Studies of Nambu-Bracket in the M-theory

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1. Recent Remarkable progress in the M-theory

Recent progress in the 'M-theory'

(Bagger-Lambert, Phys. Rev. D 77, 065008 (2008))

(A. Gustavsson, Nucl.Phys.B811:66-76,2009)

BLG model

First model dealing with Multiple membranes

Studies of this model

↓

the deeper understanding about the fundamental degree of freedom in M-theory

Special property of BLG model.

N=8 3-d SUSY 'gauge' model described by the **Lie 3-algebra**

$$T^a, T^b, T^c \in V_3 \Rightarrow [T^a, T^b, T^c] = f^{abc}{}_d T^d \in V_3$$

$\Longleftrightarrow$ Usual quantum field theory: described by **Lie-algebra**

$$T^a, T^b \in V_2 \Rightarrow [T^a, T^b] \in V_2$$

What is Lie 3-algebra ?

•Extension of the Lie algebra

(1)Lie algebra (Lie ring): V_2

<u>Product</u> (commutation relation): $[\,,\,] : V_2 \times V_2 \to V_2$

$$T^a, T^b \in V_2 \Rightarrow [T^a, T^b] \in V_2$$

Product is **defined with 2-components**

(2)Lie 3-algebra (not ring): V_3

<u>3-Product':</u> $[\,,\,,\,] : V_3 \times V_3 \times V_3 \to V_3$

$$T^a, T^b, T^c \in V_3 \Rightarrow [T^a, T^b, T^c] = f^{abc}{}_d T^d \in V_3$$

Product is **defined with 3-components not 2 !**

Field contents of BLG model

X_a^I :8 component of the scalar fields.

I :8 transverse directions in 11-d target space

a :gauge indices

Ψ_a :16 components Majorana spinor with SO(8)

Chirality condition $\Gamma_-\Psi_a = -\Psi_a, \quad \Gamma_+\Psi_a = 0.$

$$\Gamma_\pm \equiv \frac{1 \pm \Gamma}{2}. \qquad \Gamma \equiv \Gamma_{012} = 1 \otimes \bar{\gamma}_9,$$

$\tilde{A}_\mu{}^b{}_a$:(1+2)-d gauge field with 2 gauge indices.

Action

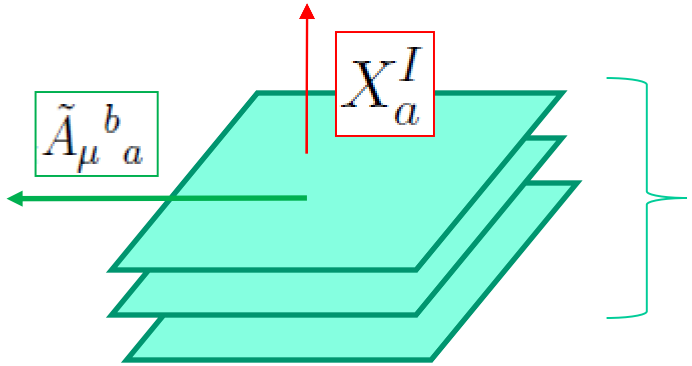
$$S = \int d^3x\, \mathcal{L},$$

$$\mathcal{L} = -\frac{1}{2}\langle D^\mu X^I, D_\mu X^I\rangle + \frac{i}{2}\langle\bar{\Psi}, \Gamma^\mu D_\mu \Psi\rangle + \frac{i}{4}\langle\bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle - V(X) + \mathcal{L}_{CS}.$$

$$\mathcal{L}_{CS} = \frac{1}{2}\varepsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right).$$

$$V(X) = \frac{1}{12}\langle[X^I, X^J, X^K], [X^I, X^J, X^K]\rangle.$$

$$(D_\mu X^I(x))_a = \partial_\mu X^I_a(x) - \tilde{A}_\mu{}^b{}_a(x) X^I_b(x), \quad \tilde{A}_\mu{}^b{}_a \equiv A_{\mu cd} f^{cdb}{}_a,$$



Multiple M2-branes in (1+10) dimensions.

2-2. An M5-brane with Nambu-bracket in the BLG model

The BLG model involves not only multiple membranes but also an Single M5-brane.

M5-brane: **(1+5) dimensional extended object allowed to exist in the M-theory.**

Ho-Matsuo (JHEP 0806:105,2008)

Ho-Imamura-Matsuo-Shiba (JHEP 0808:014,2008)

2-3.How the M5-brane shows up ?

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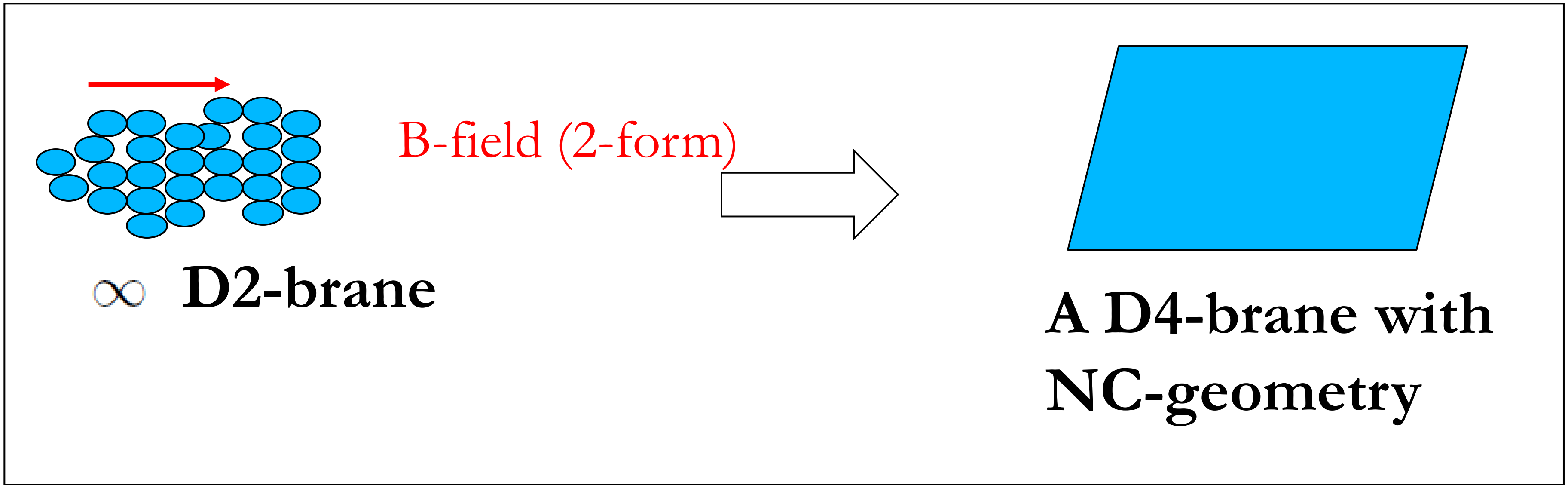
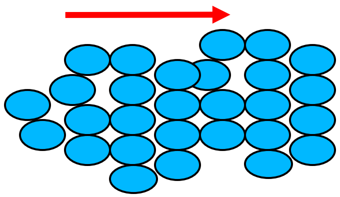
↓

3-dimensional version of the Myers effects !!



Myers effect in the D-brane system

In string theory

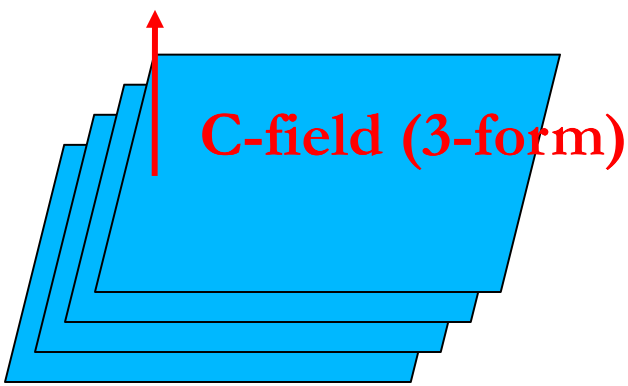


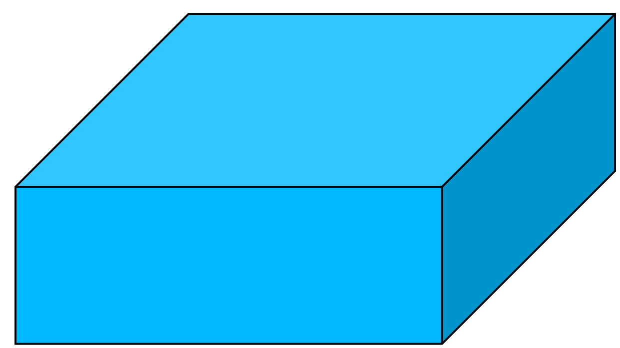
∞ D2-brane

B-field (2-form)

A D4-brane with NC-geometry

In BLG model





∞ Membranes

A M5-brane with ‘NC-geometry’

(3-dimensional version)

2-4.Geometry along the extended directions by the Myers effect

(1) D-brane case

Non-commutative geometry

$[X^I, X^J] = i\theta^{IJ}$ $i\theta^{IJ}$: Non-commutative parameter

This is allowed only in the case the # of D-brane is infinite (U(N) $N \to \infty$))

(2) M-brane case

Nambu-bracket structure

$$\{y^\mu, y^\nu, y^\rho\} = C\epsilon^{\mu\nu\rho} = C^{\mu\nu\rho}$$

This is allowed only in the case the # of M-brane is infinite

**Situation of the M5-brane theory with Nambu-bracket is
very similar to D-brane theories
with Non-commutative geometry,**

So there would be deep relationships between these two,

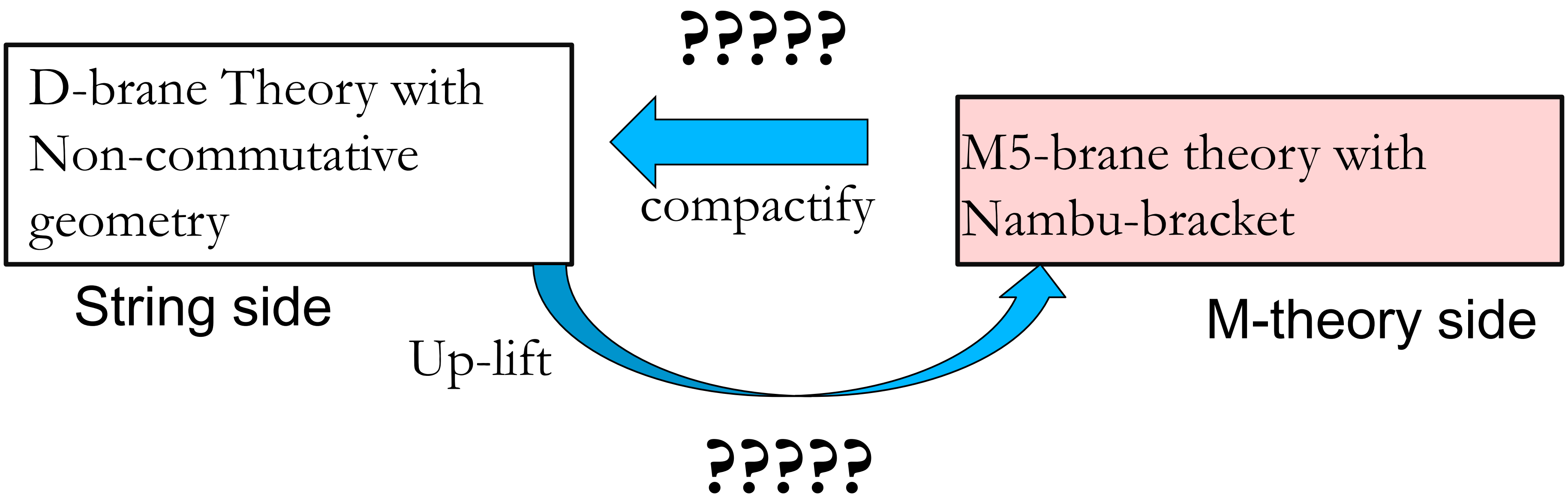
D-brane Theory with Non-commutative geometry

String side

M5-brane theory with Nambu-bracket

M-theory side

In particular,



It is very worthwhile to study Nambu-Bracket to understand the M-theory origin of the Non-commutative gauge theory on the D-brane.

Contents of this talk

Contents of this talk

3. Brief review of the Nambu-Bracket

4. Recent investigation about the Nambu-Bracket

3. Brief review of the Nambu-Bracket

Nambu-Poisson structure in the Lie 3-algebra

***dimension of Lie 3-algebra = ∞**

→ **Lie 3-algebra can be written by the Fourier modes along the internal 3-directions**

$$T^a, T^b \rightarrow \cos \vec{p_a}\cdot\vec{y}, \quad \sin \vec{p_b}\cdot\vec{y},$$

(as well as) $e^{i\vec{p}\cdot\vec{x}} = \cos\vec{p}\cdot\vec{x} + i\sin\vec{p}\cdot\vec{x}$

(something like root)

We denote the basis of Lie 3-algebra with Fourier mode as $T^a \rightarrow \chi^a(y) = e^{i\vec{p}_a\cdot\vec{y}}$

$$p^a = 0, \pm 1, \pm 2, \ldots \pm \infty$$

Field variable $f(x) = f^a(x)T^a \to f(x,y) = f^a(x)\chi^a(y)$

*3-product becomes Nambu-Poisson bracket.

$$[T^a, T^b, T^c] \to \{\chi^a, \chi^b, \chi^c\} = \epsilon^{\mu\nu\rho}\partial_\mu\chi^a\partial_\nu\chi^b\partial_\rho\chi^c$$

*Inner product is

$$\mathrm{Tr}(T^aT^b) \to \langle\chi^a, \chi^b\rangle \equiv \int d^3y(\chi^a)^*(y)\chi^b(y)$$

It seems to enhance

(1+2) dimensional world-volume

→ (1+(2+3)) dimensional world-volume

But the kinetic term along the 3 enhanced direction is still lacking. $\longrightarrow$ Not real space yet

How the internal 3-direction shows up as real world-volume by inducing the kinetic term along the directions ?

A. Expanding the field X^I around the background

$$X^I(x,y) = y^I + \sum_a X^I_a(x)\chi^a(y)$$

Then the potential term serves the kinetic term

$$\{X^I, X^J, X^K\}^2 \sim \left(\epsilon^{\mu\nu\rho}\delta^I_\mu\delta^J_\nu\partial_\rho X^K\right)^2$$

(1+2) dimensional theory is really enhanced to (1+5) dimensional theory.

How the internal 3-direction shows up as real world-volume by inducing the kinetic term along the directions ?

A. Expanding the field X^I around the background

$$X^I(x,y) = y^I + \sum_a X^I_a(x)\chi^a(y)$$

Then the potential term serves the kinetic term

$$\{X^I, X^J, X^K\}^2 \sim \left(\epsilon^{\mu\nu\rho}\delta^I_\mu\delta^J_\nu\partial_\rho X^K\right)^2$$

The source of the C-field

Action

$$S = \int d^3x\, \mathcal{L},$$

$$\mathcal{L} = -\frac{1}{2}\langle D^\mu X^I, D_\mu X^I\rangle + \frac{i}{2}\langle\bar{\Psi}, \Gamma^\mu D_\mu \Psi\rangle + \frac{i}{4}\langle\bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle - V(X) + \mathcal{L}_{CS}.$$

$$V(X) = \frac{1}{12}\langle[X^I, X^J, X^K], [X^I, X^J, X^K]\rangle.$$

Nambu-Poisson structure

Expand around the background

$$V(X) = \frac{1}{12}\langle[X^I, X^J, X^K], [X^I, X^J, X^K]\rangle.$$

→ **Serves kinetic terms**

$$\{X^I, X^J, X^K\}^2 \sim \left(\epsilon^{\mu\nu\rho}\delta^I_\mu \delta^J_\nu \partial_\rho X^K\right)^2$$

Trace changed to integration over enhanced direction

$$\langle , \rangle \to \int d^3y$$

M5-brane action shows up !!

$$S = \frac{T_6}{T_{str}^2} \int d^3x d^3y \, \mathcal{L}$$

$$\mathcal{L} = \mathcal{L}_X + \mathcal{L}_{\rm pot} + \mathcal{L}_\Psi + \mathcal{L}_{\rm int} + \mathcal{L}_{\rm CS}$$

$$\begin{aligned}
\mathcal{L}_X + \mathcal{L}_{\rm pot} &= -\frac{1}{2}(\mathcal{D}_\mu X^i)^2 - \frac{1}{2}(\mathcal{D}_{\dot\mu} X^i)^2 - \frac{1}{4}\mathcal{H}^2_{\lambda\dot\mu\dot\nu} - \frac{1}{12}\mathcal{H}^2_{\dot\mu\dot\nu\dot\rho} \\
&\quad -\frac{1}{2g^2} - \frac{g^4}{4}\{X^{\dot\mu}, X^i, X^j\}^2 - \frac{g^4}{12}\{X^i, X^j, X^k\}^2. \\
\mathcal{L}_\Psi + \mathcal{L}_{\rm int} &= \frac{i}{2}\bar\Psi\Gamma^\mu\mathcal{D}_\mu\Psi + \frac{i}{2}\bar\Psi\Gamma^{\dot\rho}\Gamma_{\dot1\dot2\dot3}\mathcal{D}_{\dot\rho}\Psi \\
&\quad + \frac{ig^2}{2}\bar\Psi\Gamma_{\dot\mu i}\{X^{\dot\mu}, X^i, \Psi\} + \frac{ig^2}{4}\bar\Psi\Gamma_{ij}\{X^i, X^j, \Psi\}
\end{aligned}$$

$$\{y^{\mu}, y^{\nu}, y^{\rho}\} = C\epsilon^{\mu\nu\rho} = C^{\mu\nu\rho}$$

C-field

3-dimensional version of the B-field

Expansion around this → **Kinetic term around the extra 3-direction**

C-field plays the role to enhance the extra 3-directions

Note !!

New field ingredient in the M5-brane theory

2-form gauge fields show up

$$X^{\dot{\mu}}(x,y) = y^{\dot{\mu}} + b^{\dot{\mu}}(x,y), \quad b_{\dot{\mu}\dot{\nu}} = \epsilon_{\dot{\mu}\dot{\nu}\dot{\rho}} b^{\dot{\rho}}.$$

2-form fields along enhanced 3-directions

$$b_{\lambda\dot{\mu}}(x,y) = \frac{\partial}{\partial y'^{\dot{\mu}}} A_\lambda(x,y,y') \Big|_{y'=y}$$

λ :Directions along the original M2-branes extend

$\dot{\mu}$:Directions along the enhanced 3-directions

Gauge transformation laws

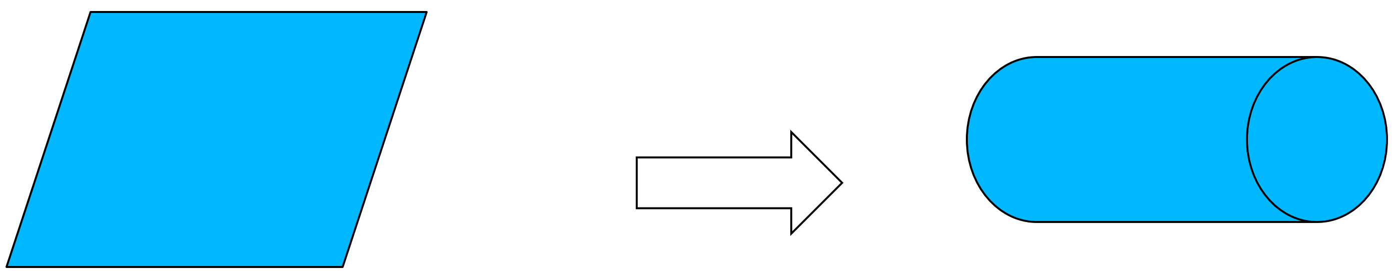
$$\begin{aligned}
\delta_\Lambda X^i &= g(\delta_\Lambda y^{\dot\rho})\partial_{\dot\rho} X^i, \\
\delta_\Lambda \Psi &= g(\delta_\Lambda y^{\dot\rho})\partial_{\dot\rho} \Psi, \\
\delta_\Lambda b_{\dot\kappa\dot\lambda} &= \partial_{\dot\kappa}\Lambda_{\dot\lambda} - \partial_{\dot\lambda}\Lambda_{\dot\kappa} + g(\delta_\Lambda y^{\dot\rho})\partial_{\dot\rho} b_{\dot\kappa\dot\lambda}, \\
\delta_\Lambda b_{\lambda\dot\sigma} &= \partial_\lambda \Lambda_{\dot\sigma} - \partial_{\dot\sigma}\Lambda_\lambda - g\delta_{\mathrm{gc}} b_{\lambda\dot\sigma}.
\end{aligned}$$

Written as 3-dimensional volume preserving diffeo.

$$\partial_{\dot\mu}\delta y^{\dot\mu} = 0,$$

Relationship between Nambu-bracket and non-commutative geometry

*Double dimensional reduction



‘M5-brane’

with Nambu-bracket

D4-brane

with non-commutative geometry

M5-brane theory with Nambu-Poisson bracket

↓ Double dimensional reduction

D4-brane theory with Poisson bracket.

M5-brane theory with Nambu-Poisson bracket

Double dimensional reduction

D4-brane theory with Poisson bracket.

(1+4) dimensional non-commutative
Supersymmetric gauge theory in $\theta \to 0$

$$[f, g]_{\text{Moyal}} = f e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu} g - g e^{\frac{i}{2}\theta^{\mu\nu}\overleftarrow{\partial}_\mu \overrightarrow{\partial}_\nu} f$$

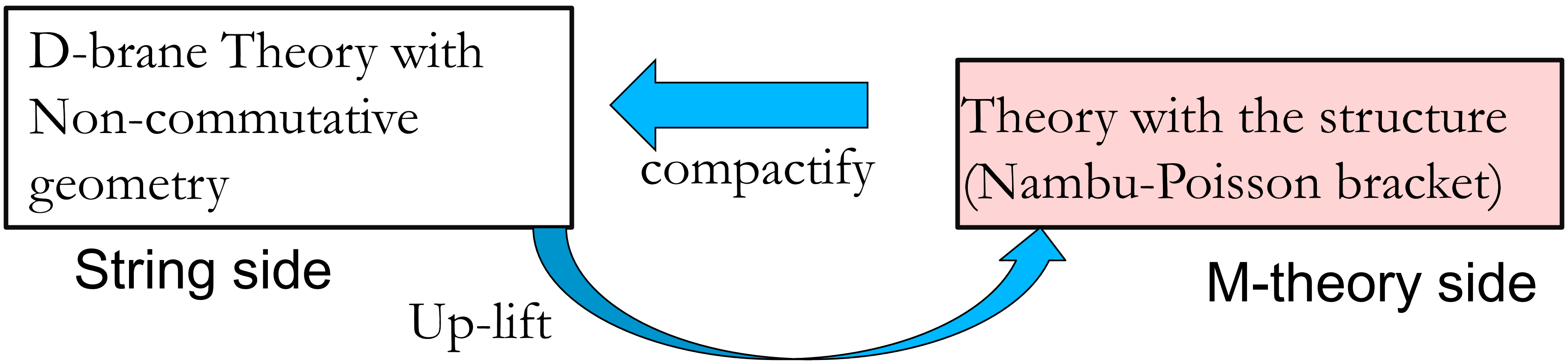
$$= \boxed{\theta^{\mu\nu}\partial_\mu f \partial_\nu g} + O(\theta^3)$$

Picking up only up to $O(\theta)$

→ Poisson limit

The important point

Nambu-bracket might capture the M-theory up-lifted non-commutative geometry



We want to study the relationship to understand the M-theory further !!

4. Recent further investigation about the Nambu Bracket.

Natural questions

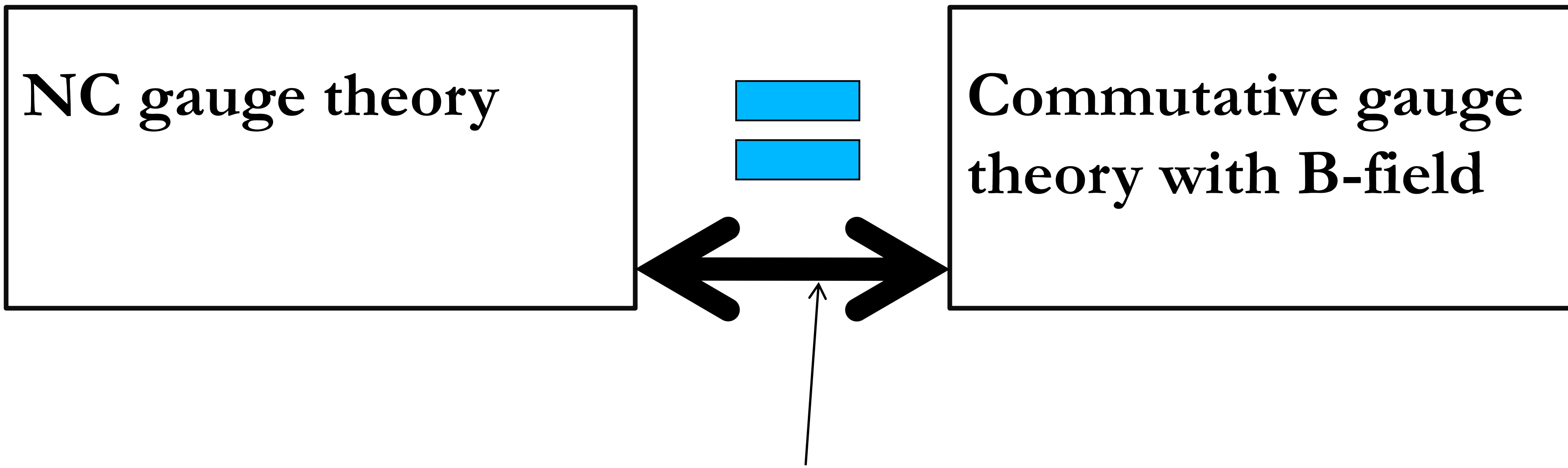
(A) **How the Seiberg-Witten map behaves in the Nambu-bracket M5-brane theory**

(B) **What is the theory recovering the full order of non-commutative gauge theory beyond the Poisson limit ?**

(A)How the Seiberg Witten map behaves in the M5-brane theory

In the D-brane theory

the NC gauge theory and the theory with B-field are equivalent



There are map connecting these two theory

Seiberg=Witten Map !!

How do we find Seiberg=Witten Map ?

$A \longrightarrow \hat{A}_\mu$

Map : $\hat{A}_\mu = \hat{A}_\mu(A)$

- $\hat{A}_\mu$:gauge field in the NC side
- A :gauge field in the theory with B-field

Note that the two theories are physically equivalent

⇒ ***Coset divided by the physically equivalence (by gauge sym) in NC side***

=

Coset divided by the physically equivalence (by gauge sym) in commutative side

$$\delta_{\hat{\lambda}} \hat{A}(A) = \hat{A}(A + \delta_\lambda A) - \hat{A}(A)$$

- $\delta_{\hat{\lambda}}$:gauge trans. in NC side
- δ_λ :gauge trans. in the theory with B-field

Solutions of the Seiberg-Witten map

$$\widehat{A}_i(A) = A_i + A'_i(A) = A_i - \frac{1}{4}\theta^{kl}\{A_k, \partial_l A_i + F_{li}\} + \mathcal{O}(\theta^2)$$

$$\widehat{\lambda}(\lambda, A) = \lambda + \lambda'(\lambda, A) = \lambda + \frac{1}{4}\theta^{ij}\{\partial_i\lambda, A_j\} + \mathcal{O}(\theta^2)$$

Such maps are obtained order by order of the θ

All order θ solution in the Poisson-Bracket theory

(Jurco,Shupp, Wess, Nucl.Phys.**B**584,784-794(2000))

This case, the map can be obtained in all order

They prepare the parameter $\theta(t) = \frac{1}{2}\theta^{ij}(t)\partial_i \wedge \partial_j,\ t \in [0,1],$
obeying the following equation,

$$\partial_t \theta(t) = -\theta(t) F \theta(t), \qquad \theta(0) = \theta,$$

$$F_{ij} = \partial_i A_j - \partial_j A_i.$$

This becomes non-commutative parameter

A_i **:Gauge field in commutative side with gauge transformation**

$$A \mapsto A + d\lambda.$$

By using this set up, we can define the map

$$A^i_\rho = \left(e^{\partial_t + \theta^{ij}(t) A_i \partial_j} - 1 \right)\Big|_{t=0} x^i$$

Gauge field in the Poisson bracket theory

Under the gauge transformation $A \mapsto A + d\lambda.$

$$A^i_\rho \overset{(\lambda)}{\mapsto} A^i_\rho + \{\tilde{\lambda}, x^i\} + \{\tilde{\lambda}, A^i_\rho\}.$$

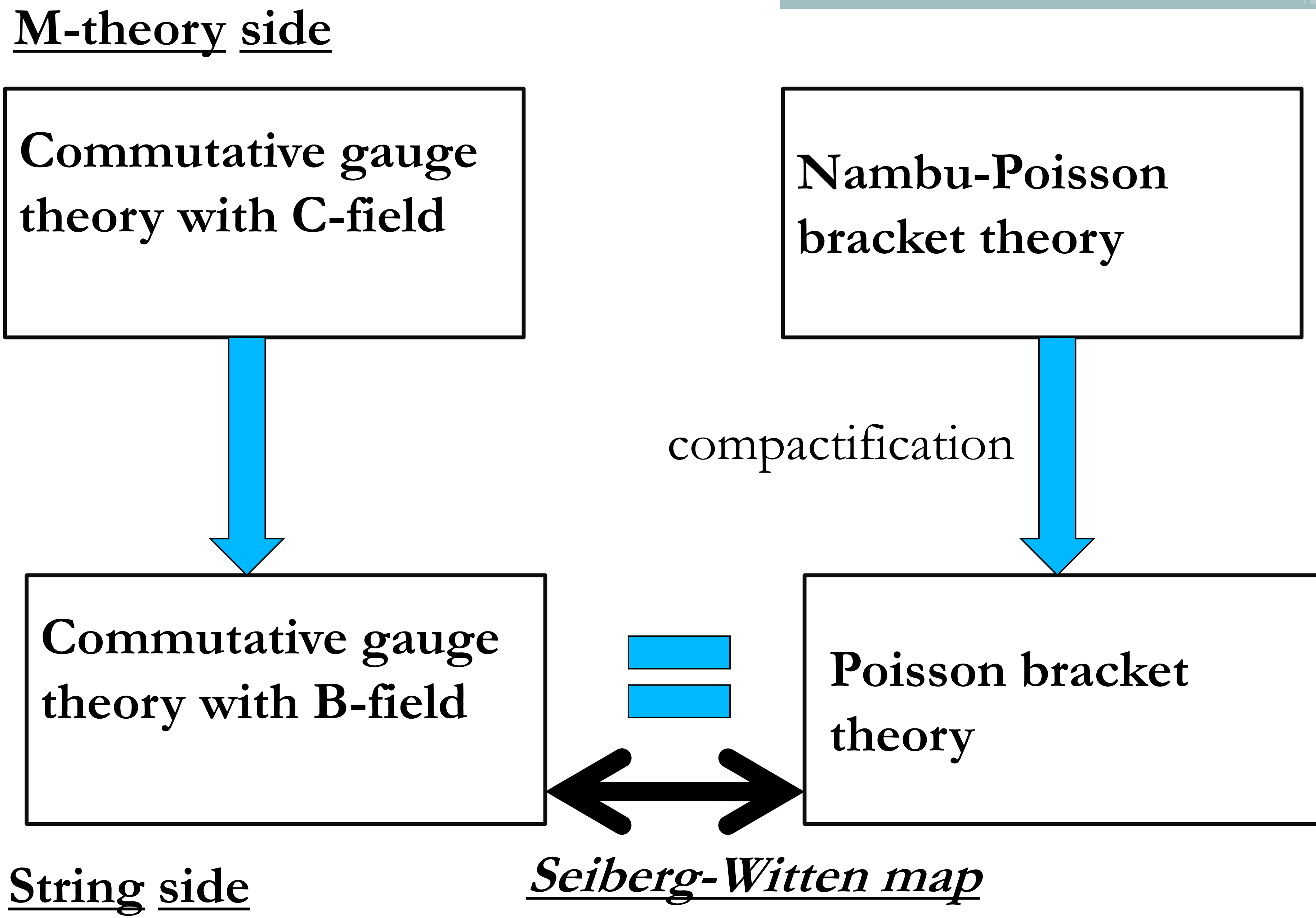
$\tilde{\lambda}$. **:Gauge parameter with area preserving diffeo**

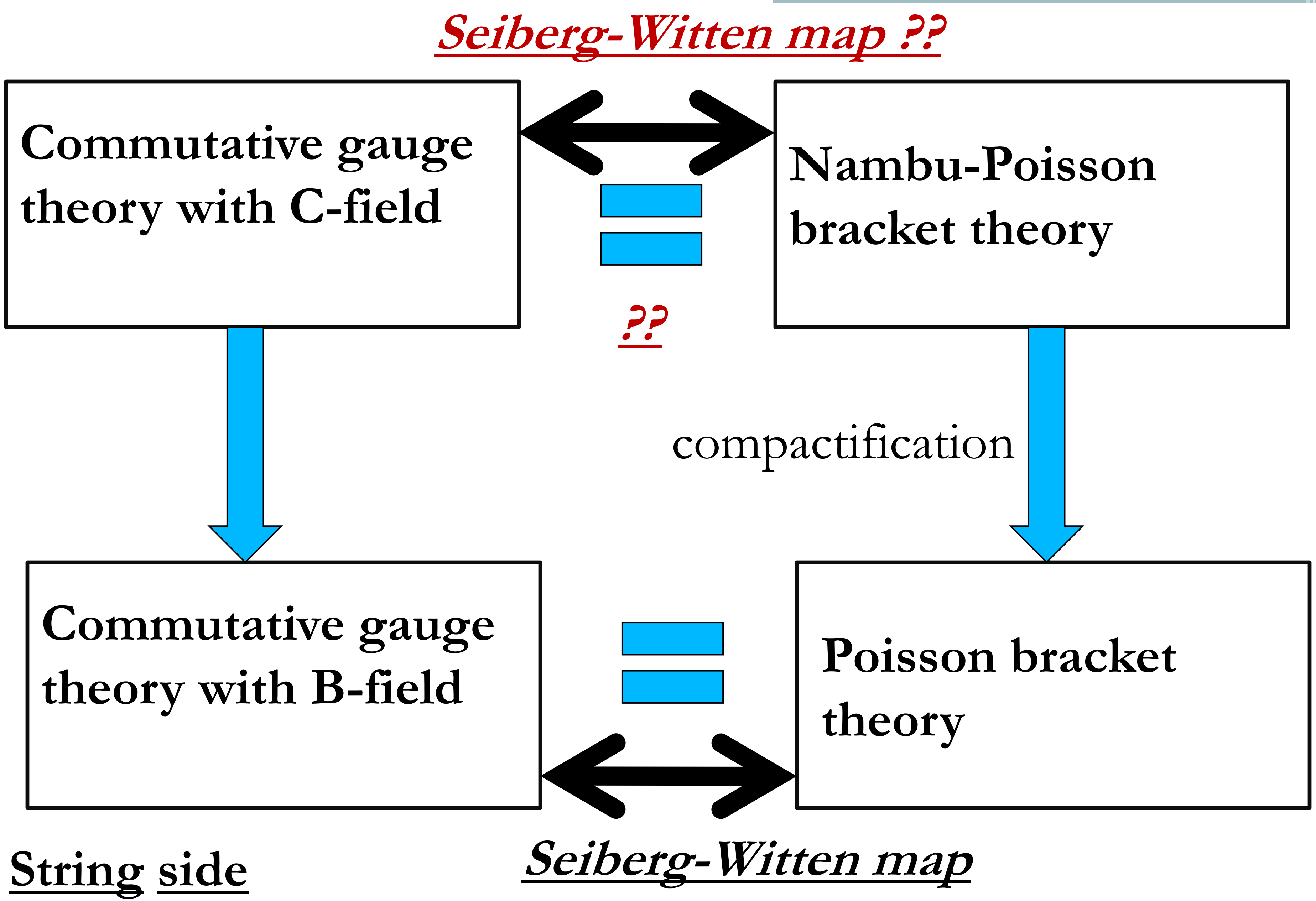
Poisson-Bracket gauge transformation !!

Consistent with

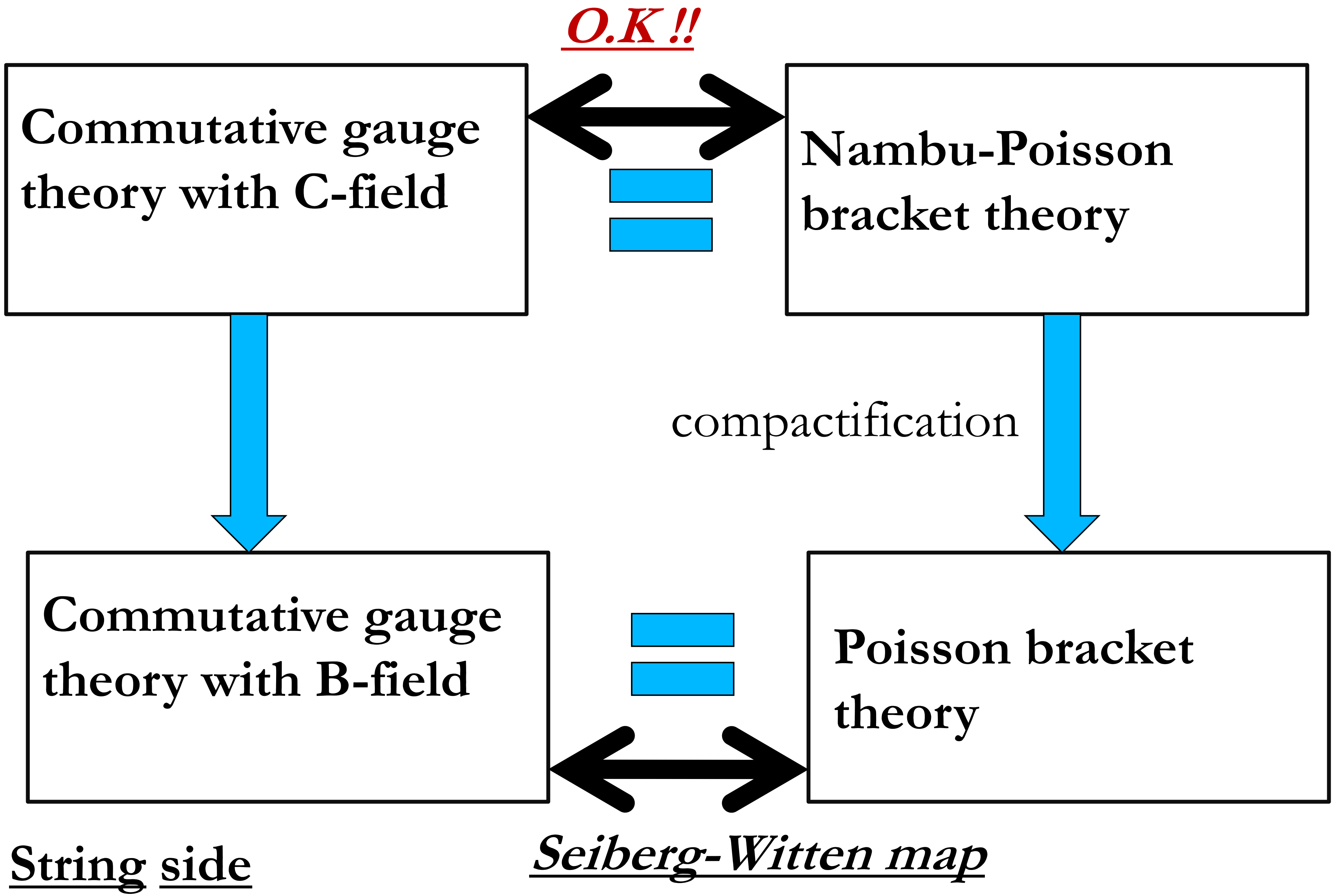
$$\delta_{\hat{\lambda}} \hat{A}(A) = \hat{A}(A + \delta_\lambda A) - \hat{A}(A)$$

Is there corresponding Seiberg-Witten map in the M-theory side ?


M-theory side
Commutative gauge theory with C-field
Nambu-Poisson bracket theory
compactification
Commutative gauge theory with B-field
=
Poisson bracket theory
String side
Seiberg-Witten map



Seiberg-Witten map ??
Commutative gauge theory with C-field
Nambu-Poisson bracket theory
??
compactification
Commutative gauge theory with B-field
Poisson bracket theory
String side
Seiberg-Witten map


O.K !!
Commutative gauge theory with C-field
Nambu-Poisson bracket theory
compactification
Commutative gauge theory with B-field
Poisson bracket theory
String side
Seiberg-Witten map

Order by order solution

Ho-Imamura-Matsuo-Shiba
(JHEP 0808:014,2008)

Gauge transformation in M5-brane in constant C-field

$$\hat{\delta}_{\hat{\lambda}}\hat{\Phi}(\Phi) = \hat{\Phi}(\Phi + \delta_\lambda \Phi) - \hat{\Phi}(\Phi),$$

Gauge transformation in Nambu-bracket

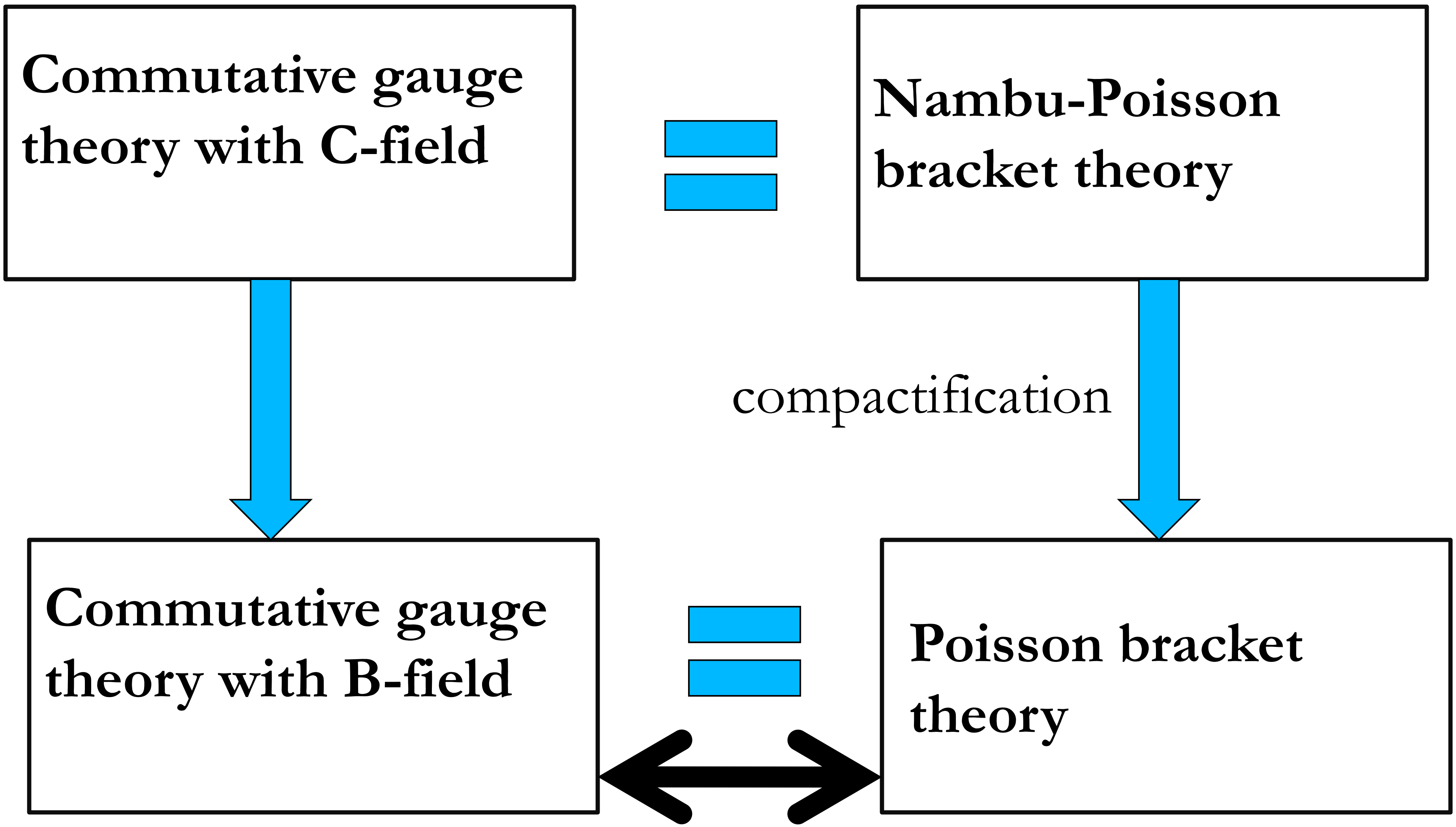
They solve the solution up to order g^1

(g**: "Non-commutative" parameter)**

$$\hat{b}^{\dot{\mu}}(b) = b^{\dot{\mu}} + \frac{g}{2} b^{\dot{\nu}} \partial_{\dot{\nu}} b^{\dot{\mu}} + \frac{g}{2} b^{\dot{\mu}} \partial_{\dot{\nu}} b^{\dot{\nu}} + \mathcal{O}(g^2),$$

$$\hat{B}_\mu{}^{\dot{\mu}}(B,b) = B_\mu{}^{\dot{\mu}} + g b^{\dot{\nu}} \partial_{\dot{\nu}} B_\mu{}^{\dot{\mu}} - \frac{g}{2} b^{\dot{\nu}} \partial_\mu \partial_{\dot{\nu}} b^{\dot{\mu}} + \frac{g}{2} b^{\dot{\mu}} \partial_\mu \partial_{\dot{\nu}} b^{\dot{\nu}} + g \partial_{\dot{\nu}} b^{\dot{\nu}} B_\mu{}^{\dot{\mu}} - g \partial_{\dot{\nu}} b^{\dot{\mu}} B_\mu{}^{\dot{\nu}} - \frac{g}{2} \partial_{\dot{\nu}} b^{\dot{\nu}} \partial_\mu b^{\dot{\mu}} + \frac{g}{2} \partial_{\dot{\nu}} b^{\dot{\mu}} \partial_\mu b^{\dot{\nu}} + \mathcal{O}(g^2),$$

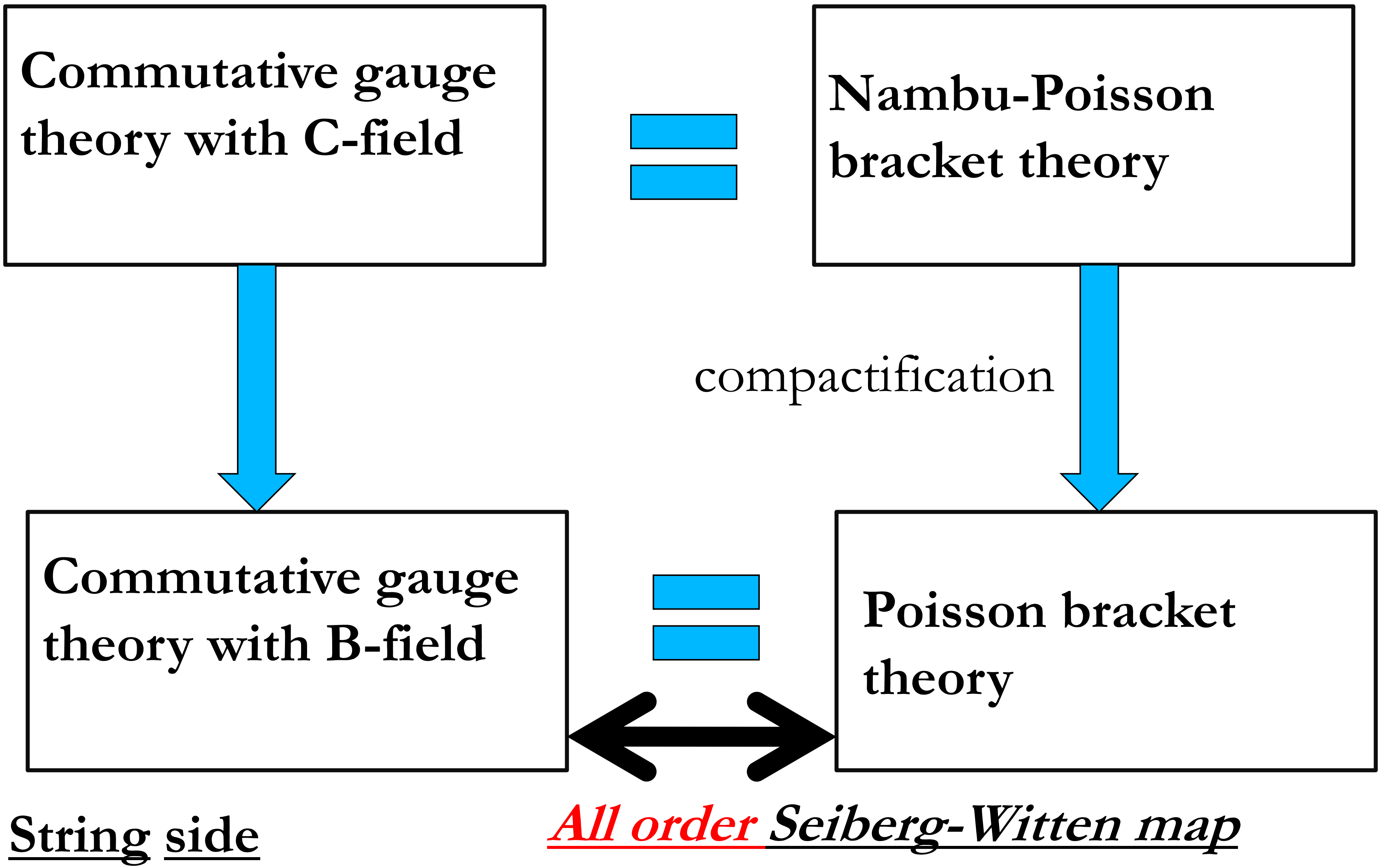
$$\hat{\kappa}^{\dot{\mu}}(\kappa, b) = \kappa^{\dot{\mu}} + \frac{g}{2} b^{\dot{\nu}} \partial_{\dot{\nu}} \kappa^{\dot{\mu}} + \frac{g}{2} (\partial_{\dot{\nu}} b^{\dot{\nu}}) \kappa^{\dot{\mu}} - \frac{g}{2} (\partial_{\dot{\nu}} b^{\dot{\mu}}) \kappa^{\dot{\nu}} + \mathcal{O}(g^2).$$



String side

Seiberg-Witten map in all order of θ

All order Seiberg-Witten map exist !



All order solution in the M-theory side

Chen, Ho, Furuuchi, T.T arXiv:1006.5291

We prepare the parameter obeying the

$$\begin{aligned}\partial_t \theta^{ijk}(t) &= \frac{1}{6}\left(\theta^{a_1 ij}(t)\theta^{a_2 a_3 k}(t) + \theta^{a_1 jk}(t)\theta^{a_2 a_3 i}(t) + \theta^{a_1 ki}(t)\theta^{a_2 a_3 j}(t)\right) H_{a_1 a_2 a_3} \\ &= \frac{1}{6}\theta^{ijk}(t)\theta^{a_1 a_2 a_3}(t) H_{a_1 a_2 a_3}.\end{aligned}$$

→ 3-form field strength

Seiberg-Witten map of each fields are

$$x^i + \hat{b}^i = e^{\partial_t + \frac{1}{2}\theta^{ijk}(t) b_{ij}\partial_k}\, x^i \Big|_{t=0}.$$

$$\begin{aligned}&\overset{\circ}{\partial}_\mu - \hat{B}_\mu{}^i\, \overset{\circ}{\partial}_i \\ &= e^A \left(\overset{\circ}{\partial}_\mu - \theta^{ijk}(t)\left(\overset{\circ}{\partial}_j\, b_{\mu k} - \frac{1}{2}\partial_\mu b_{jk}\right)\overset{\circ}{\partial}_i\right) e^{-A}\Big|_{t=0}.\end{aligned}$$

$$A \equiv \partial_t + \frac{1}{2}\theta^{ijk}(t) b_{ij}\partial_k,$$

Gauge transformations

$$\hat{b}^i(b+\delta_\Lambda b) - \hat{b}^i(b)$$
$$= \left[e^{\partial_t + \frac{1}{2}\theta^{ijk}(t)(b_{ij}+\partial_i\Lambda_j-\partial_j\Lambda_i)\partial_k} - e^{\partial_t+\frac{1}{2}\theta^{ijk}(t)b_{ij}\partial_k}\right] x^i\Big|_{t=0}.$$

for $A \equiv \partial_t + \frac{1}{2}\theta^{ijk}(t)b_{ij}\partial_k, \quad B \equiv \frac{1}{2}\theta^{ijk}(t)(\partial_i\Lambda_j - \partial_j\Lambda_i)\partial_k,$

It is $\left(e^{A+B}x^i - e^A x^i\right)\Big|_{t=0} = \left(\left[e^{A+B}e^{-A} - 1\right] e^A x^i\right)\Big|_{t=0}.$

This becomes $e^{A+B}e^{-A} - 1 = \hat{\kappa}^i(t)\partial_i + \mathcal{O}(\Lambda^2).$

By straight calculation: $\partial_i \hat{\kappa}^i = 0.$

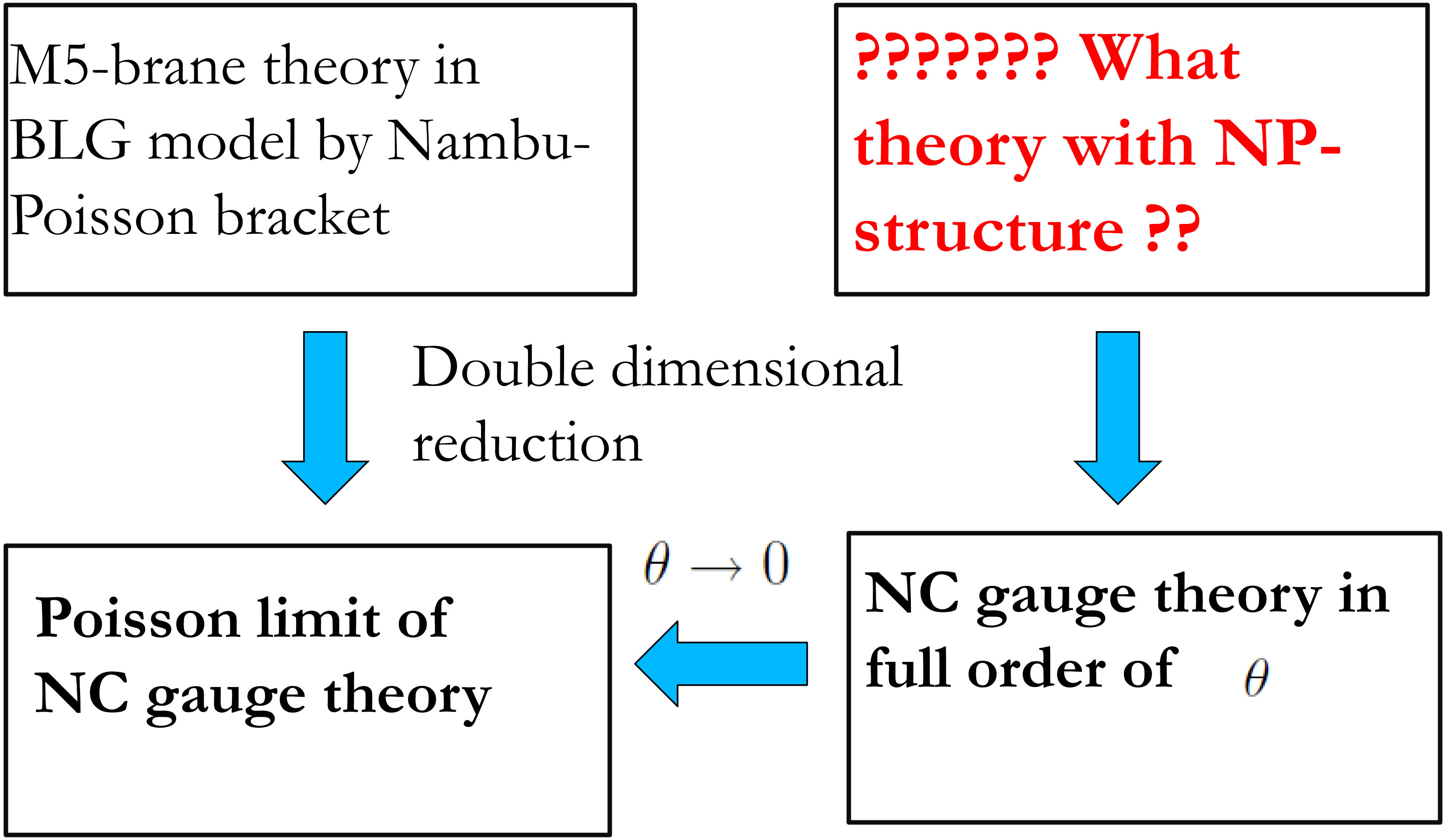
This becomes

$$\hat{b}^i(b+\delta_\Lambda b) - \hat{b}^i(b) = \hat{\kappa}^i + \hat{\kappa}^j\partial_j\hat{b}^i, = \delta_{\hat{\Lambda}}\hat{b}^i$$

Consistent with $\delta_{\hat{\lambda}}\hat{\Phi}(\Phi) = \hat{\Phi}(\Phi+\delta_\lambda\Phi) - \hat{\Phi}(\Phi),$

(B) The theory recovering the full order of Non-commutaive geometry beyond the Poisson limit ?

Chen, Ho,T.T JHEP 1003:104,(2010)


M5-brane theory in BLG model by Nambu-Poisson bracket
??????? What theory with NP-structure ??
Double dimensional reduction
Poisson limit of NC gauge theory
$\theta \to 0$
NC gauge theory in full order of θ

Necessary condition for the theory.

The Poisson theory : ignoring the higher order terms of θ in the Moyal products.

↓

The deformed theory recovering full order of Moyal product should include the higher order terms of θ

Gauge transformation laws must be modified to include higher order terms θ

The problem:

Can modified gauge transformation laws makes the closed algebra ?

$$[\delta_{\kappa}, \delta_{\kappa'}] = \delta_{\kappa''},$$

(1) We suggest the possible deformation and
(2) We tested the closure of the gauge algebra.

Ways to reproduce the full Moyal product

(1) Gauge transformation in the M-theory itself is modified to include the higher order of θ

(2) Gauge transformation in the M-theory is not deformed. But the way of the compactification is deformed to have Moyal product with higher order of θ

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By Lecomte & Roger (1996, French paper)

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Why ? **Following mathematical theorem**

There is no non-trivial deformation of the volume preserving diffeomorphism in $d \geq 3$

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<u>Chen, Ho,T.T JHEP 1003:104,(2010)</u>

(2) Gauge transfor[illegible] in the M-theory is not deformed. But th[illegible] o[illegible]he compactification is deformed to ha[illegible]e M[illegible] product with higher order of θ

If there is such deformation of the reduction, **gauge sym. with Moyal product must be embedded to Nambu-bracket theory.**

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If there is such deformation of the reduction, **gauge sym. with Moyal product must be embedded to Nambu-bracket theory.**

<u>But the embedded algebra can not be closed consistently</u>.

(2) Gauge transfor n in the M-theory is not deformed. But th w y o he compactification is deformed to ha e M product with higher order of θ

There are following two possibilities

(1) Gauge transfor ion the M-theory itself is modified to incl le t h gher order of θ

(2) Gauge transfor in the M-theory is not deformed. But th v o he compactification is deformed to ha e M product with higher order of θ

So,

M5-brane theory in BLG model by Nambu-Poisson bracket

?????? Wha… theo… w…h N P- struct…e ?

No go theorem!!

Very difficult !!

Poisson limit of NC gauge theory

$\theta \rightarrow 0$

NC gauge theory in full order of θ

Chen, Ho, T.T JHEP 1003:104, (2010)

Crucial structure of No-go theorem

Usually, the closure of the gauge algebra is helped by the associativity of ring structure.

$$(\hat{\lambda}_1 * \hat{\lambda}_2) * \hat{\lambda}_3 = \hat{\lambda}_1 * (\hat{\lambda}_2 * \hat{\lambda}_3)$$

⇒ $$[\hat{\lambda}_1, [\hat{\lambda}_2, \cdot]_{\mathrm{Moyal}}]_{\mathrm{Moyal}} - [\hat{\lambda}_2, [\hat{\lambda}_1, \cdot]_{\mathrm{Moyal}}]_{\mathrm{Moyal}} = [[\hat{\lambda}_1, \hat{\lambda}_2], \cdot]_{\mathrm{Moyal}}]_{\mathrm{Moyal}}$$

But the Lie-3-algebra as well as Nambu-bracket does not have ring structure with associativity,

⇒ This is the obstacle to make the symmetry algebra closed

Candidates can not have gauge symmetry

Summary

Summary

Nambu-Bracket structure might capture the M-theory uplifted version of the non-commutative geometries.

We have found several supporting evidences.

But there still remain so many things to be clarified

Future directions

(1) Up-lift of the full order of Moyal product

Would instruct the proper direction of the improvement of the math structure.

(2) Multiple M5-branes

Recent development by Lambert=Papageorgakis ??

Non-abelian extension of the 2-form gauge fields ??

Go higher
(Beyond Fuji ??
新高山？？)

完。 結束了