Inflationary Cosmology II

— Standard Slow-Roll Inflation —

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§1. Introduction

- Horizon problem

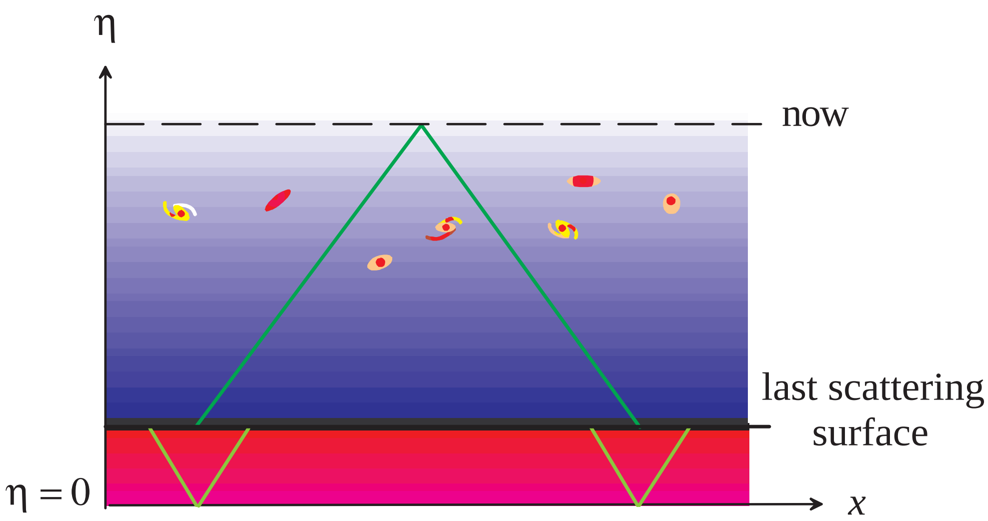
$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 \qquad + \qquad \text{Einstein eqs.}$$

$$\Rightarrow \qquad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \qquad \boxed{\rho + 3p > 0 \quad \Leftrightarrow \quad \text{decelerated expansion}}$$

If $a \propto t^n$, then $n(n-1) < 0 \quad \Rightarrow \quad 0 < n < 1$

$$ds^2 = a^2(\eta)\left(-d\eta^2 + d\vec{x}^2\right), \quad d\eta = \frac{dt}{a}.$$

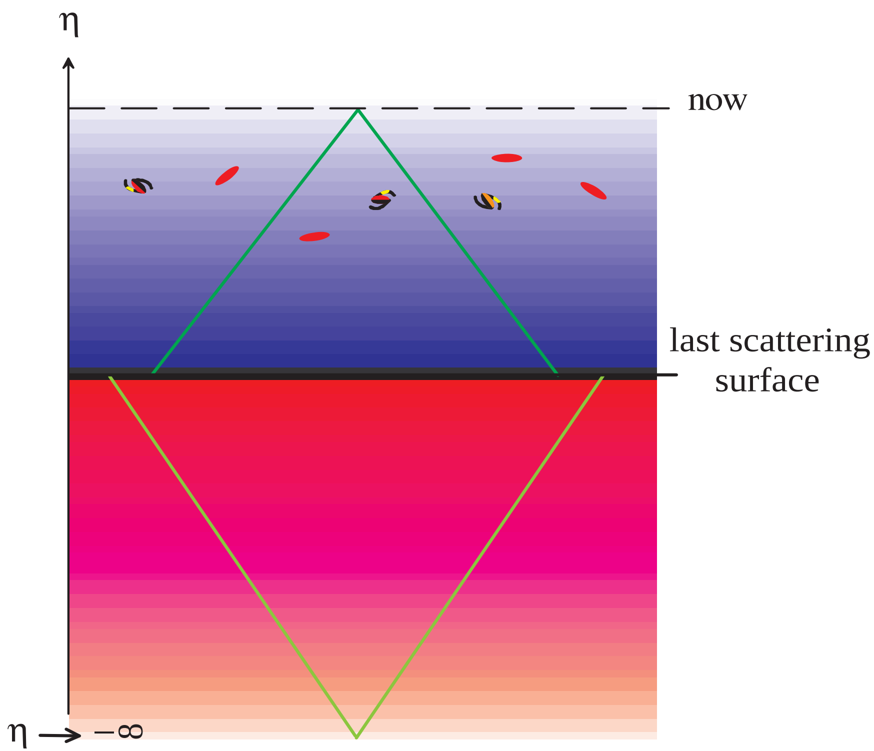
(η: conformal time $\cdots$ maintains causality)



$d\eta = \pm dx$: light ray

$$\eta = \int \frac{dt}{a} \longrightarrow 0 \quad \text{for } t \longrightarrow 0$$

- Solution to the horizon problem



Existence of a stage $a \propto t^n$ $\boxed{n > 1}$
in the early universe

$$\Leftrightarrow \quad \rho + 3p < 0$$

$$\Rightarrow \quad \int_0^t \frac{dt}{a} = \int d\eta = \infty \text{ !!}$$

- Entropy problem (= flatness problem)

Entropy within the curvature radius: $N_\gamma \sim$ conserved

$$N_\gamma = n_\gamma \left(\frac{a}{\sqrt{|K|}} \right)^3 \sim \left(\frac{T_0}{H_0} \right)^3 |1 - \Omega_0|^{-3/2} > \left(\frac{T_0}{H_0} \right)^3 \sim 10^{87}$$

$T_0 \sim 10^{-4}\text{eV} \quad H_0 \sim 10^{-33}\text{eV}$

Where does this big number come from?

"Huge entropy production in the early universe"

§2. Single-field slow-roll inflation

Universe dominated by a scalar field:

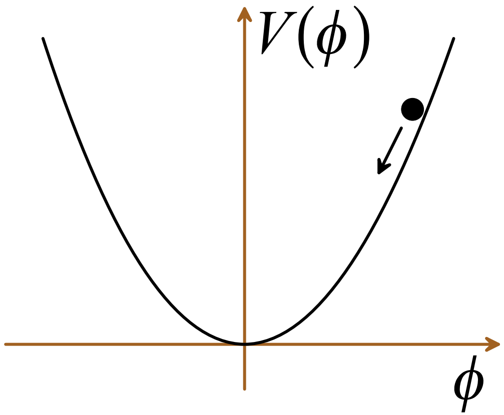
$$\begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases} \Rightarrow \quad \rho + 3p = 2(\dot{\phi}^2 - V(\phi))$$

$$\text{if} \quad \dot{\phi}^2 < V(\phi) \quad \Longrightarrow \quad \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

accelerated expansion

e.g., Chaotic inflation (or Creation of Universe from nothing)

(Linde, Vilenkin, Hartle-Hawking, $\cdots$)



$$\rho_{\rm initial} \lesssim M_{pl}^4 \approx \left(10^{19}\,\text{GeV}\right)^4$$

$$M_{pl} = 1/\sqrt{8\pi G}$$

$\cdots$ quantum gravitational

if $V''(\phi) \ll M_{pl}^2$, then $\phi \gg M_{pl}$

- Equations of motion:

$$\ddot{\phi} + \underbrace{3H\dot{\phi}}_{\text{friction}} + V'(\phi) = 0 \quad (e.g.,\ H \lesssim M_{pl} \text{ initially in chaotic inflation})$$

$$\Rightarrow \quad \boxed{\dot{\phi} \approx -\frac{V'}{3H}} \quad \text{(slow roll (1))} \quad \Leftrightarrow \quad \left|\frac{\ddot{\phi}}{3H\dot{\phi}}\right| \ll 1$$

$$\begin{cases} \dot{H} = -4\pi G(\rho + p) = -4\pi G\dot{\phi}^2 \\ H^2 = \dfrac{8\pi G}{3}\left(\dfrac{1}{2}\dot{\phi}^2 + V(\phi)\right) \end{cases}$$

$$\Rightarrow \quad \boxed{H^2 \approx \frac{8\pi G}{3}V(\phi)} \quad \text{(potential dominated (2))} \quad \Leftrightarrow \quad \left|\frac{\dot{H}}{H^2}\right| \approx \frac{3\dot{\phi}^2}{2V(\phi)} \ll 1$$

The slow-roll condition (1) is satisfied, provided that

$$\frac{\eta}{3} \equiv \frac{V''}{9H^2} \approx M_{pl}^2 \frac{V''}{3V} \ll 1\,, \quad \frac{\epsilon}{3} \equiv \frac{\dot{H}}{3H^2} \approx M_{pl}^2 \frac{V'^2}{6V^2} \ll 1$$

· Slow-roll inflation assumes that the above two are fulfilled.

η, ϵ $\cdots$ slow-roll parameters (Liddle & Lyth 1992)

(Note that the above are not necessary but sufficient conditions.)

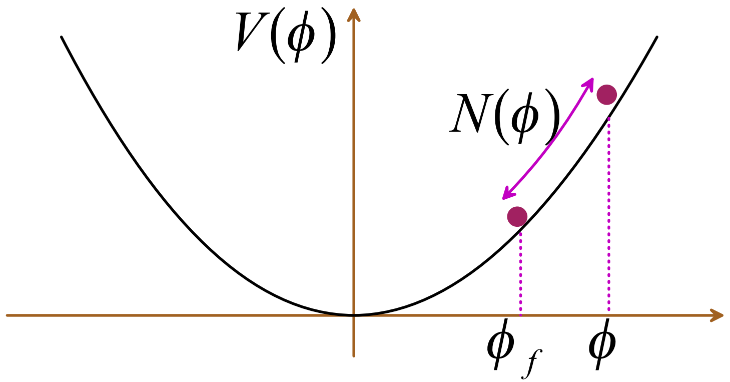
· There are models that violate either or both of the above two conditions.

(Need special care when calculating the perturbations)

- e-folding number of inflation $\quad a \propto e^{-N}$

$$N(\phi) = \int_t^{t_f} H\,dt = \int_\phi^{\phi_f} \frac{H}{\dot\phi} d\phi \approx \frac{1}{M_{pl}^2} \int_{\phi_f}^{\phi} \frac{V}{V'} d\phi = \frac{\phi^2 - \phi_f^2}{4M_{pl}^2} \sim \left(\frac{\phi}{2M_{pl}}\right)^2$$

$\Uparrow$ slow roll $\qquad \Uparrow$ $V = \frac{1}{2} m^2 \phi^2$



$$\left(\int_{\phi_f}^{\phi} \frac{V}{V'} d\phi = N(\phi) M_{pl}^2 \gg M_{pl}^2 \right)$$

problem?

For $V(\phi) \sim (10^{15}\mathrm{GeV})^4$, $N(\phi) \gtrsim 60$ solves horizon & flatness problems

$$N(\phi) \gtrsim 60 \quad \text{at} \quad \phi \gtrsim 16 M_{pl} \quad \text{for} \quad V = \frac{1}{2} m^2 \phi^2$$

Slow roll ends at $\phi \lesssim M_{pl}$ $\quad \Rightarrow \quad$ Reheating $\quad$ (entropy generation)

§3. Generation of cosmological perturbations

Action: $$S = \int d^4x \sqrt{-g} \left(\frac{M_{pl}^2}{2} R - \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right).$$

Cosmological perturbations are generated from quantum (vacuum) fluctuations of the inflaton ϕ and the metric $g_{\mu\nu}$.

- Scalar-type (density) perturbations
  - $g_{\mu\nu}$ and ϕ:

$$ds^2 = a^2 \Big[-(1+2A) d\eta^2 - 2\partial_i B \, d\eta dx^j + \Big((1+2\mathcal{R})\delta_{ij} + 2\partial_i \partial_j H_T \Big) dx^i dx^j \Big],$$
$$\phi(t, x^i) = \phi(t) + \chi(t, x^i)$$

A : Lapse function ($\sim$ time coordinate) perturbation

B : Shift vector ($\sim$ space coordinate) perturbation

Scalar perturbation has 2 degrees of coordinate gauge freedom.

$\mathcal{R}$: Spatial curvature (potential) perturbation $\left(\delta \overset{(3)}{R} = -\frac{4}{a^2} \overset{(3)}{\Delta} \mathcal{R} \right)$

H_T : Shear of the metric ($\sim$ traceless part of the extrinsic curvature)

No dynamical degree of freedom in the metric itself.

⋆ Action expanded to 2nd order (in the Hamiltonian form)

Makino & MS (1991), Garriga, Montes, MS & Tanaka (1998)

$$S_2 = \int d\eta\, d^3x \left(\sum_a P_a\, Q'_a - \mathcal{H}_s - A\, C_A - B\, C_B \right)$$

$$\mathcal{H}_s = \frac{1}{2a^2} P_\chi^2 - 4\pi G \phi' P_{\mathcal{R}} \chi + \cdots\,, \quad {}' = d/d\eta\,,$$
$$C_A = \phi'\, P_\chi + \cdots \quad \text{(Hamiltonian constraint)}\,,$$
$$C_B = P_{H_T} \quad \text{(Momentum constraint)}\,,$$
$$Q_a = \{\mathcal{R}, H_T, \chi\}, \quad P_a = \{\text{Momentum conjugate to } Q_a\}\,.$$

· Gauge transformation $[\xi^\mu = (T, \partial_i L)\,]$ is generated by C_A and C_B:

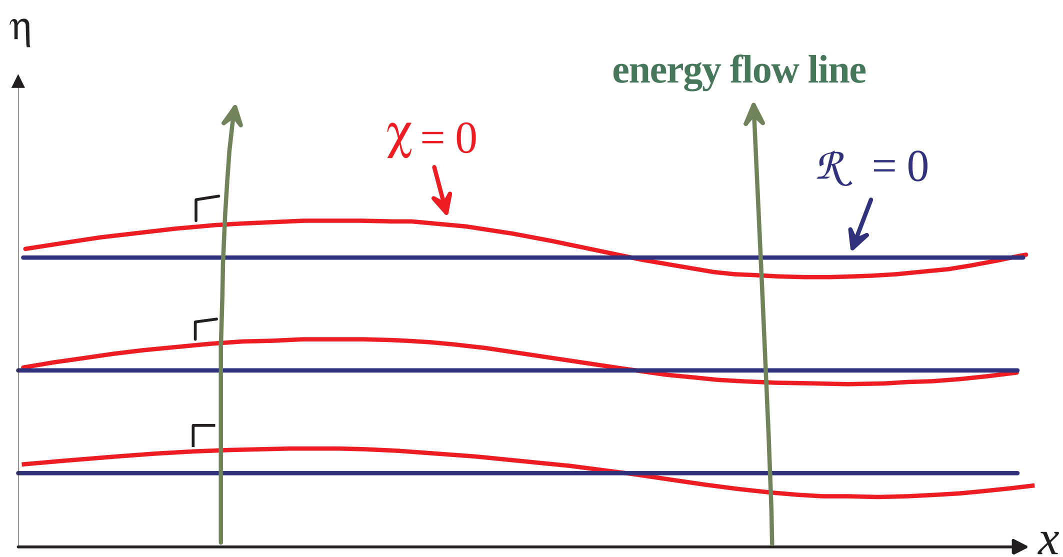
$$\delta_g Q = \left\{ Q, \int (T C_A + L C_B)\, d^3x \right\}_{P.B.}$$

- Reduction to unconstrained variables *a lá* Faddeev-Jackiw (1988)

1. Solve $C_A = \phi' P_\chi + \cdots = 0$ with respect to P_χ and insert it into S_2.
   Also, insert $C_B = P_{H_T} = 0$ into S_2.
2. The resulting S_2 is a functional of $\{P_\mathcal{R}, \mathcal{R}, \chi\}$: $\quad S_2^* = S_2^*\left[P_\mathcal{R}, \mathcal{R}, \chi\right]$
3. Since S_2 is gauge-invariant, S_2^* must be written solely in terms of gauge-invariant variables. Indeed, we find

$$S_2^* = S_2^*\left[P_c, \mathcal{R}_c\right]\,; \quad P_c \equiv P_\mathcal{R} + \frac{a^2}{4\pi G\phi'}\overset{(3)}{\Delta}\chi\,, \quad \mathcal{R}_c \equiv \mathcal{R} - \frac{\mathcal{H}}{\phi'}\chi$$

   This is in fact the same as choosing $\chi = 0$ gauge (called 'comoving' slicing).
   i.e., $\mathcal{R}_c$ is the curvature perturbation on the comoving hypersurface.



⋆ S_2^* in the Lagrangian form:

$$S_2^* = \int d\eta\, d^3x\, \frac{a^2\phi'^2}{2\mathcal{H}^2}\left(\mathcal{R}_c'^2 - (\nabla\mathcal{R}_c)^2\right); \quad \mathcal{H} \equiv \frac{a'}{a} = a\,H$$

Equation of motion (for Fourier modes: $\overset{(3)}{\Delta} \to -k^2$)

$$\mathcal{R}_c'' + 2\frac{z'}{z}\mathcal{R}_c' + k^2\mathcal{R}_c = 0\,; \quad z \equiv \frac{a\phi'}{\mathcal{H}} = \frac{a\dot{\phi}}{H}\ (\sim a \text{ for slow-roll inflation})\,.$$

For $k < \mathcal{H}$ ($\Leftrightarrow\ k/a < H$),

$$\mathcal{R}_c' \propto \begin{cases} z^{-1} & \sim \text{decaying mode} \\ 0 & \sim \text{growing mode} \end{cases}$$

· Growing mode of $\mathcal{R}_c$ stays constant on super-horizon scales.

· This holds for adiabatic perturbations in general cosmological models.
(i.e., the existence of a constant mode)

But this does not mean that adiabatic $\mathcal{R}_c$ is constant on super-horizon scales

unless the decaying mode decays fast enough after horizon-crossing.

- Inflaton perturbation on flat slicing

Alternatively, in terms of χ on $\mathcal{R}=0$ hypersurface (flat slicing),

$$\chi_F \equiv \chi - \frac{\phi'}{\mathcal{H}}\mathcal{R} = -\frac{\phi'}{\mathcal{H}}\mathcal{R}_c$$

$$S_2^* = S_2^*[\chi_F] = \int d\eta\, d^3x\, \frac{a^2}{2}\left(\chi_F'^{\,2} - (\nabla\chi_F)^2 - a^2 m_{eff}^2\,\chi_F^2\right);$$

$$m_{eff}^2 = -\frac{\left\{a^2\left(\phi'/\mathcal{H}\right)'\right\}'}{a^4\left(\phi'/\mathcal{H}\right)} = \partial_\phi^2 V + \frac{2}{M_{pl}^2}\frac{d}{dt}\left(\frac{V}{H}\right)$$

$\chi_F \sim$ minimally coupled almost massless scalar in de Sitter space

$\because\ \partial_\phi^2 V \ll H^2$, $2M_{pl}^{-2}(V/H)^{\dot{}} \approx 6\dot{H} \ll H^2$ for slow-roll inflation.

(N.B. the sufficient conditions for slow roll were $\partial_\phi^2 V \ll 3H^2$ and $\dot{H} \ll 3H^2$.)

· de Sitter approximation for the background:

$$H = \text{const.}\,, \quad a(\eta) = \frac{1}{-H\eta} \quad (-\infty < \eta < 0)$$

This is a good approximation for $k > \mathcal{H}$ (sub-horizon scale) modes

- Canonical quantization

$$\pi(\eta,\vec{x}) = \frac{\delta S_2^*[\chi_F]}{\delta \chi_F'(\eta,\vec{x})}\,, \quad [\chi_F(\eta,\vec{x}), \pi(\eta,\vec{x}')] = i\delta(\vec{x}-\vec{x}')$$

$$\Rightarrow \quad \hat{\chi}_F = \int \frac{d^3k}{(2\pi)^{3/2}} \left(\hat{a}_{\vec{k}}\, \chi_k(\eta)\, e^{i\vec{k}\cdot\vec{x}} + \text{ h.c.} \right); \quad [\hat{a}_{\vec{k}},\, \hat{a}^\dagger_{\vec{k}'}] = \delta(\vec{k}-\vec{k}')$$

$$\chi_k'' + 2\mathcal{H}\chi_k' + \left(k^2 + m_{eff}^2 a^2\right)\chi_k = 0\,; \quad \chi_{\vec{k}}\,\bar{\chi}_{\vec{k}}' - \chi_{\vec{k}}'\,\bar{\chi}_{\vec{k}} = \frac{i}{a^2}$$

$$\Leftrightarrow \quad \ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m_{eff}^2\right)\chi_k = 0\,; \quad \chi_{\vec{k}}\,\dot{\bar{\chi}}_{\vec{k}} - \dot{\chi}_{\vec{k}}\,\bar{\chi}_{\vec{k}} = \frac{i}{a^3}$$

(in terms of the cosmic proper time t)

slow roll $\Rightarrow$ $m_{eff}^2 \ll H^2$ $\sim$ massless

de Sitter approximation:

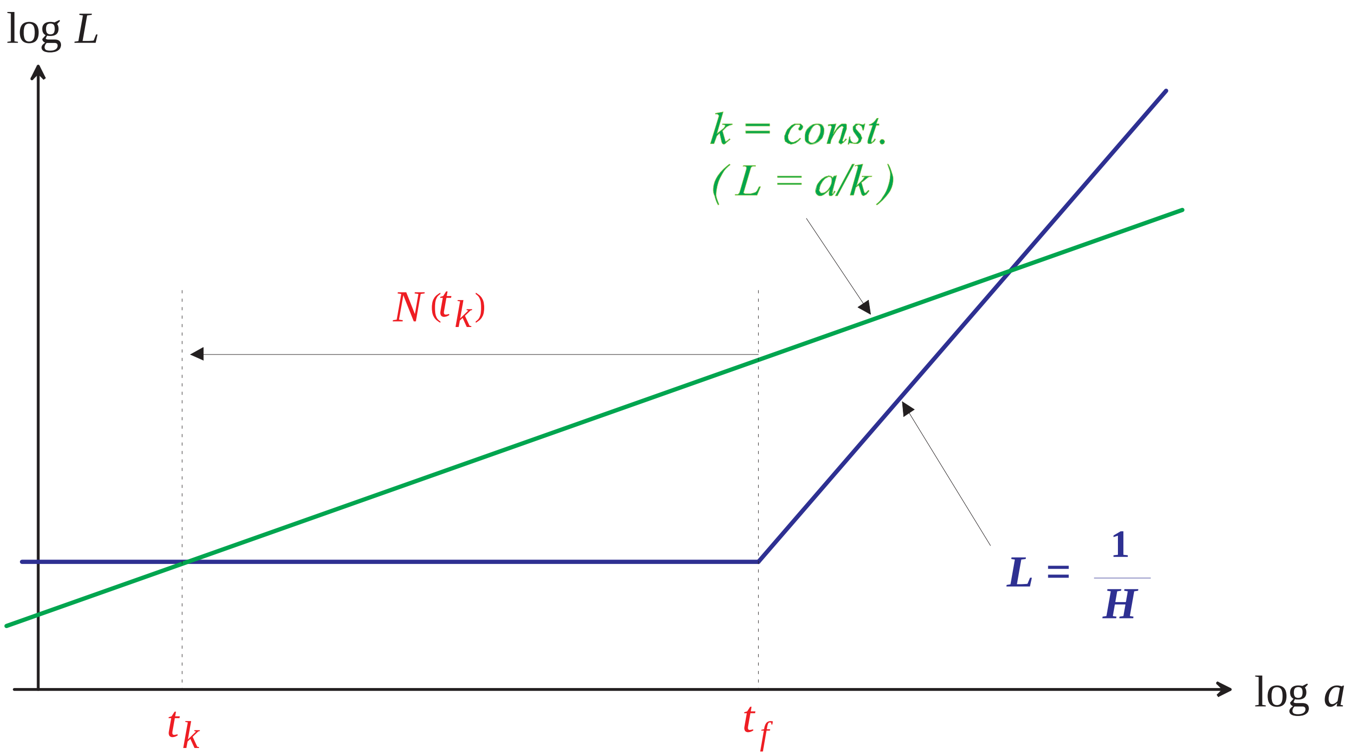
$$\Rightarrow \quad \chi_k \approx \frac{H}{(2k)^{3/2}}(i - k\eta)\, e^{-i\eta} \quad \begin{cases} \xrightarrow[k/\mathcal{H}\to\infty]{} \dfrac{1}{\sqrt{2k}a} e^{-ik\eta} \\[2ex] \xrightarrow[k/\mathcal{H}\to 0]{} \dfrac{H}{\sqrt{2k^3}} e^{-i\alpha_k} \end{cases}$$

$$\langle\delta\phi^2\rangle_k\big|_{\text{on flat slice}} = \langle\chi_F^2\rangle_k \equiv \frac{4\pi k^3}{(2\pi)^3}|\chi_k|^2 \to \left(\frac{H}{2\pi}\right)^2 \quad \text{for} \quad k \lesssim \mathcal{H}$$

· de Sitter approximation breaks down at $k \ll \mathcal{H}$.
i.e., the time-variation of χ_k on super-horizon scales cannot be neglected.

· However, the corresponding mode of $\mathcal{R}_c$ becomes constant on super-horizon scales.

$$\Rightarrow \quad \mathcal{R}_{c,k}(\eta) \approx \mathcal{R}_{c,k}(\eta_k) = -\frac{\mathcal{H}}{\phi'}\chi_k(\eta_k) \approx \frac{H^2(t_k)}{\sqrt{2k^3}\,\dot{\phi}(t_k)}e^{-i\alpha_k}\,.$$



$t = t_k \quad \Leftrightarrow \quad \eta = \eta_k \quad \Leftrightarrow \quad k = \mathcal{H}(\eta_k)$ $\cdots$ horizon crossing time

- Curvature perturbation spectrum (say, at $\eta = \eta_f$)

$$\langle \mathcal{R}_c^2 \rangle_k \equiv \frac{4\pi k^3}{(2\pi)^3} P_{\mathcal{R}_c}(k;\eta) = \frac{4\pi k^3}{(2\pi)^3} \left|\mathcal{R}_{c,k}(\eta)\right|^2 = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 \bigg|_{t=t_k}$$

Since $dN = -H dt$,

$$\frac{\partial N}{\partial \phi} = -\frac{H}{\dot{\phi}} \quad \Rightarrow \quad \langle \mathcal{R}_c^2 \rangle_k = \left(\frac{\partial N}{\partial \phi}\frac{H}{2\pi}\right)^2 \bigg|_{t=t_k} = \left(\frac{\partial N}{\partial \phi}\delta\phi\right)^2 \bigg|_{t=t_k} \text{ on flat slice}$$

That is, for single-field slow-roll inflation,

$$\mathcal{R}_c = \delta N|_{t=t_k} = \frac{\partial N}{\partial \phi}\delta\phi\bigg|_{t=t_k} \quad \left(\delta\phi = \frac{H}{2\pi}\right) \quad \text{on flat slice}$$

The knowledge of the homogeneous background is sufficient to predict the perturbation spectrum: δN-formula

δN-formalism can be extended to the multi-component/nonlinear case.

If $\langle \mathcal{R}_c^2 \rangle_k \propto k^{n_S - 1}$ $\qquad n_S = 1$: scale-invariant (Harrison-Zeldovich) spectrum

- Tensor-type perturbations

$$ds^2 = -dt^2 + \; a^2(t)\,(\delta_{ij} + h_{ij})\,dx^i dx^j$$

h_{ij} $\cdots$ Transverse-Traceless

$$\begin{aligned}\delta^2 S_G \;&= \frac{1}{64\pi G}\int d^4x\,a^3\left(\dot{h}_{ij}^2 - \frac{1}{a^2}(\nabla h_{ij})^2\right)\\ &= \frac{1}{2}\int d^4x\,a^3\left(\dot{\varphi}_{ij}^2 - \frac{1}{a^2}(\nabla\varphi_{ij})^2\right)\;; \quad \varphi_{ij} := \frac{1}{\sqrt{32\pi G}}h_{ij}\end{aligned}$$

$\varphi_{ij} \sim$ massless scalar (2 degrees of freedom)

$$\langle\varphi_{ij}^2\rangle_k = \; 2\times\left(\frac{H}{2\pi}\right)^2$$

$$\Rightarrow\; \frac{4\pi k^3}{(2\pi)^3}P_T(k) \equiv \langle h_{ij}^2\rangle_k = 2\times 32\pi G\times\left(\frac{H}{2\pi}\right)^2 = \frac{8}{\pi}\frac{H^2}{m_{pl}^2}$$

$\uparrow$ contribute to CMB anisotropy

$$\frac{T}{S} = \frac{\text{tensor}}{\text{scalar}} \sim \frac{\langle h_{ij}^2\rangle}{\langle\mathcal{R}_c^2\rangle} \equiv \frac{P_T(k)}{P_S(k)} = 24\left.\frac{\dot{\phi}^2}{V}\right|_{k_0=aH} \qquad \text{slow roll} \quad \Rightarrow \quad \frac{T}{S} \ll 1\,.$$

$$\frac{T}{S} \sim 0.13 \quad \text{for} \quad V = \frac{1}{2}m^2\phi^2 \qquad \text{(small but non-negligible)}$$

§4. Power Spectrum

∗ scalar-type (curvature) perturbation

$$n_S \equiv 1 + \frac{d\ln[P_{\mathcal{R}}(k)k^3]}{d\ln k}.$$

$$k = a(t_k)H \quad \rightarrow \quad d\ln k = \frac{da}{a} + \frac{dH}{H} \approx \frac{da}{a} = \left.\frac{d}{Hdt}\right|_{t=t_k}.$$

For slow-roll inflation,

$$n_S - 1 = \frac{d}{Hdt}\ln[P_{\mathcal{R}}(k)k^3] = \frac{d}{Hdt}\left(\ln H^4 - \ln\dot{\phi}^2\right) \approx M_{pl}^2\left(2\frac{V''}{V} - 3\frac{V'^2}{V^2}\right).$$

For a chaotic type potential, $V \propto \phi^p$ $(p > 0)$, Liddle & Lyth (1992)

$$n_S = 1 - p(p+2)\frac{M_{pl}^2}{\phi^2} = 1 - \frac{p+2}{2N(\phi)} < 1 \quad \Leftarrow \quad \text{redder for larger } p$$

∗ tensor-type perturbation

$$n_T \equiv \frac{d\ln[P_T(k)k^3]}{d\ln k} = \frac{d}{Hdt}\ln[P_T(k)k^3] = \frac{d}{Hdt}\ln H^2 = 2\frac{\dot{H}}{H^2}$$

$$\approx -3\frac{\dot{\phi}^2}{V} = -\frac{1}{8}\frac{P_T(k)}{P_S(k)} \quad \Leftarrow \quad \text{consistency relation!}$$

- Other Models

  ∗ power-law inflation

$$V(\phi) \propto \exp[\lambda\phi/m_{pl}] \ \leftarrow \ \text{dilaton in string theories ?}$$

$$a \propto t^{\alpha} \quad (\alpha = \frac{16\pi}{\lambda^2})$$

$$\Rightarrow \quad n_S < 1\,, \qquad \frac{T}{S} \gtrsim 0.1$$

  ∗ hybrid inflation ← supergravity-motivated ?

e.g., $$V(\phi, \psi) = \frac{1}{4\lambda}\left(M^2 - \lambda\psi^2\right)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\psi^2$$

$a \propto e^{Ht}$, $\quad H^2 \approx \frac{8\pi G}{3}V_0$ when $\psi = 0$, $\phi > M/g$.

$$\Rightarrow \quad n_S > 1\,, \qquad \frac{T}{S} \ \text{can be large or small.}$$

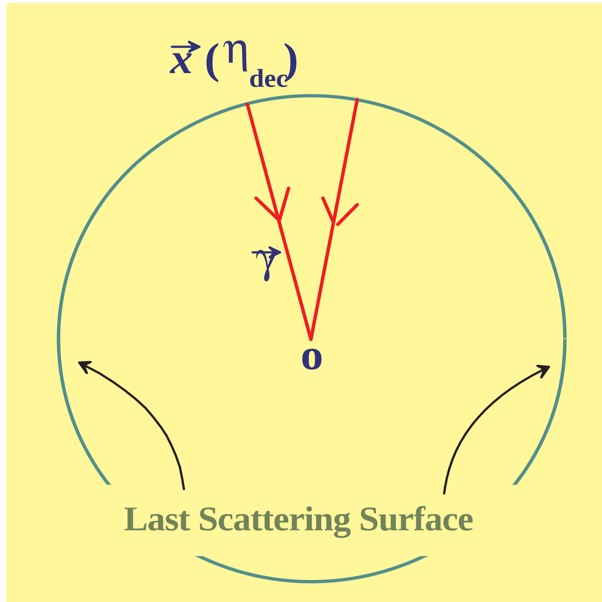
§5. Large angle CMB anisotropy

$$\left(\frac{\delta T}{T}\right)(\vec{\gamma},\eta_0) = (\Theta_s + \Psi)(\eta_{\rm dec}, \vec{x}(\eta_{\rm dec})) + \int_{\eta_{\rm dec}}^{\eta_0} d\eta\, 2\partial_\eta \Psi(\eta, \vec{x}(\eta)) + \text{Doppler} + \cdots$$

(Sachs-Wolfe) (Integrated Sachs-Wolfe)

Θ_s : temperature fluctuation on Newtonian (longitudinal) slices

Ψ : Newton potential



For a dust-dominated universe at decoupling,

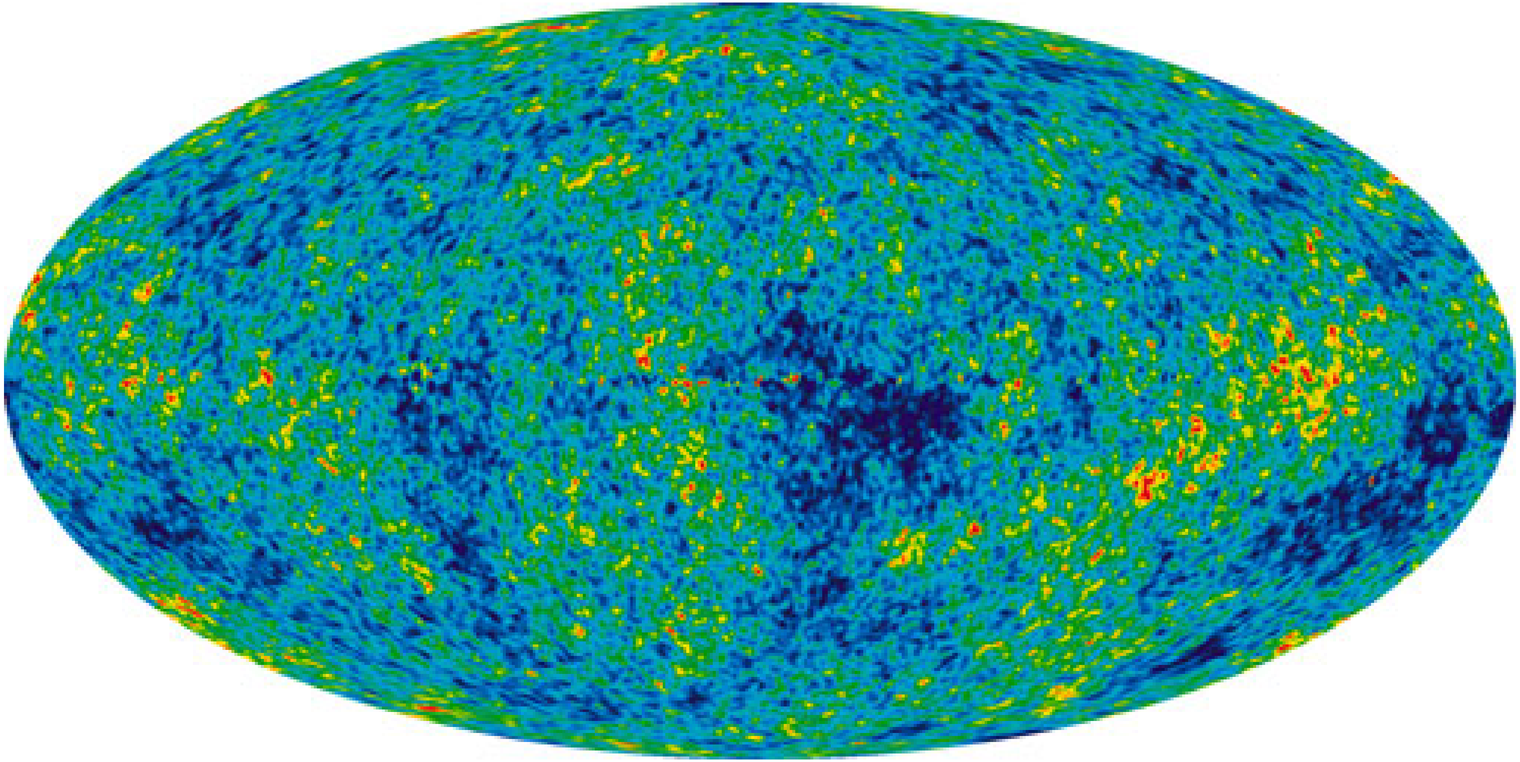
$$\text{SW: } \Theta_s + \Psi = -\frac{1}{5}\mathcal{R}_{c*} - \frac{2}{5}S_{\rm dr}\,,$$

$$= \begin{cases} \frac{1}{3}\Psi & \text{for adiabatic} \\ 2\Psi & \text{for isocurvature} \end{cases}$$

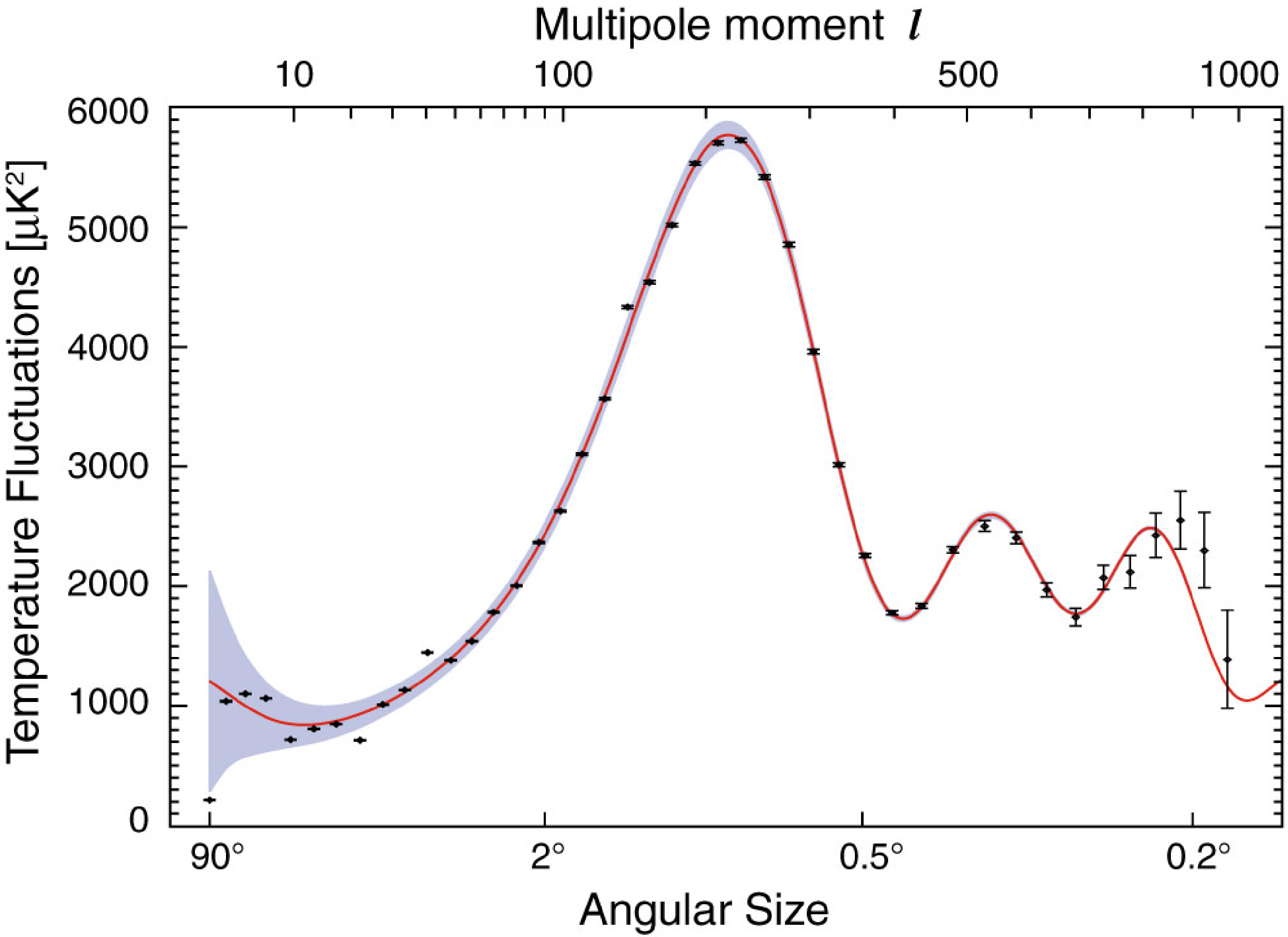
ISW: $\partial_\eta \Psi \approx 0$

$\mathcal{R}_{c*}$: primordial adiabatic curvature perturbation

$$S_{\rm dr} = \frac{\delta\rho_{\rm d}}{\rho_{\rm d}} - \frac{3}{4}\frac{\delta\rho_{\rm r}}{\rho_{\rm r}} \quad \sim \text{ entropy perturbation}$$



WMAP 7 year Full Sky Map



WMAP 5 year Angular Power Spectrum

- Observational implications of Large-angle CMB anisotropy

COBE (1996), WMAP 1st year (2003) $\sim$ 7 year (2008)

$$\left\langle \left(\frac{\delta T}{T}\right)^2 \right\rangle \sim 10^{-10} \quad \text{at } \theta \sim 10^\circ$$

$$\Downarrow$$

$$\langle \Psi^2 \rangle_k \sim 10^{-10} \quad \text{at} \quad \frac{k_0}{a_0} = H_0 \sim \frac{1}{\text{present horizon scale}} \qquad \left(H_0^{-1} \sim 3000\,\mathrm{Mpc} \sim 10^{28}\mathrm{cm}\right)$$

For $V = \frac{1}{2} m^2 \phi^2$,

$$\langle \Psi^2 \rangle_{k_0} \approx \left(\frac{3}{5}\right)^2 \langle \mathcal{R}_c^2 \rangle_{k_0} = \left(\frac{3}{5}\right)^2 \left(\frac{H^2}{2\pi \dot{\phi}}\right)^2 \Bigg|_{\frac{k_0}{a}=H} \approx 5 \frac{m^2}{M_{pl}^2} \left(\frac{N(\phi)}{60}\right)^2 \Bigg|_{\frac{k_0}{a}=H}$$

$$\Rightarrow \quad m \sim 10^{13}\mathrm{GeV}\,, \quad V \sim (10^{16}\mathrm{GeV})^4$$

- power-law index: $n_S \approx 0.96$ for $V = \frac{1}{2} m^2 \phi^2$

Observation: $n_{\mathrm{WMAP},1^{yr}} = 0.99 \pm 0.04 \quad \Rightarrow \quad n_{\mathrm{WMAP},7^{yr}} = 0.962 \pm 0.013$

Consistent with the simplest (quadratic potential) chaotic inflation model.
Note that the value n_{WMAP} is a result of Bayesian (likelihood) analysis.
Other models are not excluded at all.

§6. Summary on single-field slow-roll inflation

- The (adiabatic) growing mode of the curvature perturbation on comoving slices $\mathcal{R}_c$ stays constant super-horizon scales.
  - $\mathcal{R}_c = \delta N(\phi)$ at $\phi = \phi(t_k)$ in the slow-roll case.
  - $\mathcal{R}_c$ may vary in time if the slow-roll condition is violated.
  - Slow-roll models predict almost scale-invariant spectrum, but other spectral shapes are possible.
  - The simplest chaotic inflation model with quadratic potential is marginally consistent with current observational data (WMAP 7 year).
- Tensor perturbations may or may not be non-negligible.