Wall-crossing, free fermions and matrix models

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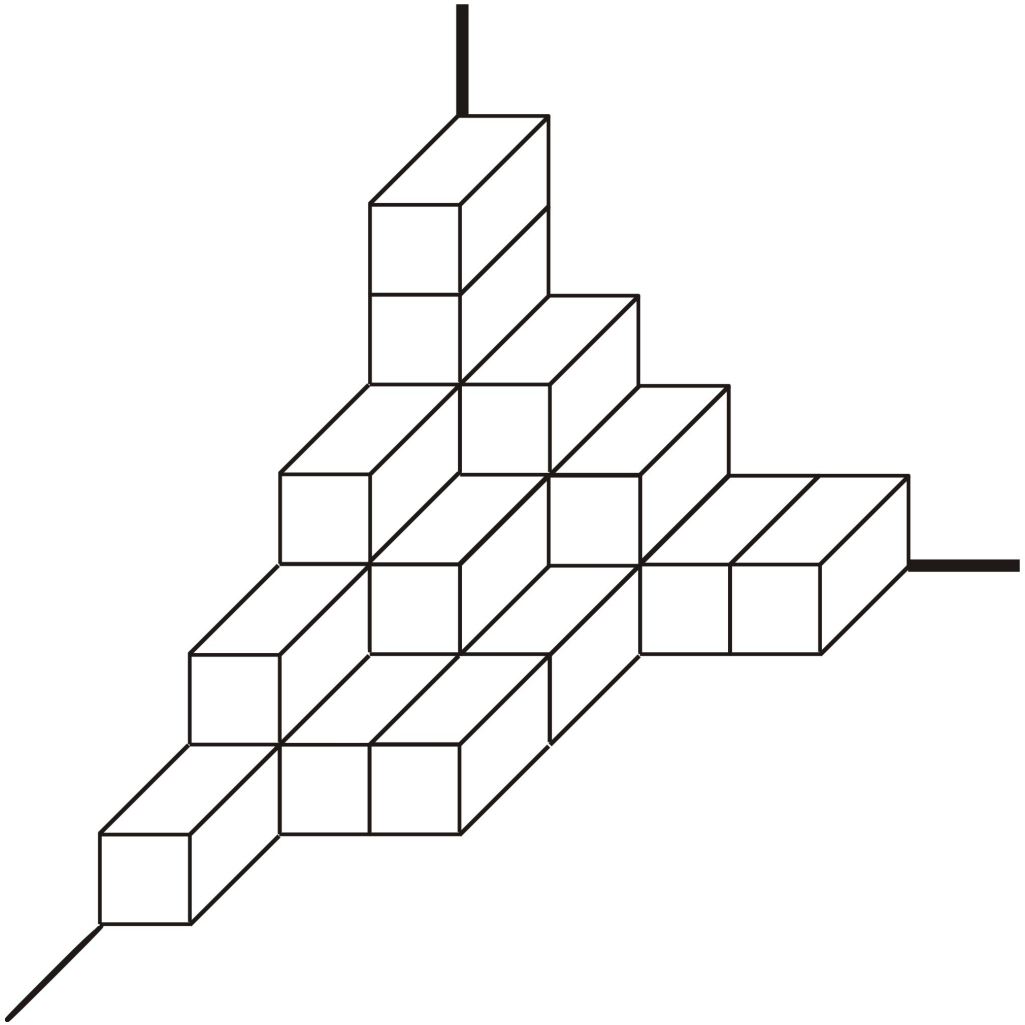
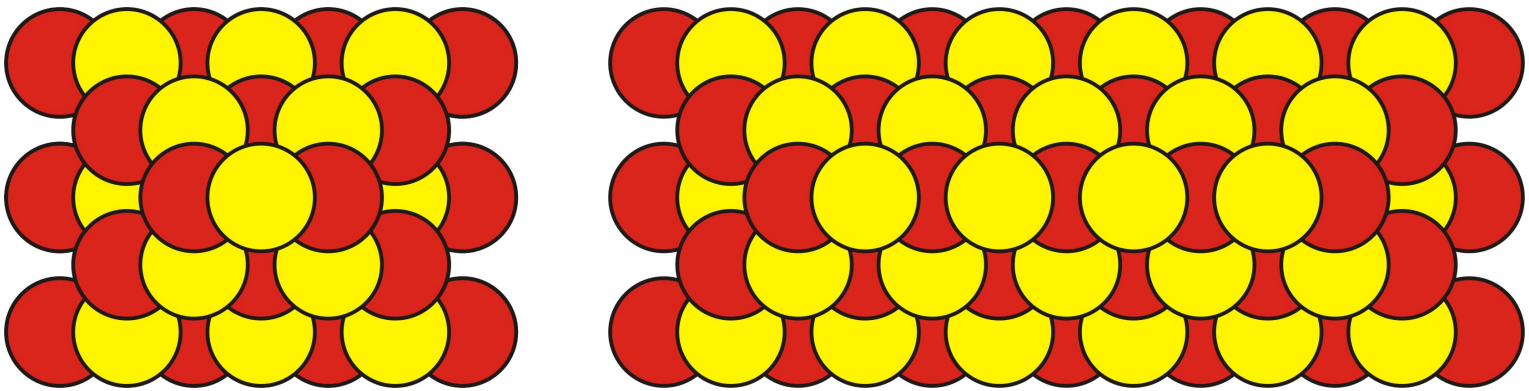
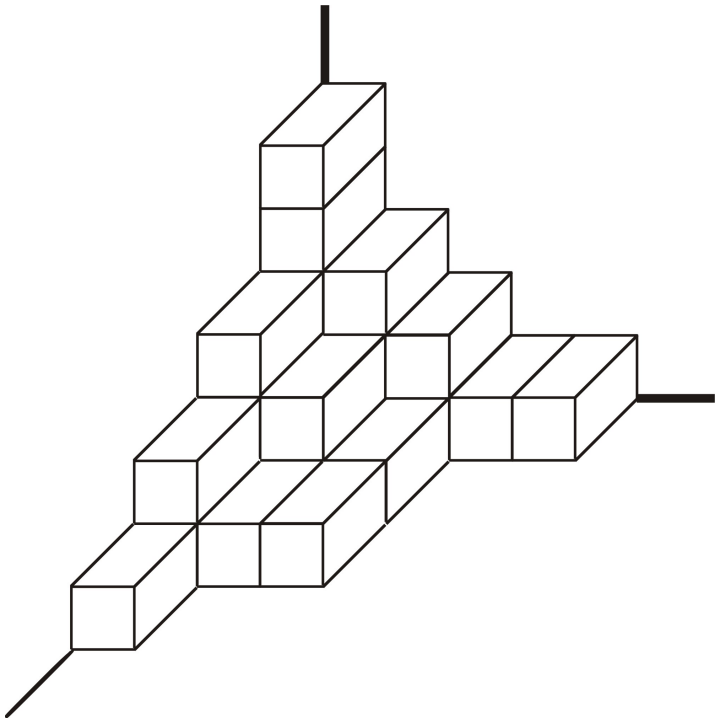
Crystal model for $\mathbb{C}^3$

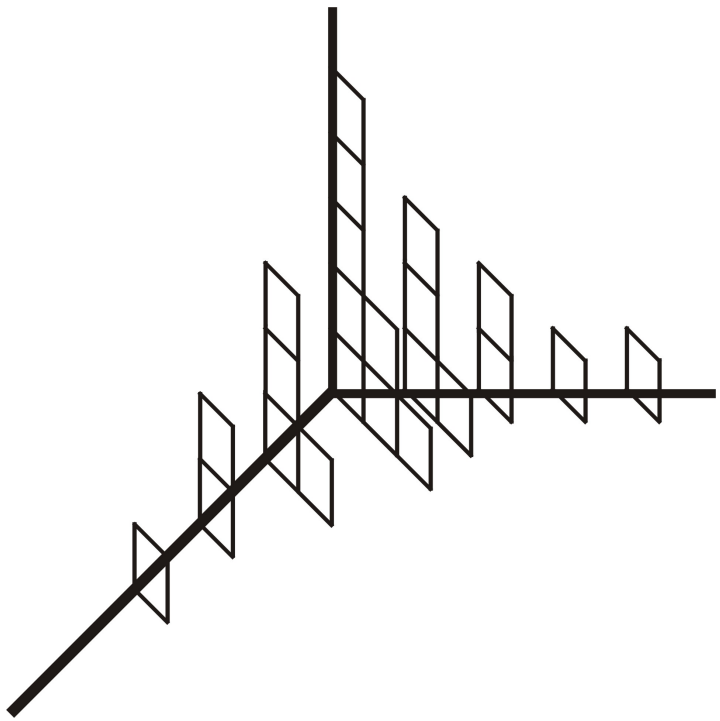


Crystal models for the conifold

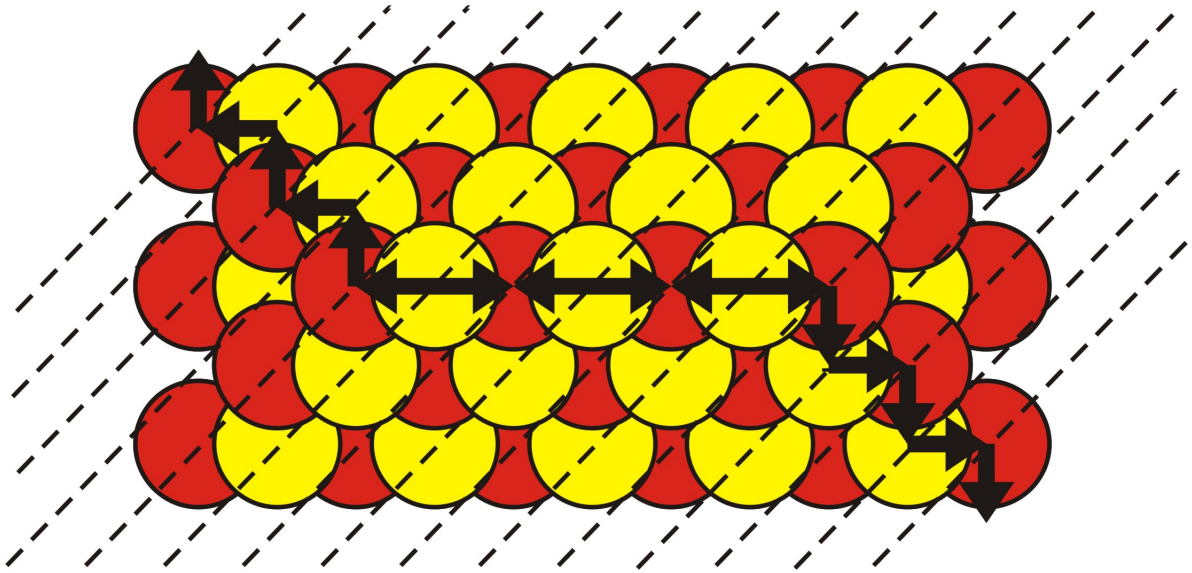


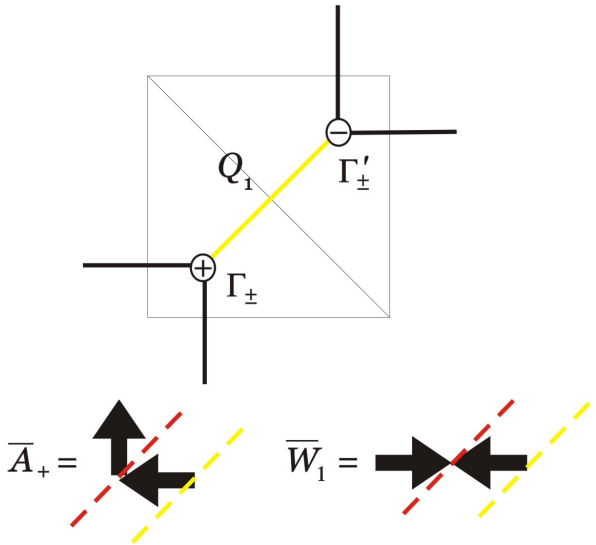
Slicing the crystal and free fermions

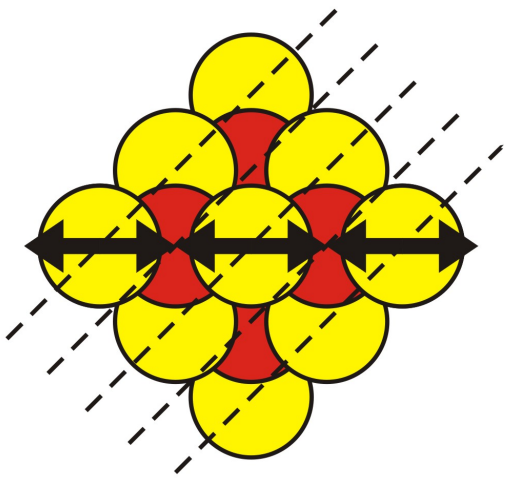




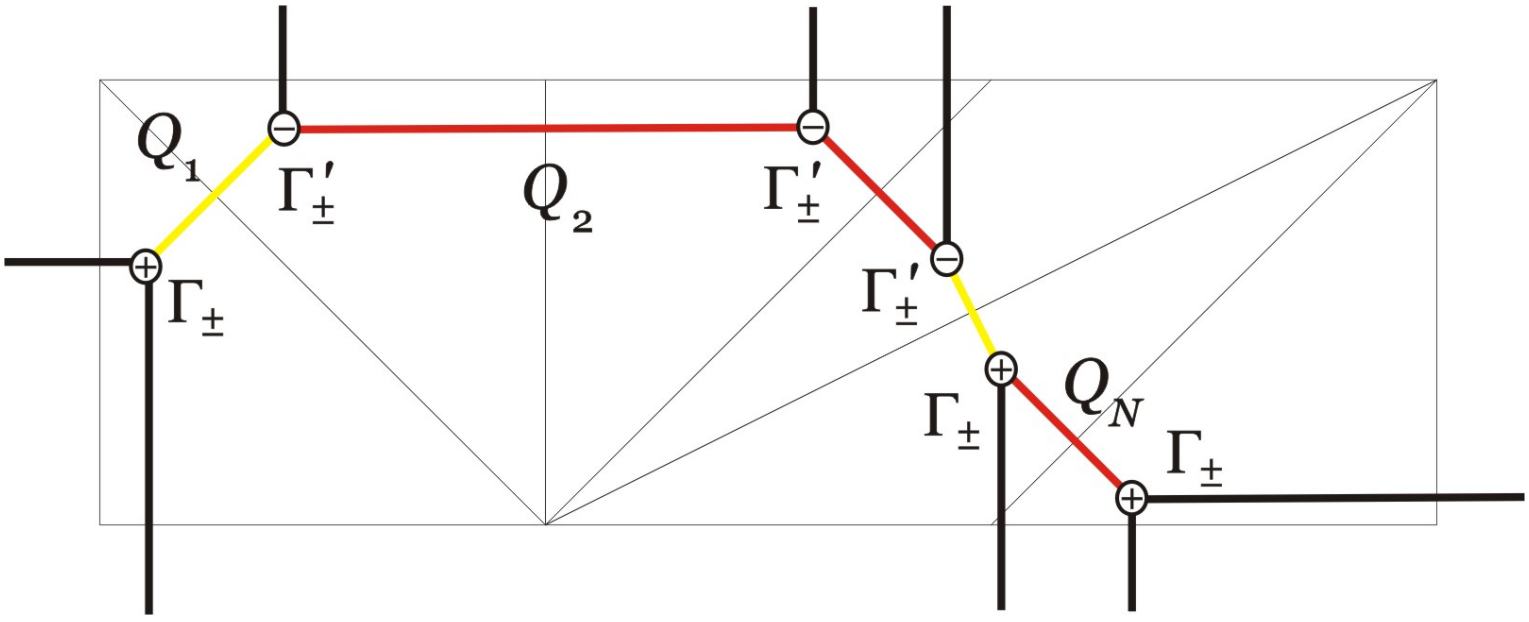
Slicing the conifold pyramid





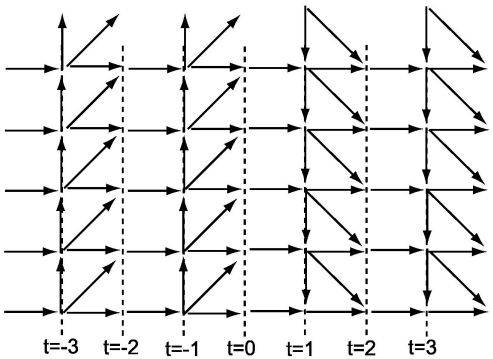


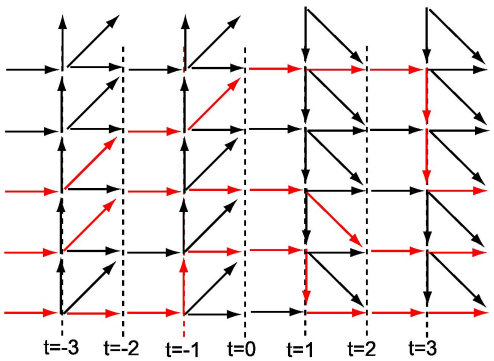
Generalized Donaldson-Thomas invariants from fermions

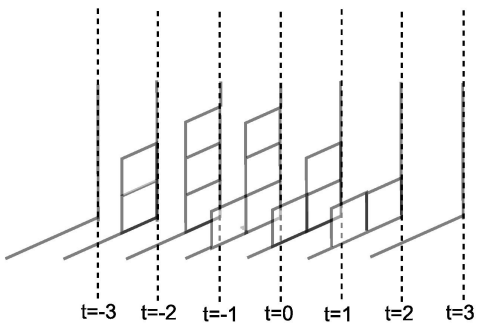


$$Z_n^{BPS} = Z_{top}(Q)Z_{top}(Q^{-1})|_{chamber} = \langle \Omega_+ | \overline{W}_p^n | \Omega_- \rangle$$

Non-intersecting paths

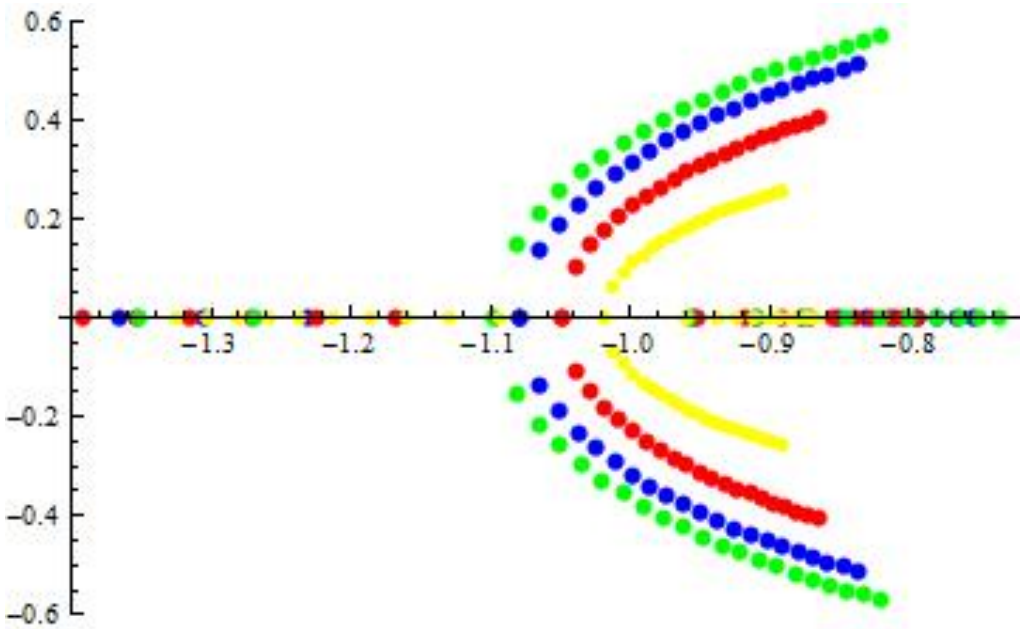




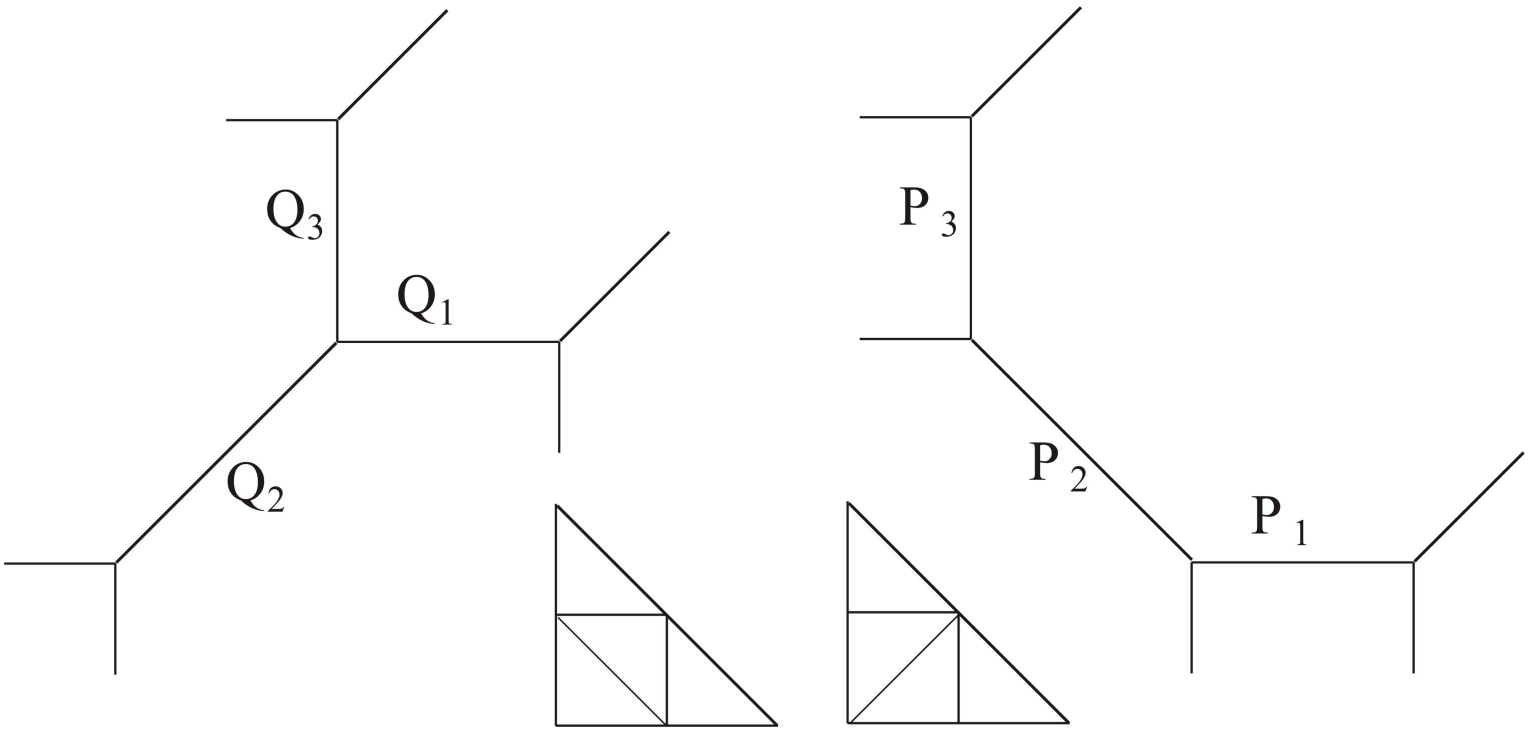


Eigenvalues

$$\frac{\sqrt{a+Q}-\sqrt{b+Q}}{\sqrt{a+Qe^{\tau}}-\sqrt{b+Qe^{\tau}}}e^{-\frac{\tau+T}{2}}=\sqrt{Q}=\frac{\sqrt{(a+Q)b}-\sqrt{(b+Q)a}}{\sqrt{(a+Qe^{\tau})b}-\sqrt{(b+Qe^{\tau})a}}e^{\frac{\tau+T}{2}}$$



$$a,b=-1+\epsilon^2\frac{(1-\mu)(1-\mu\epsilon^2)+(1-Q)(1+\mu\epsilon^2-2\mu)}{(1-\mu\epsilon^2)^2}+$$

$$\pm 2i\epsilon\frac{\sqrt{(1-Q)(1-\epsilon^2)(1-\mu)(1-Q\mu\epsilon^2)}}{(1-\mu\epsilon^2)^2}$$

Resolutions of $\mathbb{C}^3/\mathbb{Z}_2 \times \mathbb{Z}_2$