Inhomogeneous Lambda-CDM cosmology

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Plan

- Brief motivation for considering inhomogeneity
- Asymptotics and `late time expansion'   (expansion about dS)
- Resummation of expansion   (expansion about FLRW)
- Averaging
- Late time observations - redshift, luminosity distance

Motivations

- Last decade has seen interesting discovery of late time acceleration probably due to Lambda
- Last decade has also seen interest in going beyond PT about FLRW, and challenging the accuracy of using an FLRW background
    - second order PT and non-gaussianity
    - claims that dark matter/energy are artefacts of nonlinearity
    - formal interest: how to ‘average’ cosmology and develop RG
- Develop methods that allow interpretation of late time observation which only depend on late time physics (ie. not dependent on inflationary initial conditions, weakly dependent on Einstein equations themselves...)

Late time expansion

- Method introduced by Starobinsky in context of inflation ['82]
- Analogous to Graham-Fefferman expansion for hyperbolic space, and its generalization to AdS in 'holographic RG'
- Take Einstein equations with perfect dust fluid (CDM on large scales) and Lambda. Write the metric as;

$$ds^2 = \frac{1}{y^2}\left(-\frac{dy^2}{H^2} + g_{ij}(x,y)dx^i dx^j\right)$$

- Have taken normal coordinates to const y surfaces. Note: const x^i curves are geodesic in normal coordinates. Since dust follows geodesics we may choose the y surfaces to fix const x^i curves to comove with the dust.
- For regular $g_{ij}(x,y)$ have conformal boundary at $y = 0$

Late time expansion...

- Then the stress tensor is: $$T_{yy} = \frac{1}{H^2 y^2}(\Lambda + \rho)\ , \quad T_{yi} = 0\ , \quad T_{ij} = -\frac{\Lambda}{y^2} g_{ij}$$
    - cf. synch gauge
- Note: this choice is deviation from previous work and gives large simplication
- The Einstein equations are:

$$\dot{g}_{ij} = \partial_y g_{ij}$$
$$\ddot{g}_{ij} = \partial_y^2 g_{ij}$$
$$\dot{g} = g^{ij}\dot{g}_{ij}$$

    - Eqn for density $$\ddot{g} - \frac{1}{y}\dot{g} - \frac{1}{2}\dot{g}_{ij}\dot{g}^{ij} = -\frac{1}{H^2 y^2}\rho$$
    - Constraint $$\nabla^j \dot{g}_{ij} - \nabla_i \dot{g} = 0$$
    - Tensor eqn $$\ddot{g}_{ij} - \frac{2}{y}\dot{g}_{ij} + \frac{1}{8} g_{ij}\left(\dot{g}_{mn}\dot{g}^{mn} - \dot{g}^2\right) - \dot{g}_{im}\dot{g}_j{}^m + \frac{1}{2}\dot{g}\,\dot{g}_{ij} + \frac{2}{H^2}\left(R_{ij} - \frac{1}{4} g_{ij} R\right) = 0$$

Late time expansion...

- First eqn simply determines the dust density ρ

- Consider first tensor equation. This may be solved as expansion in 'y':

$$g_{ij}(y,x) = \bar{g}_{ij}(x) + y^2 a_{ij}^{(0)}(x) + y^3 h_{ij}(x) + y^4 a_{ij}^{(1)}(x) + y^5 a_{ij}^{(2)}(x) + \ldots$$

- Frobenius expansion about y=0 (not Taylor expansion). Here $\bar{g}_{ij}(x)$ is the conformal boundary metric, and $h_{ij}(x)$ can be thought of as 'extrinsic curvature' of conf boundary.

- All other terms in expansion, $a_{ij}^{(n)}(x)$, determined in terms of $\bar{g}, h$ and we note that: $$a_{ij}^{(0)} = \frac{1}{H^2}\left(\bar{R}_{ij} - \frac{1}{4}\bar{g}_{ij}\bar{R}\right)$$

- Indicies above raised/lowered wrt $\bar{g}$ so $a_{ij}^{(0)}(x)$ `lives' on (as do $a_{ij}^{(n)}(x)$)

Late time expansion...

- Now consider the constraint equation: $\nabla^j \dot{g}_{ij} - \nabla_i \dot{g} = 0$
- Define: $\Phi_i \equiv \bar{\nabla}^j \dot{g}_{ij} - \bar{\nabla}_i \dot{g}$
- Tensor equation implies this quantity evolves in y as $\partial_y \Phi_i = \left(\frac{2}{y} - \frac{1}{2}\dot{g} \right) \Phi_i$
- Integrate to give: $\Phi_i(y,x) = A_i(x)\, y^2 e^{-\frac{1}{2}\int_0^y d\tilde{y}\, \dot{g}(\tilde{y},x)}$ for some constants $A_i(x)$
- Evaluate Φ on solution expansion; $\Phi_i(y,x) = 3\left(\bar{\nabla}^j h_{ij} - \bar{\nabla}_i h \right) y^2 + O(y^3)$
- By comparison we observe the constraint implies: $\bar{\nabla}^j h_{ij} - \bar{\nabla}_i h = 0$
- Note this constraint, applied to data h, then holds for all of expansion.

Late time expansion...

- Dust equation then determines: $$\frac{\rho}{H^2} = -3y^3h + \frac{3}{2}y^5a^{(0)}h + O(y^6)$$
- Note the physical requirement that the trace $h < 0$
- Physically the late time expansion captures cosmologies (or patches of them) where Lambda comes to dominate at late times.
    - Appears to be observed in our universe on large scales
- We see such solutions of Lambda-CDM are characterized by a 3-metric $\bar{g}_{ij}$ and a tensor h_{ij} living on that 3-metric and obeying $\bar{\nabla}^j h_{ij} - \bar{\nabla}_i h = 0$
- Note the scaling symmetry; $y \to \lambda\, y \qquad \bar{g}_{ij} \to \lambda^2 \bar{g}_{ij} \qquad h_{ij} \to h_{ij}/\lambda$

Higher terms

- Define operator $\mathcal{O}$ on tensor T_{ij} obeying $\bar{\nabla}^j T_{ij} - \bar{\nabla}_i T = 0$

$$\begin{aligned} \mathcal{O}_{ij}{}^{mn} T_{mn} &= -\frac{1}{2}\bar{\nabla}^2 T_{ij} + \frac{1}{2}\bar{\nabla}_i \partial_j T - \bar{R}_i{}^m{}_j{}^n T_{mn} \\ &\quad + \bar{R}_{m(i} T_{j)}{}^m - \frac{1}{4}\bar{R} T_{ij} + \frac{1}{4}\bar{g}_{ij}\bar{R}_{mn} T^{mn} \end{aligned}$$

- Then some higher terms are:

$$\begin{aligned} a_{ij}^{(1)} &= a_{im}^{(0)} a^{(0)m}_{j} - \frac{1}{2} a^{(0)} a_{ij}^{(0)} - \frac{1}{8}\bar{g}_{ij}\left(a^{(0)mn} a_{mn}^{(0)} - (a^{(0)})^2\right) \\ &\quad - \frac{1}{2H^2}\mathcal{O}_{ij}{}^{mn} a_{mn}^{(0)} \\ a_{ij}^{(2)} &= \frac{3}{5}\left(a_{im}^{(0)} h_j{}^m + a_{jm}^{(0)} h_i{}^m\right) - \frac{3}{10}\left(h a_{ij}^{(0)} + a^{(0)} h_{ij}\right) \\ &\quad - \frac{3}{20}\bar{g}_{ij}\left(a_{mn}^{(0)} h^{mn} - a^{(0)} h\right) - \frac{1}{5H^2}\mathcal{O}_{ij}{}^{mn} h_{mn} \end{aligned}$$

Comments

- This expansion is an expansion about dS. Take $\bar{g}_{ij} = \delta_{ij}$ and $h_{ij} = 0$ and the result is flat sliced dS.
- Deformations about flat dS are controlled by curvatures of $\bar{g}_{ij}$, h_{ij} and its derivatives all dimensionalized by H
- Expect good convergence where geometry on const y slice is close to flat dS
- Since we are in a regime where Lambda dominates, may expect reasonable convergence.
- Have ignored radiation. Would enter at $O(y^4)$ order. Due to radiation-matter epoch occurring far before matter-Lambda, coefficients in expansion will be unnaturally small - reasonable study $O(y^4)$ order and higher ignoring radiation.

Comments

- Consider general FLRW. Take; $\bar{g}_{ij} = \Omega_{ij}$ with $\bar{R}_{ij} = 2\,k\,\bar{g}_{ij}$ and homogeneous
- Then Ω_{ij} is sphere metric (k>0), is flat δ_{ij} (k=0) or hyperbolic metric (k<0)
- Then we require: $h_{ij} = -\rho_0/(3H)^2\Omega_{ij}$ to give FLRW data, with $\rho \sim \rho_0 y^3 + \dots$
- Note that whilst terms in expansion must be small, this is not equivalent to cosmological PT.
- For example: take $\bar{g}_{ij}$ to be a squashed sphere metric. Taking the radius of curvature to be large so that $y_{now}^2 \bar{R}_{ij} \ll H^2$ this deformation is well described in our expansion today. However, for strong squashing this cannot be described by PT.

Resummation

- Previous late time soln is expansion about flat dS. This is not ideal since our solution doesn't treat the FLRW solution exactly, even though we know this!

- Consider flat FLRW:

$$\begin{aligned} g_{ij} &= \left(1 - \tfrac{\rho_0}{12H^2} y^3\right)^{4/3} \delta_{ij} \\ &\simeq \left(1 - \tfrac{\rho_0}{9H^2} y^3 + \tfrac{\rho_0^2}{648H^4} y^6 + \tfrac{\rho_0^3}{34992H^6} y^9 + \ldots\right) \delta_{ij} \end{aligned}$$

$$\rho \sim \rho_0 y^3 + \ldots$$

- Surprisingly this converges back to big bang, $y_{BB} = \left(\frac{12H^2}{\rho_0}\right)^{1/3}$

- However, for $\rho_0/12H^2 \sim O(1)$ may expect slow convergence.

- Even if we are in Lambda dominated epoch, high redshift SN are not.

- Encouraging as, for weak inhomog. may expect convergence far back!

Resummation...

- Consider general FLRW. Take; $g_{ij} = a^2(y)\Omega_{ij} \quad a(0) = 1,$ so; $\bar{g}_{ij} = \Omega_{ij}$
- Then $h_{ij} = -\rho_0/(3H)^2\Omega_{ij}$ gives FLRW data, with $\rho \sim \rho_0 y^3 + \dots$
- Usual analysis allows determination of function $a(y;k,\rho_0)$ `exactly' from odes

$$\begin{aligned} \frac{\partial_y^2 a}{a} - \frac{2}{y}\frac{\partial_y a}{a} + \frac{1}{2}\left(\frac{\partial_y a}{a}\right)^2 + \frac{k}{2H^2a^2} &= 0 \\ \frac{\partial_y^2 a}{a} - \frac{1}{y}\frac{\partial_y a}{a} &= -\frac{\rho}{6y^2H^2} \end{aligned}$$

- Expanding soln with $\rho \sim \rho_0 y^3 + \dots$ can straightforwardly be computed.

$$\int_0^{\left(\frac{a(y;k,\rho_0)}{y}\right)^{3/2}} \frac{dx}{\sqrt{\frac{1}{3}\rho_0 - k\,x^{2/3} + H^2\,x^2}} = -\frac{3}{2H}\log\frac{y}{y_0}$$

Resummation...

- For a tensor T_{ij} denoted $\tilde{T}_{ij} \equiv T_{ij} - \frac{1}{3}\bar{g}_{ij}T$ as its anistropic component.
- Now resum as; $ds^2 = \frac{1}{y^2}\left(-\frac{dy^2}{H^2} + a^2(y; \frac{1}{6}\bar{R}, -\frac{1}{3H^2}h)\hat{g}_{ij}(x,y)dx^i dx^j\right)$

$$\hat{g}_{ij}(y,x) \quad = \quad \bar{g}_{ij}(x) + y^2 b^{(0)}_{ij}(x) + y^3\tilde{h}_{ij}(x) + y^4 b^{(1)}_{ij}(x) + y^5 b^{(2)}_{ij}(x) + \ldots$$

- Where again, $\bar{\nabla}^j h_{ij} - \bar{\nabla}_i h = 0$, and the pair $(\bar{g}, h)$ characterize the solution
- Higher terms:

$$\begin{aligned} H^2 b^{(0)}_{ij} &= \tilde{\bar{R}}_{ij} \\ H^4 b^{(1)}_{ij} &= \frac{1}{48}\bar{\nabla}^2\bar{R}\bar{g}_{ij} - \frac{1}{16}\bar{\nabla}_i\partial_j\bar{R} + \frac{1}{4}\bar{\nabla}^2\tilde{\bar{R}}_{ij} \\ &\quad -\frac{1}{6}\bar{R}\tilde{\bar{R}}_{ij} - \frac{1}{2}\tilde{\bar{R}}_{im}\tilde{\bar{R}}^m{}_j + \frac{1}{4}\tilde{\bar{R}}_{mn}\tilde{\bar{R}}^{mn}\bar{g}_{ij} \\ H^2 b^{(2)}_{ij} &= \frac{1}{30}\bar{\nabla}^2 h - \frac{1}{10}\bar{\nabla}_i\partial_j h + \frac{1}{10}\bar{\nabla}^2\tilde{h}_{ij} \\ &\quad -\frac{7}{30}h\tilde{\bar{R}}_{ij} - \frac{13}{120}\bar{R}\tilde{h}_{ij} + \frac{3}{5}\tilde{\bar{R}}_{m(i}\tilde{h}_{j)}{}^m \end{aligned}$$

Resummation...

- Seemingly trivial, but the expansion of $\hat{g}_{ij}(y,x)$ in y is an expansion about FLRW. Taking $\bar{g}_{ij} = \Omega_{ij}$ and $h_{ij} = -\rho_0/(3H)^2\Omega_{ij}$ gives $\hat{g}_{ij}(y,x) = \bar{g}_{ij}(x)$

- Truncating $\hat{g}_{ij}(y,x)$ at some order in y now exactly treats the `homogeneous' dust and curvature components. All terms in the expansion parameterize deformations in inhomogeneity and/or anisotropy

- Speed of convergence of series is now determined by inhomog/anisotropy only. Presumably convergence is good where constant y slices geometrically are similar to FLRW slices.

Averaging

- We may think of the data $(\bar{g}, h)$ with $\bar{\nabla}^j h_{ij} - \bar{\nabla}_i h = 0$ geometrically.
- Take a geometry defined by $\bar{g}$ and a perturbation of that geometry $\delta\bar{g}$
- Define a quantity $u_i \equiv \bar{\nabla}^j \delta\bar{g}_{ij} - \frac{1}{2}\bar{\nabla}_i \delta g$; harmonic gauge if $u_i = 0$
- (Expect) we may uniquely choose harmonic gauge for perturbation s.t. $u_i = 0$ which is achieved by a diffeo generated by ξ^i which obeys $\bar{\nabla}^j \bar{\nabla}_j \xi_i + \bar{R}_{ij}\xi^j = H_i$
- Then any pair $(\bar{g}, \delta\bar{g})$ is equivalent to the pair $(\bar{g}, h)$ taking $h_{ij} = \delta\bar{g}_{ij} - \frac{1}{4}\bar{g}_{ij}\delta\bar{g}$
- Physical dust constraint $h < 0$ is $\delta\bar{g} < 0$ in harmonic gauge.
- Since $\delta\left(\sqrt{\det \bar{g}_{ij}}\right) = \frac{1}{2}\sqrt{\det \bar{g}_{ij}}\delta g$ then our perturbation must *locally decrease volume* in harm gauge.

Averaging...

- Canonical method for smoothing a geometry is Ricci flow: $\frac{d}{d\tau}\bar{g}_{ij}(\tau) = -2\bar{R}_{ij}[\bar{g}(\tau)]$
- Consider a metric $\bar{g}$ and a nearby metric $\bar{g} + \epsilon\delta\bar{g}$
- If we flow both, we find; $\frac{d}{d\tau}\delta\bar{g}_{ij} = \triangle_L \delta g_{ij} - 2\bar{\nabla}_{(i}u_{j)}$ with $\delta\bar{g}_{ij}(0) = \delta\bar{g}_{ij}$

$$\triangle_L \delta g_{ij} \equiv \bar{\nabla}^2\delta\bar{g}_{ij} + 2\bar{R}_{i\ j}^{\ m\ n}\delta\bar{g}_{mn} - 2\bar{R}_{(i}^{\ \ m}\delta\bar{g}_{j)m}$$

- Hence we may canonically smooth a pair $(\bar{g}, \delta\bar{g})$ by simultaneously flowing $\bar{g}$ by Ricci flow, and $\delta\bar{g}$ by its linearization (in the background of $\bar{g}$).
- To smooth our pair $(\bar{g}, h)$ we think of it as a pair, $(\bar{g}, \delta\bar{g})$, then flow for some time τ and then convert back to $(\bar{g}, h)$.

Example

- Flat FLRW $\bar{g}_{ij} = \delta_{ij}$ and $\delta\bar{g}_{ij} = \tilde{\rho}\delta_{ij}$ fixed point for const $\tilde{\rho}$
- Well known that 3-flat space is stable under Ricci flow.
- Consider perturbation: $\delta\bar{g}_{ij} = \tilde{\rho}\,\delta_{ij} + E_{ij}$ where $\tilde{\rho}$ non-constant function and also E_{ij} is traceless and chosen so that $\bar{\nabla}^j E_{ij} = \frac{1}{2}\bar{\nabla}_i\tilde{\rho}$ implying $u_i = 0$
- Take; $\tilde{\rho} \to \mathrm{constant}$ and $E_{ij} \to 0$ as, $|x| \to \infty$
- Then about flat space $\bar{g}_{ij} = \delta_{ij}$ one finds; $\frac{d}{d\tau}\tilde{\rho} = \bar{\nabla}^2\tilde{\rho}\,,$ $\frac{d}{d\tau}E_{ij} = \bar{\nabla}^2 E_{ij}$ and in fact the condition $u_i = 0$ is preserved by the flow.
- Thus expect flat FLRW is a *stable* fixed point of the averaging flow

Late time observation - WARNING!

- Here we will consider that our universe can now, and for reasonable time in past, be described on large scales (~ 1/H) by our solution.
- Standard cosmology where on large scales universe is very close to FLRW is confirmed by CMB to very high precision. Determines all large scales to high precision using PT about flat FLRW.
- Therefore it is unlikely we can learn anything new by considering late time measurements and our expansion.
- However, many assumptions go into standard calculation - metric closeness to FLRW, inflation initial conditions. It may prove useful to confirm aspects of this picture from measurements (eg. SN) using only late time assumptions.
  - ie. derive closeness to FLRW from first principles.

Late time observation...

- Consider a comoving source at (y_e, x_e^i) and a comoving observer at $(y_o, 0)$
- At late time the observer meets conf. boundary at $x^i = 0$
- Consider the observer looking in a past direction parameterized by the unit norm vector $\bar{v}^i$
- We may solve null geodesic equations, and compute redshift along the curve, and also the luminosity distance of the source.

$$\bar{V}^{mn} \equiv \bar{v}^m \bar{v}^n - \frac{1}{4}\bar{g}^{mn}(0)$$

$$1 + Z = \frac{y_e}{y_o}\Big(1 - \frac{(y_e^2 - y_o^2)}{2H^2}\bar{V}^{mn}\bar{R}_{mn} - \frac{1}{2}\Big(\frac{(y_e - y_o)^2(2y_e + y_o)}{3H^3}\bar{V}^{mn}\bar{v}^k\bar{\nabla}_k\bar{R}_{mn} - \left(y_o^3 - y_e^3\right)\bar{v}^i\bar{v}^j h_{ij}\Big) + \mathcal{O}\left(y_e^4, y_e^3 y_o \ldots y_o^4\right)\Big)\Big|_{x^i=0}$$

- Find expansion in y_e, y_o, and answer depends on expansion of $\bar{g}$ about $x^i = 0$

Late time observation...

- For luminosity distance, D_L , we find similar type of expression. Inverting previous redshift formula to get y_e in terms of Z , we find the relation;

$$\bar{V}^{mn} \equiv \bar{v}^m \bar{v}^n - \frac{1}{4}\bar{g}^{mn}(0)$$

$$D_L^2 = \frac{(1+Z)^2 Z^2}{H^2}\Bigg(1 + \frac{2(1+Z)}{H^2} y_o^2 \bar{V}^{mn}\bar{R}_{mn} + \frac{Z(1+Z)}{H^3} y_o^3 \bar{V}^{mn}\bar{v}^k \bar{\nabla}_k \bar{R}_{mn} + \frac{3(1+Z)(2+Z)}{2} y_o^3 h_{ij}\bar{v}^i \bar{v}^j + \frac{Z^2(2+Z)}{4} y_o^3 trh + \mathcal{O}\left(y_o^4\right)\Bigg)\Bigg|_{x^i=0}$$

- Note we may choose ‘local inertial coords’;

$$\bar{g}_{ij}(x) = \delta_{ij} - \frac{1}{3}\bar{R}_{ikjl}\Big|_{x^i=0} x^k x^l - \frac{1}{6}\left(\bar{\nabla}_k \bar{R}_{iljm}\right)\Big|_{x^i=0} x^k x^l x^m + \mathcal{O}\left(x\right)^4$$

- And recall in 3d; $\bar{R}_{ijkl} = 2\left(\bar{g}_{i[k}\bar{R}_{l]j} - \bar{g}_{j[k}\bar{R}_{l]i}\right) - \bar{R}\,\bar{g}_{i[k}\bar{g}_{l]j}$

- We see terms in expansion related to how $\bar{g}$ varies away from $x^i = 0$

Late time observation with resummation

- Resummed expression:

$$D_L^2(Z) = g\left(y_o, Z; \frac{\bar{R}}{6}, -3H^2 h\right)^2 \Bigg(\quad 1 + \tfrac{2(1+Z)}{H^2} y_o^2 \bar{v}^i \bar{v}^j \bar{\tilde{R}}_{ij} + \tfrac{Z(1+Z)}{H^3} y_o^3 \left(\bar{v}^m \bar{v}^n \bar{v}^i \bar{\nabla}_i \bar{\tilde{R}}_{mn} + \tfrac{1}{12} \bar{v}^i \bar{\nabla}_i \bar{R}\right)$$

$$+ \tfrac{3(1+Z)(2+Z)}{2} y_o^3 \bar{v}^i \bar{v}^j \tilde{h}_{ij} + O(y_o^4)\Bigg)\Bigg|_{x^i=0}$$

- Where FLRW function $g\left(y_o, Z; k, \rho_0\right)$ determined by;

$$g(y_o, Z; k, \rho_0) \equiv (1+Z)\frac{a(y_o; k, \rho_0)}{y_o} r(Z; k, \rho_0),$$

$$r(y_o, Z; k, \rho_0) \equiv \frac{1}{\sqrt{k}} \sin\left[\sqrt{k} \int_{y_o}^{y^*(Z)} \frac{dy}{Ha(y; k, \rho_0)}\right]$$

where $1 + Z = \dfrac{y^*(Z)}{y_o} \dfrac{a(y_o; k, \rho_0)}{a(y^*(Z); k, \rho_0)}$

Late time observation...

- Luminosity distance can determine some, but not all of data $(\bar{g}, h)$

$$D_L^2(Z) = g\left(y_o, Z; \frac{\bar{R}}{6}, -3H^2 h\right)^2 \Bigg(\quad 1 + \tfrac{2(1+Z)}{H^2} y_o^2 \bar{v}^i \bar{v}^j \tilde{\bar{R}}_{ij} + \tfrac{Z(1+Z)}{H^3} y_o^3 \left(\bar{v}^m \bar{v}^n \bar{v}^i \bar{\nabla}_i \tilde{\bar{R}}_{mn} + \tfrac{1}{12} \bar{v}^i \bar{\nabla}_i \bar{R} \right)$$
$$+ \tfrac{3(1+Z)(2+Z)}{2} y_o^3 \bar{v}^i \bar{v}^j \tilde{h}_{ij} + O(y_o^4) \Bigg) \Bigg|_{x^i=0}$$

- The data, $\bar{R}_{ij}, h_{ij}, \nabla_{(i}\bar{R}_{jk)}$ at observer can (in principle) be determined by measuring, eg. standard candle SN in all directions, $\bar{v}^i$

- However, at this order we cannot determine asymmetric parts of $\nabla_i \bar{R}_{jk}$ such as $\nabla_{[i}\bar{R}_{j]k}$

- Whether these can be determined by higher orders is interesting question.

Summary

- New way to characterize inhomogenous Lambda-CDM universe *such as our own*, in terms of final data $(\bar{g}, \delta\bar{g})$, with $\delta\bar{g}$ locally volume decreasing.

- Assumes only matter content, Lambda and perfect dust fluid (CDM), and final dominance of Lambda.

- Uses method of Starobinsky/Holo RG together with resummation to express solutions as deformations about FLRW. Method is non-perturbative in metric deformation.

- Natural way to average these cosmologies, with Ricci flow giving flow in this space of solutions. Shown flat FLRW stable fixed pt of this flow.

- Natural way to observe characterizing data; eg. by SN and luminosity distance. No assumption about initial data - eg. form of inflation, PT etc...

Outlook

- Expect this method works generally for any late time acceleration. Hence can possibly generalize the method to use dark energy. Also may include radiation fluid.

- What characterizing data can be extracted from observation in principle?

- What can be determined in practice? Have studied SN data...

- Most important question: how do we connect CMB initial conditions to this late time data. Obvious in linear theory - but what about non-linear theory?