Stability of spherical symmetric spacetimes in Modified Gravity

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Introduction

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- Motivated different approaches

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- New gravity and new matter: Extended Brans-Dicke, Galileon

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- Interesting phenomenology [Review by ADF, Tsujikawa '10]

General MGM

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- Two extra new scalar fields: F, ξ

MGM and PT

- Gauss-Bonnet gravity

$$S = \frac{1}{2} M_{\rm pl}^2 \int d^4x \sqrt{-g}[R + \xi\, G - V(\xi)].$$

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- Instability $c_s^2 < 0$ on FLRW before DE
  [ADF, Tsujikawa '10]

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- In other backgrounds, both propagate

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- Look for behavior of perturbations

Black hole stability ADF, Suyama, Tanaka '11

- Study vacuum spher. symm. $(z = \cos\theta)$

$$ds^2 = -Adt^2 + dr^2/B + \frac{r^2\,dz^2}{1-z^2} + r^2(1-z^2)d\phi^2$$

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- Expand action at second order

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- Introduce Lagr. multipl. $Q = \dot{h}_1 - h_0' + 2h_0/r$

- Result

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- Radial speed

$$c_{\rm odd}^2 = \frac{1}{A} \frac{AF - 2BA'\xi'}{F - 2B'\xi' - 4B\xi''}$$

Even modes

- $\delta g_{00} = -AH_0 Y_{l0}$, $\delta g_{01} = H_1 Y_{l0}$, $\delta g_{rr} = H_2 Y_{l0}/B$, $\delta g_{rz} = \alpha \partial_z Y_{l0}$, $F = F(r) + \delta F \, Y_{l0}$, $\xi = \xi(r) + \delta\xi \, Y_{l0}$

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- Integrate out H_2 and H_0

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- $\det K_{ij} \to 0$, as $B \to 1 + Cr^2$: (a)dS.

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- Require that, with $\bar{G} \equiv 16 r_s^2 / r^6$

$$-1 < \tanh 2b = -\frac{2\sqrt{\bar{G}} U_{,F\xi}}{U_{,\xi\xi} + \bar{G} U_{,FF}} < 1$$

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- If $M_{\text{cutoff}} \sim L_{\text{exp}}^{-1}$ viable if

$$|9\bar{G}\,\det(f_{,ij})| = |m_+^{-2} m_-^{-2}| \ll L_{\text{exp}}^4$$
$$|3(f_{,RR} - \bar{G} f_{,GG})| = |m_+^{-2} + m_-^{-2}| \ll L_{\text{exp}}^2$$

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- $m_+^2 = \mathcal{O}(10^{22.7-2.7p}\mathrm{cm})^{-2} \gg (1\mathrm{AU})^{-2}$, if $p \geq 4$

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- If $|f_{,RR}| < |\bar{G}f_{,GG}|$, problem: $m_+ \leftrightarrow m_-$, $m_- \ll m_+$

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