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Introduction to AdS/CMT

① What is AdS/CMT

② Holographic metal and SC

③ Holographic SC / Insulator Transition

Hartnoll 0903.3246, Horowitz 1002.1722 → ① and ②.

Nishioka-Ryu-TT 0911.0962
Horowitz-Way 1007.3714 } → ③.

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① What's AdS/CMT.

Finite temperature AdS/CFT.

$$\text{Gravity on } AdS_{d+1}\ BH \xrightarrow{AdS/CFT} CFT_d \text{ at finite Temp}$$

with T_{BH} (Hawking Temp). T_{CFT}.

⇓ ⇓

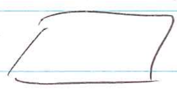
GR description (No stringy correction) — strongly coupled theory.

∃ Many cond. mat systems which are strongly coupled

Ex.
- High Tc cuprates.
- Heavy fermion system
- Graphene
- FQHE.
- ⋮

Can we apply AdS/CFT?

2+1 dim



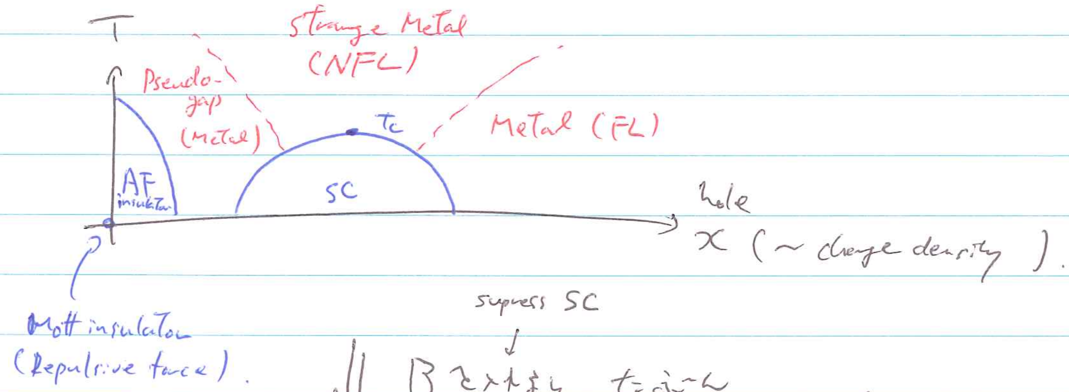
Fermi surface is point
⇓
not screening for coulomb
⇓
strong interaction.

Basic concepts of AdS/CMT.

(1) often, criticality is assumed.

why?
- (i) AdS/CFT
- (ii) universality class.
- (iii) Many examples in cond-mat.

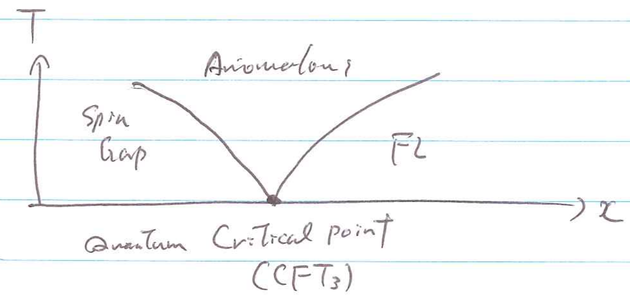
Ex. high Tc cuprates. ($La_{2-x}Sr_xCuO_4$). x: charge density.



suppress SC

⇓ B をいれると、たぶん

(Recently Sachdev).

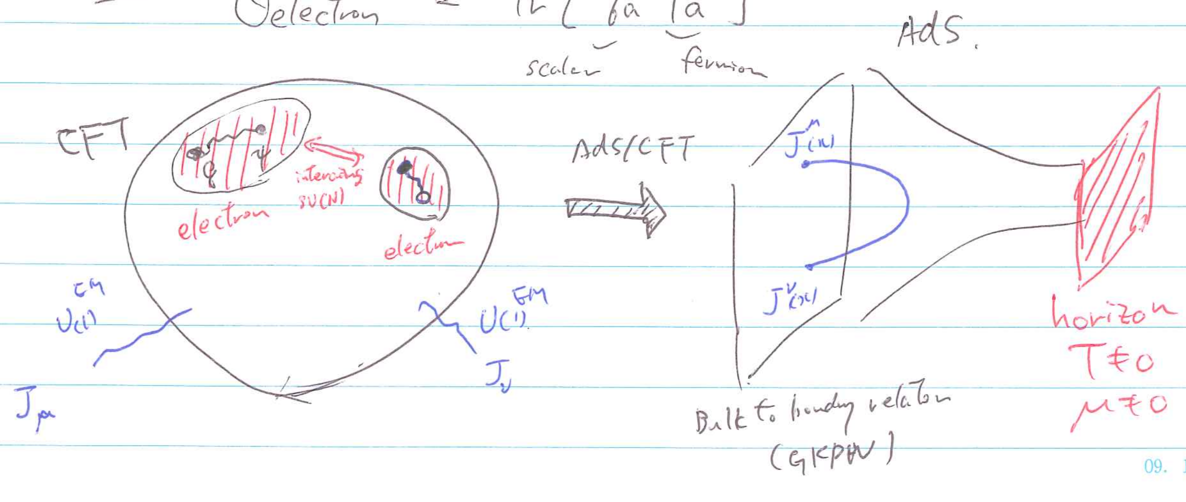


(2) Electrons are singlets of SU(N) gauge theory.

AdS/CFT: CFT = Large N gauge theory.

U(1)EM ⟶ global sym of CFT. : $J_\mu^{U(1)}$

<u>Ex.</u> $O_{electron} = Tr[\underbrace{\phi_a}_{scalar} \underbrace{\psi_a^\alpha}_{fermion}]$



Ex. of Emergent gauge sym in cond-mat.

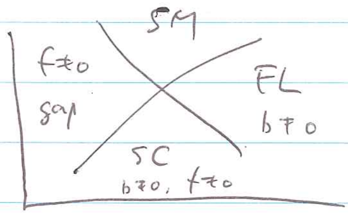
(i) slave boson method (RVB theory)

$$c_{i\downarrow}^{\dagger} = b_{ia}^{\dagger}\psi_{ia}, \quad c_{i\uparrow}^{\dagger} = b_{ia}^{\dagger}\tilde{\psi}_{ia}$$

$a = 1, 2 \longrightarrow SU(2)$ gauge symmetry.

holon: (b_{i1}, b_{i2}).

spinon: $(\psi_{i1}, \psi_{i2}) \equiv (f_{i\uparrow}, f_{i\downarrow}^{\dagger})$



(ii) FQHE at $\nu = \frac{5}{2}$

Moore-Read state (Pfaffian state).

$$\Phi(z_i) = Pf\left(\frac{1}{z_i - z_j}\right) \prod_{i<j} (z_i - z_j)^2 \exp\left(-\sum_i |z_i|^2\right).$$

$\Downarrow$

Non-abelian anyon $[\sigma]$

Edge states : $[I] + [\sigma] + [\varepsilon]$

Ising model $c = 1/2$.

WZW $SU(2)$ $k=2$

"

3D $SU(2)$ Chern-Simons theory $k=2$.

② Holographic metal and SC

(2-1). holographic metal.

Consider a charged AdS BH in 4D

Einstein-Maxwell theory

$$S_{EM} = \int dx^4 \sqrt{-g} \left[\frac{1}{4}\left(R + \frac{6}{L^2}\right) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \right]$$

$$\begin{cases} ds^2 = \dfrac{L^2}{z^2}\left(-f(z)dt^2 + \dfrac{dz^2}{f(z)} + dx^2 + dy^2\right) \\ f(z) = 1 - \left(1 + \dfrac{\mu^2 z_+^2}{L^2}\right)\left(\dfrac{z}{z_+}\right)^3 + \dfrac{z_+^2 \mu^2}{L^2}\left(\dfrac{z}{z_+}\right)^4 \\ A_t = \mu\left(1 - \dfrac{z}{z_+}\right). \qquad A_t(z = z_+) = 0. \end{cases}$$

$$T_{BH} = \frac{1}{4\pi z_+}\left(3 - \frac{z_+^2 \mu^2}{L^2}\right)$$

$$T_{BH} = 0 \text{ at } 3 = \frac{z_+^2 \mu^2}{L^2} \longrightarrow \text{Extremal BH.}$$

charge density

$$\rho = -\left.\frac{\delta S}{\delta A_t}\right|_{\text{on-shell}} = -A_t'\Big|_{bdy} = \frac{\mu}{z_+}.$$

$$\left(\text{Similarly, } \overset{(\text{current})}{j_x} = -\frac{\delta S}{\delta A_x} = A_x' = 0\right)$$

$z \to 0$ (bdy limit)

$$\begin{cases} A_t = \mu - \rho z + \cdots \\ A_x = A_x^{(0)} + j_x \cdot z + \cdots \end{cases} \qquad (A_t(z = z_+) = 0)$$

Conductivity

$$\text{perturbations} \begin{cases} g_{tx} = g_{tx}(\omega, z)\, e^{-i\omega t} \\ A_x = A_x(\omega, z)\, e^{-i\omega t} \end{cases}$$

EOM: $\left(f(z) A_x'\right)' + \left(\dfrac{\omega^2}{f(z)} - \dfrac{4\mu^2 z^2}{L^2 z_+^2}\right) A_x = 0$. ← Einstein-Maxwell.

$$\text{Boundary condition} \begin{cases} A_x(z) \xrightarrow{z \to z_+} (\text{const}) \times (z - z_+)^{-i\frac{\omega}{4\pi T}} \quad (\text{ingoing}) \\ A_x(z) \xrightarrow{z \to 0} A_x^{(0)} + J_x z + \cdots \end{cases}$$

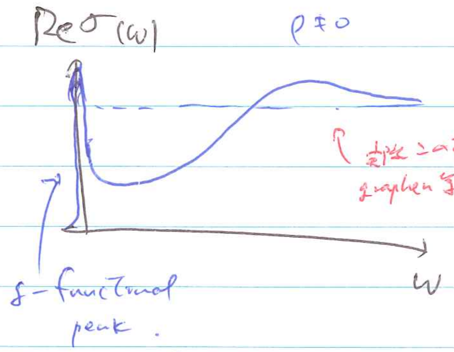
(ingoing bc → retarded correlator)
(outgoing bc → advanced correlator)

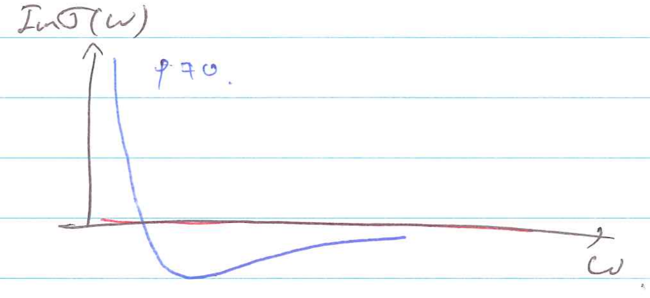
This relation is fixed by ingoing bc.

Conductivity

$$\sigma_{xx}(\omega) = \frac{\dot{J}^x(\omega)}{E_x(\omega)} = \frac{J^x(\omega)}{-i\omega A_x^{(0)}(\omega)} \longrightarrow \text{generally imaginary.}$$

Mathematica, Maple で計算.

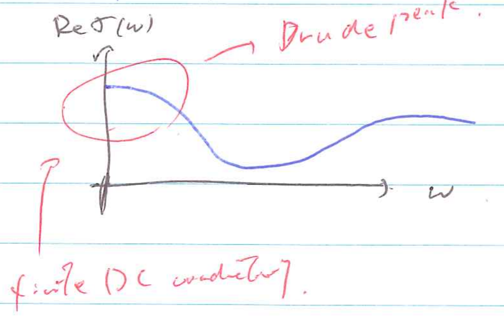


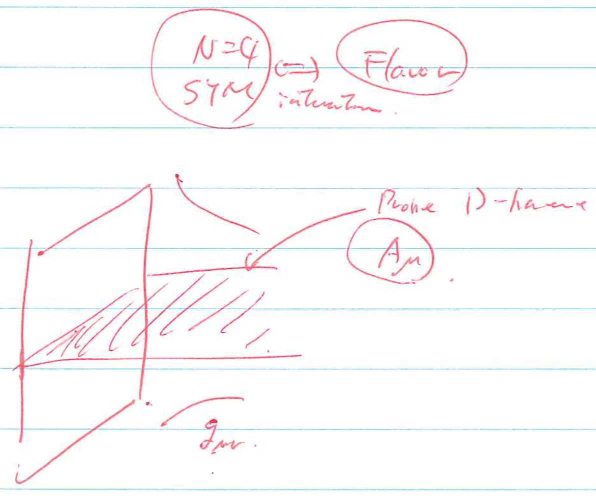


$\mathrm{Re}\,\sigma(\omega) \sim \rho\,\delta(\omega)$.

$\omega = 0$ DC conductivity $= \infty$. (due to the absence of impurity)

cf. Probe D-brane calculations





(2-2) Holographic superconductor

Add a complex charged (q) scalar field ϕ, ϕ^*.

$$S = S_{EM} + \int d^4x \sqrt{-g}\left(-|(\partial_\mu - iqA_\mu)\phi|^2 - m^2|\phi|^2\right).$$

$$V_{eff} = \underbrace{g^{tt} q^2 A_t A_t |\phi|^2}_{\text{near horizon region}} + m^2|\phi|^2.$$

$$- q^2 \frac{z_+}{L^2}\mu^2.$$

$$\Downarrow$$

if $q\mu$ is large enough, we have $\langle\phi\rangle \neq 0$!

$$T_{BH} \sim \frac{1}{z_+},$$

$$\therefore\ T_c \sim \frac{q\mu}{L} \sim \frac{\sqrt{q\rho}}{L}$$

for simplicity.
(choose $m^2L^2 = -2 \geq -\frac{9}{4}$.)

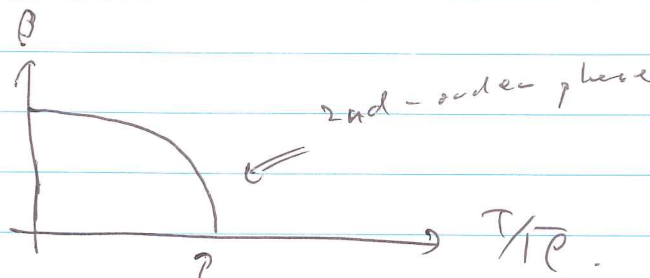
(a) Probe approx.

$q\mu$ = fixed, $q \gg 1$.

$\Rightarrow$ we can neglect backreactions!

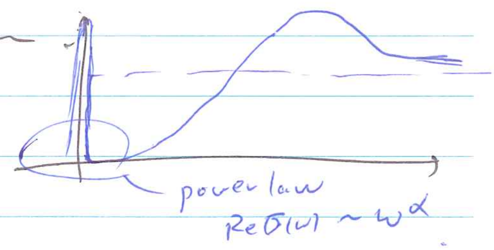
$$\begin{cases} \phi(z) \xrightarrow{z\to 0} \underset{\text{source}}{\alpha z} + \underset{\langle O\rangle}{\beta z^2} + \cdots \quad \Delta = 2. \\ A_t(z) \xrightarrow{z\to 0} \mu - \rho z + \cdots \end{cases} \qquad (A_t(z=z_+) = 0).$$

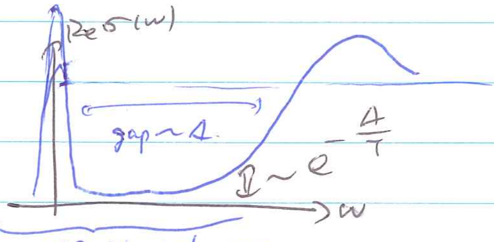
we impose $\alpha = 0$ $\longrightarrow$ normalizable ~~sol~~ solution!



$\beta \sim \sqrt{T_c - T}$
(Mean field theory).

$T\to 0$





The δ-func is stable!

(b) extremal BH.

$T=0$: $z_+^2 \mu^2 = 3L^2$ (extremal)

$$ds^2 = \underbrace{\frac{L^2}{6} ds^2_{AdS_2}}_{\Downarrow} + \frac{L^2}{z_+^2}(dx^2 + dy^2).$$

BF bound for AdS_2 $\qquad\qquad\qquad$ AdS_4 BF bound.

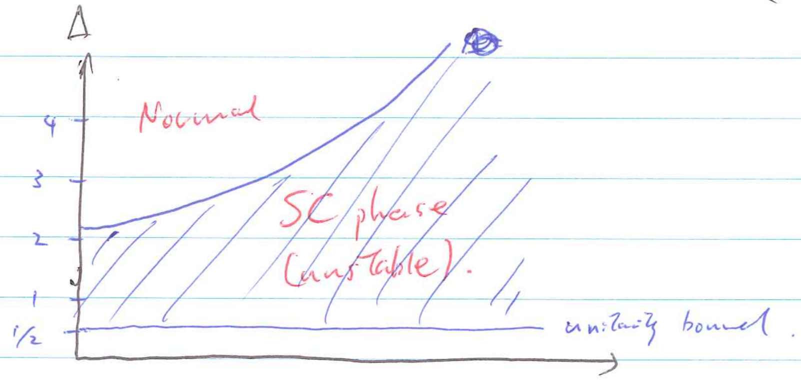
$$\boxed{m^2L^2 \geq -\frac{3}{2}} \longleftarrow \boxed{m^2L^2 \geq -\frac{9}{4}}.$$

more strict!

charge effect

$$\Rightarrow V_{eff} = (g^2 A_t^2 g^{tt} + m^2)|\phi|^2 \xrightarrow{AdS_2 \text{ limit}} (m^2 - \frac{g^2}{2})|\phi|^2.$$

$$m^2 - \frac{g^2}{2} \geq -\frac{3}{2L^2} \quad \text{(Denef-Hartnoll '09)}$$



(3) Holographic Superconductor / Insulator phase Transition.

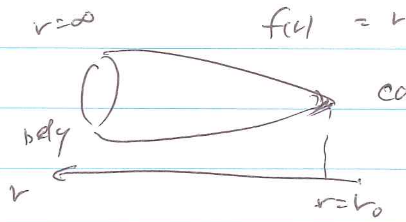
(3-1) Probe Analysis (0911.0962)

High Tc ⟶ ∃ Mott insulator phase.

Consider AdS$_5$ soliton

$$ds^2 = L^2 \frac{dr^2}{f(r)} + r^2(\underbrace{-dt^2 + dx^2 + dy^2}_{2+1\ \dim}) + f(r)d\chi^2. \qquad \chi \sim \chi + \frac{\pi L}{r_0}$$

$$f(r) = r^2 - \frac{r_0^2}{r^2}$$



The dual gauge theory
⟶ (2+1) dim SU(N) YM
confining gauge theory ⟶ mass gap phase

(ϕ, ϕ^*) ⟶ charge q $\quad m^2 = -\frac{15}{4L^2}$

$S = S_E + S_M + S_{scalar}$

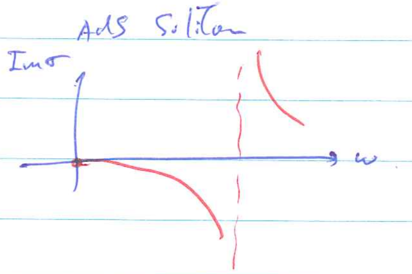
$$\phi \underset{r\to\infty}{\sim} \frac{\alpha}{r^{3/2}} + \frac{\beta}{r^{5/2}} + \cdots \qquad \text{b.c. } \alpha = 0.$$

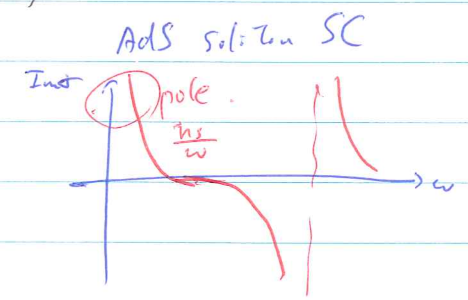
$z = \frac{1}{r}$. We can show: $q\mu$ is large enough $\beta \neq 0$.

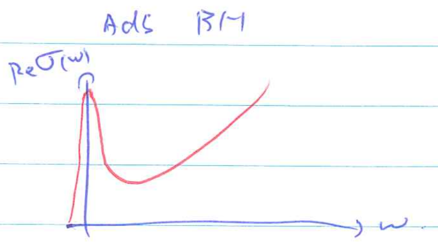
∃ confinement / deconfinement transition
⇕
AdS soliton ⟷ AdS BH.

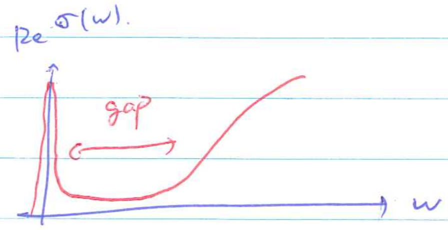
‖

4 different phases!

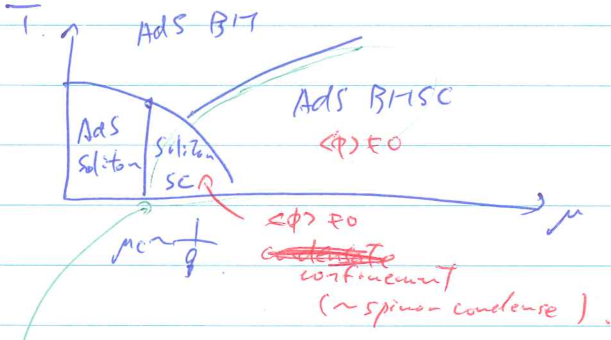








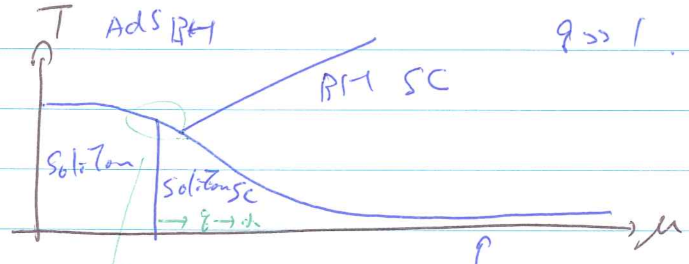
(Naive) Phase diagram from Probe Appx.



⟷ very similar to High Tc phase diagram.

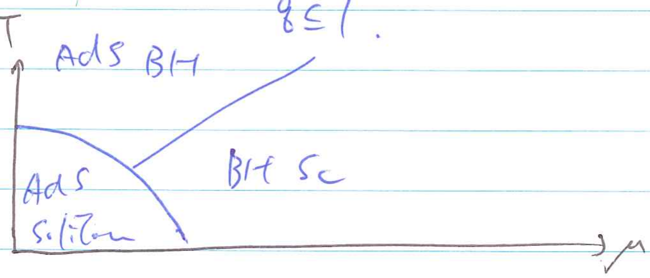
⟨ϕ⟩ が出た probe 近似なので この辺りはよくない。

(3-2) backreacted Analysis (1009. 3718).



q を小さくしていくと ここの部分が広がったところで

T→0 で BH SC は singular になるので energy 的に不利になる。と予想されている。



(AdS Soliton SC は出ない)