 light yr
(300Mpc)

Observable Universe ~ 10^{10} light yr

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Cosmological Perturbation Theory

- extremely brief (and biased) history -

general formulation

Lifshitz-Khalatnikov (1963)

synchronous gauge

Gerlach-Sengupta (1979)

gauge-invariant (for spherically sym space)

Bardeen (1980)

gauge-invariant (for isotropic space)

Kodama-Sasaki(1984)

gauge-inv (for isotropic n-dim space, multi-fluid/scalar)

inflation

Starobinsky (1980)

R^2 model/gravitational wave spectrum

Mukhanov (1981)

scalar power spectrum for R^2 model

Linde (1982)

slow-roll inflation (new inflation)

Mukhanov (1985), Sasaki (1986)

inflaton quantization/scalar power spectrum

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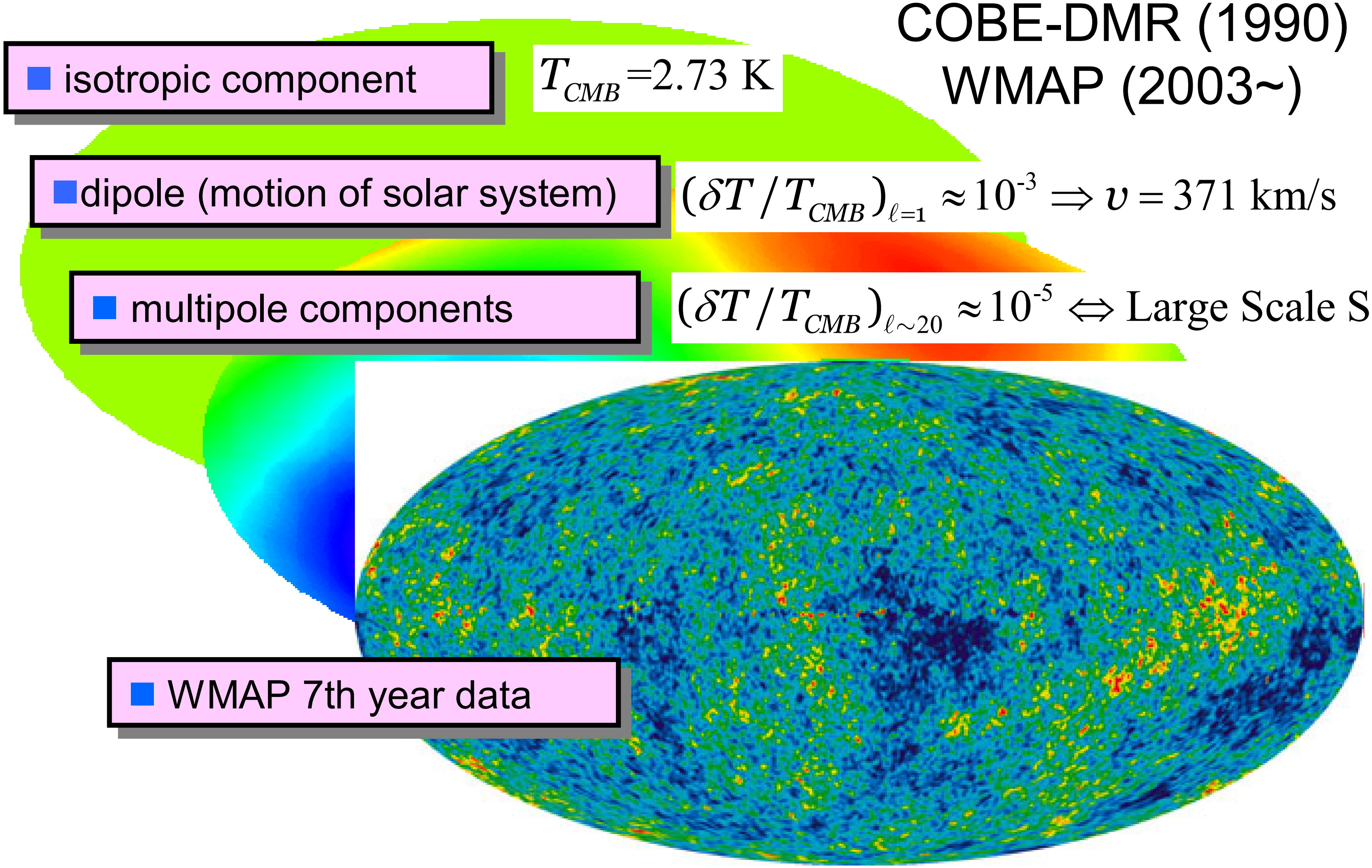
CMB Full Sky Map

COBE-DMR (1990)
WMAP (2003~)



- isotropic component

$T_{CMB} = 2.73\ \mathrm{K}$

- dipole (motion of solar system)

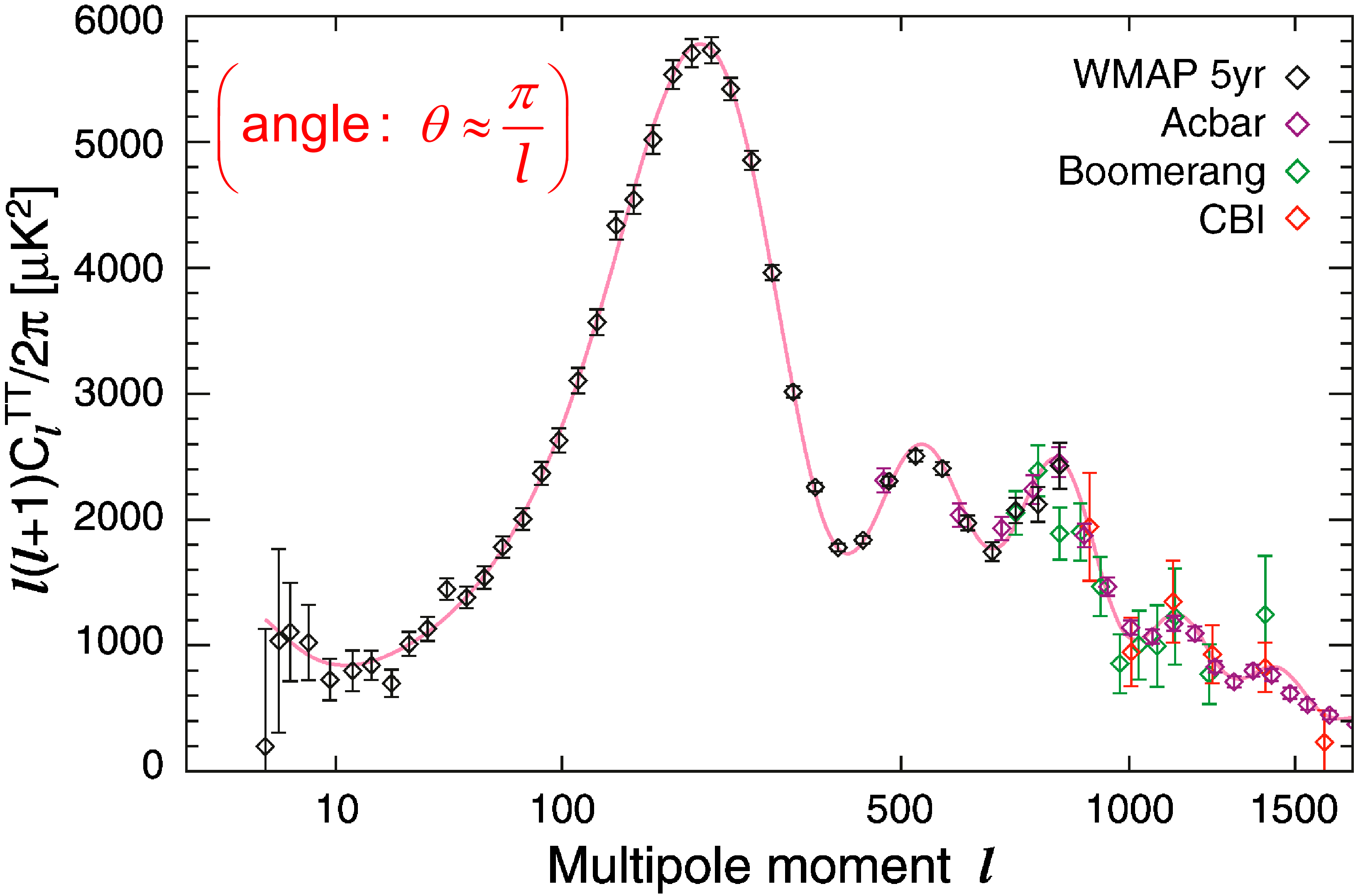
$(\delta T / T_{CMB})_{\ell=1} \approx 10^{-3} \Rightarrow \upsilon = 371\ \mathrm{km/s}$

- multipole components

$(\delta T / T_{CMB})_{\ell \sim 20} \approx 10^{-5} \Leftrightarrow \mathrm{Large\ Scale\ Structure}$

- WMAP 7th year data

CMB Anisotropy Spectrum



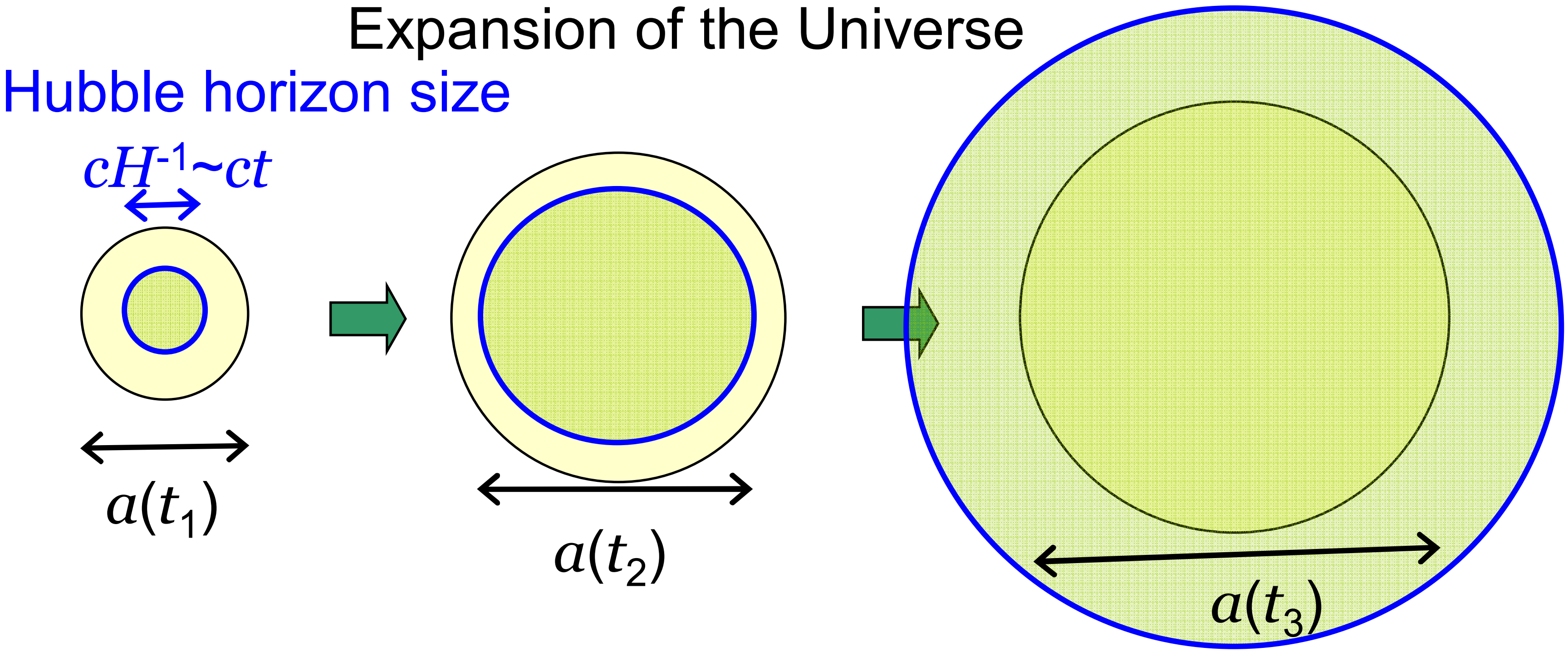
Horizon Problem

Why detection of $\delta T/T$ at $\theta > 10^{\circ}$ ($\sim \pi/20$) so important?

- Because in the standard Friedmann universe, the size of causal volume (horizon size) grows like ~ ct.

In fact, the original horizon problem is why the universe could have been so homogeneous on scales much greater than the horizon size.

Origin of Horizon Problem



Expansion of the Universe

Hubble horizon size

$cH^{-1} \sim ct$

$a(t_1)$

$a(t_2)$

$a(t_3)$

$a(t) \propto t^{1/2}$ for hot bigbang universe

Horizon grows faster than the cosmic expansion in the standard Friedmann (Bigbang) Universe

Horizon Problem

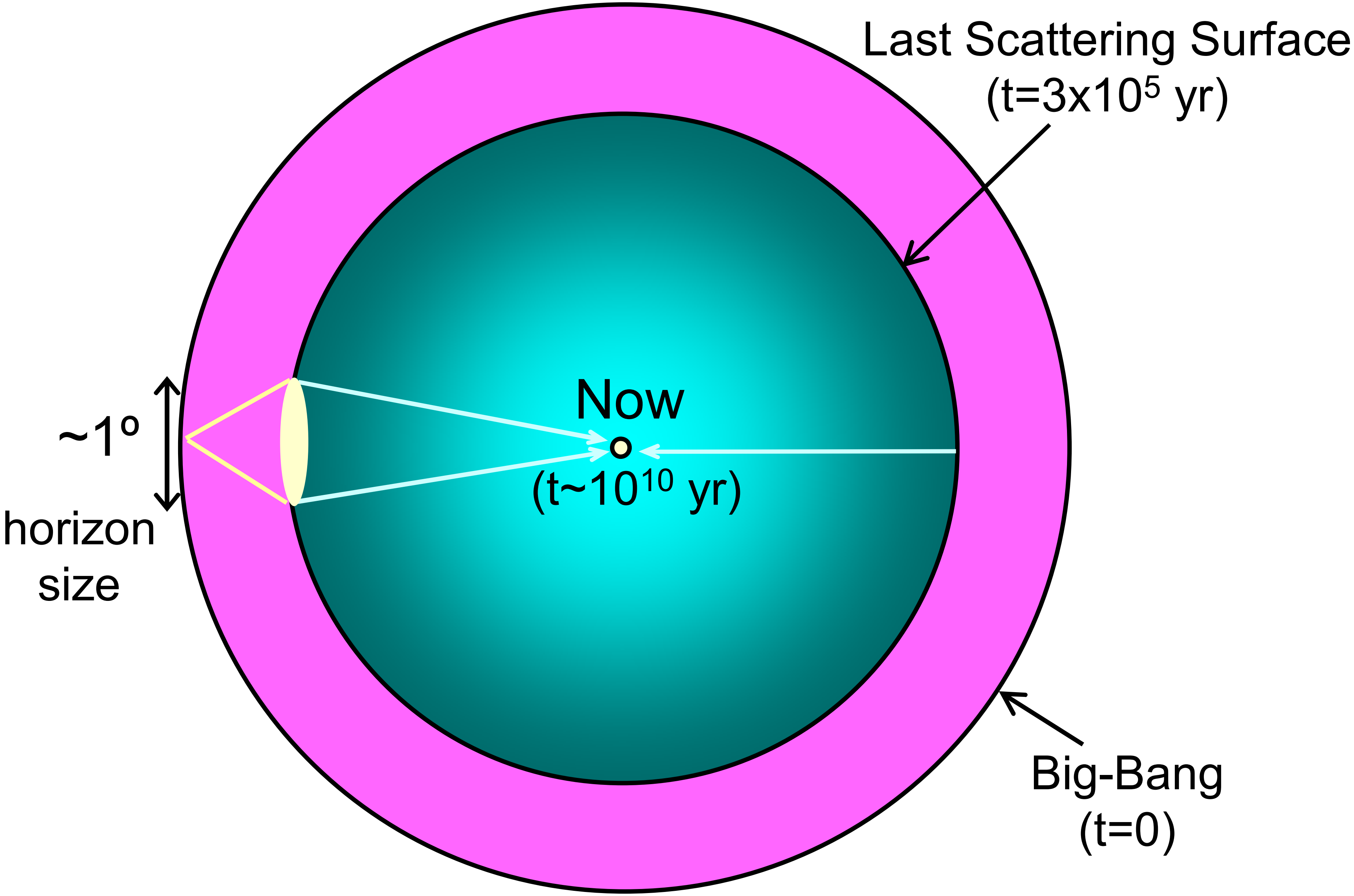
Why the detection of $\delta T/T$ at $\theta > 10^{\circ}$ was so important?

- Because in the standard Friedmann universe, the size of causal volume (horizon size) grows like ~ ct.

In fact, the original horizon problem is why the universe could have been so homogeneous on scales much greater than the horizon size.

- The angle sustaining the horizon size at LSS is ~ 1°.
- Thus, any causal, physical process cannot produce correlation on scales $\theta > 1^{\circ}$.
- But $(\delta T/T)_{\theta > 10^{\circ}} \neq 0$ means there exists non-zero correlation.

There are ~10^4 causally independent patches on LSS



Last Scattering Surface

($t=3x10^5$ yr)

~1°

horizon size

Now

(t~10^{10} yr)

Big-Bang

($t=0$)

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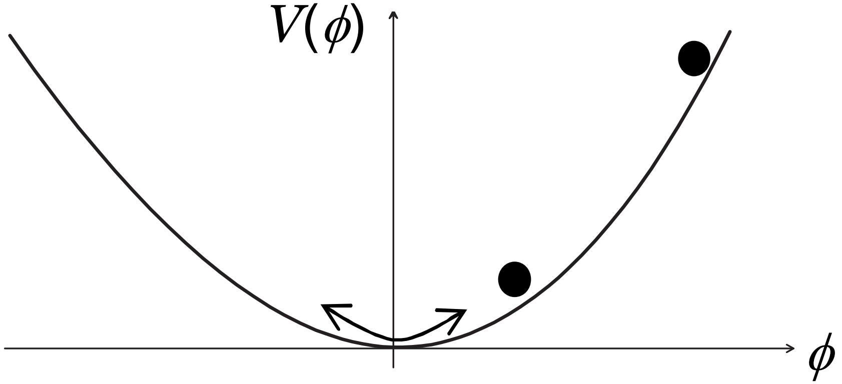
Inflationary Universe

Universe dominated by a scalar (inflaton) field

For sufficiently flat potential:

$$H^2 \approx \frac{8\pi G}{3} V(\phi) \quad \left(\Leftarrow \frac{1}{2}\dot{\phi}^2 \ll V(\phi) \right)$$

$$\Rightarrow \quad \frac{|\dot{H}|}{H^2} = \frac{3\dot{\phi}^2}{2V(\phi)} \ll 1$$



- H is almost constant ~ exponential expansion = inflation
- ϕ slowly rolls down the potential: slow-roll (chaotic) inflation

Linde (1983)

- Inflation ends when ϕ starts damped oscillation.

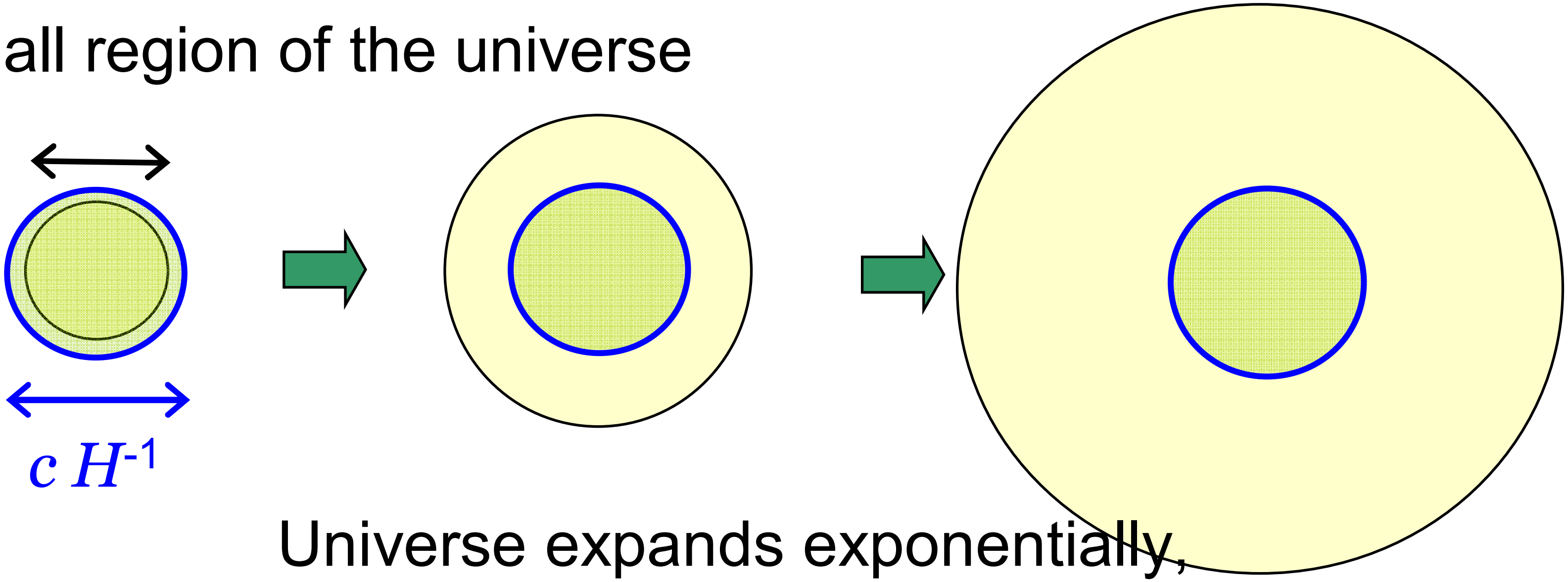
⇨ ϕ decays into thermal energy (radiation)

Birth of Hot Bigbang Universe

Hubble horizon during Inflation

$a(t) \sim e^{Ht}$; $H \sim$const.

A small region of the universe

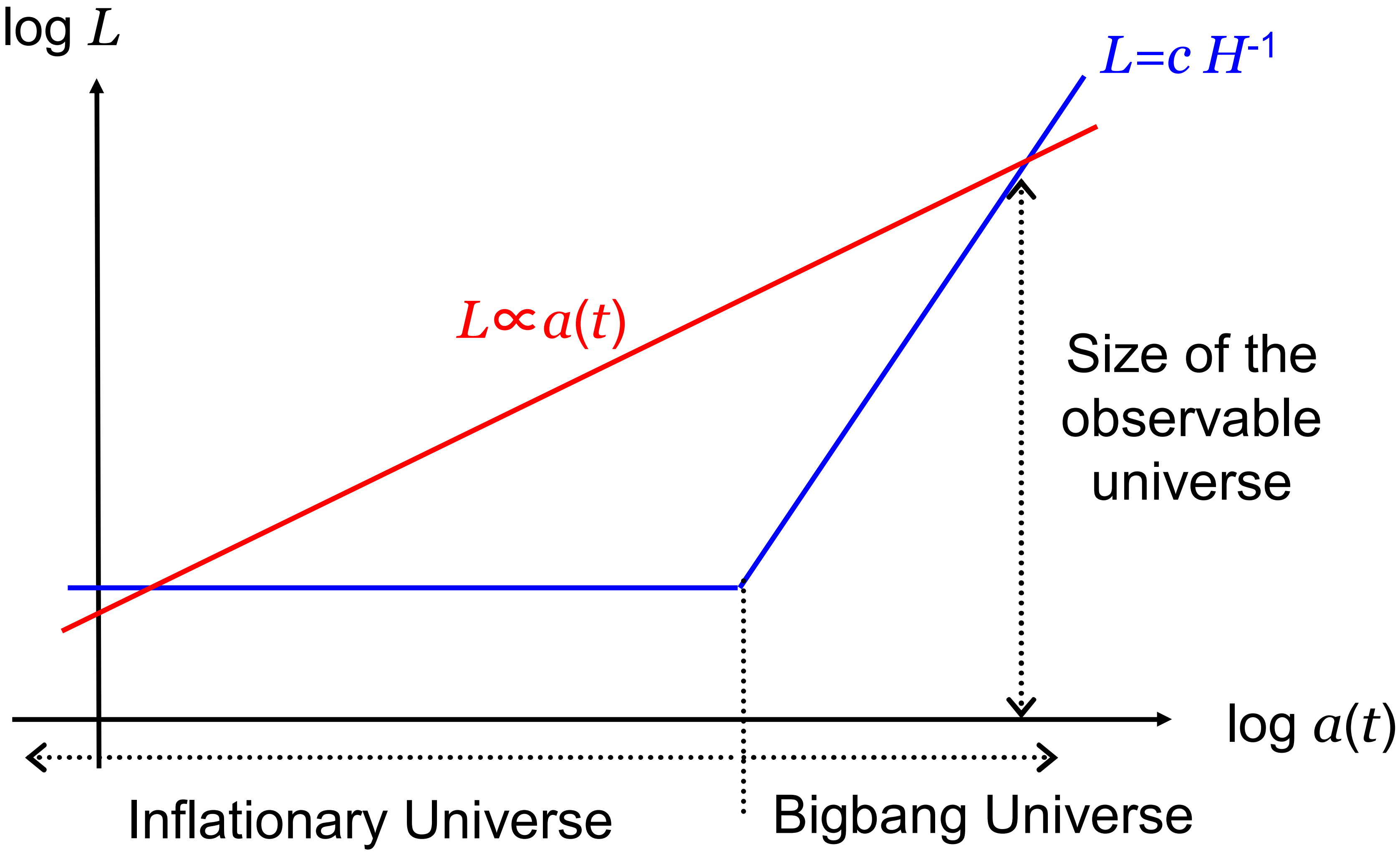


Universe expands exponentially,
while the Hubble horizon size remains almost constant.

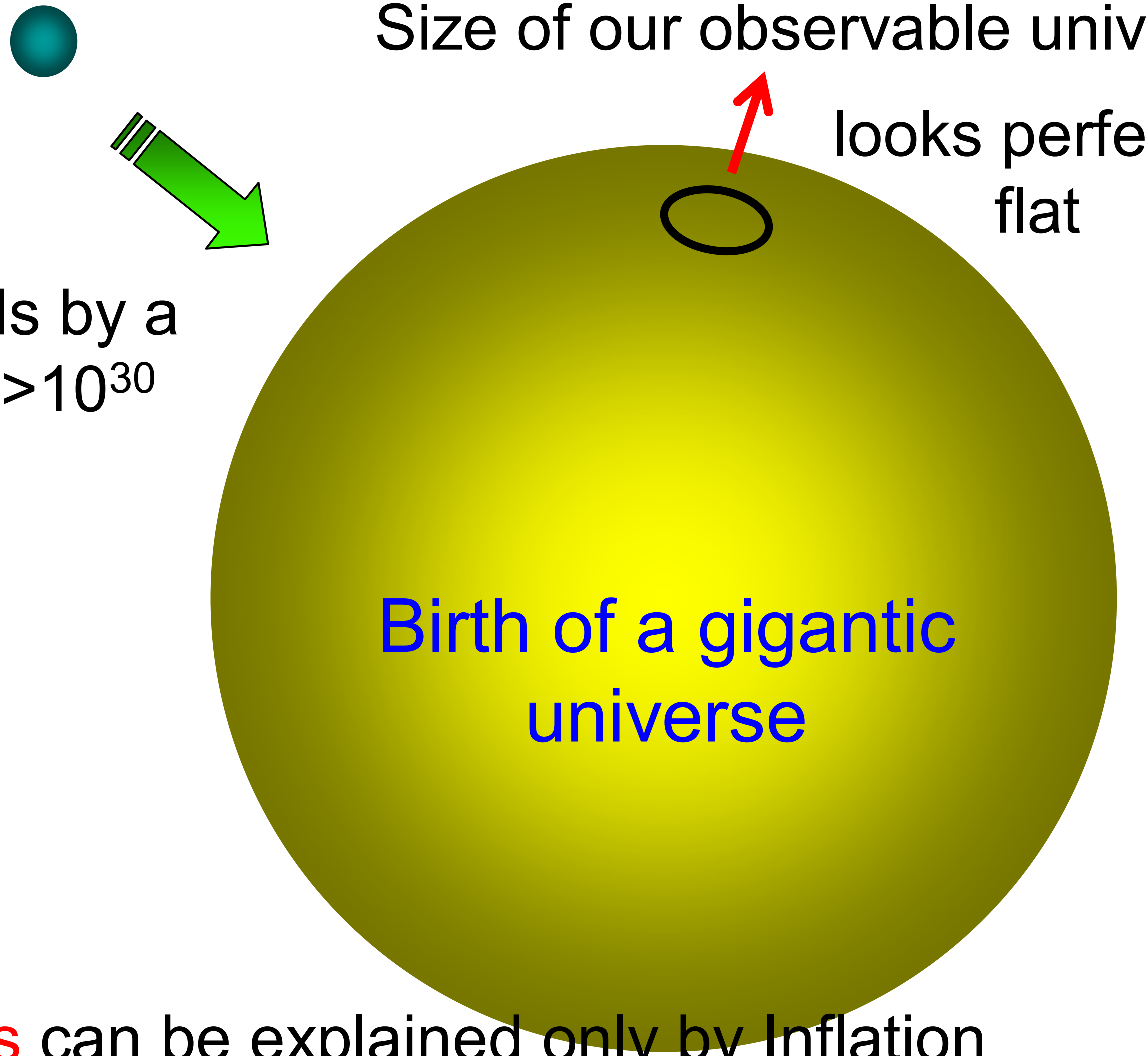
An initially tiny region can become much larger than the entire observable universe

→ solves the horizon problem.

Length Scales of the Inflationary Universe



Flatness of the Universe



small universe

Size of our observable universe

looks perfectly flat

expands by a factor >10^{30}

Birth of a gigantic universe

Flatness can be explained only by Inflation

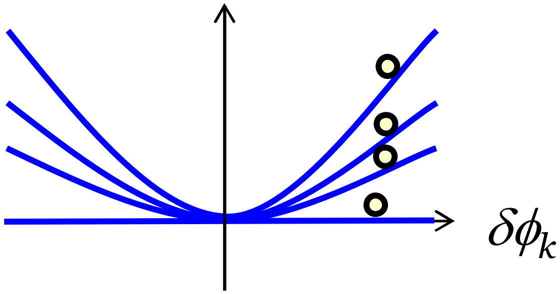
Seed of Cosmological Perturbations

Zero-point (vacuum) fluctuations of ϕ: $\delta\phi = \sum_k \delta\phi_k(t) e^{ik\cdot x}$

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + \omega^2(t)\delta\phi_k = 0 \; ; \quad \omega^2(t) = \frac{k^2}{a^2(t)} \equiv \left(\frac{2\pi c}{\lambda(t)}\right)^2$$

physical wavelength $\lambda(t) \propto a(t)$

harmonic oscillator with friction term and time-dependent ω



$\delta\phi_k \rightarrow$ const.

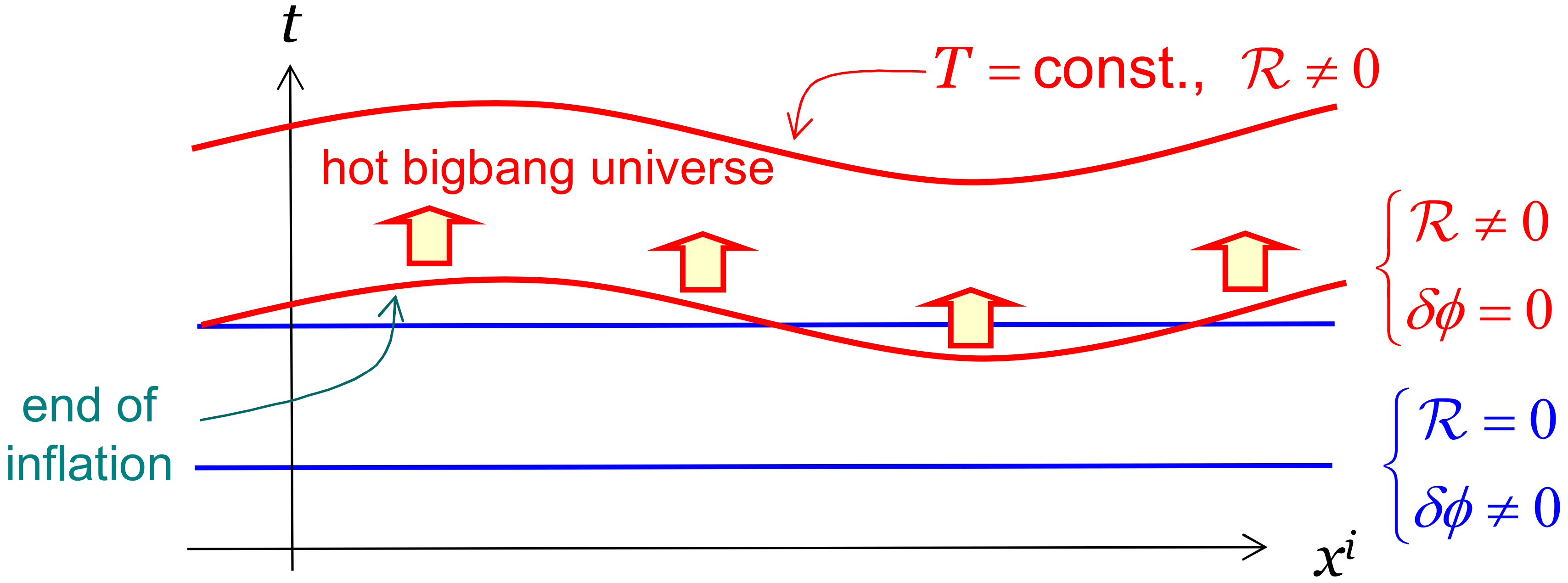
··· frozen when $\lambda > c\,H^{-1}$
(on superhorizon scales)

gravitational wave modes also satisfy the same eq.

Generation of Curvature Perturbations

curvature perturbation $\mathcal{R}$ ≈ gravitational potential Ψ

- $\delta\phi$ is frozen on “flat” ($\mathcal{R}$=0) 3-surface (t=const. hypersurface)
- Inflation ends/damped osc starts on ϕ=const. 3-surface.



Theoretical Predictions

- Amplitude of curvature perturbation:

$$\mathcal{R} = \left.\frac{H^2}{2\pi\dot{\phi}}\right|_{k/a=H}$$

Mukhanov (1985), Sasaki (1986)

- Power spectrum index:

$$M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \sim 2.4\times 10^{18}\,\mathrm{GeV}\text{: Planck mass}$$

$$\frac{4\pi k^3}{(2\pi)^3} P_{\mathcal{R}}(k) = A k^{n_S-1} \ ; \ n_S - 1 = M_{pl}^2 \left(2\frac{V''}{V} - 3\frac{V'^2}{V^2} \right)$$

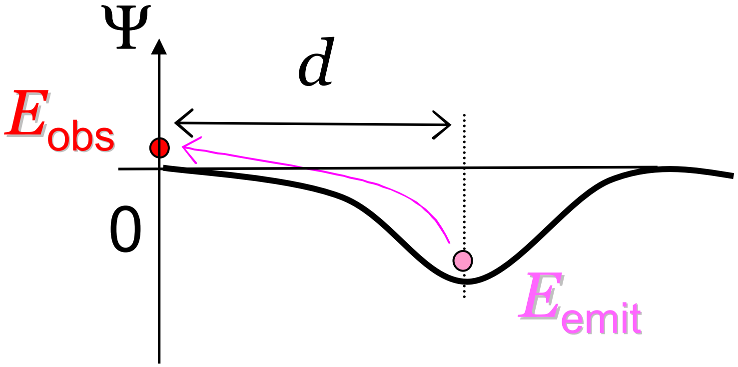
- Tensor (gravitational wave) spectrum:

$$\frac{4\pi k^3}{(2\pi)^3} P_T(k) = A k^{n_T} \ ; \ n_T = -3\frac{\dot{\phi}^2}{V} = -\frac{1}{8}\frac{P_{\mathcal{R}}(k)}{P_T(k)}$$

Liddle-Lyth (1992)

CMB Anisotropy from Curvature Perturbations

- Photons climbing up from gravitational potential well are redshifted.



For Planck distribution,

$$\frac{\Delta T}{T}(\vec{n}) \equiv \frac{T_{\text{obs}}}{T_{\text{emit}}} - 1 = \Psi(\vec{x}_{\text{emit}})$$

$$\vec{x}_{\text{emit}} = \vec{n}\,d\ ;\quad \vec{n} = \text{line of sight}$$

c=1 units

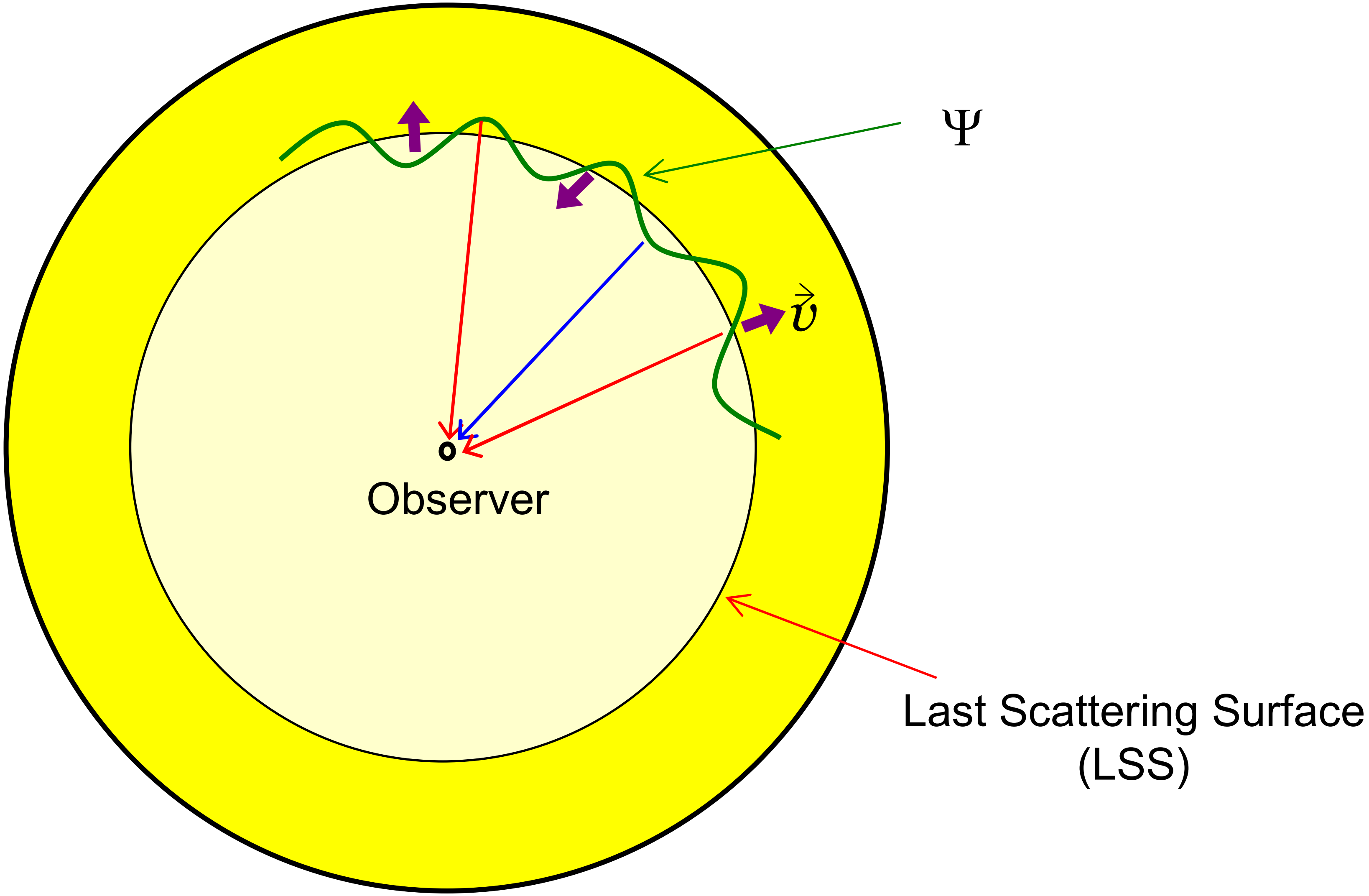
- In an expanding universe, this is modified to be $\frac{\Delta T}{T}(\vec{n}) = \frac{1}{3}\Psi(\vec{x}_{\text{emit}})$

Sachs-Wolfe effect

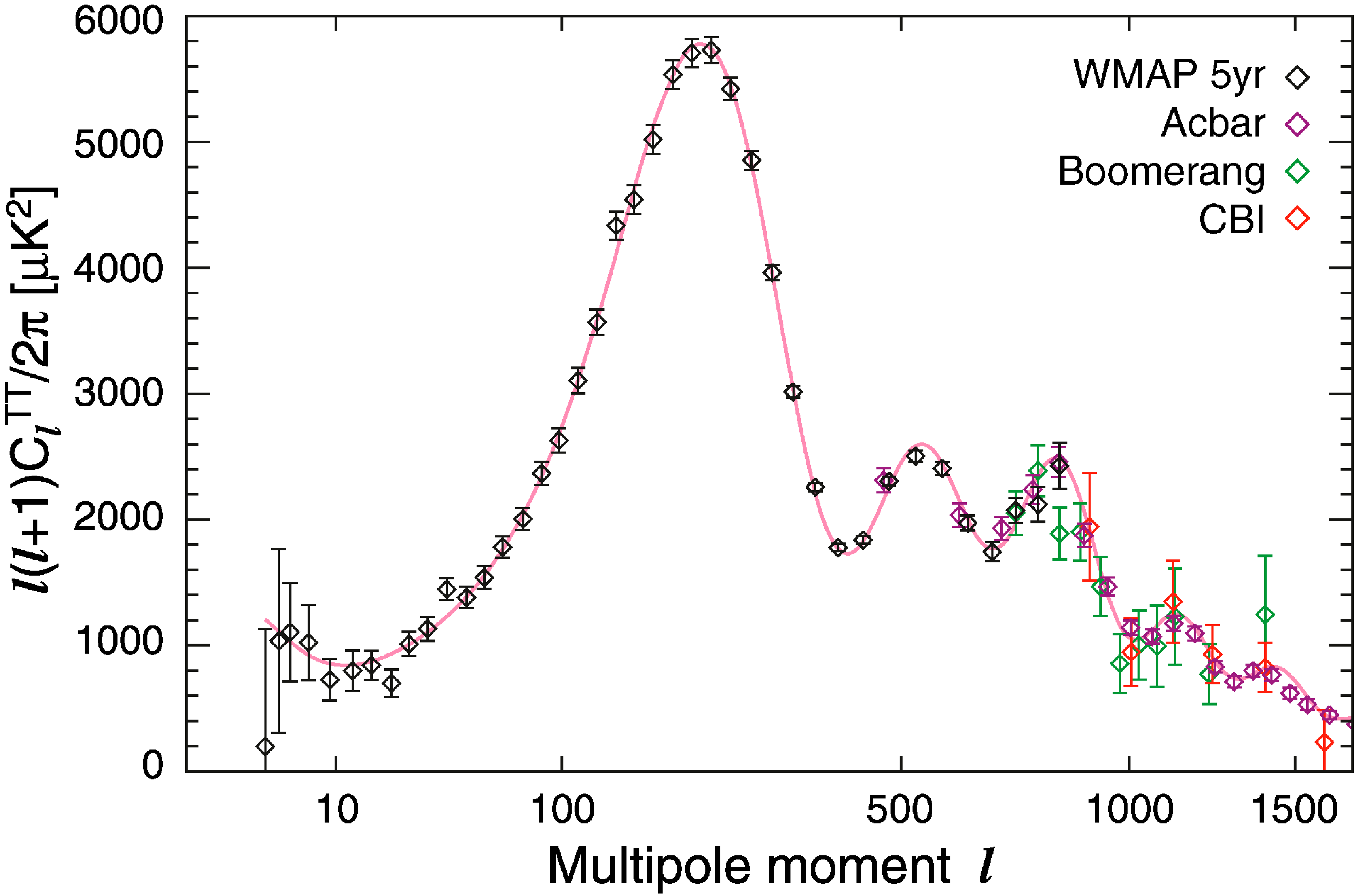
- There is also the standard Doppler effect:

$$\frac{\Delta T}{T}(\vec{n}) = -\vec{n}\bullet\vec{v}(\vec{x}_{emit})$$

$$\frac{\Delta T}{T}(\vec{n}) = \frac{1}{3}\Psi(\vec{x}_{\rm LSS}) - \vec{n}\bullet\vec{v}(\vec{x}_{\rm LSS}) + \cdots (\text{minor corrections})$$



CMB Anisotropy Spectrum



- Amplitude of curvature perturbation:

$$\mathcal{R} = \left.\frac{H^2}{2\pi\dot{\phi}}\right|_{k/a=H}$$

Mukhanov (1985), Sasaki (1986)

$$\mathcal{R}_{\text{COBE}} \sim 10^{-5} \;\Rightarrow\; V^{1/4}(\phi) \sim 10^{16}\,\text{GeV}$$

- Power spectrum index:

$$M_{pl} \equiv \frac{1}{\sqrt{8\pi G}} \sim 2.4\times 10^{18}\,\text{GeV: Planck mass}$$

$$\frac{4\pi k^3}{(2\pi)^3} P_{\mathcal{R}}(k) = Ak^{n_S-1} \;;\; n_S - 1 = M_{pl}^2\left(2\frac{V''}{V} - 3\frac{V'^2}{V^2}\right)$$

$n_{S,\text{WMAP}} - 1 = -0.040 \pm 0.013 \;\Leftrightarrow\; n_S - 1 \sim -0.04$ for a typical model

- Tensor (gravitational wave) spectrum:

$$\frac{4\pi k^3}{(2\pi)^3} P_T(k) = Ak^{n_T} \;;\; n_T = -3\frac{\dot{\phi}^2}{V} = -\frac{1}{8}\frac{P_{\mathcal{R}}(k)}{P_T(k)}$$

Liddle-Lyth (1992)

to be observed by PLANCK/CMBPOL/...

Understanding of the Origin of Large Scale Structure