Gauge/String Duality and High Energy Scattering

Chung-I Tan, Brown U.
August 7, 2010, Summer Institute 2010
Fuji Calm

- talk based on
- R. Brower, J. Polchinski, M. Strassler, and C-I Tan, hep-th/0603115, hep-th/0707.2408, hep-th/0710.4378;
- R. Brower, S. Mathur, and C-I Tan, hep-th/0003115, hep-th/9908196
- R. Brower, M. Djuric and C-I Tan, hep-th/0812.0354
- R. Brower, M. Djuric, I. Sarcevic and C-I Tan, hep-ph/1007.2259

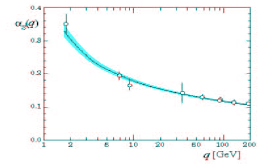
Outline

- Scales in QCD--brief history of "QCD string"
- QCD "Closed String" as Metric Fluctuations in AdS space
  - Graviton is a Regge cut in AdS
  - Pomeron as a Reggeized Massive Graviton
  - Pomeron Vertex Operator
  - Transverse AdS_3 and High Energy Scattering
- Anti-Symmetric Forms -- Odderon
- Beyond Graviton exchange -- Eikonalization
- Deep Inelastic Scattering at Small-x
- Summary

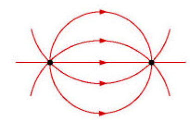
I. Scalar Dependence of QCD and History of Hadron Scattering at High Energies

Asymptotic Freedom

perturbative



$$\alpha_s(q) \equiv \frac{\bar{g}(q)^2}{4\pi} = \frac{c}{\ln(q/\Lambda)} + \dots$$



$r < 0.1\ fm$

Confinement

non-perturbative

$r >> 1\ fm$

Force at Long Distance--Constant Tension/Linear Potential, Coupling increasing, Quarks and Gluons strongly bound <==> "Stringy Behavior"

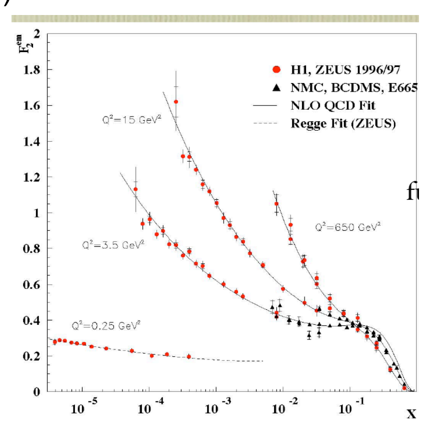
Test of Perturbative QCD-- Deep Inelastic Scattering (DIS)

Anomalous Dimension of

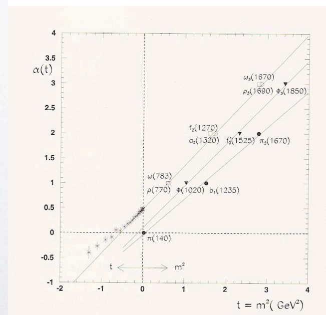
Leading twist operator

DGLAP evolution

$$\mathrm{tr}(F_{+\mu} D_+^{J-2} F_+{}^{\mu})$$



Regge Behavior and Regge Trajectory



$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

7

Genesis of String Theory

- Duality between direct-channel resonances and Regge behavior at high energies

$$\sum_r \frac{g_r^2(t)}{s-(Mr-i\Gamma_r)^2} \simeq \beta(t)(-\alpha' s)^{\alpha(t)}$$

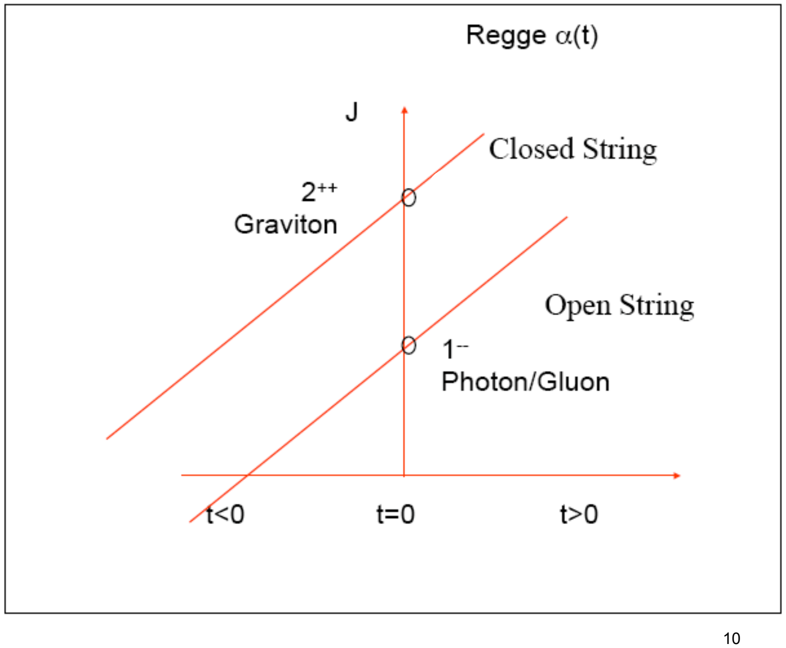
- Expressed mathematically (Veneziano)

$$A_{\pi^+\pi^-\to\pi^+\pi^-}(s,t) = g_0^2 \frac{\Gamma(1-\alpha_\rho(t))\Gamma(1-\alpha_\rho(s))}{\Gamma(1-\alpha_\rho(t)-\alpha_\rho(s))}$$

- Interpret as quantum theory of open string.

Genesis of String Theory
continued

- This is not the end of the story.
- Unitarity requires closed string.
- Virasoro amplitude:



$$A(s,t,u) = \beta \frac{\Gamma(1-\alpha(s)/2)\Gamma(1-\alpha(t)/2)\Gamma(1-\alpha(u)/2)}{\Gamma(1-(\alpha(t)+\alpha(u))/2)\Gamma(1-(\alpha(s)+\alpha(u))/2)\Gamma(1-(\alpha(t)+\alpha(s))/2)}$$

Birth of Classic String Theory!

Death and Resurrection of QCD string

(i) ZERO MASS STATE (gauge/graviton)

(ii) SUPER SYMMETRY

(iii) EXTRA DIMENSION 4+6 = 10

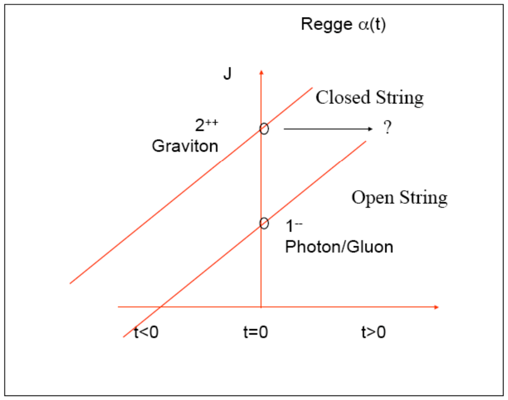
(iv) NO HARD PROCESSES! (totally wrong dynamics)

Stringy Rutherford Experiment

At Wide Angle: s,-t,-u >> 1/α'

$$A_{closed}(s,t) \to \exp\left[-\frac{1}{2}\alpha'(s\ln s + t\ln t + u\ln u)\right]$$

To get back to QCD: Need to give mass to Graviton



4-Dim Massive Graviton

5-Dim Massless Mode:

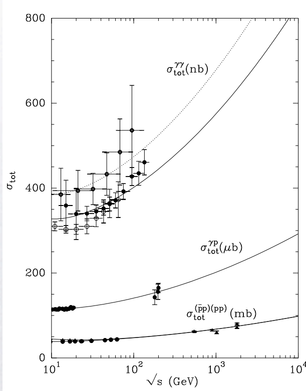
$$0 = E^2 - (p_1^{\,2} + p_2^{\,2} + p_3^{\,2} + p_r^{\,2})$$

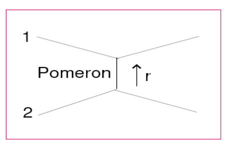
If, due to Curvature in fifth-dim, $p_r^{\,2} \neq 0$,

Four-Dimensional Mass:

$$E^2 = (p_1^{\,2} + p_2^{\,2} + p_3^{\,2}) + M^2$$

Total Cross Sections





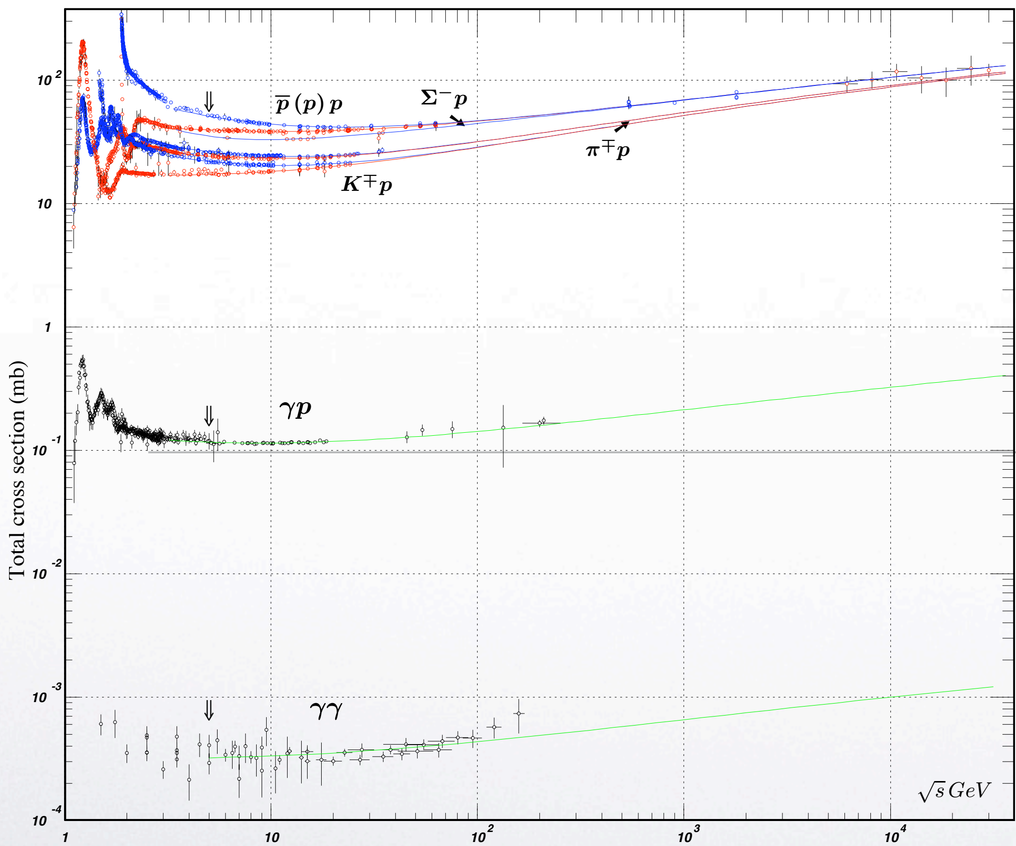
$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

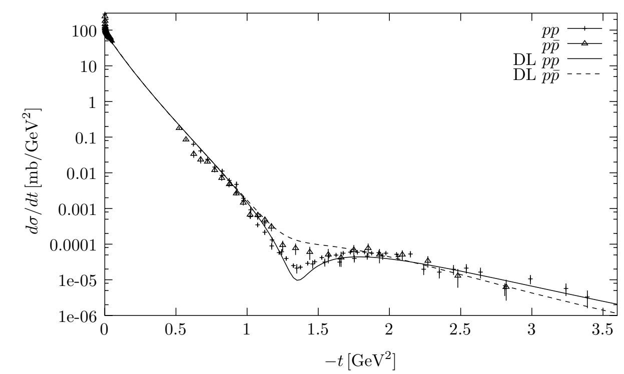
$$\sigma_{total} \sim \mathcal{A}(s,0)/s \sim S^{J(0)-1} \sim s^{\alpha(0)-1}$$

$$\alpha(0) > 1$$

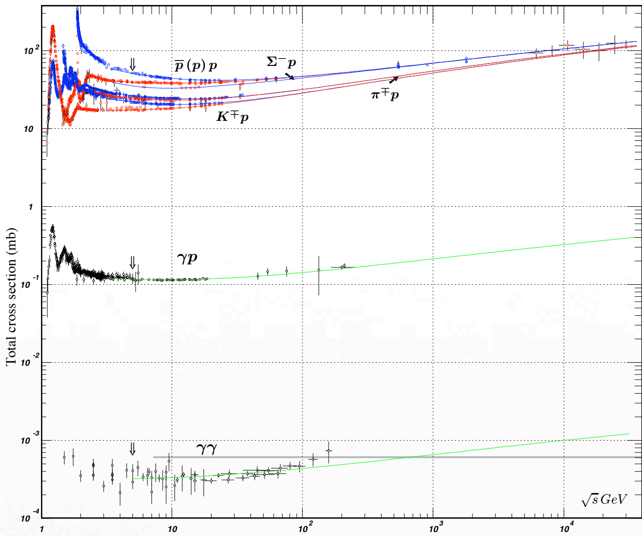
(IR) Pomeron as Closed String??

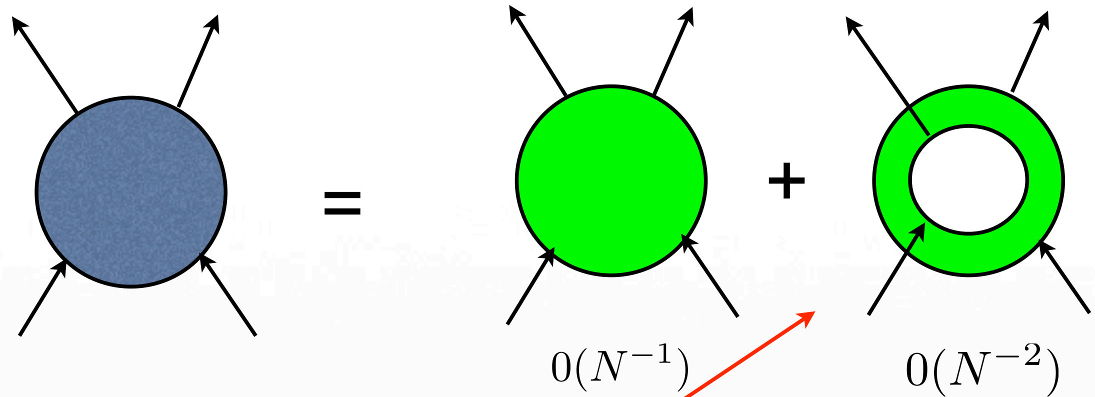
← *AdS?? Graviton??*


Total cross section (mb)
$\bar{p}\,(p)\,p$
$\Sigma^- p$
$\pi^\mp p$
$K^\mp p$
γp
$\gamma\gamma$
$\sqrt{s}\,GeV$


pp
$p\bar{p}$
DL pp
DL $p\bar{p}$
$d\sigma/dt$ [mb/GeV^2]
$-t$ [GeV^2]

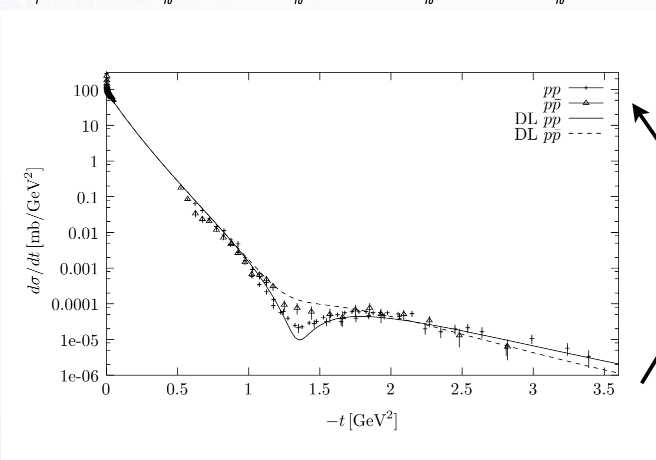
Open-string Scattering

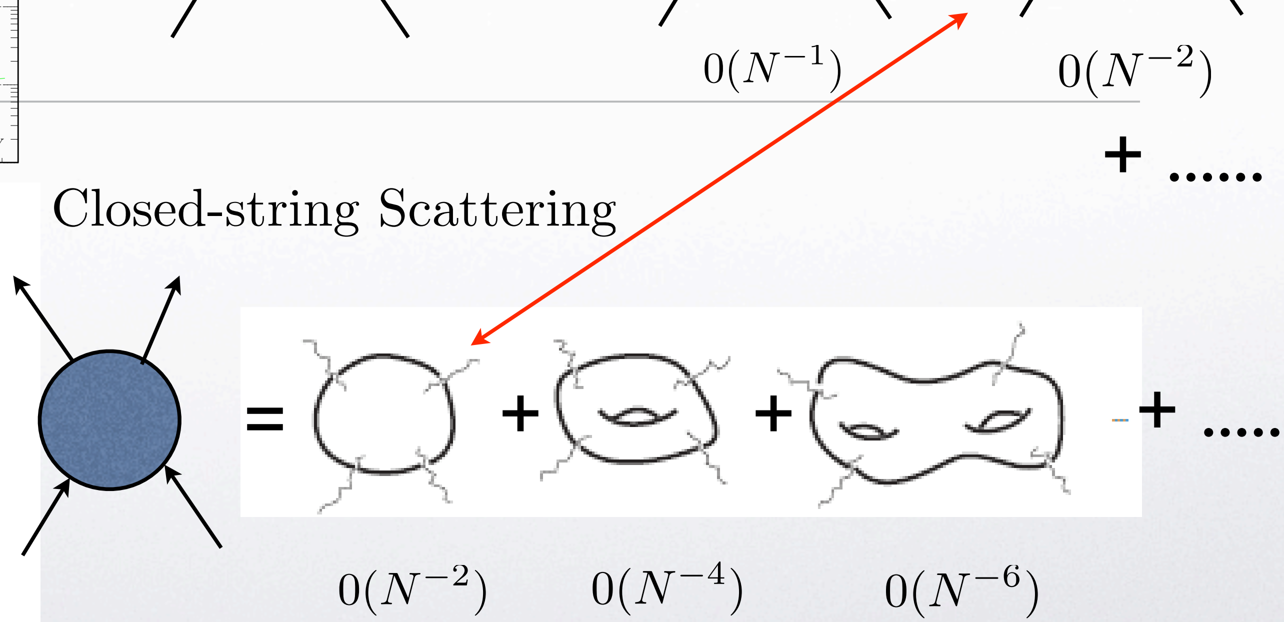




+ ......

Closed-string Scattering





In this talk, will focus on "closed strings" only. For "open-string" in AdS/CFT, e.g., mesons and baryons, see talks by Koji Hashimoto and others.

What is the (bare) Pomeron anyway?

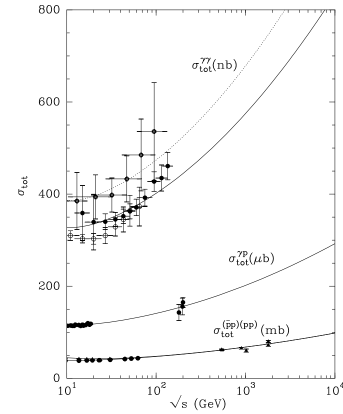
Definition:

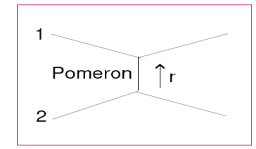
The Pomeron ´ the vacuum exchange contribution to scattering at high energies at leading order in $1/N_c$ expansion.

$$A(s,t) = g_s^2 A_1(s,t,\lambda) + g_s^4 A_2(s,t,\lambda) + \cdots$$

Where $\lambda = g^2 N_c$ & $g_s = 1/N_c$

Total Cross Sections





$$\mathcal{A} \sim s^{J(t)} = s^{\alpha(0)+\alpha' t}$$

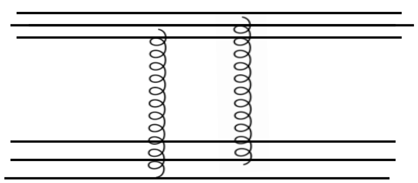
$$\sigma_{total} \sim \mathcal{A}(s,0)/s \sim S^{J(0)-1} \sim s^{\alpha(0)-1}$$

$$\alpha(0) > 1$$

(IR) Pomeron as Closed String??

Two gluon exchange (Low-Nussinov Pomeron!)

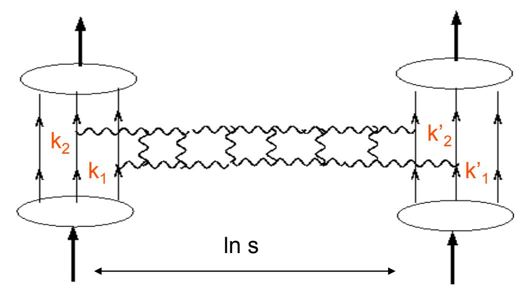
- $J_{cut} = 2\,(J-1) + 1 = 1$



F.E. Low. Phys. Rev. D 12 (1975), p. 163.
S. Nussinov. Phys. Rev. Lett. 34 (1975), p. 1286.

BFKL: Balitsky & Lipatov; Fadin,Kuraev,Lipatov'75

$t = -(k_1 + k_2)^2$ →



$$\lambda = g^2 N_c \sim 0$$

- ❑ Sum diagrams 1^{st} order in $g^2 N_c$ & all orders $(g^2 N_c \log s)^n$
- ❑ BKFL equation for 2 "reggized" gluon ladder is L = 2 SL(2,C) spin chain to one loop order .
- ❑ Accidentally "planar" diagrams (e.g. $N_c = 1$) and conformal.

2-GLUONS in 4d = GRAVITON in 5d

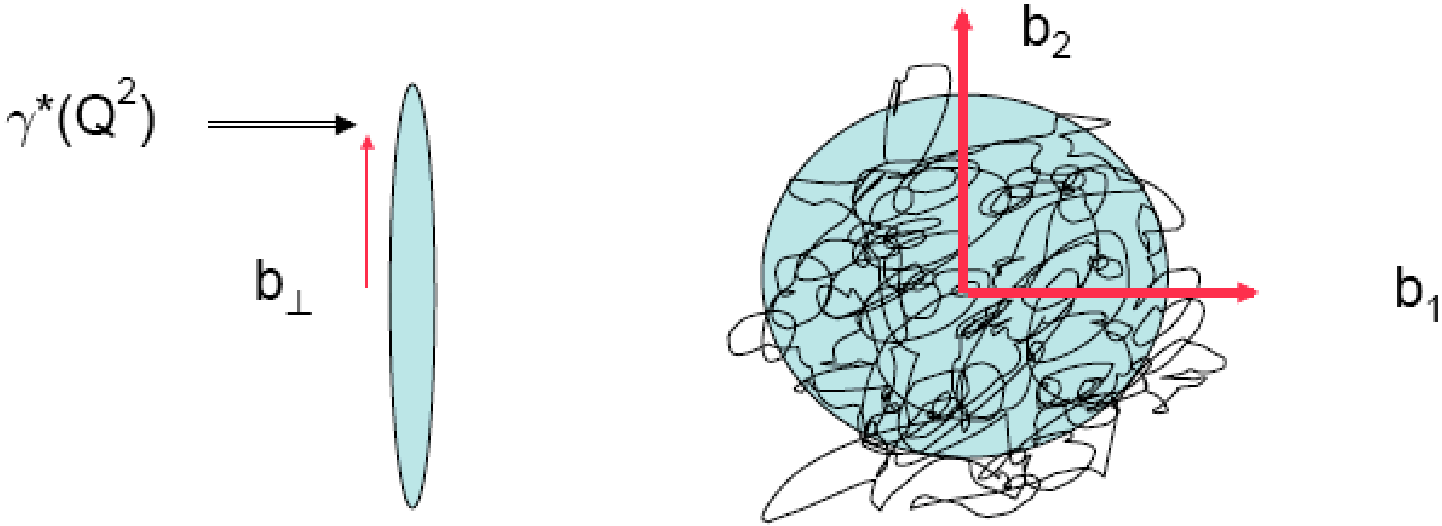
?

CFT = AdS

Emergence of 5-dim AdS-Space

Let $z = 1/r$, $0 < z < z_0$, where $z_0 \sim 1/\Lambda_{qcd}$

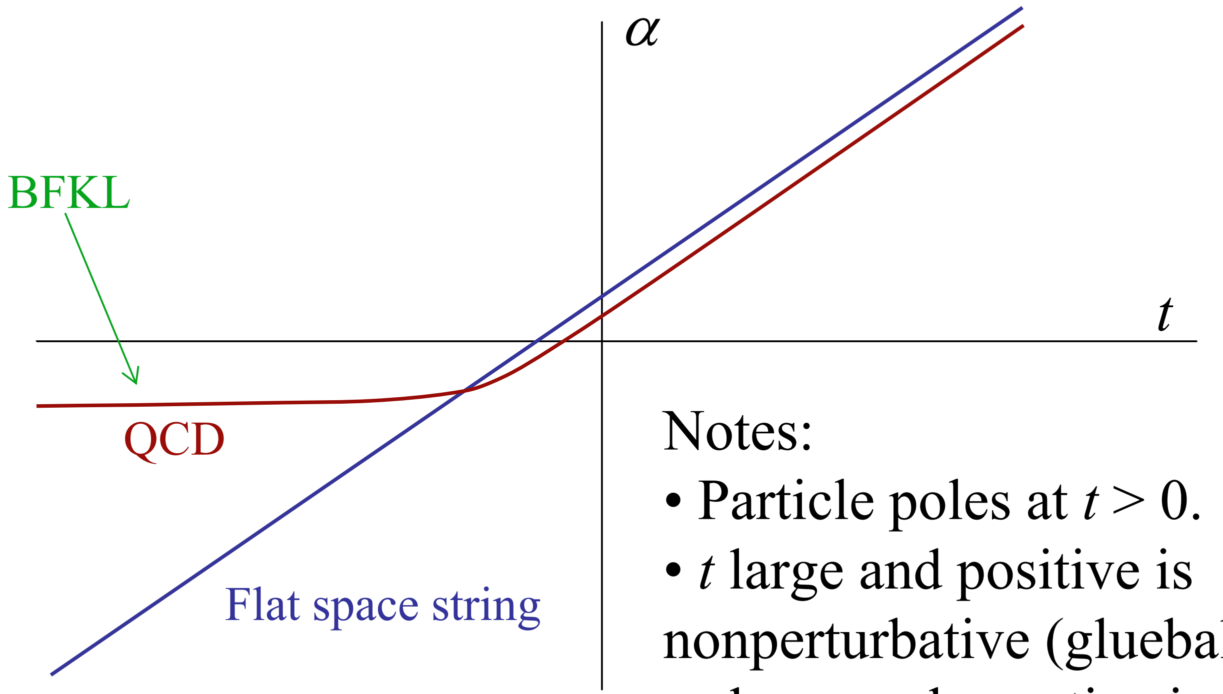
"Fifth" co-ordinate is size **z / z'** of proj/target



5 kinematical Parameters:

2-d Longitudinal	$p^\pm = p^0 \pm p^3 \simeq \exp[\pm \log(s/\Lambda_{qcd})]$
2-d Transverse space:	$x'_\perp - x_\perp = b_\perp$
1-d Resolution:	$z = 1/Q$ (or $z' = 1/Q'$)

Trajectories are different in QCD:



Notes:

- Particle poles at $t > 0$.
- t large and positive is nonperturbative (glueballs).
- t large and negative is perturbative (BFKL).

What can we learn from AdS/QCD?

II: Gauge/String Duality

QCD Pomeron as “metric fluctuations” in AdS

- Strong <==> Weak duality
- Geometry of AdS/CFT and Scale Invariance
- High Energy Scattering
- Confinement and Glueball Spectrum
- Pomeron as Reggeized Massive Graviton

IIa: Degrees of Freedom

Weak Coupling:

Gluons and Quarks: $A_\mu^{ab}(x), \psi_f^a(x)$

Gauge Invariant Operators: $\bar{\psi}(x)\psi(x),\ \ \bar{\psi}(x)D_\mu\psi(x)$

$S(x) = TrF_{\mu\nu}^2(x),\ \ O(x) = TrF^3(x)$

$T_{\mu\nu}(x) = TrF_{\mu\lambda}(x)F_{\lambda\nu}(x),\ \ etc.$

$$\mathcal{L}(x) = -TrF^2 + \bar{\psi}\not{D}\psi + \cdots$$

Strong Coupling:

Metric tensor: $G_{mn}(x) = g_{mn}^{(0)}(x) + h_{mn}(x)$

Anti-symmetric tensor (Kalb-Ramond fields): $b_{mn}(x)$

Dilaton, Axion, etc. $\phi(x),\ a(x),\ etc.$

Other differential forms: $C_{mn\cdots}(x)$

$$\mathcal{L}(x) = \mathcal{L}(G(x), b(x), C(x), \cdots)$$

$\mathcal{N} = 4$ SYM Scattering at High Energy in StrongCoupling

$$\langle e^{\int d^4x \phi_i(x) \mathcal{O}_i(x)} \rangle_{CFT} = \mathcal{Z}_{string}\left[\phi_i(x,z)|_{z\sim 0} \to \phi_i(x)\right]$$

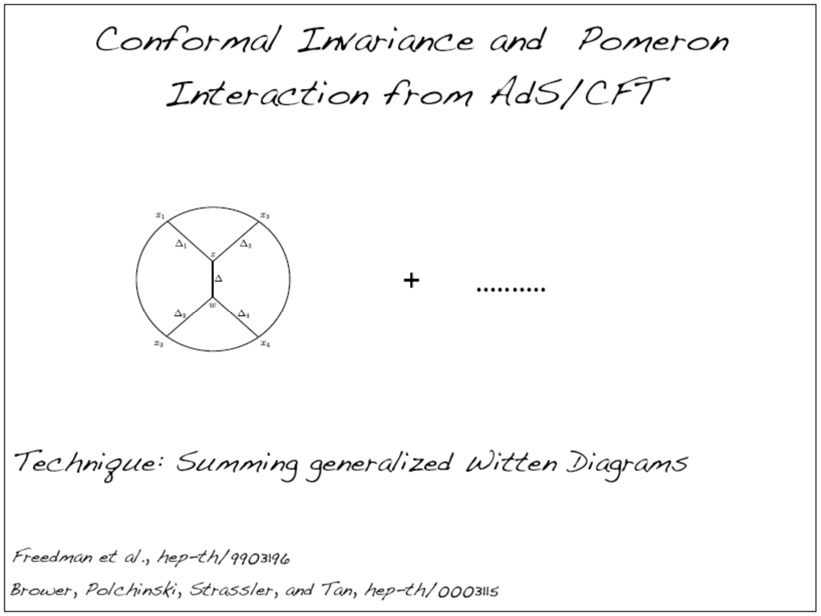
Bulk Degrees of Freedom from type-IIB Supergravity on AdS_5:

- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

$$\lambda = g^2 N_c \to \infty$$

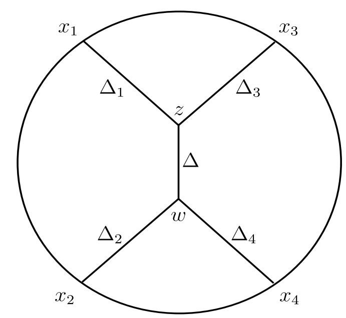
Supergravity limit

- Strong coupling
- Conformal
- Pomeron as Graviton in AdS



- Draw all "Witten-Feynman" Diagrams in AdS5,
- High Energy Dominated by Spin-2 Exchanges:

$$p_1 + p_2 \to p_3 + p_4$$



One Graviton Exchange at High Energy

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde{\Phi}_\Delta(p_1^2,z)\tilde{\Phi}_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde{\Phi}_\Delta(p_2^2,z')\tilde{\Phi}_\Delta(p_4^2,z')$$

$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

- Strong Coupling Pomeron has $J=2$
- Need to consider λ finite.
- For QCD, needs confinement to introduce a scale.

$$\lambda = g^2 N_c \to \infty$$

Geometry of AdS/CFT and Scale Invariance

What is the curved space?

Maldacena: UV (large r) is (almost) an $AdS_5 \times X$ space

$$ds^2 = r^2 dx_\mu dx^\mu + \frac{dr^2}{r^2} + ds_X^2$$

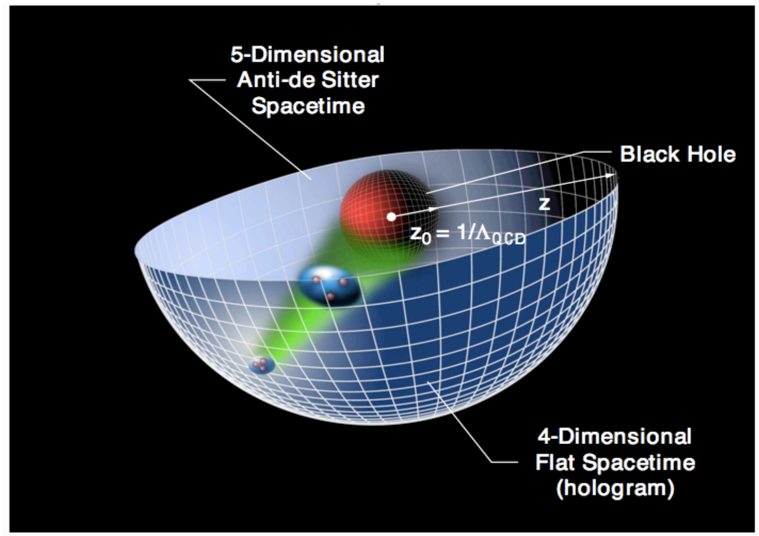
Captures QCD's approximate UV conformal invariance

$$x \to \zeta x \ , \ r \to \frac{r}{\zeta} \qquad \text{(recall } r \sim \mu\text{)}$$

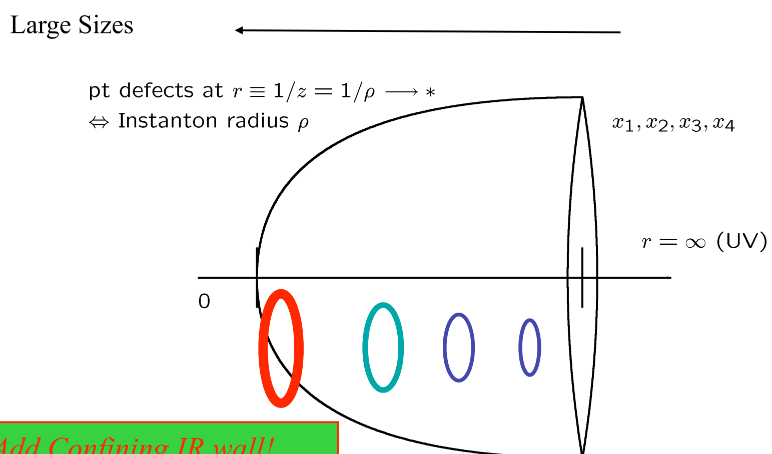
Confinement: IR (small r) is cut off in some way

$$r \sim \mu > r_{min} \sim \Lambda_{QCD}$$

For Pomeron: *string theory* on cut-off AdS_5 (X plays no role)



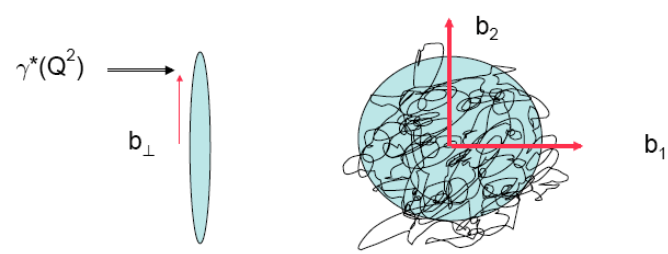
Cutoff AdS_5



Add Confining IR wall!

z=1/r,

"Fifth" co-ordinate is size z / z' of proj/target



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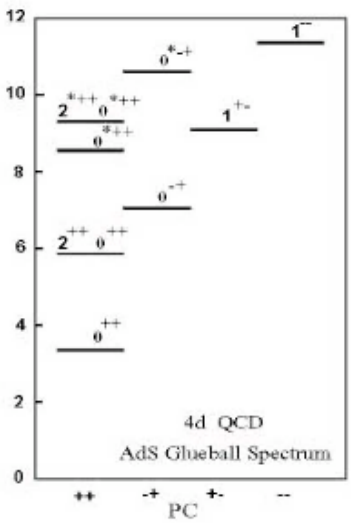
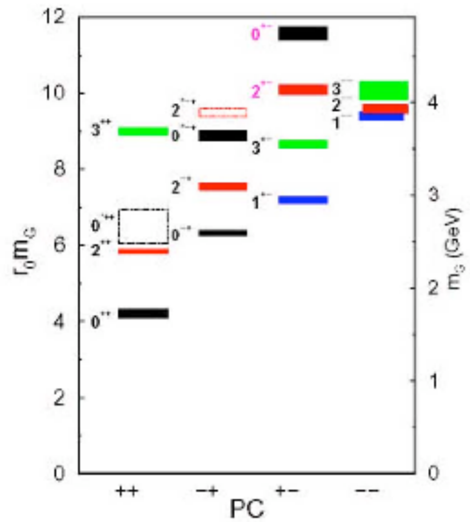
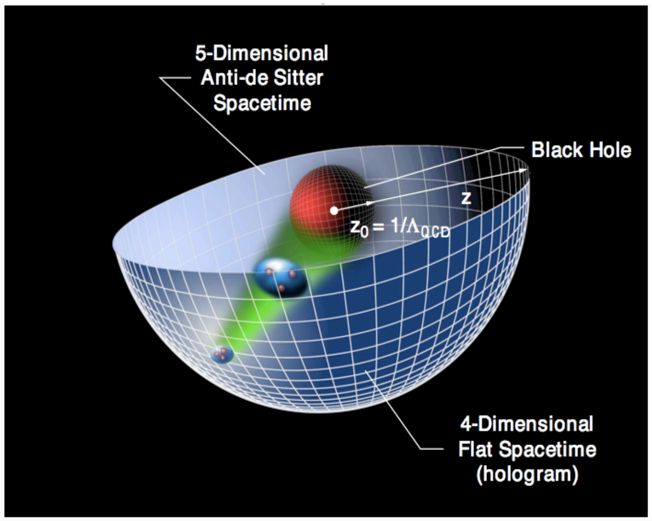
Confinement Deformation: Glueball Spectrum







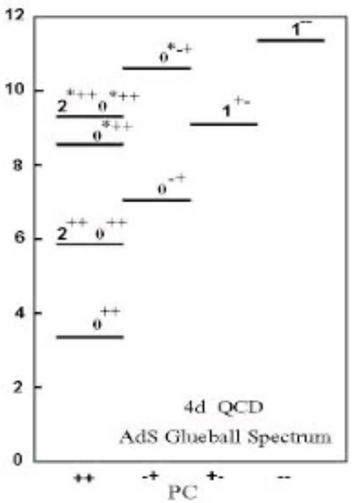
Four-Dimensional Mass:

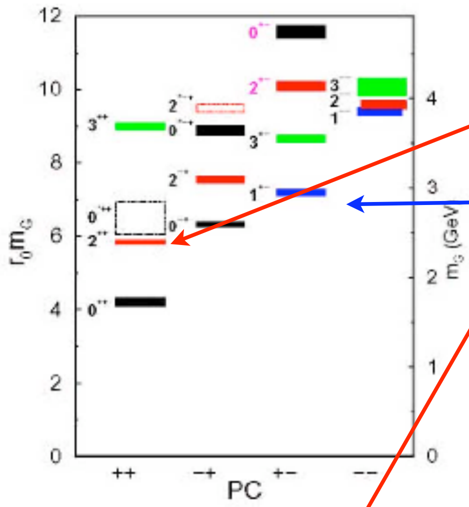
$$E^2 = (p_1^2 + p_2^2 + p_3^2) + M^2$$

5-Dim Massless Mode:

$$0 = E^2 - (p_1^2 + p_2^2 + p_3^2 + p_r^2)$$

Confinement Deformation: Glueball Spectrum





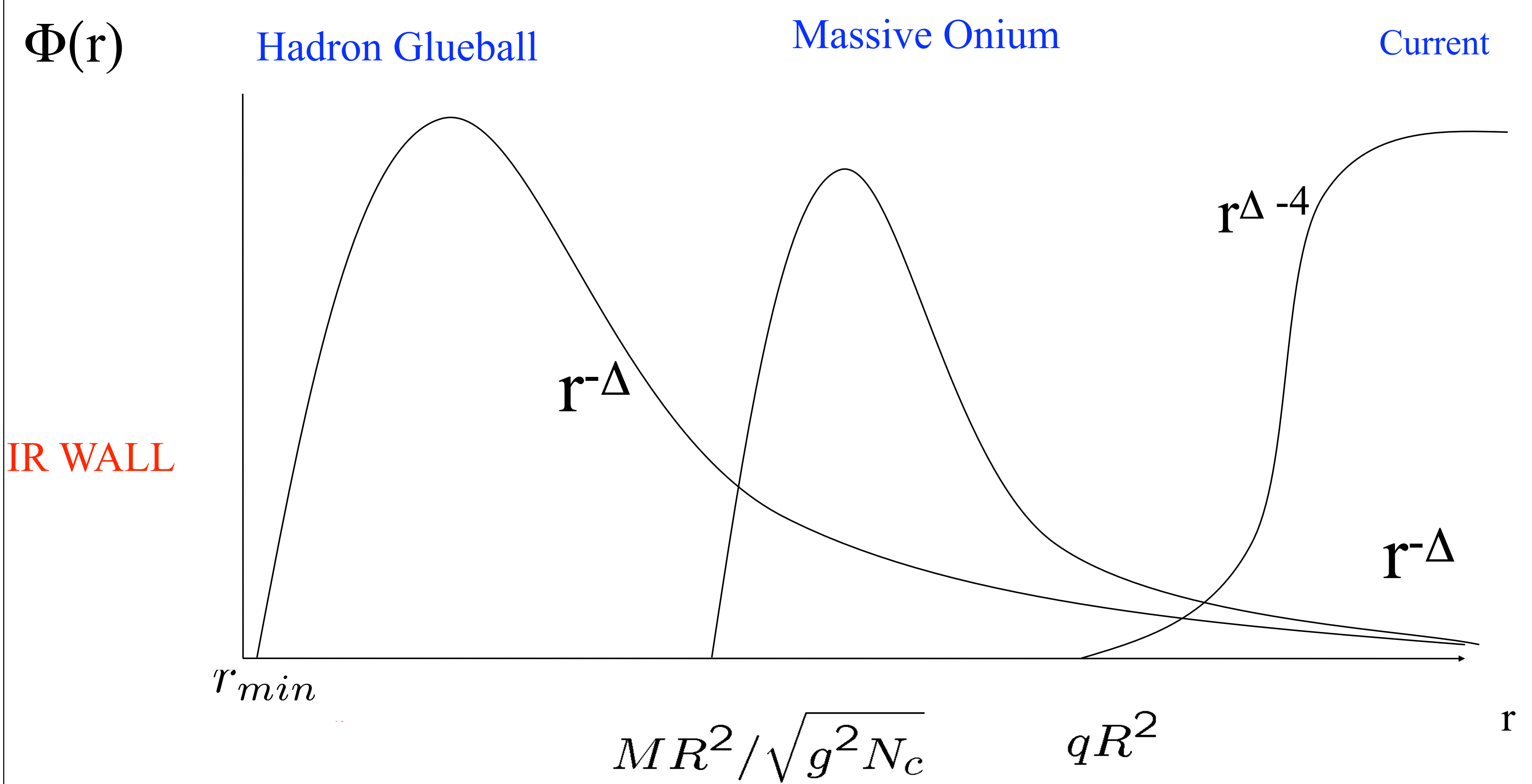
Pomeron trajectory

Odderon trajectory

States from 11-d G_{MN}				States from 11-d A_{MNL}		
$G_{\mu\nu}$	$G_{\mu,11}$	$G_{11,11}$	m_0 (Eq.)	$A_{\mu\nu,11}$	$A_{\mu\nu\rho}$	m_0 (Eq.)
G_{ij} 2^{++}	C_i $1^{++}_{(-)}$	ϕ 0^{++}	4.7007 (T_4)	B_{ij} 1^{+-}	C_{123} $0^{+-}_{(-)}$	7.3059 (N_4)
$G_{i\tau}$ $1^{-+}_{(-)}$	C_τ 0^{-+}		5.6555 (V_4)	$B_{i\tau}$ $1^{--}_{(-)}$	$C_{ij\tau}$ 1^{--}	9.1129 (M_4)
$G_{\tau\tau}$ 0^{++}			2.7034 (S_4)	G^α_α State 0^{++}		10.7239 (L_4)

Table 1: IIA Classification for QCD_4. Subscripts to J^{PC} designate $P_\tau = -1$.

Approx. Scale Invariance and the 5th dimension



==> Hard Scattering (Polchinski-Strassler)

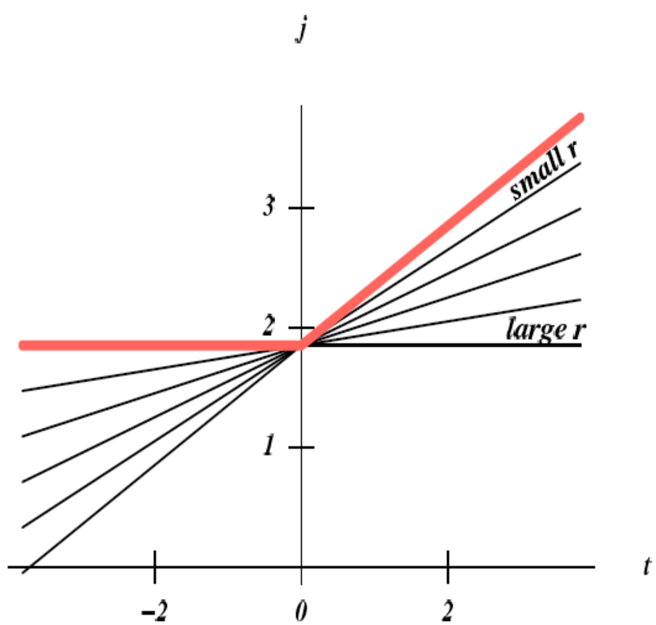
IIb: Pomeron as Diffusion in AdS

Conformal Pomeron in Target Space:

Ultra-local approximation in AdS:

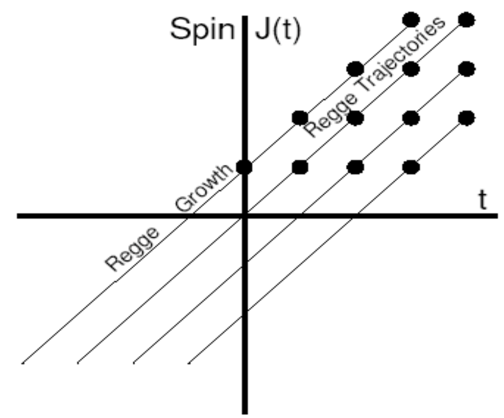
$$\tilde{s} = \frac{R^2}{r^2} s \ , \quad \tilde{t} = \frac{R^2}{r^2} t \ , \qquad \alpha'_{\rm eff}(r) = \frac{R^2 \alpha'}{r^2}$$

$$\mathcal{T}_{10}^{(\pm)}(\tilde{s}, \tilde{t}) \sim f^{(\pm)}(\alpha' \tilde{t})(\alpha' \tilde{s})^{\alpha_{\pm}(0) + \alpha' \tilde{t}/2} \sim s^{\alpha_{\pm}(0) + \alpha'_{eff}(r) t/2}$$



Flat Space String Scattering -- Regge Behavior

$$\mathrm{Im} A \sim \sum_i s^{J_i(t)}$$



$$J(t) = \alpha(t) = \alpha_0 + \alpha' t$$

$$t \leftrightarrow \nabla_b^2$$

$$G(s; \vec{b}, \vec{b}') \longleftrightarrow \langle\, \vec{b} \mid s^{2+\alpha'\nabla_b^2/2} \mid \vec{b}' \,\rangle$$

$$\sim s^{\alpha_0} \frac{\exp\left[-|\vec{x}|^2/\alpha' \ln s\right]}{\sqrt{\ln s}}$$

$\Leftarrow$ Diffusion in Impact Space

Diffusion in AdS

AdS, C=+1:

$$\alpha' \tilde{t} \to \alpha' \Delta_P \equiv \frac{\alpha' R^2}{r^2} \nabla_b^2 + \alpha' \Delta_{\perp P}$$

$$s^{2+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i} s^j G(j)$$

with

$$G(j) = \frac{1}{j - 2 - \alpha' \Delta_P / 2}$$

Effective Schrodinger Equation:

$$(j - 2 - \alpha' \Delta_P/2) G(j; z, z', t) = \delta(z - z')$$

$$At \;\; t = 0 \;\; and \;\; z = e^{-u}$$

$$\left[-\partial_u^2 + 4 + 2\sqrt{\lambda}(j-2) \right] = e^u \delta(u - u')$$

Fixed cut in J-plane:

Weak coupling:
(BFKL)

$$j_0 = 1 + \frac{4 \ln 2}{\pi} \alpha N$$

Strong coupling: $j_0 = 2 - \frac{2}{\sqrt{\lambda}}$

Comparison of strong vs weak coupling kernel at t=0

Strong Coupling:

$$\mathcal{K}(r,r',s) = \frac{s^{j_0}}{\sqrt{4\pi\mathcal{D}\ln s}} e^{-(\ln r - \ln r')^2/4\mathcal{D}\ln s}$$

Diffusion in "warped co-ordinate"

$$j_0 = 2 - \frac{2}{\sqrt{g^2N}} + O(1/g^2N) \qquad \mathcal{D} = \frac{1}{2\sqrt{g^2N}} + O(1/g^2N) \quad .$$

Weak Coupling:

$$K(s,k_\perp,k'_\perp) \approx \frac{s^{\alpha(0)-1}}{\sqrt{\pi\ln s}} e^{-[(\ln k'_\perp - \ln k_\perp)^2/4\mathcal{D}\ln s]}$$

$$j_0 = 1 + \ln(2) g^2N/\pi^2 \qquad \mathcal{D} = \frac{14\zeta(3)}{\pi} g^2N/4\pi^2.$$

Pomeron Propagator at Finite Coupling λ :

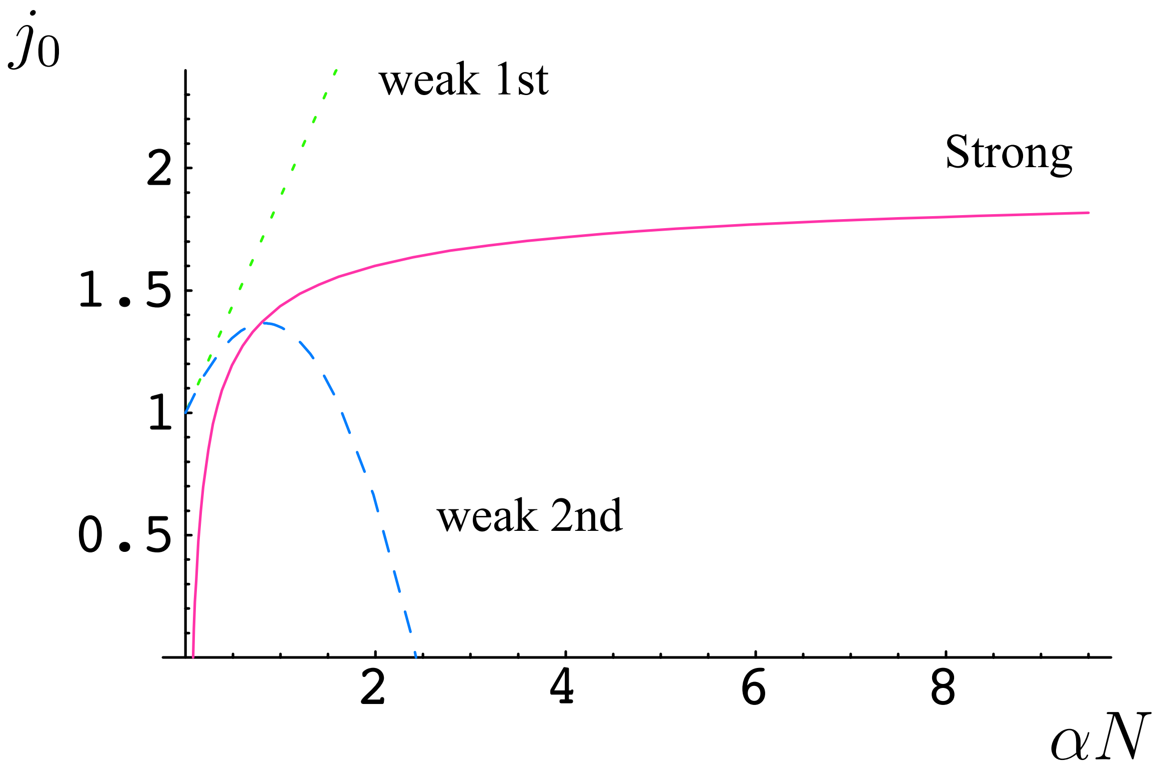
Due to Diffusion in AdS

- Pomeron becomes cut at

$$j_0 = 2 - 2/\sqrt{\lambda}$$

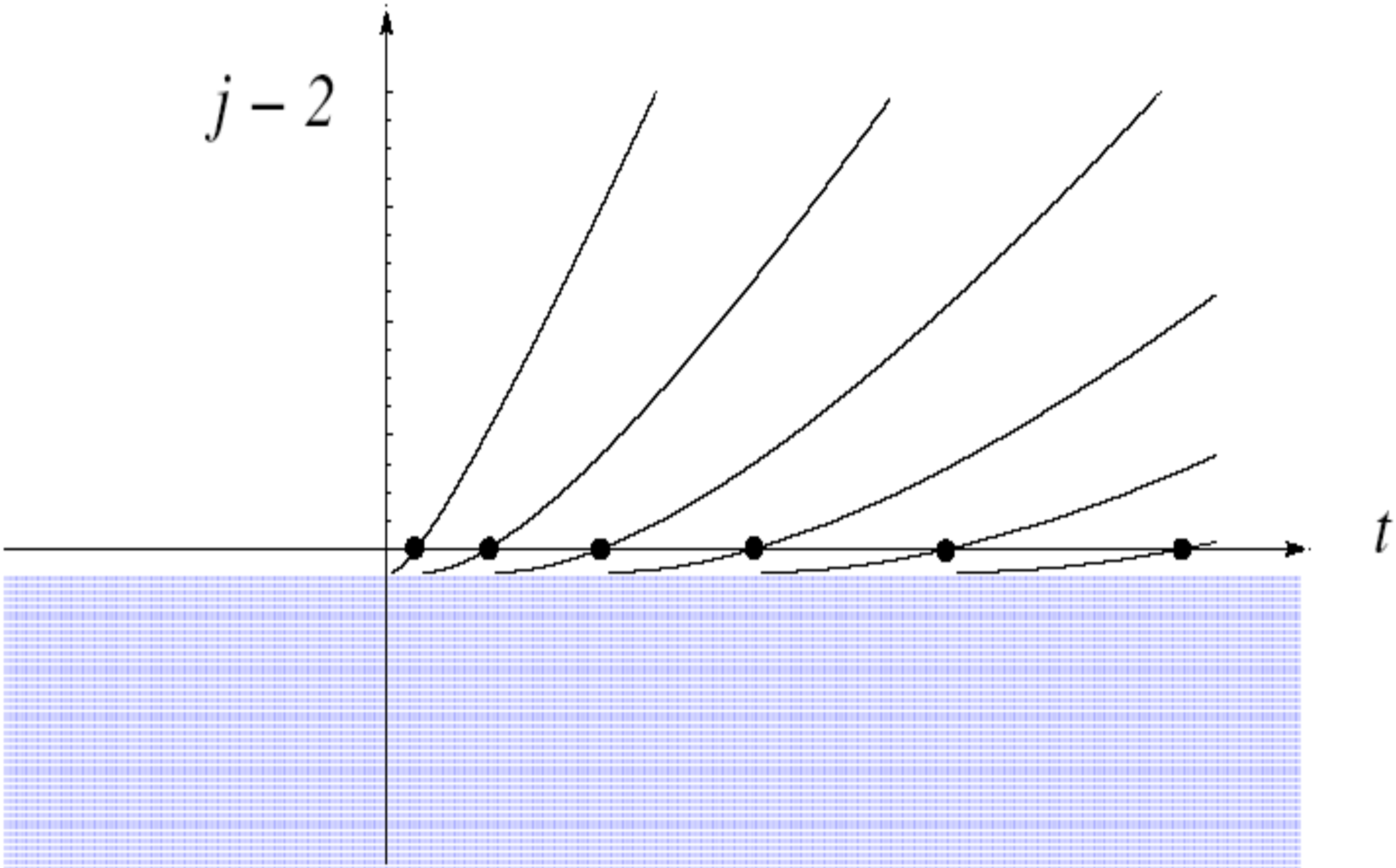
- Conformal: No scale and No Regge trajectory

$\mathcal{N} = 4$ Strong vs Weak BFKL



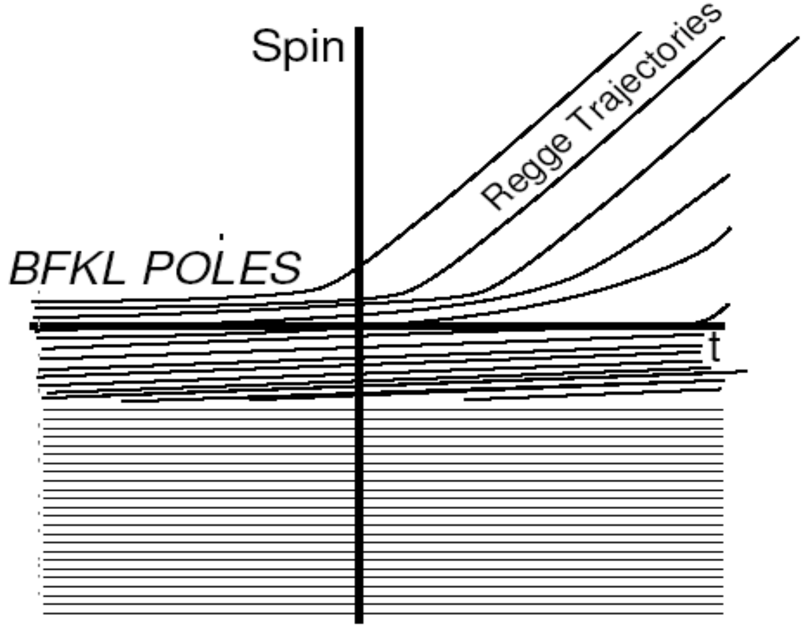
Hardwall Spectrum:

solving an effective Schrodinger equation



Pomeron in QCD

Running UV, Confining IR (large N)

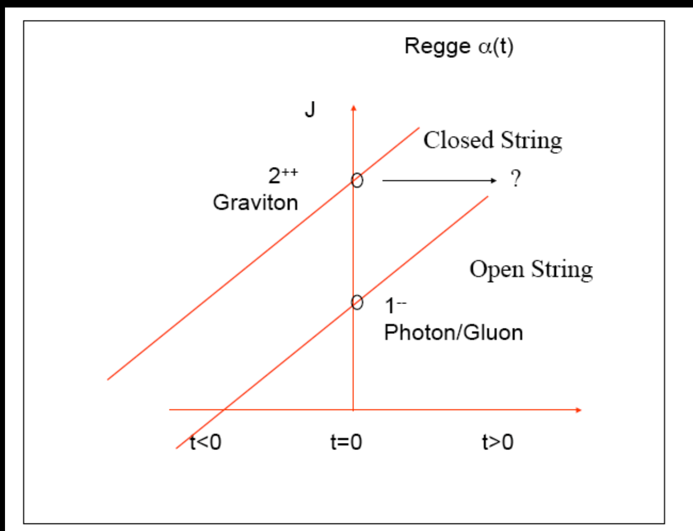


The hadronic spectrum is little changed, as expected.

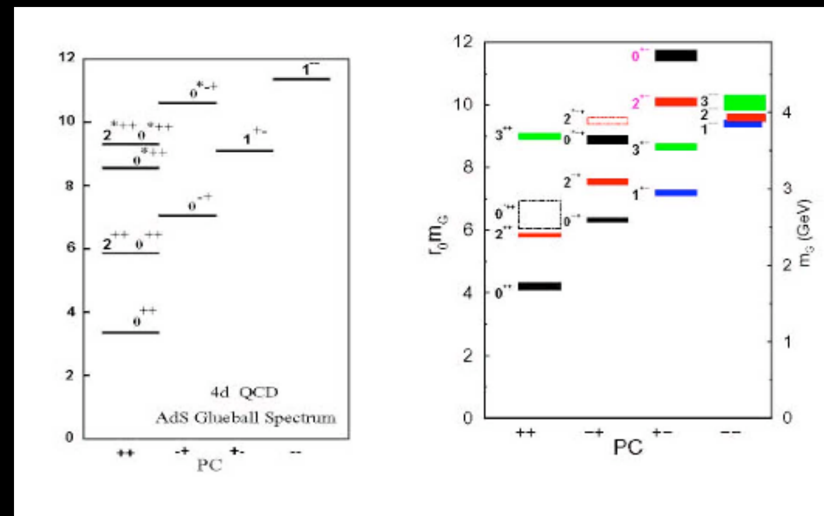
The BFKL cut turns into a set of poles, as expected.

QCD Pomeron <====> Graviton (metric) in AdS

Flat-space String



Confinement



Conformal Invariance

Fixed cut in J-plane:

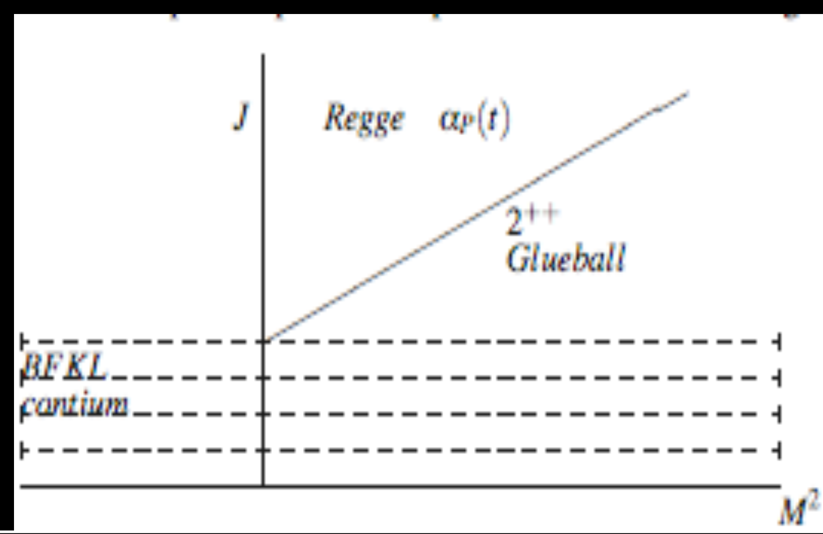
Weak coupling:
(BFKL)

$$j_0 = 1 + \frac{4\ln 2}{\pi}\alpha N$$

Strong coupling:

$$j_0 = 2 - \frac{2}{\sqrt{\lambda}}$$

Pomeron in AdS Geometry



IIc: String Theoretic Approach:

$$OPE ==> Pomeron\ \ Vertex\ \ Operator$$

$$(L-1)V_P = (\bar{L}-1)V_P = 0$$

Pomeron Vertex Operator Approach:

work by Brower, Polchinski, Strassler and Tan. First we'll briefly describe flat space scattering.

- At tree level, string theory scattering amplitude is given by an integral over vertex operators

$$A_n \sim \int d^2 w_2 d^2 w_3 \cdots d^2 w_{n-2} < V_1 V_2 \cdots V_n >$$

- We will be interested in 2-2 scattering, where this is given by

$$A_4 = \int d^2 w < V_1(0) V_2(w, \bar{w}) V_3(1) V_4(\infty) >$$

$$Using\ OPE,\ and\ imposing$$

$$(L-1)V_p = (\bar{L}-1)V_p = 0$$

$$A_4 = \int d^2w < V_1(0)V_2(w,\bar{w})V_3(1)V_4(\infty) >$$

- BPST showed that in the Regge limit of $s \to \infty$ and $s \gg t$ we can calculate the scattering amplitude by introducting a 'Pomeron vertex operator'

$$A_4 \sim < V_1 V_2 V_P^- >< V_P^+ V_3 V_4 >$$

Introduction to High Energy Scattering in String Theory

Flat Space continued

- Here
$$V_P^{\pm} = (\frac{2}{\alpha'}\partial X^{\pm}\bar{\partial}X^{\pm})^{1+\frac{\alpha' t}{4}} e^{\mp ikX}$$
- This simplifies calculations, and leads to an interpretation of scattering being mediated by Pomeron exchange.
- This was derived in light cone coordinates, where in the Regge limit we can separate the states into the ones with a large $+$ component and the ones with a large $-$ component.

Introduction to High Energy Scattering in String Theory

Flat Space continued

- Here

$$V_P^{\pm} = (\frac{2}{\alpha'}\partial X^{\pm}\bar{\partial} X^{\pm})^{1+\frac{\alpha' t}{4}} e^{\mp ikX}$$

- However, flat space string theory is not enough for a connection with QCD.
- This is where the AdS/CFT correspondence comes in.

The AdS/CFT Correspondence

The metric for AdS space is

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + d\Omega_5$$

We can introduce a new coupling λ, where

$$\lambda \equiv \frac{R^4}{\alpha'^2}$$

The correspondence relates λ to the Yang-Mills coupling constant via the relation

$$\lambda = g_{YM}^2 N_c,$$

therefore we see that λ is the 't Hooft coupling.

$$(L-1)V_P = (\bar{L}-1)V_P = 0$$

The basic idea is the same as in flat space.

- We begin by introducing the AdS space Pomeron vertex operator

$$V_P(j,\pm) = (\partial X^{\pm}\bar{\partial}X^{\pm})^{\frac{j}{2}} e^{\mp ikX}\phi_j(z)$$

- We see that we now have a wave function that depends on the AdS coordinate z. For the Pomeron this function is

$$\phi_{+j}(z) \sim z^{2-j} K_{2i\nu}(|t|^{\frac{1}{2}}\, z)$$

- With this in mind, we can express the amplitude as

$$A_4 \sim \int \frac{dj}{2\pi i} \int d\nu \frac{\nu \sinh 2\pi\nu}{\pi} \frac{\Pi(j)s^j}{j - j_0 + \rho\nu^2}$$

$$\times < V_1 V_2 V_P(j,\nu,k,-) >< V_P(j,\nu,k,+) V_3 V_4 >$$

where $\rho = \frac{2}{\sqrt{\lambda}}$ and $j_0 = 2-\rho$. V_i are the state dependent vertex operators.

“2-Gluons” = “Graviton”

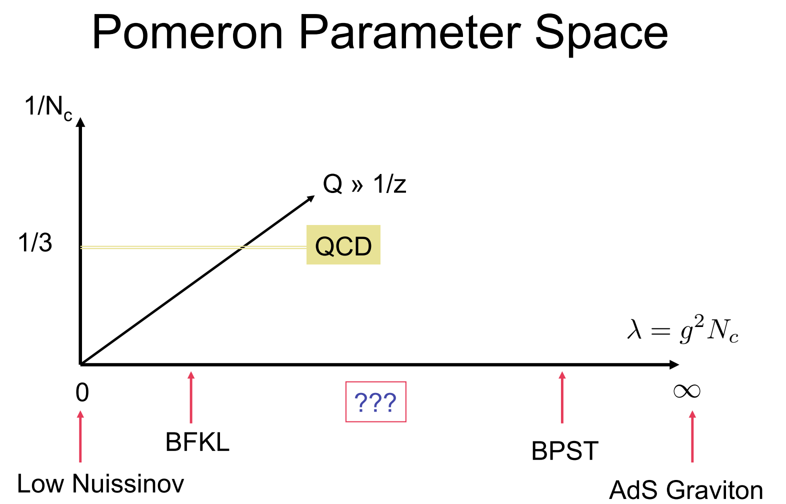
In gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously.

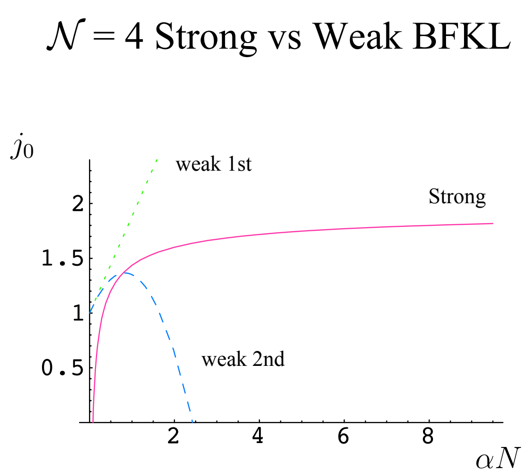
Pomeron can be associated with a Reggeized Massive Graviton.

Both the IR (soft) Pomeron and the UV (BFKL) Pomeron are dealt in a unified single step.

R. Brower, J. Polchinski, M. Strassler, and C-I Tan,
“The Pomeron and Gauge/String Duality”, (hep-th/0603115.)

Gauge/String Duality: QCD at Strong Coupling





- C=+1: Pomeron <=> Graviton:

$$\alpha_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$$

$$(symmetric\ tensor : g_{\mu\nu})$$

- C=-1: Odderon <=> Kalb-Ramond

$$\alpha_0^{(-)} = 1 - m_{ads}^2/2\sqrt{\lambda} + O(1/\lambda)$$

$$(anti - symmetric\ tensor : b_{\mu\nu})$$

- New Questions: New realization of conformal inv., Confinement, Unitarity, Saturation, Confinement, Froissart, etc.?

IId. Conformal Invariance at HE and Graviton

* Reduction to AdS_3

* New Realization of Conformal Invariance

  - Conformal limit: $\Delta(J)$ *curve*

  - Confinement:

$Symmetry \leftrightarrow Isometry$

full $O(4,2)$ conformal group as isometries of AdS_5

15 generators: $P_\mu, M_{\mu\nu}, D, K_\mu$

collinear group $SL_L(2,R) \times SL_R(2,R)$ used in DGLAP

generators: $D \pm M_{+-}\ ,\ P_\pm\ ,\ K_\mp$

$SL(2,C)$ Möbius invariance

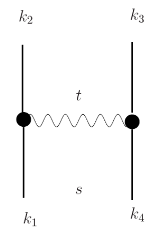
generators: $iD \pm M_{12}\ ,\ P_1 \pm iP_2\ ,\ K_1 \mp iK_2$

isometries of the Euclidean (transverse) AdS_3 subspace of AdS_5

$$propagator = (J - M_{+-})^{-1}$$

Lorentz boost, $\exp[-yM_{+-}]$

$$ds^2 = R^2[dz^2 + dwd\bar{w}]/z^2$$



AdS_3 is the hyperbolic space H_3. Indeed $SL(2,C)$ is the subgroup generated by all elements of the conformal group that commute with the boost operator, M_{+-} and as such plays the same role as the little group which commutes with the energy operator P_0.

$$J_0 = w\partial_w + \frac{1}{2}z\partial_z \quad , \quad J_- = -\partial_w \quad , \quad J_+ = w^2\partial_w + wz\partial_z - z^2\partial_{\bar{w}}$$

$$\bar{J}_0 = \bar{w}\partial_{\bar{w}} + \frac{1}{2}z\partial_z \quad , \quad \bar{J}_- = -\partial_{\bar{w}} \quad , \quad \bar{J}_+ = \bar{w}^2\partial_{\bar{w}} + \bar{w}z\partial_z - z^2\partial_w$$

$$M_{+-} = 2 - H_{+-}/(2\sqrt{\lambda}) + O(1/\lambda) \qquad H_{+-} = -z^3\partial_z z^{-1}\partial_z - z^2\nabla^2_{x_\perp} + 3 \; .$$

$$[H_{+-} + 2\sqrt{\lambda}(j-2)]G_3(j,v) = z^3\delta(z-z')\delta^2(x_\perp - x'_\perp)$$

Finite Strong Coupling Pomeron Propagator--

Conformal Limit

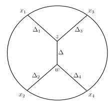
- Spin 2 and Reduction to AdS_3
- Spin 2 -------> J by Using Complex angular momentum representation

One Graviton in Momentum Representation at High Energy

$$J = 2$$

$$T^{(1)}(p_1,p_2,p_3,p_4) = g_s^2 \int \frac{dz}{z^5} \int \frac{dz'}{z'^5}\, \tilde{\Phi}_\Delta(p_1^2,z)\tilde{\Phi}_\Delta(p_3^2,z)\mathcal{T}^{(1)}(p_i,z,z')\tilde{\Phi}_\Delta(p_2^2,z')\tilde{\Phi}_\Delta(p_4^2,z')$$

$p_1 + p_2 \to p_3 + p_4$



$$\mathcal{T}^{(1)}(p_i,z,z') = (z^2 z'^2 s)^2 G_{++,--}(q,z,z') = (zz's)^2 G^{(5)}_{\Delta=4}(q,z,z')$$

Reduction to AdS-3 at High Energy for Near Forward Scattering

* momentum transfer q is transverse:

$$(zz')G^{(3)}_{\Delta=3}(x^\perp,z,z') = \int \frac{dq^\perp}{(2\pi)^2} e^{ix^\perp q^\perp} G^{(5)}_{\Delta=4}(q^\pm = 0, q^\perp, z, z')$$

* AdS-3 Propagator:

$$\mathcal{K}(s,x^\perp,z,z') = (zz's)^2 (zz')G^{(3)}_3(x^\perp,z,z')$$

$$\{-\partial_z z^{-1}\partial_z - z^{-1}\partial^2_{x^\perp} + 3z^{-3}\}G^{(3)}_3(x_\perp,x'_\perp,z,z') \;=\; \delta(z-z')\delta^{(2)}(x_\perp - x'_\perp)$$

* Isometry of Euclidean AdS-3 is SL(2C) --- the same symmetry group as BFKL kernel (spin-chains): 2-Gluons = Graviton

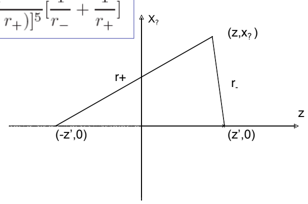
AdS-3 Propagator in Conformal Limit

$$G_3^{(3)}(x_\perp, z, z') = \frac{1}{4\pi}\frac{1}{[y+\sqrt{(y^2-1)}]^2\sqrt{y^2-1}}$$

$$y = \frac{z^2+z'^2+(x-x')^2}{2zz'}$$

$$r_\pm = \sqrt{(z \pm z')^2 + x_\perp^2} = \sqrt{(2zz')(y \pm 1)}$$

$$G_3^{(3)}(x_\perp, z, z') = \frac{(2zz')^3}{\pi}\frac{1}{[(r_- + r_+)]^5}[\frac{1}{r_-}+\frac{1}{r_+}]$$



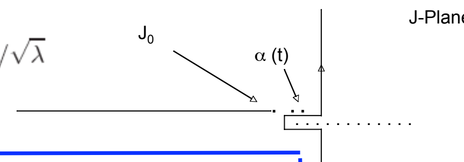
Randall-Sundrum with vanishing
Direchlet bdry condition at z=0

Complex j-Plane:

$$\mathcal{T}^{(1)}(p_i, z, z') = \int \frac{dj}{2\pi i}\frac{(1+e^{-i\pi j})}{\sin \pi j}(\tilde{s})^j\, G^{(5)}(j, q, z, z')$$

Integration Contour for Mellin Transform

$$j_0 = 2 - 2/\sqrt{\lambda}$$



$$\{2\sqrt{\lambda}(j-2) - z^5\partial_z z^{-3}\partial_z - z^2 t\}G^{(5)}_{\Delta(j)}(j, q, z, z') = z^5\delta(z-z')$$

Reduction to AdS-3:

$$G^{(5)}_\Delta(j, q^\pm = 0, q^\perp, z, z') \to (zz')G^{(3)}_{(\Delta-1)}(j, q_\perp, z, z')$$

$$\mathcal{K}(j, x^\perp - x'^\perp, z, z') = (zz'/R^4)G_3(j, v)$$

AdS_3 Green's function which has a simple closed form,

$$G_3(j,v) = \frac{1}{4\pi}\frac{\left[1+v+\sqrt{v(2+v)}\right]^{(2-\Delta_+(j))}}{\sqrt{v(2+v)}}\,.$$

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Impact Representation:

$$T^{(1)}(s; x_\perp - y_\perp) = (1/2\pi)^2 \int d^2 q_\perp e^{i(x_\perp - y_\perp)\cdot q_\perp} T^{(1)}(s, -q_\perp^2)$$

$$T^{(1)}(s; x_\perp - y_\perp) = g_s^2 \int \frac{dz dz'}{z^5 \ z'^5} \tilde{\Phi}_\Delta(p_1^2, z) \tilde{\Phi}_\Delta(p_3^2, z') \mathcal{K}(s, x_\perp - y_\perp, z, z') \tilde{\Phi}_\Delta(p_2^2, z') \tilde{\Phi}_\Delta(p_4^2, z')$$

j-plane Representation:

$$\mathcal{K}(s, x_\perp - y_\perp, z, z') = (zz') \int \frac{dj}{2\pi i} \frac{(1 + e^{-i\pi j})}{\sin \pi j} (\tilde{s})^j \ G^{(3)}_{\Delta_2}(j, x_\perp - y_\perp, z, z')$$

Reduction to AdS-3:

$$G^{(3)}_{\Delta_2}(j, x_\perp - y_\perp, z, z') = \frac{1}{(2\pi)^2} \int d^2 q_\perp e^{i(x_\perp - y_\perp)\cdot q_\perp} \tilde{G}^{(3)}_{\Delta_2}(j, -q_\perp^2, z, z')$$

D.E. for Propagator:

$$\{2\sqrt{\lambda}(j-2) - z^3 \partial_z z^{-1} \partial_z - z^2 \partial^2_{x\perp} + 3\} G^{(3)}_{(\Delta(j)-1)}(x_\perp, x'_\perp, z, z') \quad = \quad z^3 \delta(z - z') \delta^{(2)}(x_\perp - x'_\perp)$$

Strong Coupling Pomeron Propagator--

Conformal Limit

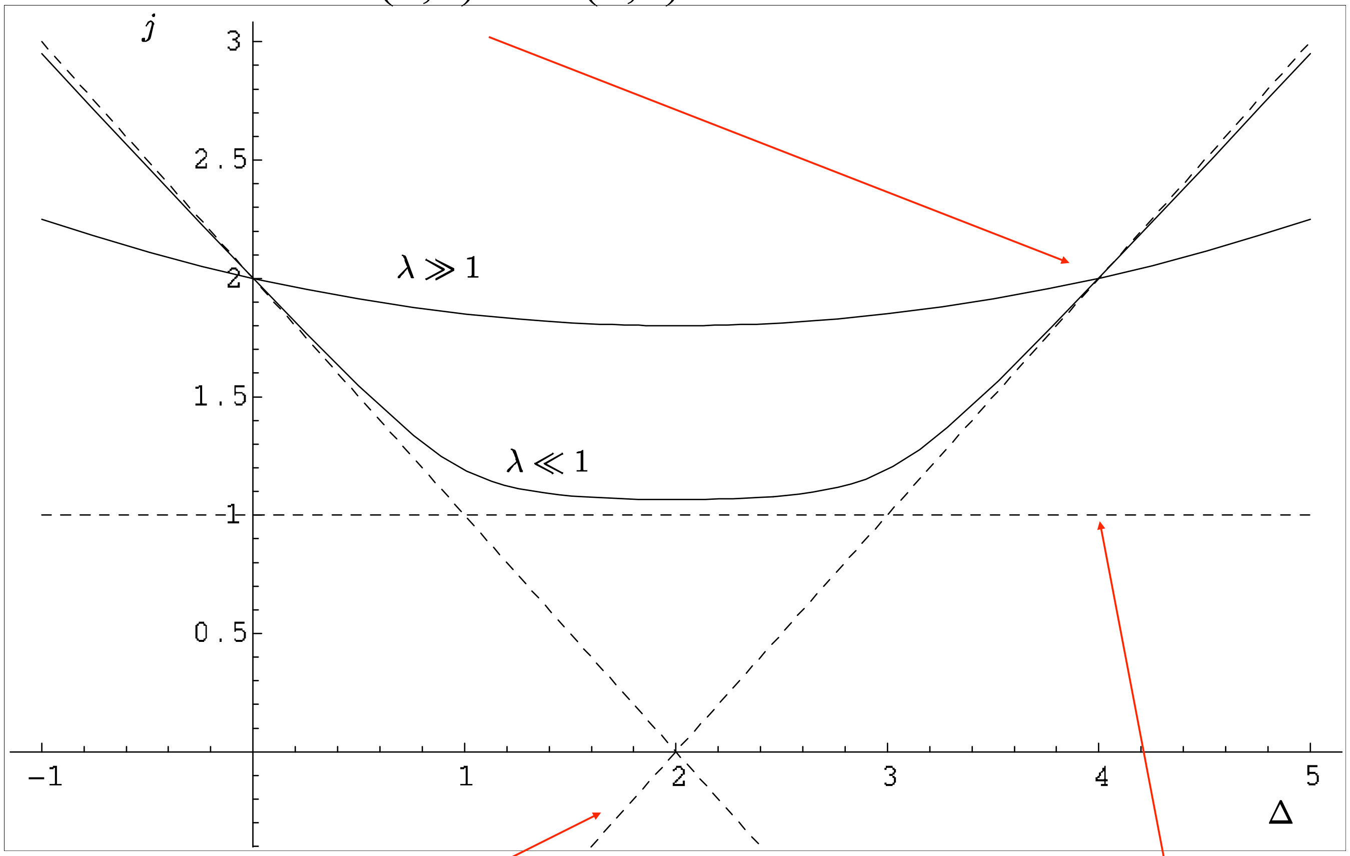
- Use J-dependent Dimension

$$\Delta : \quad 4 \to \Delta(J) = 2 + [2\sqrt{\lambda}(J - J_0)]^{1/2} = 2 + \sqrt{j}$$

- BFKL-cut: $J_0 = 2 - \dfrac{2}{\sqrt{\lambda}}$

Spin-Dimension Curve

(4,2) and (0,2) have zero anomalous dimension



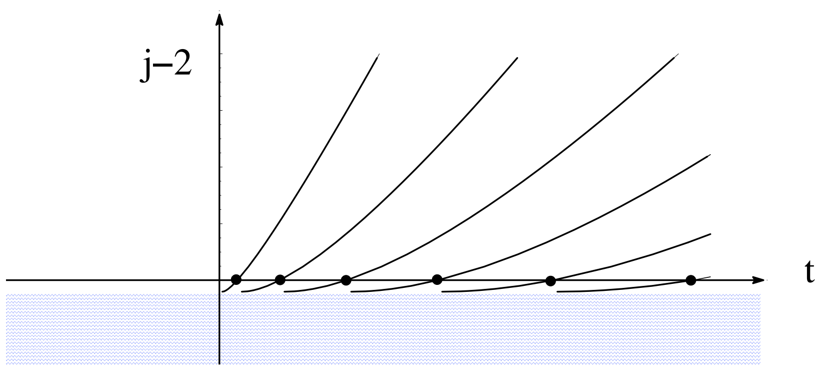
λ = 0 Anomalous
Dim=0

λ = 0, BFKL

inversion symmetry: Δ ➔ 4 - Δ

With Confinement

- discrete spectrum



Cutoff at large b:

Conformal:

$$\mathcal{K}(j, x_\perp - x'_\perp, z, z') \sim [(x_\perp - x'_\perp)^2]^{-1-\sqrt{c(j-j_0)}}$$

$$\mathcal{K}(j_0, x_\perp - x'_\perp, z, z') \sim \frac{1}{(x_\perp - x'_\perp)^2}$$

Confining:

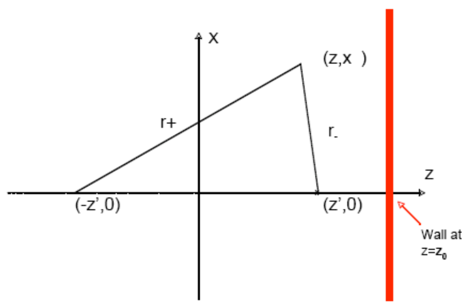
$$\begin{aligned}\mathcal{K}(j, x_\perp - x'_\perp, z, z') &\simeq \frac{|d_0|^2 J_{\sqrt{j}}(m_0 z) J_{\sqrt{j}}(m_0 z')}{2\pi} K_0(m_0|x_\perp - x'_\perp|) \\ &\simeq \frac{|d_0|^2 J_{\sqrt{j}}(m_0 z) J_{\sqrt{j}}(m_0 z')}{2\pi} e^{-m_0|x_\perp - x'_\perp|}\end{aligned}$$

$$\mathcal{K}(j_0, x_\perp - x'_\perp, z, z') \simeq \frac{|d_0|^2 J_0(m_0 z) J_0(m_0 z')}{2\pi} e^{-m_0|x_\perp - x'_\perp|}$$

AdS Graviton Propagator in Confined Background

* Infinite number of Images charges

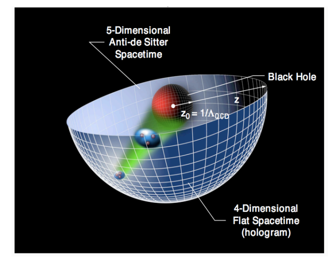
* Easier using spectral representation in momentum



Pomeron Propagator in momentum representation --with or without Confinement

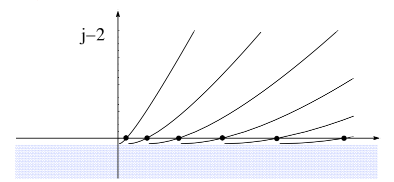
Spectral Rep. in Conformal limit:

$$G(j,t,z,z') = \int_0^\infty dk\; k\, \frac{J_{\sqrt{j}}(kz) J_{\sqrt{j}}(kz')}{k^2 - t}$$



Spectral Rep. with Confinement

$$G(j,q,z,z') = \sum_n \frac{\Phi_n(z)\Phi_n(z')}{m_n^2(j) - t}$$



Ref. Brower, Polchinski, Strassler, Tan, het-0603115

III: Odderon in AdS

Massless modes of a closed string theory:

metric tensor,	$G_{mn} = g^0_{mn} + h_{mn}$
Kolb-Ramond anti-sym. tensor,	$b_{mn} = -b_{nm}$
dilaton, etc.	$\phi, \chi, \cdots$

$\mathcal{N} = 4$ SYM Scattering at High Energy

AdS_5 boundary, $z \to 0$,

$$\langle e^{\int d^4x \phi_i(x) O_i(x)} \rangle_{CFT} = Z_{string}\left[\phi_i(x,z)|_{z\sim 0} \to \phi_i(x)\right],$$

Bulk Degrees of Freedom from Supergravity:

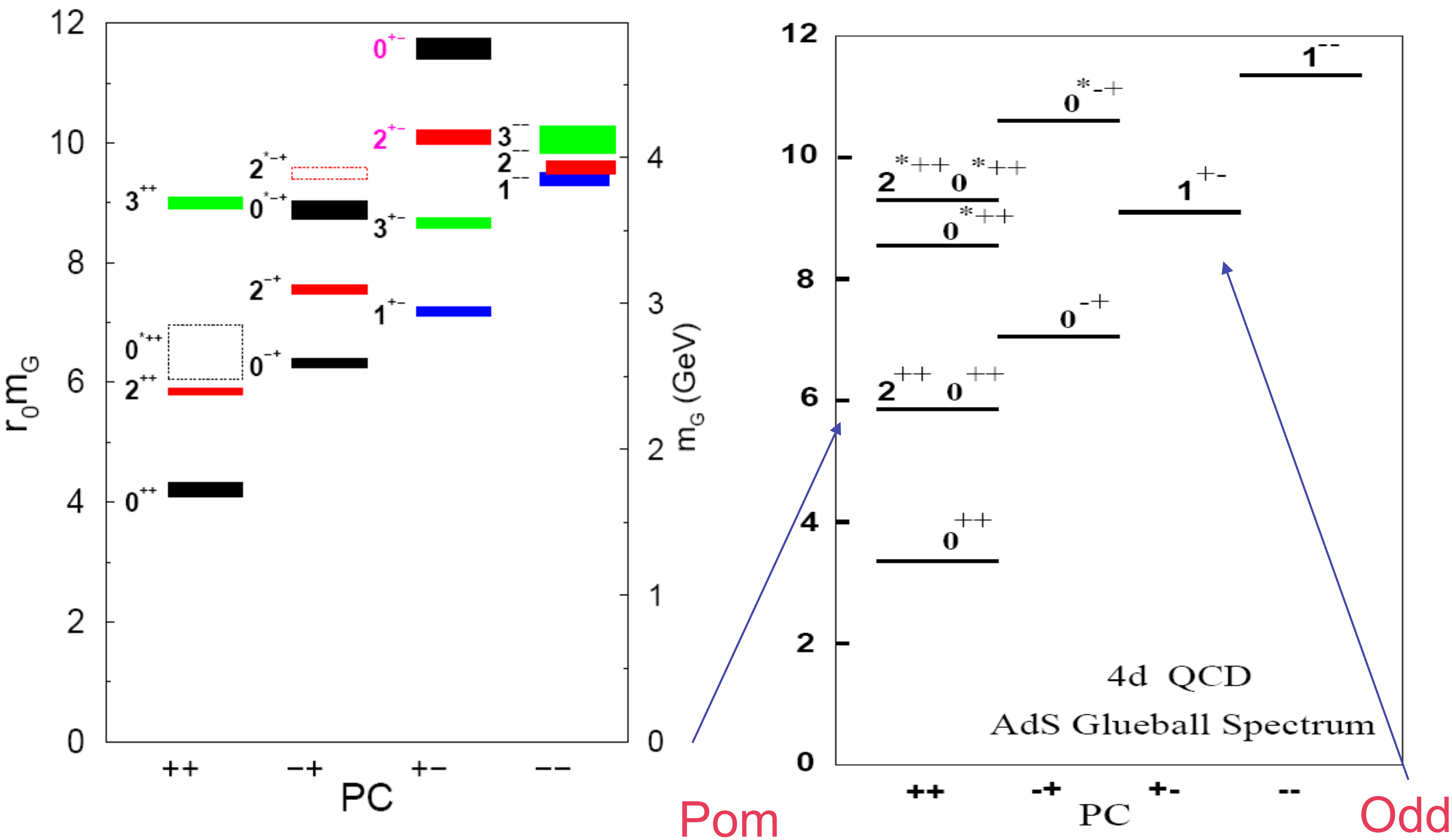
- metric tensor: G_{MN}
- Kalb-Ramond 2 Forms: B_{MN}, C_{MN}
- Dilaton and zero form: ϕ and C_0

Born-Infeld Action

$$S = \int d^4x \det[G_{\mu\nu} + e^{-\phi/2}(B_{\mu\nu} + F_{\mu\nu})] + \int d^4x (C_0 F \wedge F + C_2 \wedge F + C_4))$$

Dimension	State J^{PC}	Operator	Supergravity
$\Delta = 4$	0^{++}	$Tr(FF) = \vec{E}^a \cdot \vec{E}^a - \vec{B}^a \cdot \vec{B}^a$	ϕ
$\Delta = 4$	2^{++}	$T_{ij} = E_i^a \cdot E_j^a + B_i^a \cdot B_j^a - \text{trace}$	G_{ij}
$\Delta = 4$	0^{-+}	$Tr(F\tilde{F}) = \vec{E}^a \cdot \vec{B}^a$	C_0
$\Delta = 6$	1^{+-}	$Tr(F_{\mu\nu}\{F_{\rho\sigma}, F_{\lambda\eta}\}) \sim d^{abc}F^aF^bF^c$	B_{ij}
$\Delta = 6$	1^{--}	$Tr(\tilde{F}_{\mu\nu}\{F_{\rho\sigma}, F_{\lambda\eta}\}) \sim d^{abc}\tilde{F}^aF^bF^c$	$C_{2,ij}$

Confinement gives a discrete spectrum of Glueballs:
Lattice Data vs AdS IIA Gravity dual Gauge ($\alpha' = 0$)



flat-space expectation

$$F_{\bar{c}b\to\bar{a}d} \equiv \bar{F} = F^+ + F^- \qquad [\sigma_T(\bar{a}b) + \sigma_T(ab)] \sim (2/s)\ \mathrm{Im}\ F^+$$

$$F_{ab\to cd} \equiv F = F^+ - F^- \qquad [\sigma_T(\bar{a}b) - \sigma_T(ab)] \sim (2/s)\ \mathrm{Im}\ F^-$$

$$\mathcal{T}_{10}^{(+)}(s,t) \to f^{(+)}(\alpha' t)\left[\frac{(-\alpha' s)^{2+\alpha' t/2} + (\alpha' s)^{2+\alpha' t/2}}{\sin\pi(2+\alpha' t/2)}\right]$$

$$\mathcal{T}_{10}^{(-)}(s,t) \to f^{(-)}(\alpha' t)\left[\frac{(-\alpha' s)^{1+\alpha' t/2} - (\alpha' s)^{1+\alpha' t/2}}{\sin\pi(1+\alpha' t/2)}\right]$$

$$\alpha_+(t) = 2 + \alpha' t/2 \qquad\qquad \alpha_-(t) = 1 + \alpha' t/2$$

Massless Modes at t=0

Massless Modes in Flat-Space String

$$|I, J; k\rangle = a^{\dagger}_{1,I}\tilde{a}^{\dagger}_{1,J}|NS\rangle_L|NS\rangle_R|k\rangle$$

$$|h\rangle = \sum_{I,J} h^{IJ}|I, J; k\rangle \quad , \quad |B\rangle = \sum_{I,J} B^{IJ}|I, J; k\rangle \quad , \quad |\phi\rangle = \sum_{I,J} \eta^{IJ}|I, J; k \perp\rangle$$

fluctuations of the metric G_{MN}

anti-symmetric Kalb-Ramond background B_{MN}

dilaton, ϕ

Flat-Space String Theory

$$T_{10}^{(+)}(s,t) \to f^{(+)}(\alpha' t)\left[\frac{(-\alpha' s)^{2+\alpha' t/2} + (\alpha' s)^{2+\alpha' t/2}}{\sin \pi(2+\alpha' t/2)}\right] \qquad \alpha_+(t) = 2 + \alpha' t/2 \,.$$

$$|I, J; k\rangle = a^\dagger_{1,I}\tilde{a}^\dagger_{1,J}|NS\rangle_L|NS\rangle_R|k\rangle$$

$$|h\rangle = \sum_{I,J} h^{IJ}|I, J; k\rangle \quad , \quad |B\rangle = \sum_{I,J} B^{IJ}|I, J; k\rangle \quad , \quad |\phi\rangle = \sum_{I,J} \eta^{IJ}|I, J; k \perp\rangle \,.$$

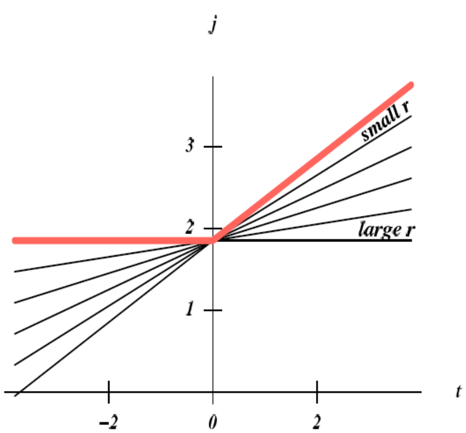
$$T_{10}^{(-)}(s,t) \to f^{(-)}(\alpha' t)\left[\frac{(-\alpha' s)^{1+\alpha' t/2} - (\alpha' s)^{1+\alpha' t/2}}{\sin \pi(1+\alpha' t/2)}\right] \qquad \alpha_-(t) = 1 + \alpha' t/2 \,.$$

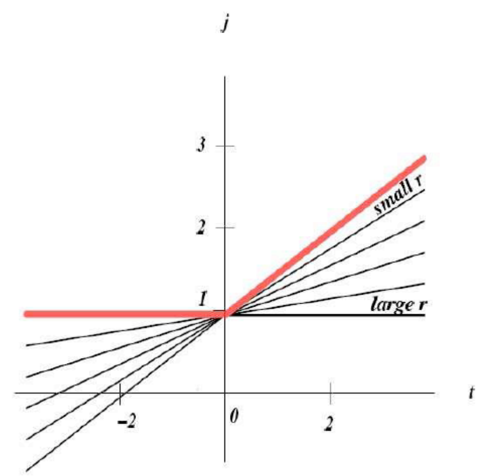
Conformal Pomeron and Odderon in Target Space:

Ultra-local approximation:

$$\tilde{s} = \frac{R^2}{r^2} s \ , \quad \tilde{t} = \frac{R^2}{r^2} t \ , \qquad \alpha'_{\rm eff}(r) = \frac{R^2 \alpha'}{r^2}$$

$$\mathcal{T}_{10}^{(\pm)}(\tilde{s}, \tilde{t}) \sim f^{(\pm)}(\alpha' \tilde{t})(\alpha' \tilde{s})^{\alpha_\pm(0) + \alpha' \tilde{t}/2} \sim s^{\alpha_\pm(0) + \alpha'_{eff}(r) t/2}$$





Diffusion in AdS

Flat Space: $t \to \nabla_b^2$

$$\tau = \log(\alpha' s) \qquad \langle\, \vec{b} \,|\, (\alpha' s)^{\alpha_\pm(0) + \alpha' t/2} \,|\, \vec{b}' \,\rangle \to (\alpha' s)^{\alpha_\pm(0)} \frac{e^{-(\vec{b}-\vec{b}')^2/(2\alpha'^2\tau)}}{\tau^{(D-2)/2}}$$

AdS5, C=+1:

$$\alpha' \tilde{t} \to \alpha' \Delta_P \equiv \frac{\alpha' R^2}{r^2} \nabla_b^2 + \alpha' \Delta_{\perp P}$$

$$\tilde{s}^{2+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i} \frac{\tilde{s}^j}{j - 2 - \alpha' \Delta_P / 2}$$

AdS5, C=-1:

$$\tilde{s}^{1+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i}\, \tilde{s}^j\, G^{(-)}(j) = \int \frac{dj}{2\pi i} \frac{\tilde{s}^j}{j - 1 - \alpha' \Delta_O / 2}$$

Diffusion in AdS

Flat Space: $t \to \nabla_b^2$

$$\tau = \log(\alpha' s) \qquad \langle\, \vec{b} \,|\, (\alpha' s)^{\alpha_{\pm}(0) + \alpha' t/2} \,|\, \vec{b}' \,\rangle \to (\alpha' s)^{\alpha_{\pm}(0)} \frac{e^{-(\vec{b}-\vec{b}')^2/(2\alpha'^2 \tau)}}{\tau^{(D-2)/2}}$$

AdS5, C=+1:

$$\alpha' \tilde{t} \to \alpha' \Delta_P \equiv \frac{\alpha' R^2}{r^2} \nabla_b^2 + \alpha' \Delta_{\perp P}$$

$$\tilde{s}^{2+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i} \frac{\tilde{s}^j}{j - 2 - \alpha' \Delta_P/2}$$

AdS5, C=-1:

$$\tilde{s}^{1+\alpha'\tilde{t}/2} = \int \frac{dj}{2\pi i}\, \tilde{s}^j\, G^{(-)}(j) = \int \frac{dj}{2\pi i} \frac{\tilde{s}^j}{j - 1 - \alpha' \Delta_O/2}$$

$$G^{(+)}(j) = \frac{1}{j - 2 - \alpha' \Delta_2 / 2}$$

$$\Delta_2 h_{MN} = 0$$

$$G^{(-)}(j) = \frac{1}{j - 1 - (\alpha'/2R^2)(\Box_{Maxwell} - m^2_{AdS,i})}$$

$$(\Box_{Maxwell} - (k+4)^2) B^{(1)}_{IJ} = 0 \ , \quad (\Box_{Maxwell} - k^2) B^{(2)}_{IJ} = 0$$

$$m^2_{AdS,1} = 16 \ , \quad m^2_{AdS,2} = 0$$

$$(1/2\sqrt{\lambda})\left\{-z\partial_z z\partial_z + z^2 t + m_{\pm}^2(j)\right\} G^{(\pm)}(z,z';j,t) = z\;\delta(z-z')$$

$$m_{+}^2(j) = 2\sqrt{\lambda}(j-2) + 4$$

$$m_{-}^2(j) = 2\sqrt{\lambda}(j-1) + m_{AdS,i}^2$$

Gauge/String Duality: Conformal Limit

- C=+1: Pomeron <===> Graviton

$$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda) \ .$$

- C=-1: Odderon <===> Kalb-Ramond Field

$$j_0^{(-)} = 1 - m_{AdS}^2/2\sqrt{\lambda} + O(1/\lambda) \ .$$

	Weak Coupling	Strong Coupling
$C = +1$	$j_0^{(+)} = 1 + (\ln 2)\ \lambda/\pi^2 + O(\lambda^2)$	$j_0^{(+)} = 2 - 2/\sqrt{\lambda} + O(1/\lambda)$
$C = -1$	$j_{0,(1)}^{(-)} \simeq 1 - 0.24717\ \lambda/\pi + O(\lambda^2)$ $j_{0,(2)}^{(-)} = 1 + O(\lambda^3)$	$j_{0,(1)}^{(-)} = 1 - 8/\sqrt{\lambda} + O(1/\lambda)$ $j_{0,(2)}^{(-)} = 1 + O(1/\lambda)$

Table 1: Pomeron and Odderon intercepts at weak and strong coupling.

J-Plane Structure

$$(1/2\sqrt{\lambda})\left\{-z\partial_z z\partial_z + z^2 t + m_{\pm}^2(j)\right\} G^{(\pm)}(z, z'; j, t) = z\,\delta(z - z')$$

$$G^{(\pm)}(z, z'; j, t) = \frac{2}{\sqrt{\lambda}\pi^2}\int_{-\infty}^{\infty} d\nu\, \nu \sinh 2\pi\nu \frac{K_{2i\nu}(|t|^{1/2}e^{-u})K_{-2i\nu}(|t|^{1/2}e^{-u'})}{j - j_0^{\pm} + \mathcal{D}\nu^2}$$

$$G^{(\pm)}(z, x^{\perp}, z', x'^{\perp}; j) = \frac{1}{4\pi zz'}\frac{e^{(2-\Delta^{(\pm)}(j))\xi}}{\sinh \xi}\,.$$

$$v = \frac{(x^{\perp} - x'^{\perp})^2 + (z - z')^2}{2zz'}$$

$$\Delta^{(\pm)}(j) = 2 + \sqrt{2}\,\lambda^{1/4}\sqrt{(j - j_0^{(\pm)})}$$

Formal Treatment via OPE

- Flat Space Pomeron Vertex Operator

$$\mathcal{V}_P^{\pm} = (2\partial X^{\pm}\bar{\partial} X^{\pm}/\alpha')^{1+\alpha' t/4} e^{\mp ik\cdot X} \ .$$

- Flat Space Odderon Vertex Operator

$$\mathcal{V}_O^{\pm} = (2\epsilon_{\pm,\perp}\partial X^{\pm}\bar{\partial} X^{\perp}/\alpha')(2\partial X^{\pm}\bar{\partial} X^{\pm}/\alpha')^{\alpha' t/4} e^{\mp ik X}$$

- Pomeron Vertex Operator in AdS

$$\mathcal{V}_P(j,\nu,k,\pm) \sim (\partial X^{\pm}\overline{\partial} X^{\pm})^{\frac{j}{2}} e^{\mp ik\cdot X} e^{(j-2)u} K_{\pm 2i\nu}(|t|^{1/2} e^{-u})$$

- Odderon Vertex Operator in AdS

$$\mathcal{V}_O(j,\nu,k,\pm) \sim (\partial X^{\pm}\overline{\partial} X^{\perp} - \partial X^{\perp}\overline{\partial} X^{\pm})(\partial X^{\pm}\overline{\partial} X^{\pm})^{\frac{j-1}{2}} e^{\mp ik\cdot X} e^{(j-1)u} K_{\pm 2i\nu}(|t|^{1/2} e^{-u})$$

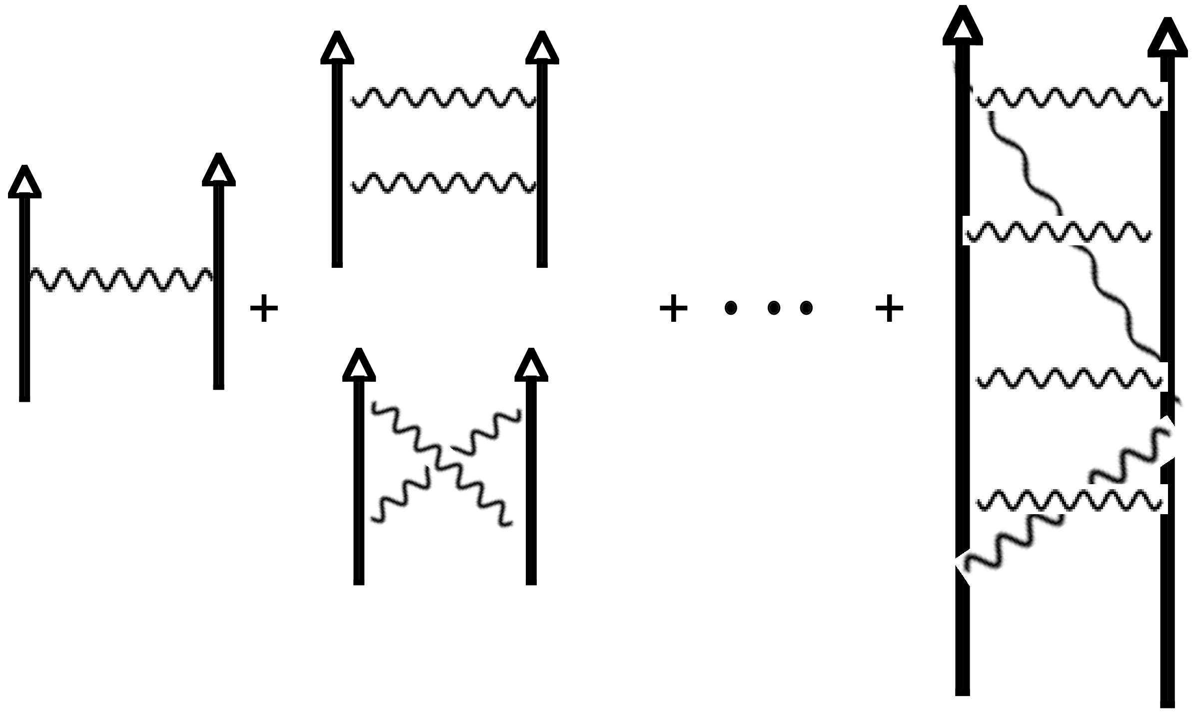
IV. Beyond Pomeron

- Sum over all Pomeron graph (string perturbative, $1/N^2$)
- Eikonal summation in AdS_3
- Constraints from Conformal Invariance, Unitarity, Analyticity, Confinement, Universality, etc.
- Froissart Bound?
- “non-perturbative” (e.g., blackhole production)

Eikonal Expansion

$$A_1(s,t=-q_\perp^2) \simeq 2s\int d^2b e^{-iqb}\chi(s,b) = 2s\chi(s,q_\perp)$$

Born term



"sum" to get $$A_{eikonal}(s,t) = -2is\int d^2b e^{-iqb}[e^{i\chi(s,b)} - 1] \ ,$$

- # Eikonal Sum: derived both via Cheng-Wu or by Shock-wave method

$$A_{2\to2}(s,t) \simeq -2is \int d^2b\, e^{-ib^\perp q_\perp} \int dz dz' P_{13}(z) P_{24}(z') \left[e^{i\chi(s,b^\perp,z,z')} - 1\right]$$

transverse AdS_3 space !!

$$P_{13}(z) = (z/R)^2 \sqrt{g(z)} \Phi_1(z)\Phi_3(z) \qquad P_{24}(z) = (z'/R)^2 \sqrt{g(z')} \Phi_2(z')\Phi_4(z')$$

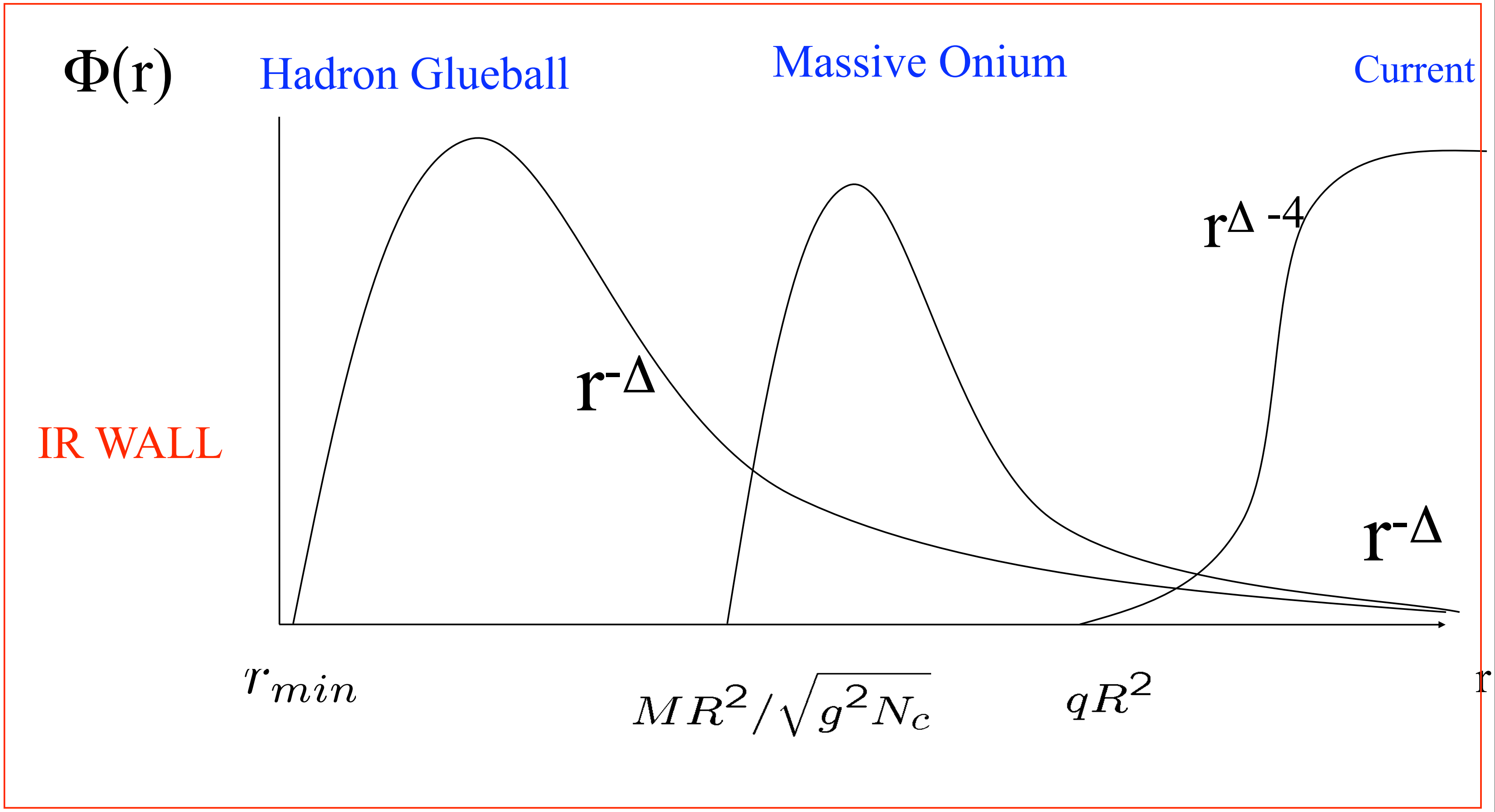
$$\chi(s, x^\perp - x'^\perp, z, z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathcal{K}(s, x^\perp - x'^\perp, z, z')$$

- Saturation:

$$\chi(s, x^\perp - x'^\perp, z, z') \;=\; O(1)$$

- Universality:

- Universality:
  By choosing wave functions, Φ, can treat DIS, Higgs Production, Proton-Proton, etc., on equal footing.



Saturation: $\chi(s, x^\perp - x'^\perp, z, z') = O(1)$

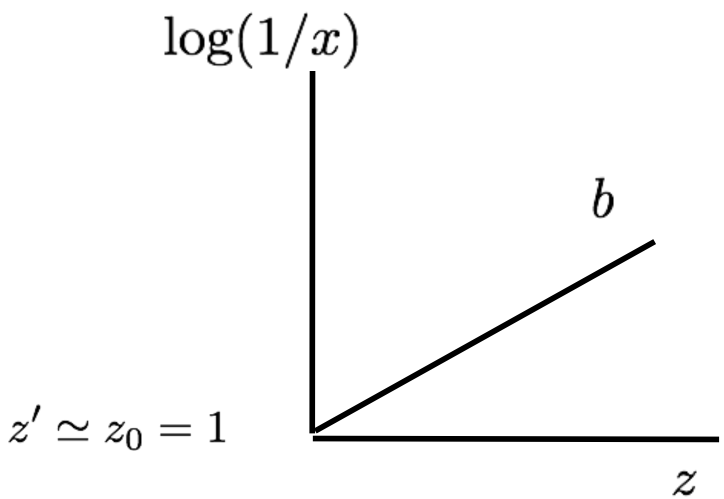
- Phase space:

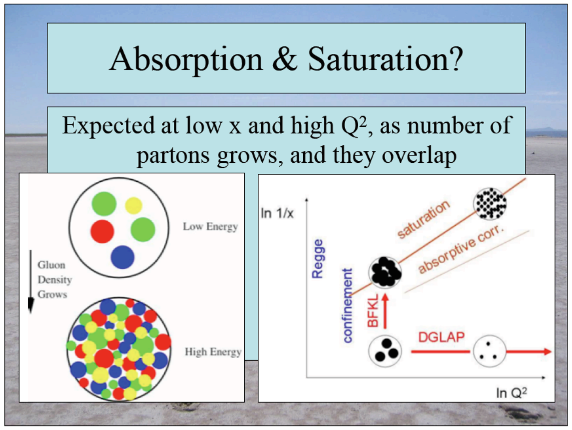
$$s \leftrightarrow 1/x$$
$$x_\perp \leftrightarrow impact\ space$$
$$z \leftrightarrow 1/Q^2 \leftrightarrow virtuality$$

- Conformal Invariance:

$$\chi(s, x^\perp - x'^\perp, z.z') \to G(s, v)$$

$$v = \frac{(x^\perp - x'^\perp)^2 + (z - z')^2}{2zz'}$$





Scattering in Conformal Limit:

Use the condition: $\chi(s, x^\perp - x'^\perp, z, z') = O(1)$

Elastic Ring: $b_{\rm diff} \sim \sqrt{zz'}\ (zz's/N^2)^{1/6}$

No Froissart $\sigma_{total} \sim s^{1/3}$

Inner Absorptive Disc:

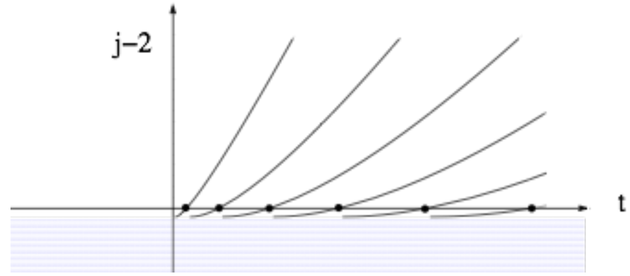
$$b_{\rm black} \sim \sqrt{zz'}\ \frac{(zz's)^{(j_0-1)/2}}{\lambda^{1/4}N} \qquad b_{\rm black} \sim \sqrt{zz'}\ \left(\frac{(zz's)^{j_0-1}}{\lambda^{1/4}N}\right)^{1/\sqrt{2\sqrt{\lambda}}(j_0-1)}$$

Inner Core: “black hole” production ?

Unitarity, Confinement and Froissart Bound

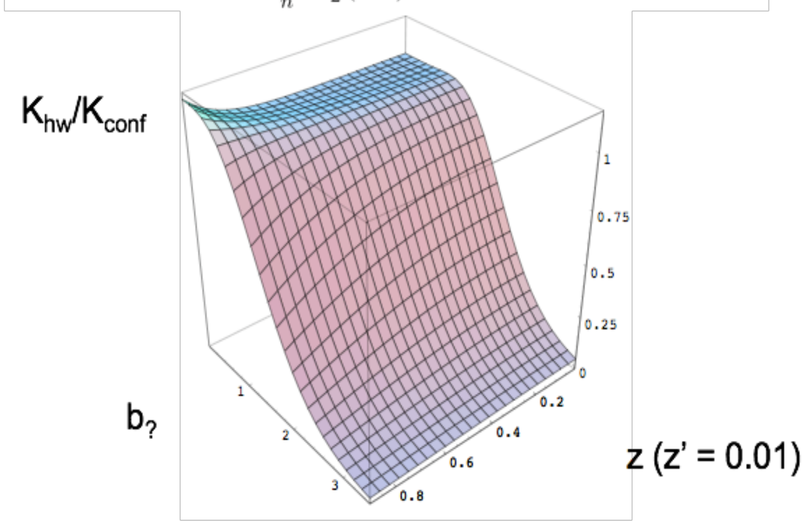
With Confinement:

discrete spectrum



Kernel for hardwall at z =1

$$K_{hw}(x_\perp, zz') \sim \frac{k_5^2 s^2}{zz'} \sum_n \frac{2}{J_2^2(m_n)} J_2(m_n z) K_0(m_n|x_\perp|) J_2(m_n z')$$



$$lim_{\Lambda\to 0} K_{hw}(x_\perp/\Lambda, z/\Lambda, z'/\Lambda) \sim \frac{\kappa_5^2 s^2}{zz'} \sum_n \frac{2}{y+\sqrt{y^2-1}} 4\pi\sqrt{y^2-1}$$

Mass of the lightest tensor Glueball provides scale

$$e^{-m_0 b}/\sqrt{m_0 b}$$

Elastic Ring:

$$b_{\rm diff} \simeq \frac{1}{m_0} \log(s/N^2\Lambda^2) + \dots$$

Absorptive Disc:

Inner Core:

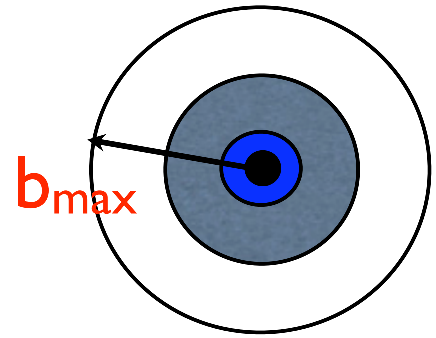
Saturation of Froissart Bound

- The Confinement deformation gives an exponential cutoff for $b > b_{max} \sim c \log(s/s_0)$,
- Coefficient $c \sim 1/m_0$, m_0 being the mass of lightest tensor glueball.
- There is a shell of "conformal region" of width

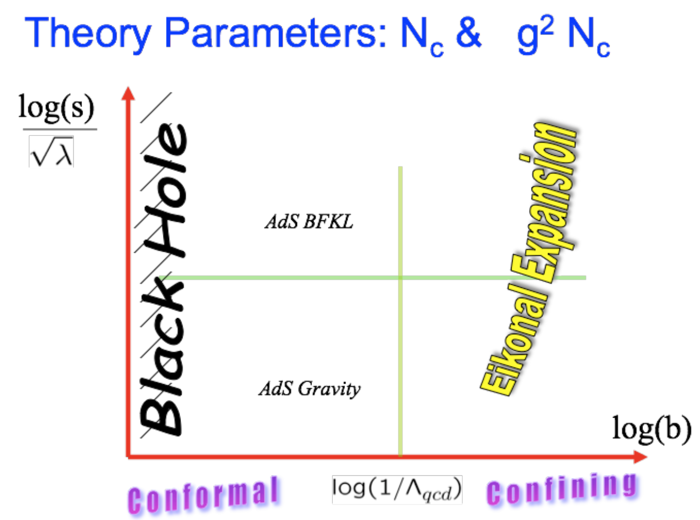
$$\Delta b \sim \log(s/s_0)$$

Froissart is respected and saturated.

Disk picture



b_{max} determined by confinement.



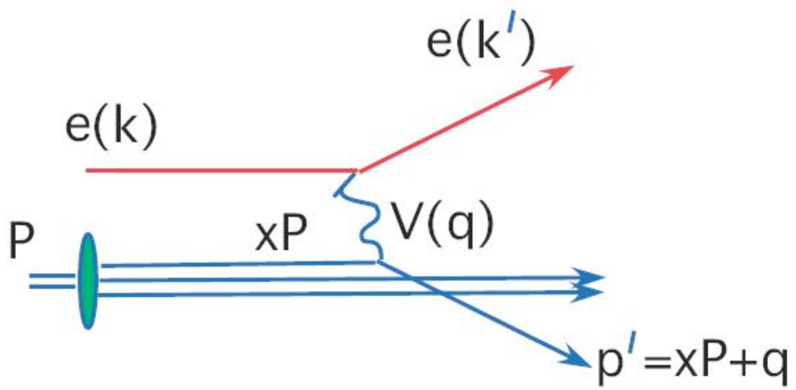
V. Deep Inelastic Scattering (DIS)

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- Phenomenological consequences, Diffractive Higgs production at LHC (in progress).

DIS

General Setup

Let us look in a little more detail at DIS.



The basic kinematical variables we need for describing this process are

- the center of mass energy

$$s = -(P+q)^2 > 0$$

- the virtual photon mass squared:

$$-Q^2 = q^2 = q^\mu q_\mu = (k-k')^2 < 0$$

- the scaling variable

$$0 < x \approx \frac{Q^2}{s} < 1$$

General Setup

The cross section

We can write the cross section for this process in the form

$$\frac{d\sigma^2}{dxdQ^2} = \frac{2\pi\alpha^2}{xQ^4}(Y_+ F_2 - x^2 F_L)$$

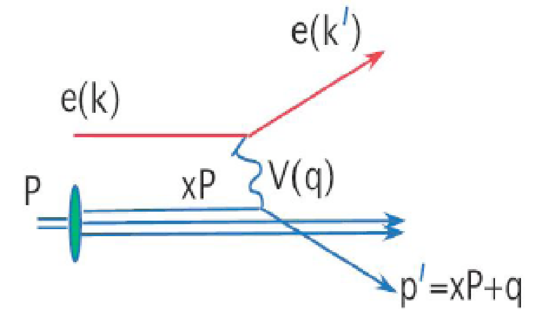
$$Y_+ = 1 + (1-x)^2,$$

In parton model, it is customary to introduce quark and gluon distribution functions:

- $F_2(x, Q^2) = x \sum_q e_q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]$
- $F_L(x, Q^2) \sim F_2 - xg(x, Q^2)$
- F_2 is what we get from most experiments, since F_L vanishes at LO in pQCD.
- It is also customary to express F_2 as, $\sigma = \sigma_T + \sigma_L$,

$$\boxed{F_2(x, Q^2) = \frac{Q^2}{4\pi^2\alpha}\sigma(x, Q^2)}.$$

Deep Inelastic Scattering (DIS)



$$F_2(x, Q2) = \frac{Q^2}{4\pi^2 \alpha_{em}} \left[\sigma_T(\gamma^* p) +_L (\gamma^* p)\right]$$

$$x \equiv \frac{Q^2}{s}$$

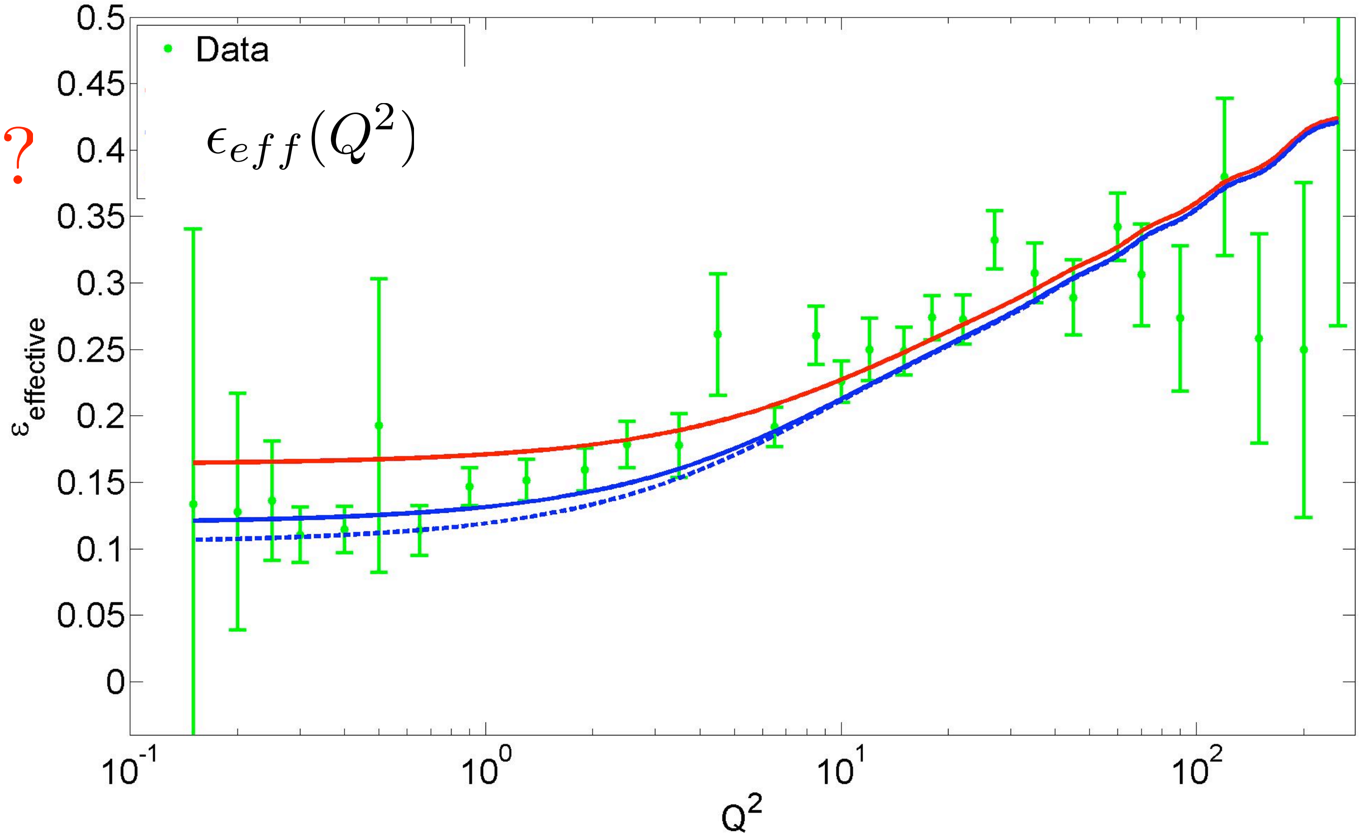
$Small\ x:\ \frac{Q^2}{s} \to 0$

Optical Theorem

$$\sigma_{total}(s, Q^2) = (1/s)\text{Im } A(s, t = 0; Q^2)$$

$$F_2(x, Q^2) \sim (1/x)^{\epsilon_{effective}}$$

Puzzles?



Questions on HERA DIS small-x data:

- Why $\alpha_{eff} = 1 + \epsilon_{eff}(Q^2)$?
- Confinement? (Perturbative vs. Non-perturbative?)
- Saturation? (evolution vs. non-linear evolution?)

Recall that, for two-to-two scattering involving on-shell hadrons, the amplitude in an eikonal sum can be expressed as

$$A(s,t) = 2is \int d^2b e^{i\vec{q}\cdot\vec{b}} \int dz dz' P_{13}(z) P_{24}(z') \{1 - e^{i\chi(s,b,z,z')}\},$$

where, for scalar glueball states,

$$P_{ij}(z) = \sqrt{-g(z)}(z/R)^2 \phi_i(z)\phi_j(z)$$

involves a product of two external normalizable wave functions. To first order in the eikonal,

$$A_4(s,t) \simeq \int d^2b e^{-i\mathbf{b}\mathbf{q}_\perp} \int dz dz' P_{13}(z) P_{24}(z') \, (2s\chi) \, ,$$

where

$$\chi(s,b,z,z') = \frac{g_0^2 R^4}{2(zz')^2 s} \mathcal{K}(s,b,z,z')$$

$\mathcal{K}$ is the BPST Pomeron kernel.

High Energy Scattering and DIS in String Theory
AdS space continued

- We are interested in calculating the structure function $F_2(x, Q^2)$, which is simply the cross section for an off-shell photon. Using the optical theorem we obtain

$$\sigma_{tot} \simeq 2 \int d^2b \int dz dz' P_{13}(z) P_{24}(z') \; Im \; \chi$$

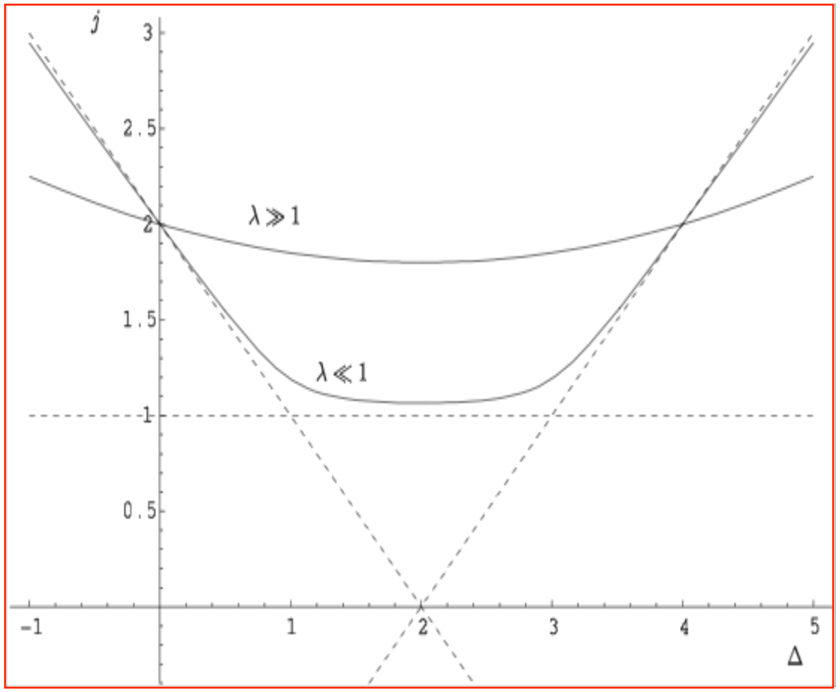
- For DIS, P_{13} should present a photon on the boundary that couples to a spin 1 current in the bulk. This current then propagates through the bulk, and scatters off the target.
- The wave function, in the conformal limit, is

$$P_{13}(z) = \frac{1}{z}(Qz)^4 (K_0^2(Qz) + K_1^2(Qz))$$

- For the proton, one for now treats it as a glueball of mass $\sim \Lambda = 1 / Q'$, which in string theory appears as a Kaluza-Klein mode of the massless dilaton, after the compactification of S^5.

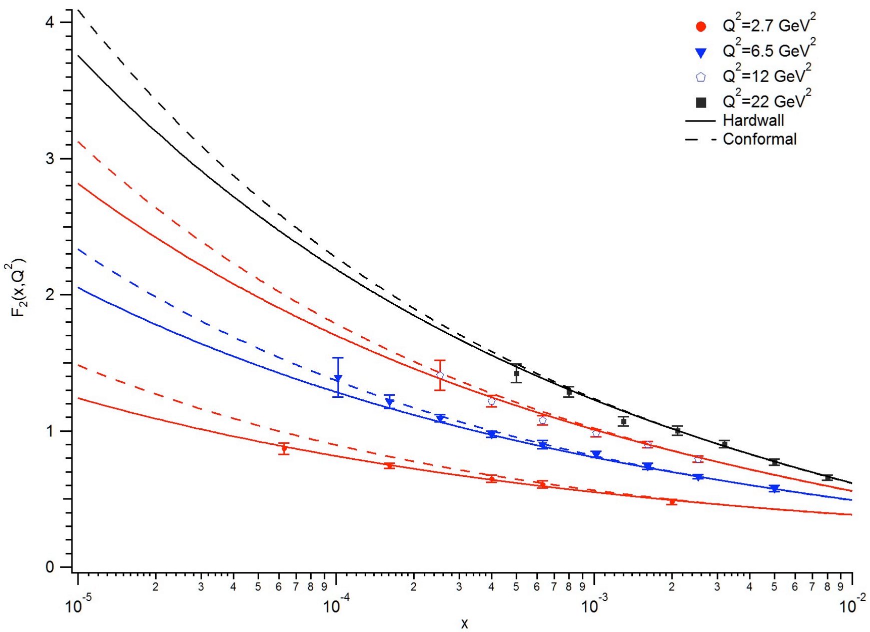
Moments and Anomalous Dimension

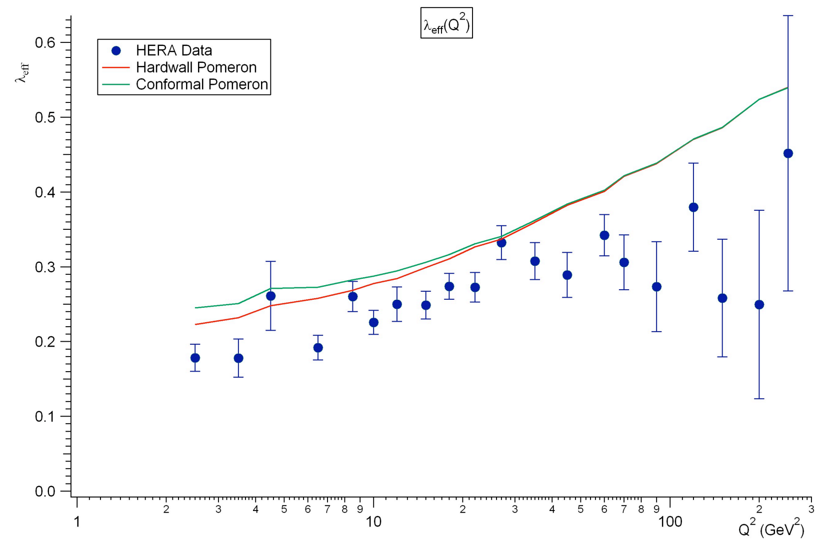
$$M_n(Q^2) = \int_0 dx\, x^{n-2} F_2(x, Q^2) \quad \longrightarrow \quad Q^{4-\Delta^{(+)}(n)}$$

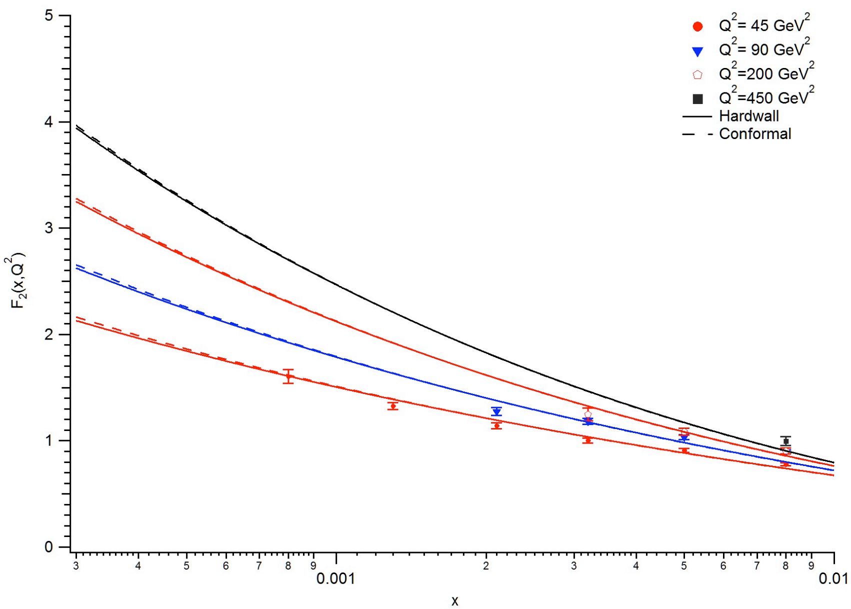


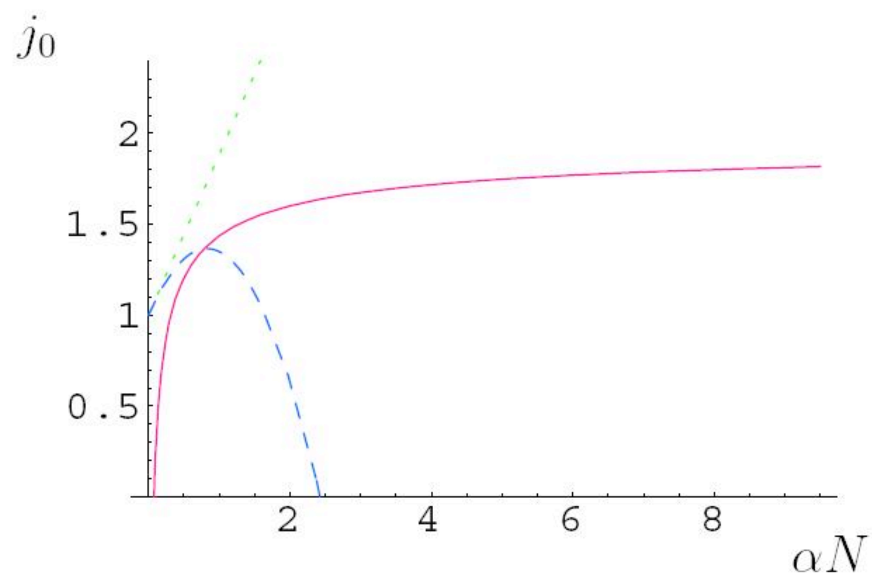
$$\Delta^{(\pm)}(j) = 2 + \sqrt{2}\,\lambda^{1/4}\sqrt{(j - j_0^{(\pm)})}$$

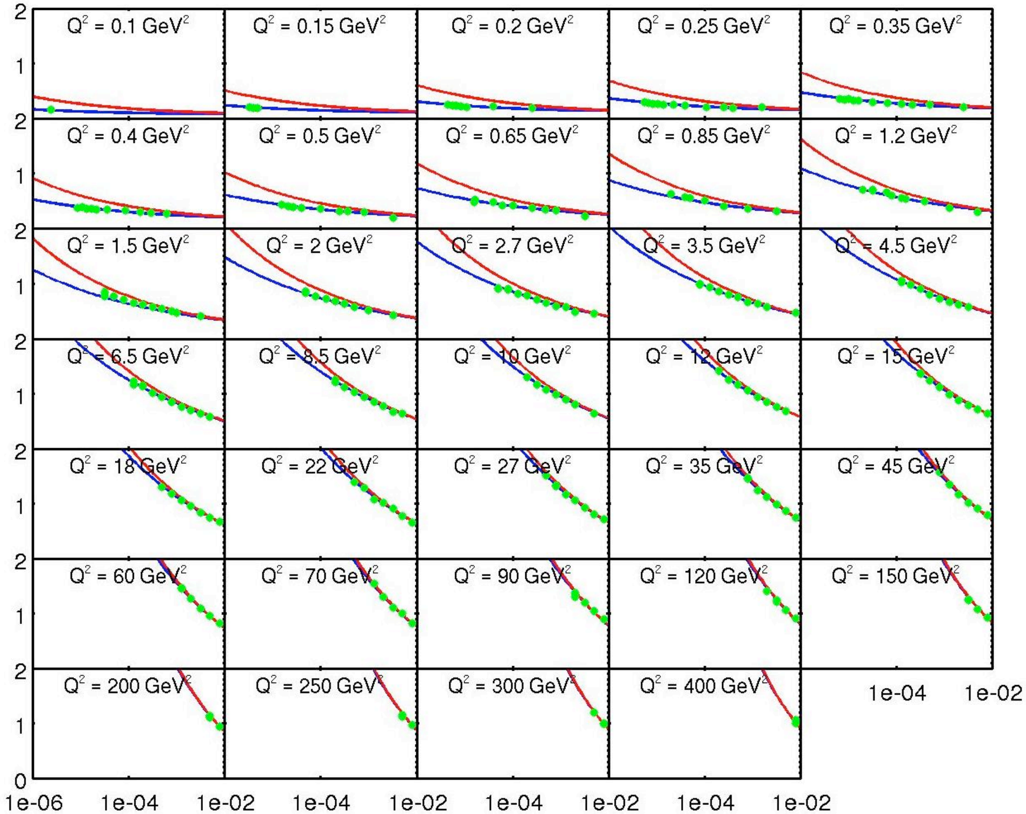
DIS after AdS/CFT

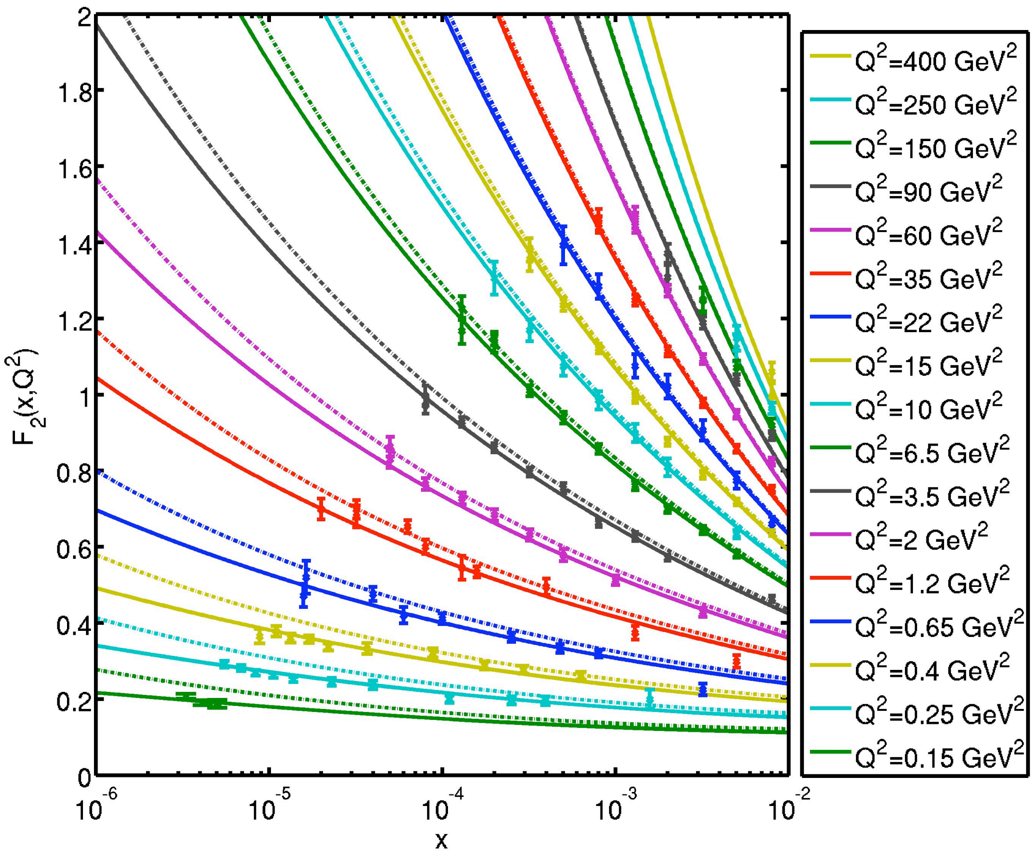




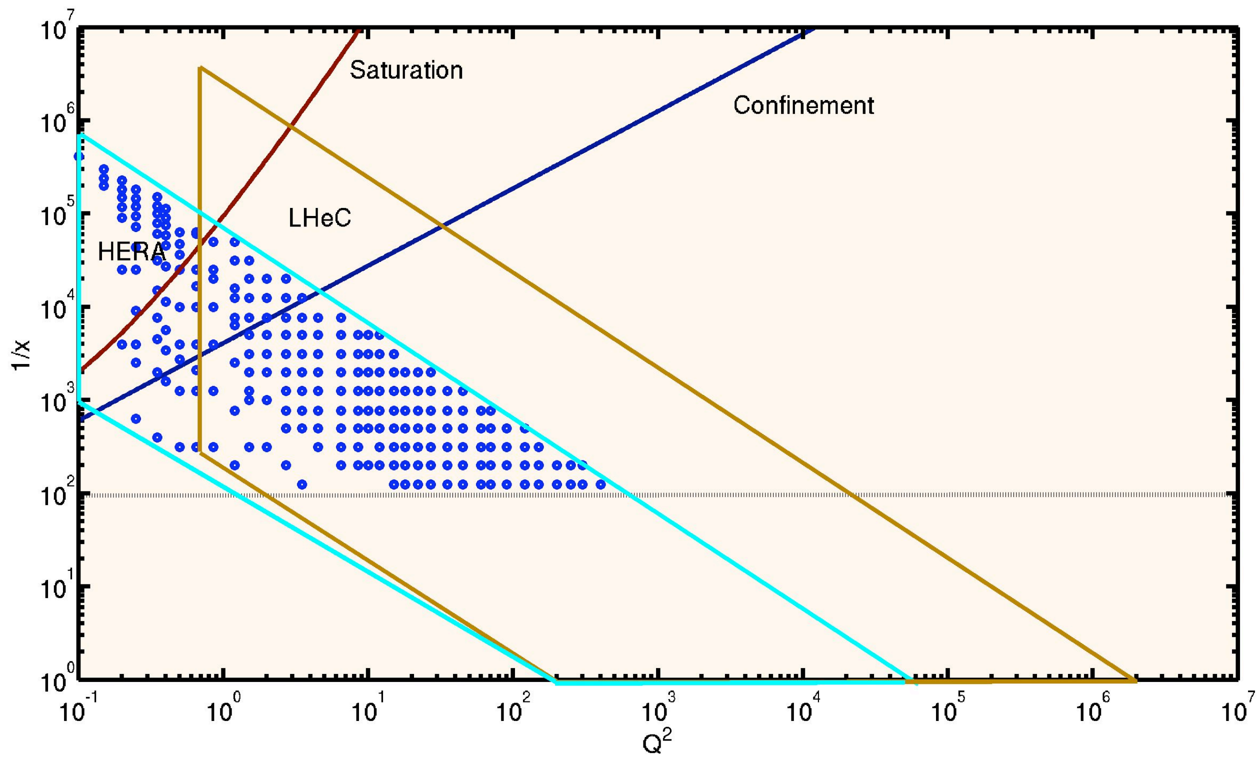





Q² = 0.1 GeV²
Q² = 0.15 GeV²
Q² = 0.2 GeV²
Q² = 0.25 GeV²
Q² = 0.35 GeV²
Q² = 0.4 GeV²
Q² = 0.5 GeV²
Q² = 0.65 GeV²
Q² = 0.85 GeV²
Q² = 1.2 GeV²
Q² = 1.5 GeV²
Q² = 2 GeV²
Q² = 2.7 GeV²
Q² = 3.5 GeV²
Q² = 4.5 GeV²
Q² = 6.5 GeV²
Q² = 8.5 GeV²
Q² = 10 GeV²
Q² = 12 GeV²
Q² = 15 GeV²
Q² = 18 GeV²
Q² = 22 GeV²
Q² = 27 GeV²
Q² = 35 GeV²
Q² = 45 GeV²
Q² = 60 GeV²
Q² = 70 GeV²
Q² = 90 GeV²
Q² = 120 GeV²
Q² = 150 GeV²
Q² = 200 GeV²
Q² = 250 GeV²
Q² = 300 GeV²
Q² = 400 GeV²
0
1
2
1e-06
1e-04
1e-02



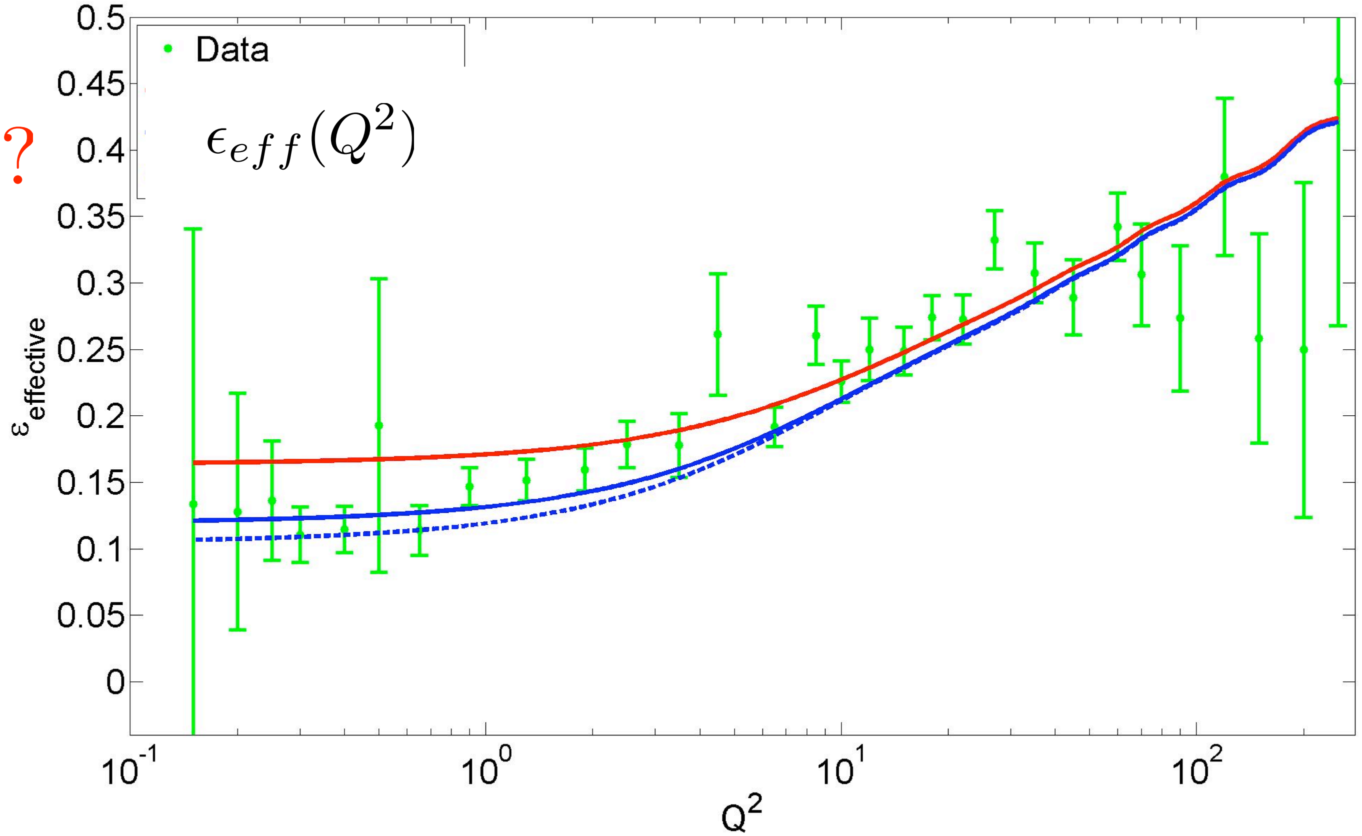
F_2(x,Q^2)
x
Q^2=400 GeV^2
Q^2=250 GeV^2
Q^2=150 GeV^2
Q^2=90 GeV^2
Q^2=60 GeV^2
Q^2=35 GeV^2
Q^2=22 GeV^2
Q^2=15 GeV^2
Q^2=10 GeV^2
Q^2=6.5 GeV^2
Q^2=3.5 GeV^2
Q^2=2 GeV^2
Q^2=1.2 GeV^2
Q^2=0.65 GeV^2
Q^2=0.4 GeV^2
Q^2=0.25 GeV^2
Q^2=0.15 GeV^2



Saturation
Confinement
LHeC
HERA
1/x
Q^2

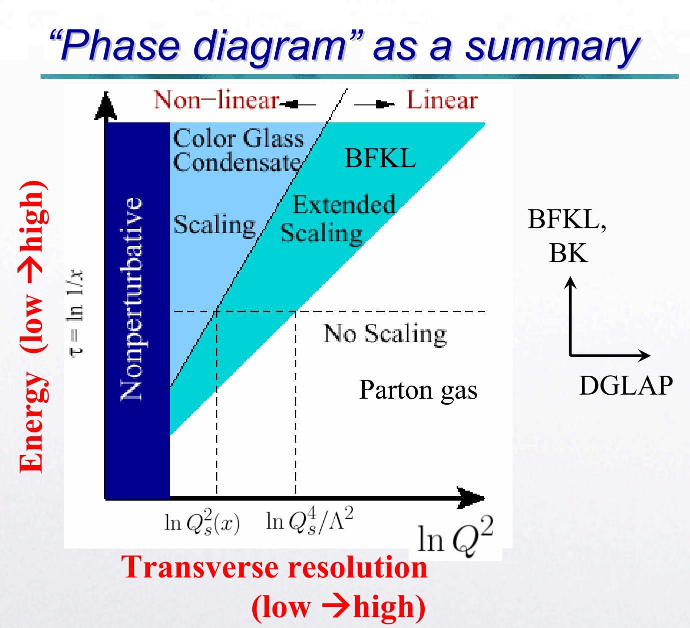
$$F_2(x,Q^2) \sim (1/x)^{\epsilon_{effective}}$$

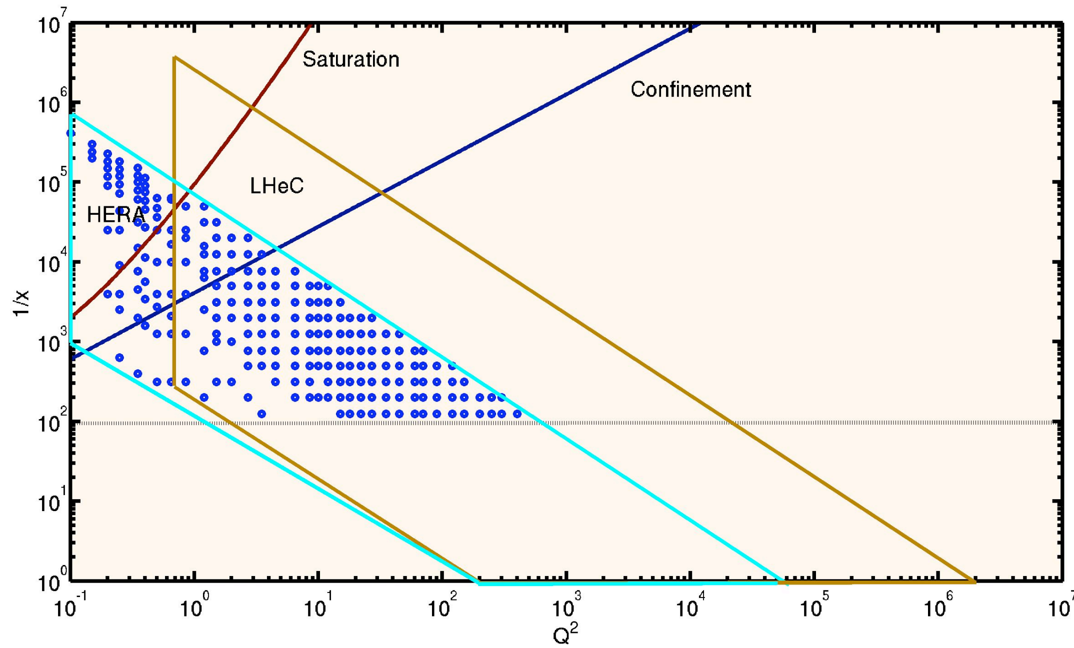
Puzzles?



Standard expectation
(from Itakura's RIKEN lectures)

AdS/CFT expectation
(from BDST: hep-ph/1007.2259)





VI. Summary and Outlook

- Provide meaning for Pomeron non-perturbatively from first principles.
- Realization of conformal invariance beyond perturbative QCD
- New starting point for unitarization, saturation, etc.
- Phenomenological consequences, DIS at small-x, Diffractive Higgs production at LHC (in progress), etc.

The QCD Pomeron

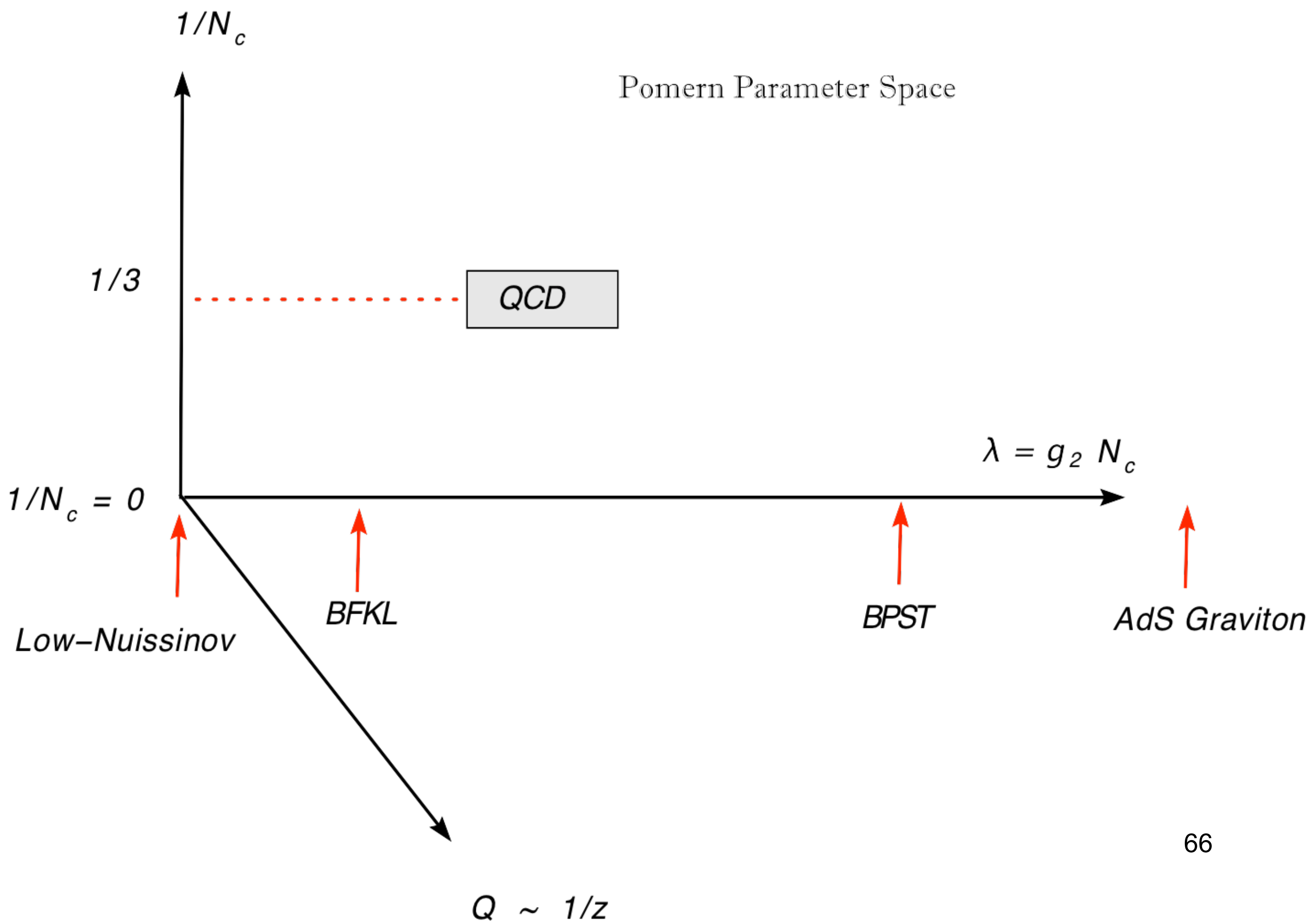
Have shown that in gauge theories with string-theoretical dual descriptions, the Pomeron emerges unambiguously.

Pomeron can be identified as Reggeized Massive Graviton.

Both the IR Pomeron and the UV Pomeron are dealt in a unified single step.

Both conceptual and practical advantages.

Diffractive Production of Higgs at LHC



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- R. Brower, M. Djuric, and C-I Tan, arXiv:0812.0354.
- Other related work, e.g., L. Cornalba, et al., (hep-th/0710.5480),
- Y. Hatta, E. Iancu, and A. H. Mueller, (hep-th/0710.2148),
- E. Levin, et al. (arXiv:0811.3586) and (arXiv:0902.3122).
- Many others.