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Anisotropic inflation and its observational predictions

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Watanabe, Kanno, JS, arXiv:0902.2833; PRL 102, 191302, 2009

Kanno, Watanabe, JS, arXiv:0908.3509; JCAP 0912:009, 2009

Watanabe, Kanno, JS, arXiv:1003.0056; Prog. Theor. Phys. 123, 1041, 2010

The nature of primordial fluctuations

The large scale structure originates from quantum fluctuations during inflation.

We need infinite number of correlation functions to characterize curvature perturbations

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = P(\mathbf{k}_1,\mathbf{k}_2)$$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = B(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\zeta(\mathbf{k}_4)\rangle = T(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)$$

$$\vdots$$

slow roll condition implies Gaussian statistics $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = P(\mathbf{k}_1,\mathbf{k}_2)$

Initial condition leads to statistical homogeneity $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1)$

cosmic no-hair suggests statistical isotropy $\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(k_1=|\mathbf{k}_1|)$

time translation invariance of deSitter yields $P(k) \approx const.$

These predictions should be robust if we do not require a percent level accuracy.

Precision tests of inflation

Precision cosmology forces us to look at fine structures of fluctuations!

Gaussian fluctuations

It is possible to have small non-gaussianity if we consider non-minimal inflationary scenarios.

Statistically homogeneous and isotropic

Almost Scale free (flat spectrum)

There should be a slight tilt because the expansion is not exactly deSitter.

Primordial GW exists independently

Logically, it is legitimate to seek a percent level deviation from the statistical homogeneity and isotropy.

Here, we concentrate on the statistical anisotropy because of the following theoretical motivation.

$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\rangle = \delta(\mathbf{k}_1+\mathbf{k}_2)P(\mathbf{k}_1)$$

Gauge kinetic function in the sky

Superstring theory —low energy→ Supergravity

- Kahler potential K $g_{i\bar{j}} = \frac{\partial^2 K}{\partial \phi^i \partial \phi^{\bar{j}}}$
- Superpotential W $D_i W = \frac{\partial W}{\partial \phi^i} + \kappa^2 \frac{\partial K}{\partial \phi^i} W$
- Gauge kinetic function f

$$S = \int d^4x \left[\sqrt{-g} R + g_{i\bar{j}} \partial^\mu \phi^i \partial_\mu \phi^{\bar{j}} - e^{\kappa^2 K} g^{i\bar{j}} \left(D_i W D_j \bar{W} - 3\kappa^2 |W|^2 \right) - \frac{1}{4} \mathrm{Re}\, f_{ab}(\phi^i) F^{a\mu\nu} F^b_{\mu\nu} + \cdots \right]$$

Inflation gives the cosmological tests for K and W.

So far, the roll of vector fields in inflationary scenario has been overlooked.
If we take into account this term, the statistical anisotropy can be expected!!

The purpose of this talk is to convince you that

- **anisotropic inflation is naturally realized in supergravity**
- **statistical anisotropy is easily produced in supergravity**
- **there is cross correlation between scalar and tensor perturbations**
- **inflation can constrain the gauge kinetic functions!**

Plan of my talk

1. Inflation with a gauge kinetic function
2. Cosmological perturbation theory
   in a simple Bianchi universe
3. The nature of primordial fluctuations
   in anisotropic inflation
4. Future issues

Inflation with a gauge kinetic function

A simple model

Action

gauge kinetic function

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\frac{1}{2}\left(\partial_\mu\phi\right)^2-V(\phi)-\frac{1}{4}f^2(\phi)F_{\mu\nu}F^{\mu\nu}\right]$$

Scalar   Vector

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$$

For homogeneous background, the time component can be eliminated by gauge transformation.
Let the direction of the vector be x - axis

$$A_\mu=(0,\ v(t),\ 0,\ 0) \qquad \phi=\phi(t)$$

Then, the metric should be Bianchi Type-I

Anisotropy

$$ds^2=-dt^2+e^{2\alpha(t)}\left[e^{-4\sigma(t)}dx^2+e^{2\sigma(t)}\left(dy^2+dz^2\right)\right]$$

Scale Factor   Plane Symmetry

The action reduces to

$$S=\int d^4x\, e^{3\alpha}\left[\frac{3}{\kappa^2}(-\dot{\alpha}^2+\dot{\sigma}^2)+\frac{1}{2}\dot{\phi}^2-V(\phi)+\frac{1}{2}f^2(\phi)e^{-2\alpha+4\sigma}\dot{v}^2\right]$$

$$\dot{v}=f^{-2}(\phi)e^{-\alpha-4\sigma}E$$

← const. of integration

Basic equations

Hamiltonian Constraint $\bullet = \partial_t$

$$\dot{\alpha}^2 = \dot{\sigma}^2 + \frac{\kappa^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{E^2}{2} f^{-2}(\phi) e^{-4\alpha-4\sigma}\right]$$

Scale factor

$$\ddot{\alpha} = -3\dot{\alpha}^2 + \kappa^2 V(\phi) + \frac{\kappa^2 E^2}{6} f^{-2}(\phi) e^{-4\alpha-4\sigma}$$

Anisotropy

$$\ddot{\sigma} = -3\dot{\alpha}\dot{\sigma} + \frac{\kappa^2 E^2}{3} f^{-2}(\phi) e^{-4\alpha-4\sigma}$$

Scalar field $' = \partial_\phi$

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - V'(\phi) + E^2 f^{-3}(\phi) f'(\phi) e^{-4\alpha-4\sigma}$$

Behavior of the vector is determined by the coupling

slow roll equations

$$\dot{\alpha}^2 = \frac{\kappa^2}{3}\left[V(\phi) + \frac{E^2}{2} f^{-2}(\phi) e^{-4\alpha-4\sigma}\right]$$

$$3\dot{\alpha}\dot{\phi} = -V'(\phi) + E^2 f^{-3}(\phi) f'(\phi) e^{-4\alpha-4\sigma}$$

$$\frac{d\alpha}{d\phi} = \frac{\dot{\alpha}}{\dot{\phi}} = -\kappa^2 \frac{V(\phi)}{V'(\phi)}$$

$$\Rightarrow \quad \alpha = -\kappa^2 \int \frac{V}{V'} d\phi$$

$$V = \frac{m^2}{2}\phi^2 \quad \Rightarrow \quad e^{-4\alpha} = e^{\kappa^2\phi^2}$$

This motivate us to take the gauge kinetic function in the following form $f(\phi) = e^{c\kappa^2\phi^2/2}$

Hamiltonian Constraint $\quad \dot{\alpha}^2 = \frac{\kappa^2}{3}\left[\frac{m^2}{2}\phi^2 + \frac{E^2}{2} e^{-c\kappa^2\phi^2-4\alpha-4\sigma}\right]$

Hence, the ratio of the energy density grows as

$$\mathcal{R} \equiv \frac{\rho_A}{\rho_\phi} = \frac{E^2 e^{-c\kappa^2\phi^2-4\alpha}}{m^2\phi^2} \propto e^{4(c-1)\alpha} \qquad c > 1$$

Attractor mechanism

Once the vector contributes the dynamics of the inflaton field, the ratio does not increase any more

$$\ddot{\phi} = -3\dot{\alpha}\dot{\phi} - m^2\phi + c\kappa^2 E^2 \phi\, e^{-c\kappa^2\phi^2 - 4\alpha - 4\sigma}$$

The opposite force to the mass term

Hence, the growth should be saturated around $c\kappa^2 E^2 e^{-c\kappa^2\phi^2 - 4\alpha} \approx m^2$

$$\mathcal{R} \equiv \frac{\rho_A}{\rho_\phi} = \frac{E^2 e^{-c\kappa^2\phi^2 - 4\alpha}}{m^2\phi^2} \quad \Longrightarrow \quad \mathcal{R} \approx \frac{1}{c\kappa^2\phi^2}$$

Typically, inflation takes place at $\kappa\phi \approx \mathcal{O}(10)$

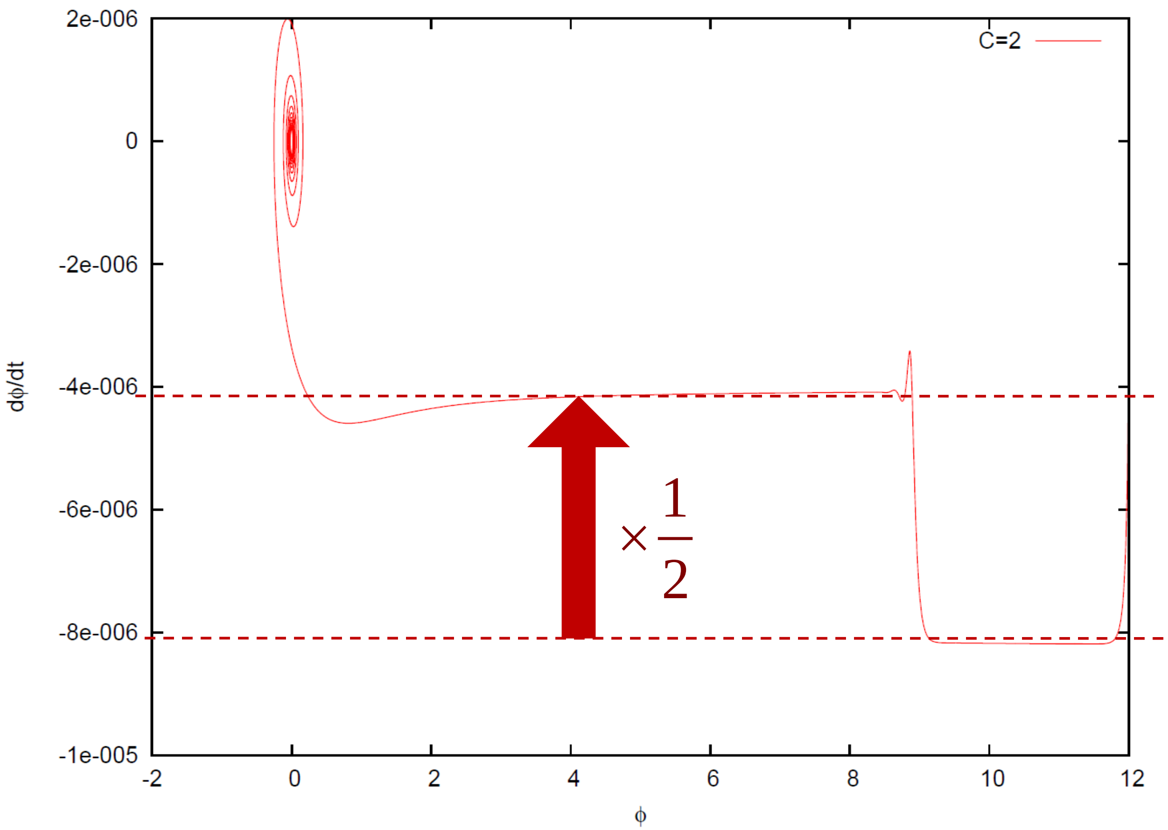
Thus, irrespective of initial conditions, we have $\mathcal{R} \approx 10^{-2}$

Phase flow: Inflaton

Scalar field

$$\begin{cases} 3\dot{\alpha}\dot{\phi} = -m^2\phi & \text{(1st inflationary phase)} \\ 3\dot{\alpha}\dot{\phi} = -\dfrac{m^2}{c}\,\phi & \text{(2nd inflationary phase)} \end{cases}$$

Numerically solution at $c = 2$ $\quad m = 10^{-5}\kappa^{-1}$ $\quad \sqrt{c}\kappa\phi_0 = 17$



The degree of Anisotropy

Assuming the slow roll, the equation for anisotropy is given by $3\dot{\alpha}\dot{\sigma} = \frac{\kappa^2 E^2}{3} e^{-c\kappa^2\phi^2 - 4\alpha}$

The degree of anisotropy is determined by

$$\frac{\Sigma}{H} \equiv \frac{\dot{\sigma}}{\dot{\alpha}} = \frac{\kappa^2 E^2 e^{-c\kappa^2\phi^2-4\alpha}}{9\dot{\alpha}^2} = \frac{2E^2 e^{-c\kappa^2\phi^2-4\alpha}}{3m^2\phi^2}$$

where we used $3\dot{\alpha}^2 = \frac{\kappa^2}{2} m^2 \phi^2$

Attractor Point

$$e^{-c\kappa^2\phi^2-4\alpha} = \frac{m^2(c-1)}{c^2\kappa^2 E^2}$$

$$\Rightarrow \quad \frac{\Sigma}{H} = \frac{2}{3}\frac{c-1}{c^2\kappa^2\phi^2}$$

The slow-roll parameter is given by $\varepsilon \equiv -\frac{\ddot{\alpha}}{\dot{\alpha}^2} = \frac{2}{c\kappa^2\phi^2}$

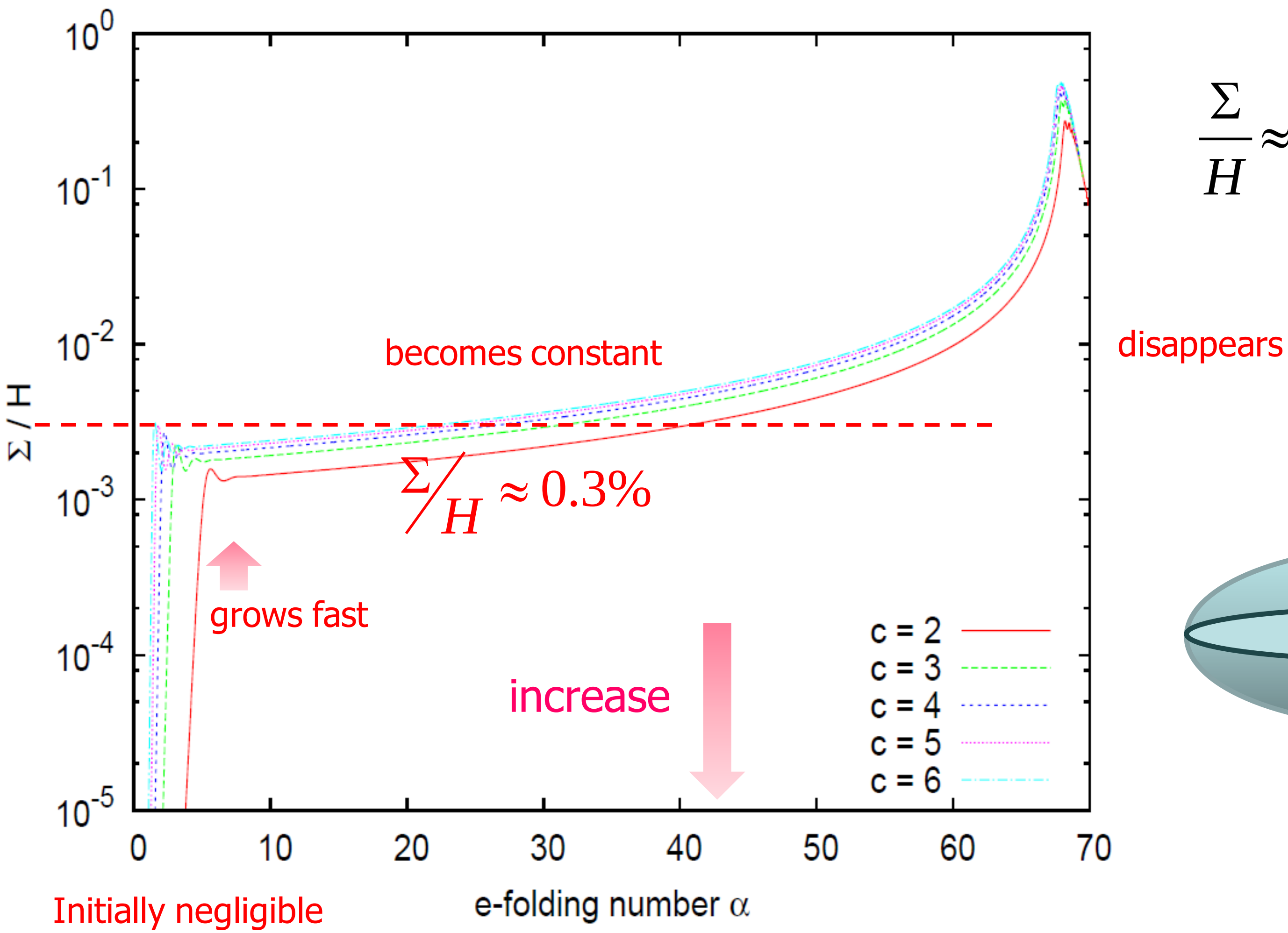
We find that the degree of anisotropy is written by the slow-roll parameter.

$$\frac{\Sigma}{H} = \frac{1}{3}\frac{c-1}{c}\varepsilon$$

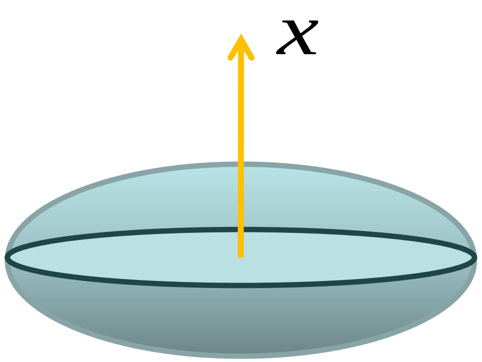
: A universal relation

Evolutions of the degree of anisotropy

Numerically solution at $\sqrt{c}\kappa\phi_0 = 17$



$$\frac{\Sigma}{H} \approx \mathcal{R}(t) = \frac{\rho_A}{\rho_\phi}$$



COSMOLOGICAL PERTURBATION THEORY IN A SIMPLE BIANCHI UNIVERSE

Flat slicing gauge in anisotropic universe

In our case, we have only 2-dimensional rotational symmetry

$$ds^2 = a^2(\eta)\left[-d\eta^2 + dx^2\right] + b^2(\eta)\left[dy^2 + dz^2\right]$$

Vector type perturbations can be characterized in a special frame as follows

$$\vec{k}_{2D} = (k_y, 0) \qquad\qquad V^i{}_{,i} = 0 \Rightarrow V_2 = 0$$

There is no tensor type perturbations in 2-d and scalar type perturbations are V_2 , components with no y, z indices, and diagonal matrix.

vector perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & b^2\beta_3 \\ * & 0 & 0 & b^2\Gamma \\ * & * & 0 & 0 \\ * & * & * & 0 \end{pmatrix} \qquad\qquad \delta A_\mu = (0,0,0,D)$$

scalar perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} -2a^2\Phi & a\beta_1 & a\beta_2 & 0 \\ * & 2a^2G & 0 & 0 \\ * & * & 2b^2G & 0 \\ * & * & * & -2b^2G \end{pmatrix} \qquad \delta A_\mu = (\delta A_0, 0, J, 0) \qquad \delta\phi$$

The blue variables are physical.

Quadratic Action

A tedious calculation gives a quadratic action for perturbations

$$S^{\text{vector}} = \int d\eta d^3x \Big[\frac{b^4}{4a^2}\beta_{3,x}^2 + \frac{b^2}{4}\beta_{3,y}^2 - \frac{b^4}{2a^2}\Gamma'\beta_{3,x} + \frac{f^2v'b^2}{a^2}\beta_3 D_{,x} - \frac{b^2}{4}\Gamma_{,y}^2 + \frac{b^4}{4a^2}\Gamma'^2 - \frac{f^2a^2}{2b^2}D_{,y}^2 - \frac{1}{2}f^2D_{,x}^2 + \frac{f^2}{2}D'^2 - \frac{f^2v'b^2}{a^2}D'\Gamma \Big]$$

$$S^{\text{scalar}} = \int d^3x d\eta \Big[\frac{b^2}{2a^2}f^2\delta A_{0,x}^2 + \frac{f^2}{2}\delta A_{0,y}^2 + \frac{b^2}{a^2}f^2v'\left(G+\Phi\right)\delta A_{0,x} - f^2J'\delta A_{0,y} - 2\frac{b^2}{a^2}ff_\phi v'\delta\phi\delta A_{0,x} + \frac{1}{4}\beta_{1,y}^2 - \frac{1}{2}\beta_{2,x}\beta_{1,y} + 2\frac{bb'}{a}\Phi_{,x}\beta_1 - \frac{b^2}{a}\phi'\delta\phi_{,x}\beta_1 + \frac{1}{4}\beta_{2,x}^2 + a\left(\frac{a'}{a}+\frac{b'}{b}\right)\beta_2\Phi_{,y} - a\left(\frac{a'}{a}-\frac{b'}{b}\right)\beta_2 G_{,y} + \frac{f^2}{a}v'\beta_2 J_{,x} - a\phi'\beta_2\delta\phi_{,y} + \frac{1}{2}f^2J'^2 - \frac{1}{2}f^2J_{,x}^2 + b^2G'^2 - a^2G_{,y}^2 - b^2G_{,x}^2 + \frac{1}{2}b^2\delta\phi'^2 - \frac{a^2}{2}\delta\phi_{,y}^2 - \frac{b^2}{2}\delta\phi_{,x}^2 - \frac{1}{2}a^2b^2V_{\phi\phi}\delta\phi^2 + \frac{b^2}{2a^2}\left(f_\phi^2 + ff_{\phi\phi}\right)v'^2\delta\phi^2 - a^2b^2V\Phi^2 + \frac{b^2}{2a^2}f^2v'^2G^2 - 2a^2b^2V\Phi G - 2bb'\Phi'G - \left(\frac{b^2}{a^2}ff_\phi v'^2 + a^2b^2V_\phi\right)\delta\phi\left(G+\Phi\right) + b^2\phi'\delta\phi'\left(G-\Phi\right) \Big]$$

Reduced Quadratic Action: Slow roll Approximation

$$-\frac{\dot{H}}{H^2}=\epsilon_H,\qquad \frac{\Sigma}{H}=\frac{1}{3}I\epsilon_H\qquad \frac{\epsilon_H'}{\epsilon_H}=2\frac{(e^\alpha)'}{e^\alpha}(2\epsilon_H-\eta_H)=2(2\epsilon_H-\eta_H)(-\eta)^{-1}$$

$$\sin\theta=\frac{k_y a}{kb}\qquad I=\frac{c-1}{c}$$

$$\begin{aligned}
S^{\text{vector}} &= \int d\eta d^3k\left[\frac{1}{2}|\bar{\Gamma}'|^2+\frac{1}{2}\left[-k^2+(-\eta)^{-2}\left\{2+3\epsilon_H+3I\epsilon_H+3I\epsilon_H\sin^2\theta\right\}\right]|\bar{\Gamma}|^2\right.\\
&\quad+\frac{1}{2}|\bar{D}'|^2+\frac{1}{2}\left[-k^2+(-\eta)^{-2}\left\{2+9\epsilon_H-3\eta_H+6I\epsilon_H\sin^2\theta\right\}\right]|\bar{D}|^2\\
&\quad\left.+\frac{\sqrt{6I\epsilon_H}}{2}(-\eta)^{-1}\sin\theta(\bar{\Gamma}'\bar{D}^*+\bar{\Gamma}^{*\prime}\bar{D})-\frac{\sqrt{6I\epsilon_H}}{2}(-\eta)^{-2}\sin\theta(\bar{\Gamma}\bar{D}^*+\bar{\Gamma}^*\bar{D})\right]
\end{aligned}$$

vector-tensor

$$\begin{aligned}
S^{\text{scalar}} &= \int d\eta d^3k\left[L^{GG}+L^{JJ}+L^{\phi\phi}+L^{\phi G}+L^{\phi J}+L^{JG}\right],\\
L^{GG} &= \frac{1}{2}|\bar{G}'|^2+\frac{1}{2}\left[-k^2+(-\eta)^{-2}\left\{2+3\epsilon_H+3I\epsilon_H+3I\epsilon_H\sin^2\theta\right\}\right]|\bar{G}|^2,\\
L^{JJ} &= \frac{1}{2}|\bar{J}'|^2+\frac{1}{2}\left[-k^2+(-\eta)^{-2}\left\{2+9\epsilon_H-3\eta_H-6I\epsilon_H\sin^2\theta\right\}\right]|\bar{J}|^2,\\
L^{\phi\phi} &= \frac{1}{2}|\delta\bar{\phi}'|^2+\frac{1}{2}\left[-k^2+(-\eta)^{-2}\left\{2+9\epsilon_H-\frac{3\eta_H}{1-I}-\frac{12I}{1-I}+\left(12I\epsilon_H+\frac{24I}{1-I}\right)\sin^2\theta\right\}\right]|\delta\bar{\phi}|^2,\\
L^{\phi G} &= -3I\sqrt{\frac{\epsilon_H}{1-I}}(-\eta)^{-2}\sin^2\theta\left(\bar{G}\delta\bar{\phi}^*+\bar{G}^*\delta\bar{\phi}\right),\\
L^{\phi J} &= \sqrt{\frac{6I}{1-I}}(-\eta)^{-1}\sin\theta\left(\delta\bar{\phi}^{*\prime}\bar{J}+\delta\bar{\phi}'\bar{J}^*\right)-\sqrt{\frac{6I}{1-I}}(-\eta)^{-2}\sin\theta\left(\delta\bar{\phi}^*\bar{J}+\delta\bar{\phi}\bar{J}^*\right),\\
L^{JG} &= -\frac{\sqrt{6I\epsilon_H}}{2}(-\eta)^{-1}\sin\theta\left(\bar{G}^{*\prime}\bar{J}+\bar{G}'\bar{J}^*\right)+\frac{\sqrt{6I\epsilon_H}}{2}(-\eta)^{-2}\sin\theta\left(\bar{G}^*\bar{J}+\bar{G}\bar{J}^*\right),
\end{aligned}$$

scalar-tensor

vector-scalar

Structure of couplings

The main features of the action can be understood by looking at the following term

Key term $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} f^2(\phi) F_{\mu\nu} F_{\alpha\beta}$$

Notice the following relations

Background quantity $$\frac{f^2 v'^2}{a^2} \approx I\varepsilon_H \qquad \frac{f_\phi}{f} \approx \frac{\kappa^2 V}{V_\phi} \approx \frac{1}{\sqrt{\varepsilon_H}} \qquad I = \frac{c-1}{c}$$

Now, we take variations

vector-tensor $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi) F_{\mu\nu}}_{f^2 v'} F_{\alpha\beta} \qquad f v' \approx \sqrt{I\varepsilon_H}$$

vector-scalar $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi)}_{f f_\phi \delta\phi} \underbrace{F_{\mu\nu}}_{v'} F_{\alpha\beta} \qquad f_\phi v' \approx \frac{f_\phi}{f} f v' \approx \sqrt{I}$$

scalar-tensor $$\sqrt{-g}\, g^{\mu\alpha} g^{\nu\beta} \underbrace{f^2(\phi)}_{f f_\phi \delta\phi} \underbrace{F_{\mu\nu} F_{\alpha\beta}}_{v'^2} \qquad f_\phi v'^2 \approx I\sqrt{\varepsilon_H}$$

The nature of primordial fluctuations in anisotropic inflation

Statistical anisotropy

In slow-roll phase($H \equiv \dot{\alpha}, \Sigma \equiv \dot{\sigma}$ are almost const.) , we have the metric:

$$ds^2 = -dt^2 + e^{2Ht}\left[e^{-4\Sigma t}dx^2 + e^{-2\Sigma t}\left(dy^2 + dz^2\right)\right]$$

Since the expansion is anisotropic, we expect statistically anisotropic fluctuations.

The power spectrum: **Ackerman et al. (2007)**

Deviation from isotropic part depends on $\hat{\mathbf{k}}$

$$P_\psi(\mathbf{k}) = P_0(k)\left[1 + g\,(\hat{\mathbf{k}}\cdot\mathbf{n})^2\right]$$

Isotropic part

may be detectable if

$$g > 0.025 \times \left(\frac{400}{\ell_{\max}}\right)^{1.27} \approx 0.3\% \quad \text{with } \ell_{\max} = 2000$$ (Planck)

Groeneboomn & Eriksen (2008)

In-In Formalism

Interaction picture

$$i\frac{\partial}{\partial\eta}|\eta\rangle = H_I|\eta\rangle\,,\quad |0\rangle = |\eta=\eta_{in}\rangle \qquad \phi_I = \bar{\mathrm{T}}\exp\left(i\int_{\eta_{in}}^{\eta} H_0 d\eta\right)\phi\,\mathrm{T}\exp\left(-i\int_{\eta_{in}}^{\eta} H_0 d\eta\right)$$

$$|\eta\rangle = \sum_{N=0}^{\infty}(-i)^N\int_{\eta_{in}}^{\eta} d\eta_N\int_{\eta_{in}}^{\eta_N} d\eta_{N-1}\cdots\int_{\eta_{in}}^{\eta_3} d\eta_2\int_{\eta_{in}}^{\eta_2} d\eta_1 H_I(\eta_N)H_I(\eta_{N-1})\cdots H_I(\eta_2)H_I(\eta_1)|0\rangle$$

$$=|0\rangle+(-i)\int_{\eta_{in}}^{\eta} d\eta_1 H_I(\eta_1)|0\rangle+(-i)^2\int_{\eta_{in}}^{\eta} d\eta_2\int_{\eta_{in}}^{\eta_2} d\eta_1 H_I(\eta_2)H_I(\eta_1)|0\rangle+\cdots$$

Bunchi-Davis vacuum

Expectation value

$$\langle\eta|X(\eta)|\eta\rangle = \langle 0|\left\{1+i\int_{\eta_{in}}^{\eta} d\eta_1 H_I(\eta_1)+i^2\int_{\eta_{in}}^{\eta} d\eta_2\int_{\eta_{in}}^{\eta_2} d\eta_1 H_I(\eta_1)H_I(\eta_2)+\cdots\right\}X(\eta)$$

$$\times\left\{1+(-i)\int_{\eta_{in}}^{\eta} d\eta_1 H_I(\eta_1)+(-i)^2\int_{\eta_{in}}^{\eta} d\eta_2\int_{\eta_{in}}^{\eta_2} d\eta_1 H_I(\eta_2)H_I(\eta_1)+\cdots\right\}|0\rangle$$

$$=\langle 0|X(\eta)|0\rangle+i\int_{\eta_{in}}^{\eta} d\eta_1\langle 0|[H_I(\eta_1),X(\eta)]|0\rangle+i^2\int_{\eta_{in}}^{\eta} d\eta_2\int_{\eta_{in}}^{\eta_2} d\eta_1\langle 0|[H_I(\eta_1),[H_I(\eta_2),X(\eta)]]|0\rangle+\cdots$$

Analytic estimates

Mode functions $\delta\phi = u(\eta)a_k + u(\eta)^* a_k^\dagger$ $u(\eta) = \frac{1}{\sqrt{2k}} e^{-ik\eta}\left(1 - \frac{i}{k\eta}\right)$

Interaction Hamiltonian $H_I = \int d^3k \left[-\sqrt{\frac{6I}{1-I}}(-\eta)^{-1}\sin\theta\left(\delta\phi^\dagger J + \delta\phi J^\dagger\right) + \cdots \right]$

Corrections

$$\frac{\delta\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle}{\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle} = \frac{i^2}{\langle 0|\delta\phi_k(\eta)\delta\phi_p(\eta)|0\rangle}\int_{\eta_{in}}^{\eta} d\eta_2 \int_{\eta_{in}}^{\eta_2} d\eta_1 \langle 0|\left[H_I(\eta_1),\left[H_I(\eta_2),\delta\phi_k(\eta)\delta\phi_p(\eta)\right]\right]|0\rangle$$

$$= \frac{24I}{1-I}\sin^2\theta \int_{\eta_{in}}^{\eta} d\eta_2 \int_{\eta_{in}}^{\eta_2} d\eta_1 \frac{8}{|u(\eta)|^2}\,\mathrm{Im}\left[-(-\eta_2)^{-1}u'(\eta_2)u^*(\eta) + (-\eta_2)^{-2}u(\eta_2)u^*(\eta)\right]$$

$$\times\,\mathrm{Im}\left[u(\eta_1)u^*(\eta_2)\left\{-(-\eta_1)^{-1}u'(\eta_1)u^*(\eta) + (-\eta_1)^{-2}u(\eta_1)u^*(\eta)\right\}\right]$$

$$\approx \frac{6I}{1-I}\sin^2\theta \int_{-1}^{\chi} d\chi_2 \int_{-1}^{\chi_2} d\chi_1 \frac{8}{\chi_1\chi_2}$$

$$\approx \frac{24I}{1-I}\sin^2\theta N^2(k)$$

Predictions of anisotropic inflation

statistical anisotropy in curvature perturbations $g_s = 24 I\, N^2(k)$

statistical anisotropy in primordial GWs $g_t = 6 I\, \varepsilon_H\, N^2(k)$

cross correlation between curvature perturbations and primordial GWs

$$\frac{\langle \zeta G \rangle}{\langle \zeta\zeta \rangle} = -24 I\, \varepsilon_H N^2(k)$$

TB correlation in CMB

small linear polarization in primordial GWs

WMAP constraint Pullen & Kamionkowski (2007) $g_s = 24 I\, N^2(k) < 0.3$

Suppose $g_s = 24 I\, N^2(k) = 0.2$ $\varepsilon_H = 0.01$, then we have

- statistical anisotropy in GWs $g_t = 10^{-3}$
- cross correlation between curvature perturbations and GWs $\frac{\langle \zeta G \rangle}{\langle \zeta\zeta \rangle} = -2 \times 10^{-3}$

Cf . current constraint $TB / TE < 10^{-2}$

The gauge kinetic functions can be constrained by observations!

Summary

If we zoom up the inflationary universe, we may see a qualitatively new phenomena!

Gaussian fluctuations

It is possible to have small non-gaussianity if we consider non-minimal inflationary scenarios.

Statistically homogeneous and isotropic

It is possible to have slight statistical anisotropy if gauge kinetic function is relevant.

Almost Scale free (flat spectrum)

There should be a slight tilt because the expansion is not exactly deSitter.

Primordial GW has correlation with curvature perturbations

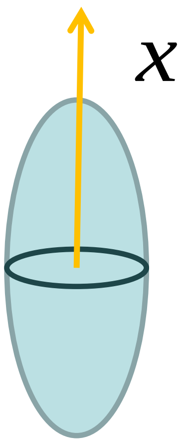
This never happens in the conventional inflation.

Future issues

More precise CMB predictions.

$$S = \int d^4x \left[\cdots - \frac{1}{4} f^2(\phi) F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} k^2(\phi) H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right]$$

$$\varepsilon^{\mu\nu\lambda\rho} A_\mu H_{\nu\lambda\rho} \; ?$$



More general Bianchi type anisotropy?