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Cosmology & String

Galileon Cosmology

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Scalar fields play an important role in cosmology

Inflation / Dark energy

In this talk, I will describe the (most general extension of the) **Galileon** and its applications to cosmology

The Galileon extends far beyond a specific scalar-field theory!

Talk Plan

1. Introduction to the *Galileon*
2. G-inflation – Inflation driven by the Galileon field –
3. Galileon models of dark energy

1

Introduction to the *Galileon*

Scalar-field Lagrangian

1st derivative

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Euler-Lagrange equation

2nd-order EOM

$$\Box\phi - V_\phi = 0$$

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi, \cdots)$$ has higher-order EOM?

— No, not necessarily

Tensor example: *Einstein-Hilbert*

$$R \supset \partial\Gamma^\lambda_{\mu\nu} \supset \partial^2 g_{\mu\nu}$$

Example:

(*) appears in an effective description of the brane-bending mode in DGP

$$\mathcal{L} \supset (\partial\phi)^2\Box\phi \qquad (*)$$

$$\rightarrow \quad \mathrm{EOM} \supset (\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)(\partial^\mu\partial^\nu\phi)$$

2nd-order EOM

The term $\partial_\mu\phi\partial^\mu\Box\phi$ is canceled out

(*) has the Galilean shift symmetry:

$$\phi \to \phi + v_\mu x^\mu + c$$

$\partial_\mu\phi \to \partial_\mu\phi + v_\mu$

looks like Galilei transformation

Look for scalar-field Lagrangians having:

(i) Galilean shift symmetry;
(ii) 2nd-order EOM

Nicolis, Rattazzi, Trincherini (2009)

Galileon (in flat space)

Only 5 possible Lagrangians that have the two properties:

$$\begin{aligned}
\mathcal{L}_1 &= \phi, \\
\mathcal{L}_2 &= (\partial\phi)^2, \\
\mathcal{L}_3 &= (\partial\phi)^2 \Box\phi, \\
\mathcal{L}_4 &= (\partial\phi)^2 \left[(\Box\phi)^2 - (\partial_\mu\partial_\nu\phi)^2 \right], \\
\mathcal{L}_5 &= (\partial\phi)^2 \left[(\Box\phi)^3 - 3\,(\Box\phi)\,(\partial_\mu\partial_\nu\phi)^2 + 2\,(\partial_\mu\partial_\nu\phi)^3 \right].
\end{aligned}$$

Deffayet, Esposito-Farese, Vikman (2009)

Covariantization

Coupling to gravity: $\partial_\mu \to \nabla_\mu$

Forget about Galilean shift symmetry,
maintain 2nd-order equations both for ϕ and $g_{\mu\nu}$

Computation example:

$$\frac{\delta}{\delta\phi}\left\{(\partial\phi)^2\left[(\Box\phi)^2-(\nabla_\mu\nabla_\nu\phi)^2\right]\right\} \supset (\cdots)^\mu\left[\Box\nabla_\mu\phi-\nabla_\mu\Box\phi\right], \longrightarrow (\cdots)^\mu R_{\mu\nu}\nabla^\nu\phi$$

$$(\cdots)\left[\Box\Box\phi-\nabla_\mu\Box\nabla^\mu\phi\right]$$

$$\longrightarrow -R^{\mu\nu}\nabla_\mu\nabla_\nu\phi-\boxed{\frac{1}{2}\nabla^\mu R\nabla_\mu\phi}$$

Add non-minimal coupling such as $[(\partial\phi)^2]^2 R$
so as to cancel higher-derivative terms

Deffayet, Esposito-Farese, Vikman (2009)

Covariant completion of Galileon

$$
\begin{aligned}
\mathcal{L}_2 &= X, \\
\mathcal{L}_3 &= X\Box\phi, \\
\mathcal{L}_4 &= \frac{X^2}{2}R + X\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right], \\
\mathcal{L}_5 &= \frac{X^2}{2}G_{\mu\nu}\nabla^\mu\nabla^\nu\phi \\
&\quad -\frac{X}{6}\left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\right].
\end{aligned}
$$

Galilean shift symmetry is now abandoned...

Non-minimal coupling to gravity

where $X := -\frac{1}{2}(\partial\phi)^2$

Question

Can we further generalize the Galileon while maintaining the 2nd-order property?

What is the most general Lagrangian of the form

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi, \partial^3\phi, \cdots; g_{\mu\nu}, \partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \cdots)$$

having 2nd-order field equations?

Answer

Generalized Galileon

Deffayet, Gao, Steer, Zahariade (2011)

Generalized Galileon

$$\mathcal{L}_1 = \phi$$

4 arbitrary functions of ϕ and X

$$\mathcal{L}_3 = X\Box\phi$$

k-inflation/k-essence

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\Big[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3\Big]$$

$$G_4G_{\mu\nu} - \frac{1}{2}G_{4X}R\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}G_{4XX}\left[(\Box\phi)^2 - (\nabla_\alpha\nabla_\beta\phi)^2\right]\nabla_\mu\phi\nabla_\nu\phi - G_{4X}\Box\phi\nabla_\mu\nabla_\nu\phi$$
$$+G_{4X}\nabla_\lambda\nabla_\mu\phi\nabla^\lambda\nabla_\nu\phi + 2\nabla_\lambda G_{4X}\nabla^\lambda\nabla_{(\mu}\phi\nabla_{\nu)}\phi - \nabla_\lambda G_{4X}\nabla^\lambda\phi\nabla_\mu\nabla_\nu\phi + g_{\mu\nu}\left(G_{4\phi}\Box\phi - 2XG_{4\phi\phi}\right)$$
$$+g_{\mu\nu}\left\{-2G_{4\phi X}\nabla_\alpha\nabla_\beta\phi\nabla^\alpha\phi\nabla^\beta\phi + G_{4XX}\nabla_\alpha\nabla_\lambda\phi\nabla_\beta\nabla^\lambda\phi\nabla^\alpha\phi\nabla^\beta\phi + \frac{1}{2}G_{4X}\left[(\Box\phi)^2 - (\nabla_\alpha\nabla_\beta\phi)^2\right]\right\}$$
$$+2\left[G_{4X}R_{\lambda(\mu}\nabla_{\nu)}\phi\nabla^\lambda\phi - \nabla_{(\mu}G_{4X}\nabla_{\nu)}\phi\Box\phi\right] - g_{\mu\nu}\left[G_{4X}R^{\alpha\beta}\nabla_\alpha\phi\nabla_\beta\phi - \nabla_\lambda G_{4X}\nabla^\lambda\phi\Box\phi\right]$$
$$+G_{4X}R_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi - G_{4\phi}\nabla_\mu\nabla_\nu\phi - G_{4\phi\phi}\nabla_\mu\phi\nabla_\nu\phi + 2G_{4\phi X}\nabla^\lambda\phi\nabla_\lambda\nabla_{(\mu}\phi\nabla_{\nu)}\phi$$
$$-G_{4XX}\nabla^\alpha\phi\nabla_\alpha\nabla_\mu\phi\nabla^\beta\phi\nabla_\beta\nabla_\nu\phi$$
$$+G_{5X}R_{\alpha\beta}\nabla^\alpha\phi\nabla^\beta\nabla_{(\mu}\phi\nabla_{\nu)}\phi - G_{5X}R_{\alpha(\mu}\nabla_{\nu)}\phi\nabla^\alpha\phi\Box\phi - \frac{1}{2}G_{5X}R_{\alpha\beta}\nabla^\alpha\phi\nabla^\beta\phi\nabla_\mu\nabla_\nu\phi$$
$$-\frac{1}{2}G_{5X}R_{\mu\alpha\nu\beta}\nabla^\alpha\phi\nabla^\beta\phi\Box\phi + G_{5X}R_{\alpha\lambda\beta(\mu}\nabla_{\nu)}\phi\nabla^\lambda\phi\nabla^\alpha\nabla^\beta\phi + G_{5X}R_{\alpha\lambda\beta(\mu}\nabla_{\nu)}\nabla^\lambda\phi\nabla^\alpha\phi\nabla^\beta\phi$$
$$-\frac{1}{2}\nabla_{(\mu}\left[G_{5X}\nabla^\alpha\phi\right]\nabla_\alpha\nabla_{\nu)}\phi\Box\phi + \frac{1}{2}\nabla_{(\mu}\left[G_{5\phi}\nabla_{\nu)}\phi\right]\Box\phi - \nabla_\lambda\left[G_{5\phi}\nabla_{(\mu}\phi\right]\nabla_{\nu)}\nabla^\lambda\phi$$
$$+\frac{1}{2}\left[\nabla_\lambda\left(G_{5\phi}\nabla^\lambda\phi\right) - \nabla_\alpha\left(G_{5X}\nabla_\beta\phi\right)\nabla^\alpha\nabla^\beta\phi\right]\nabla_\mu\nabla_\nu\phi + \nabla^\alpha G_5\nabla^\beta\phi R_{\alpha(\mu\nu)\beta} - \nabla_{(\mu}G_5G_{\nu)\lambda}\nabla^\lambda\phi$$
$$+\frac{1}{2}\nabla_{(\mu}G_{5X}\nabla_{\nu)}\phi\left[(\Box\phi)^2 - (\nabla_\alpha\nabla_\beta\phi)^2\right] - \nabla^\lambda G_5R_{\lambda(\mu}\nabla_{\nu)}\phi + \nabla_\alpha\left[G_{5X}\nabla_\beta\phi\right]\nabla^\alpha\nabla_{(\mu}\phi\nabla^\beta\nabla_{\nu)}\phi$$
$$-\nabla_\beta G_{5X}\left[\Box\phi\nabla^\beta\nabla_{(\mu}\phi - \nabla^\alpha\nabla^\beta\phi\nabla_\alpha\nabla_{(\mu}\phi\right]\nabla_{\nu)}\phi + \frac{1}{2}\nabla^\alpha\phi\nabla_\alpha G_{5X}\left[\Box\phi\nabla_\mu\nabla_\nu\phi - \nabla_\beta\nabla_\mu\phi\nabla^\beta\nabla_\nu\right.$$
$$-\frac{1}{2}G_{5X}G_{\alpha\beta}\nabla^\alpha\nabla^\beta\phi\nabla_\mu\phi\nabla_\nu\phi - \frac{1}{2}G_{5X}\Box\phi\nabla_\alpha\nabla_\mu\phi\nabla^\alpha\nabla_\nu\phi + \frac{1}{2}G_{5X}(\Box\phi)^2\nabla_\mu\nabla_\nu\phi$$
$$+\frac{1}{12}G_{5XX}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\alpha\nabla_\beta\phi)^2 + 2(\nabla_\alpha\nabla_\beta\phi)^3\right]\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2}\nabla_\lambda G_5G_{\mu\nu}\nabla^\lambda\phi$$
$$+g_{\mu\nu}\left\{-\frac{1}{6}G_{5X}\left[(\Box\phi)^3 - 3\Box\phi(\nabla_\alpha\nabla_\beta\phi)^2 + 2(\nabla_\alpha\nabla_\beta\phi)^3\right] + \nabla_\alpha G_5R^{\alpha\beta}\nabla_\beta\phi\right.$$
$$-\frac{1}{2}\nabla_\alpha\left(G_{5\phi}\nabla^\alpha\phi\right)\Box\phi + \frac{1}{2}\nabla_\alpha\left(G_{5\phi}\nabla_\beta\phi\right)\nabla^\alpha\nabla^\beta\phi - \frac{1}{2}\nabla_\alpha G_{5X}\nabla^\alpha X\Box\phi + \frac{1}{2}\nabla_\alpha G_{5X}\nabla_\beta X\nabla^\alpha\nabla^\beta\phi$$

Deffayet, Gao, Steer, Zahariade (2011)

Original derivation by Deffayet *et al.*

Start with flat space; assume

(1) $\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi)$

(2) $\mathcal{L}$ is polynomial in $\partial^2\phi$

and then covariantize

Not completely general...

Strong assumptions

Lagrangians that vanish in flat space seem missing (?)

$$\xi(\phi)\left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2\right)$$

Proof **in arbitrary dimensions**

— *D* Lagrangians in *D* dimensions

However, at least **in 4 dimensions** their result turns out to be **the most general**!

Back in 70's...

Horndeski (Lovelock's student!) determined **the most general** scalar-tensor Lagrangian of the form

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi, \partial^3\phi, \cdots ; g_{\mu\nu}, \partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \cdots)$$

that has **second-order** field equations both for ϕ and $g_{\mu\nu}$ **in 4D**

—— *The generalized Galileon was already discovered in 70's ?!*

International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363–384

Revisited recently by Charmousis *et al.* (2011)

Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

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Received: 10 *July* 1973

Not listed in SPIRES → Google

Abstract

Lagrange scalar densities which are concomitants of a pseudo–Riemannian metric-tensor, a scalar field and their derivatives of arbitrary order are considered. The most general second-order Euler–Lagrange tensors derivable from such a Lagrangian in a four-dimensional space are constructed, and it is shown that these Euler–Lagrange tensors may be obtained from a Lagrangian which is at most of second order in the derivatives of the field functions.

From Horndeski to the GG

$$\mathcal{L}_H = \delta^{\alpha\beta\gamma}_{\mu\nu\sigma}\Big[\kappa_1\nabla^\mu\nabla_\alpha\phi R_{\beta\gamma}{}^{\nu\sigma} + \frac{2}{3}\kappa_{1X}\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi$$
$$+\kappa_3\nabla_\alpha\phi\nabla^\mu\phi R_{\beta\gamma}{}^{\nu\sigma} + 2\kappa_{3X}\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\nabla^\sigma\nabla_\gamma\phi\Big]$$
$$+\delta^{\alpha\beta}_{\mu\nu}\Big[(F+2W)R_{\alpha\beta}{}^{\mu\nu} + 2F_X\nabla^\mu\nabla_\alpha\phi\nabla^\nu\nabla_\beta\phi + 2\kappa_8\nabla_\alpha\phi\nabla^\mu\phi\nabla^\nu\nabla_\beta\phi\Big]$$
$$-6(F_\phi + 2W_\phi - X\kappa_8)\Box\phi + \kappa_9$$

$$\partial_X F(\phi, X) = 2(\kappa_3 + 2X\kappa_{3X} - \kappa_{1\phi})$$

4 arbitrary functions of ϕ and X

$W(\phi)$: absorbed into redefinition of F

The two theories are equivalent!

$$K = \kappa_9 + 4X\int^X \mathrm{d}X'\left(\kappa_{8\phi} - 2\kappa_{3\phi\phi}\right),$$
$$G_3 = 6F_\phi - 2X\kappa_8 - 8X\kappa_{3\phi} + 2\int^X \mathrm{d}X'(\kappa_8 - 2\kappa_{3\phi}),$$
$$G_4 = 2F - 4X\kappa_3,$$
$$G_5 = -4\kappa_1$$

TK, Yamaguchi, Yokoyama 1105.5723

Particular cases

- $\mathcal{L}_2 = K(\phi, X)$ All the **k-inflation** models

Armendariz-Picon, Damour, Mukhanov (1999)

Example: **DBI inflation**

$$\mathcal{L} = -f(\phi)\sqrt{1 + f^{-1}(\partial\phi)^2} + f - V(\phi)$$

Silverstein & Tong (2004)

- $$\mathcal{L}_4 \quad = \quad G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$G_4 = \dfrac{M_{\rm Pl}^2}{2} \longrightarrow \mathcal{L}_4 = \dfrac{M_{\rm Pl}^2}{2}R$ Einstein-Hilbert

$G_4 = f(\phi) \longrightarrow \mathcal{L}_4 = f(\phi)R$ Familiar non-minimal coupling

Particular cases contd.

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X} \left[\cdots \right]$$

$$G_5 = -\phi \quad \longrightarrow \quad \mathcal{L}_5 = G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

Integration by parts

Sometimes used in inflation and dark energy models

e.g., Germani, Kehagias (2010)

$$\begin{aligned} K &= 8\xi^{(4)} X^2 (3 - \ln X), \\ G_3 &= 4\xi^{(3)} X (7 - 3 \ln X), \\ G_4 &= 4\xi^{(2)} X (2 - \ln X), \\ G_5 &= -4\xi^{(1)} \ln X \end{aligned}$$

Even non-minimal coupling to the Gauss-Bonnet term can be reproduced

$$\longrightarrow \quad \xi(\phi) \left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2 \right)$$

Higher-order gravity theories

- $\mathcal{L} = f(R) \longrightarrow \mathcal{L} = f(\phi) + f_\phi (R - \phi)$

f(R) models can be transformed to a scalar-tensor theory

Example: R^2 inflation $\quad R + \dfrac{R^2}{6M^2}$ Starobinsky (1980)

- $\mathcal{L} = \dfrac{R}{2} + f(\mathcal{G}) \longrightarrow \mathcal{L} = \dfrac{R}{2} + f(\phi) + f_\phi (\mathcal{G} - \phi)$

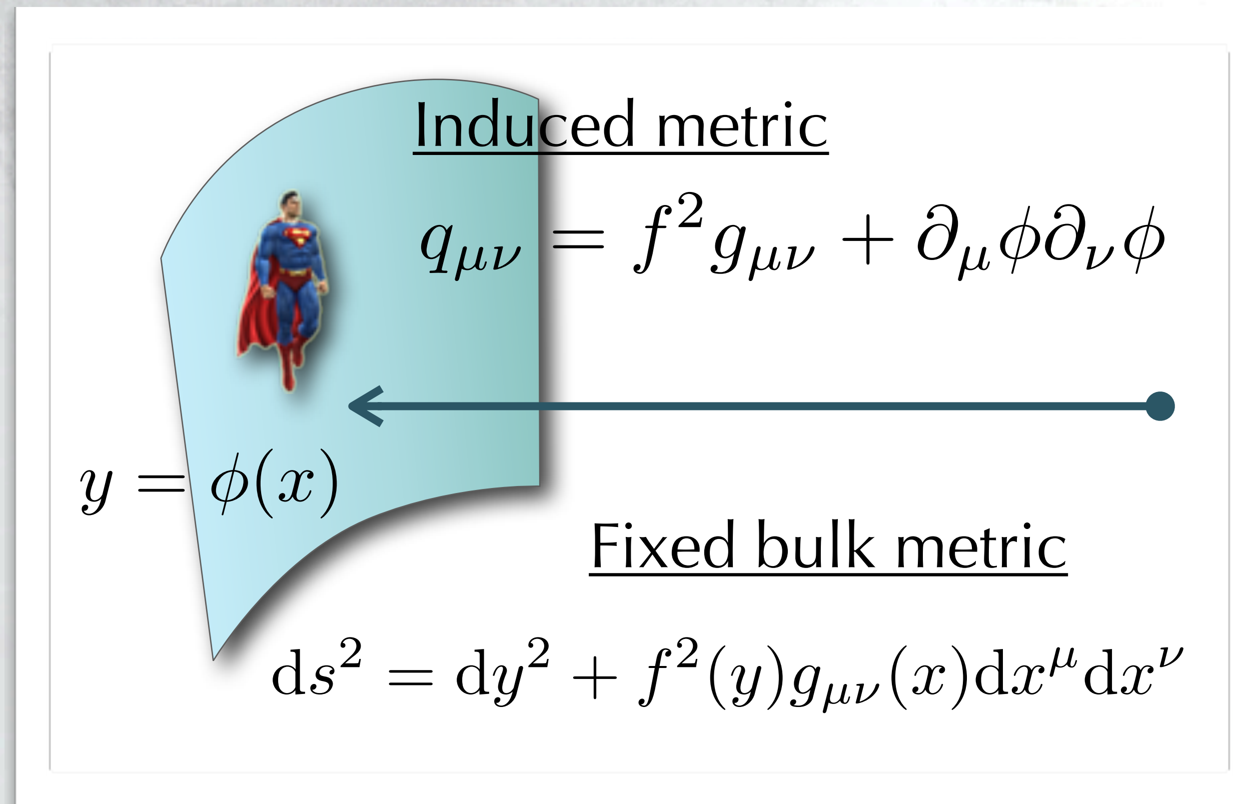
$\mathcal{G} := R^2 - 4R^2_{\mu\nu} + R^2_{\mu\nu\rho\sigma}$

This can also be transformed to a single-scalar theory, with non-minimal coupling to the Gauss-Bonnet term

de Rham, Tolley (2010)
Goon, Hinterbichler, Trodden (2011)

Galileon from higher dimensions

Particular cases of the generalized Galileon can be derived from
a probe brane embedded in a 5D bulk *a la* DBI



Probe brane action

$$S = -\lambda \int \mathrm{d}^4x \sqrt{-q}$$

DBI = particular case of $K(\phi, X)$

$$S = -\lambda \int \mathrm{d}^4x \sqrt{-g}\, f^4 \sqrt{1 + f^{-2}(\partial\phi)^2}$$

Generalize this to $S = \int \mathrm{d}^4x \sqrt{-q} F(q_{\mu\nu}, R_{\mu\nu\sigma\lambda}, K_{\mu\nu}, \nabla_\mu)$

Probe brane Lagrangian that gives **second-order** equations of motion

$$\mathcal{L} = \sqrt{-q}\left(-\lambda - M_5^3 K + \frac{M_4^2}{2} R[q] - \beta \mathcal{K}_{\rm GB}\right)$$

Gibbons-Hawking

$\sim \mathcal{L}_3$

Induced gravity

$\sim \mathcal{L}_4$

Boundary term from bulk Gauss-Bonnet

$\sim \mathcal{L}_5$

$$\mathcal{K}_{\rm GB} := -\frac{2}{3} K_{\mu\nu}^3 + K K_{\mu\nu}^2 - \frac{1}{3} K^3 - 2 G_{\mu\nu} K^{\mu\nu}$$

Brane is 4D: no higher induced Lovelock terms;
Bulk is 5D: no higher boundary terms

$$\gamma := \frac{1}{\sqrt{1+f^{-2}(\partial\phi)^2}}$$

Integration by parts

$$\sqrt{-q}K = \sqrt{-g}\left[-4ff'X + \cdots + \left(f^2 \ln\gamma\right)\Box\phi\right]$$

$$\subset K(\phi, X) \qquad \subset G_3(\phi, X)\Box\phi$$

$$\sqrt{-q}R[q] = \sqrt{-g}\left\{\cdots + \frac{f^2}{\gamma}R[g] - \gamma\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]\right\}$$

$$\subset K, G_3\Box\phi \qquad \subset G_4R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\phi\nabla_\nu\phi)^2\right]$$

$$\sqrt{-q}\mathcal{K}_{\rm GB} = \cdots$$

Minkowski bulk $f = 1,\ g_{\mu\nu} = \eta_{\mu\nu}$

Non-relativistic limit $X \ll 1$

→ Flat space Galileon

> Shuntaro's talk

Summary of Part 1

The **most general** scalar-tensor theory with **second-order** field equations is given by the **generalized Galileons** (in 4D)

$$\mathcal{L}_2 = K(\phi, X)$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2\right]$$

$$\mathcal{L}_5 = G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X}\Big[(\Box\phi)^3 -3\,(\Box\phi)\,(\nabla_\mu\nabla_\nu\phi)^2 + 2\,(\nabla_\mu\nabla_\nu\phi)^3\Big]$$

2

G-inflation

– Inflation driven by the Galileon field –

Now we have a framework to deal with

the most general single-field inflation model

Why don’t we study inflation driven by the generalized Galileon, (generalized) G-inflation?

References

“G-inflation: Inflation driven by the Galileon field”
TK, Masahide Yamaguchi, Jun'ichi Yokoyama
Phys. Rev. Lett. 105 231302 (2010) arXiv:1008.0603

“Higgs G-inflation”
Kohei Kamada, TK, Masahide Yamaguchi, Jun'ichi Yokoyama
Phys. Rev. D83 083515 (2011) arXiv:1012.4238

“Primordial non-Gaussianity from G-inflation”
TK, Masahide Yamaguchi, Jun'ichi Yokoyama
Phys. Rev. D83 103524 (2011) arXiv:1103.1740

“Generalized G-inflation: Inflation with the most general second-order field equations”
TK, Masahide Yamaguchi, Jun'ichi Yokoyama
PTP accepted, arXiv:1105.5723

2-1

Background

$$\phi(t)$$

$$a(t)$$

Background equations

$$\phi = \phi(t) \qquad \mathrm{d}s^2 = -N^2(t)\mathrm{d}t^2 + a^2(t)\mathrm{d}\mathbf{x}^2$$

$$S = \sum_{i=2}^{5} \int \mathrm{d}^4x\sqrt{-g}\mathcal{L}_i$$

Corresponding standard equations

- $\delta N \longrightarrow \sum_{i=2}^{5} \mathcal{E}_i = 0$ ↔ $\rho - 3M_{\rm Pl}^2 H^2 = 0$
- $\delta a \longrightarrow \sum_{i=2}^{5} \mathcal{P}_i = 0$ ↔ $p + M_{\rm Pl}^2\left(2\dot{H} + 3H^2\right) = 0$
- $\delta\phi \longrightarrow \dot{J} + 3HJ = P_\phi$ ↔ $\ddot{\phi} + 3H\dot{\phi} = -V_\phi$

$G_4 = M_{\rm Pl}^2/2$ reproduces Einstein gravity

$$
\begin{aligned}
\mathcal{E}_2 &= 2XK_X - K, \\
\mathcal{E}_3 &= 6X\dot\phi H G_{3X} - 2XG_{3\phi}, \\
\mathcal{E}_4 &= -6H^2G_4 + 24H^2X(G_{4X} + XG_{4XX}) - 12HX\dot\phi G_{4\phi X} - 6H\dot\phi G_{4\phi}, \\
\mathcal{E}_5 &= 2H^3X\dot\phi\,(5G_{5X} + 2XG_{5XX}) - 6H^2X\,(3G_{5\phi} + 2XG_{5\phi X})
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{P}_2 &= K, \\
\mathcal{P}_3 &= -2X\left(G_{3\phi} + \ddot\phi G_{3X}\right), \\
\mathcal{P}_4 &= 2\left(3H^2 + 2\dot H\right)G_4 - 12H^2XG_{4X} - 4H\dot XG_{4X} - 8\dot HXG_{4X} \\
&\quad -8HX\dot XG_{4XX} + 2\left(\ddot\phi + 2H\dot\phi\right)G_{4\phi} + 4XG_{4\phi\phi} + 4X\left(\ddot\phi - 2H\dot\phi\right)G_{4\phi X}, \\
\mathcal{P}_5 &= -2X\left(2H^3\dot\phi + 2H\dot H\dot\phi + 3H^2\ddot\phi\right)G_{5X} - 4H^2X^2\ddot\phi G_{5XX} \\
&\quad +4HX\left(\dot X - HX\right)G_{5\phi X} + 2\left[2\,(HX)^{\cdot} + 3H^2X\right]G_{5\phi} + 4HX\dot\phi G_{5\phi\phi}
\end{aligned}
$$

“$-G_\mu^{\ \nu}$”

Our “ $T_\mu^{\ \nu}$ ” includes “ $-G_\mu^{\ \nu}$ ”

— Distinction between gravitational and scalar-field parts is ambiguous

For $K = X - V(\phi)$

$$J \to \dot\phi$$

$$
\begin{aligned}
J &= \dot\phi K_X + 6HXG_{3X} - 2\dot\phi G_{3\phi} \\
&\quad + 6H^2\dot\phi\left(G_{4X} + 2XG_{4XX}\right) - 12HXG_{4\phi X} \\
&\quad + 2H^3X\left(3G_{5X} + 2XG_{5XX}\right) \\
&\quad - 6H^2\dot\phi\left(G_{5\phi} + XG_{5\phi X}\right)
\end{aligned}
$$

$$
\begin{aligned}
P_\phi &= K_\phi - 2X\left(G_{3\phi\phi} + \ddot\phi G_{3\phi X}\right) \\
&\quad + 6\left(2H^2 + \dot H\right)G_{4\phi} + 6H\left(\dot X + 2HX\right)G_{4\phi X} \\
&\quad - 6H^2XG_{5\phi\phi} + 2H^3X\dot\phi G_{5\phi X}
\end{aligned}
$$

$$P_\phi \to -V_\phi$$

Structure of the equations

"Friedmann equation" (00 equation)

$$\underbrace{(\cdots)}_{\sim\ \mathcal{L}_2} + \underbrace{(\cdots)H}_{\sim\ \mathcal{L}_3} + \underbrace{(\cdots)H^2}_{\sim\ \mathcal{L}_4} + \underbrace{(\cdots)H^3}_{\sim\ \mathcal{L}_5} = 0$$

"Kinetic gravity braiding"
Deffayet *et al.* (2010)

ij and scalar-field equations

$$\begin{aligned} \dot{H} &= (\cdots)\ddot{\phi} + \cdots \\ \ddot{\phi} &= (\cdots)\dot{H} + \cdots \end{aligned}$$

Not diagonal in second derivatives

In general, this mixing cannot be undone through conformal transformation

Cf. Usual (k-)inflation

T_{ij} does not contain second derivatives of ϕ

Scalar-field EOM does not contain second derivatives of $g_{\mu\nu}$

Background examples

How do we get $H \sim$ const?

(1) H is supported by potential energy;
(Slow-roll) scalar-field dynamics is modified by the G terms.

(2) H is supported by kinetic energy;
Completely different from usual slow-roll dynamics (generalization of *k-inflation*).

1. Potential-driven model

Suppose the 4 functions can be expanded in terms of X:

$$\begin{aligned} K(\phi, X) &= -V(\phi) + \mathcal{K}(\phi)X + \cdots \\ G_i(\phi, X) &= g_i(\phi) + h_i(\phi)X + \cdots \end{aligned}$$

Consider slowly rolling ϕ

Slow-roll conditions:

$$\epsilon := -\frac{\dot{H}}{H^2} \ll 1,\ \ddot{\phi} \ll H\dot{\phi},\ \dot{J} \ll HJ,\ \dot{g}_i \ll Hg_i,\ \dot{h}_i \ll Hh_i$$

"Friedmann equation"

g_4 is ϕ-dependent "$1/16\pi G$"

$$\mathcal{E}_2 = 2XK_X - K,$$
$$\mathcal{E}_3 = 6X\dot{\phi}HG_{3X} - 2XG_{3\phi},$$
$$\mathcal{E}_4 = -6H^2G_4 + 24H^2X(G_{4X} + XG_{4XX}) - 12HX\dot{\phi}G_{4\phi X} - 6H\dot{\phi}G_{4\phi},$$
$$\mathcal{E}_5 = 2H^3X\dot{\phi}(5G_{5X} + 2XG_{5XX}) - 6H^2X(3G_{5\phi} + 2XG_{5\phi X})$$

$$V(\phi) - 6H^2 g_4(\phi) \simeq 0$$

Scalar-field EOM

$$3HJ \simeq -V_\phi + 12H^2 g_{4\phi}$$

with $J \simeq (\mathcal{K} - 2g_{3\phi})\,\dot{\phi} + 6\left[Hh_3X + H^2\dot{\phi}(h_4 - g_{5\phi}) + H^3h_5X\right]$

Friction term is modified

Application:

Enhance friction so that inflation proceeds even with a steep potential

Enhancing friction

Minimal example: $\mathcal{L} = \frac{M_{\rm Pl}^2}{2} R + X - V - h_3(\phi) X \Box \phi$

$$J \simeq \left(1 + 3Hh_3\dot{\phi}\right)\dot{\phi} \simeq -V_\phi$$

$$\epsilon = -\frac{\dot{H}}{H^2} = \left(1 + 3Hh_3\dot{\phi}\right)^{-1} \times \frac{M_{\rm Pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2$$

Suppresses the slope if $Hh_3\dot{\phi} \gg 1$

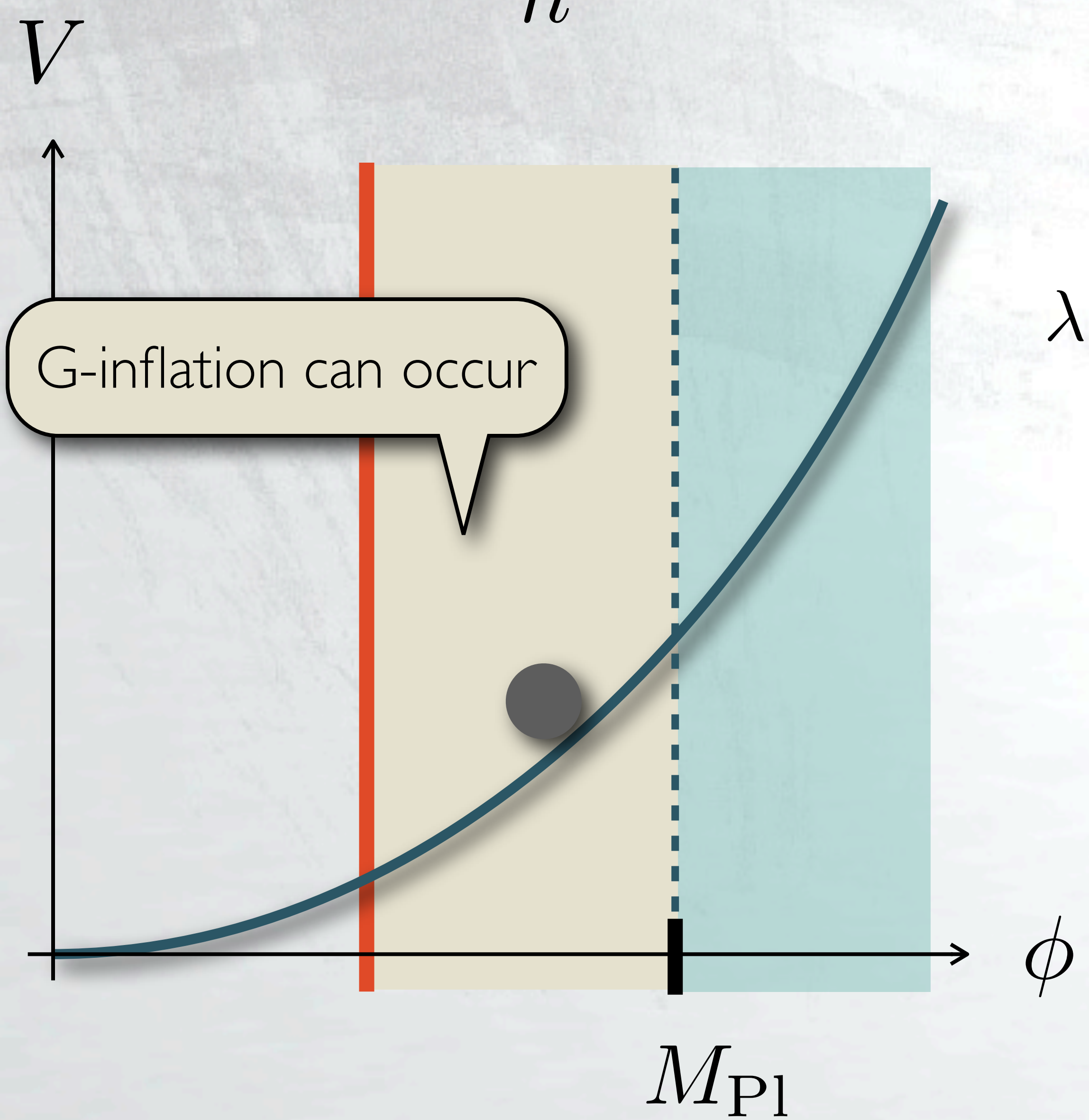
Standard expression in terms of potential

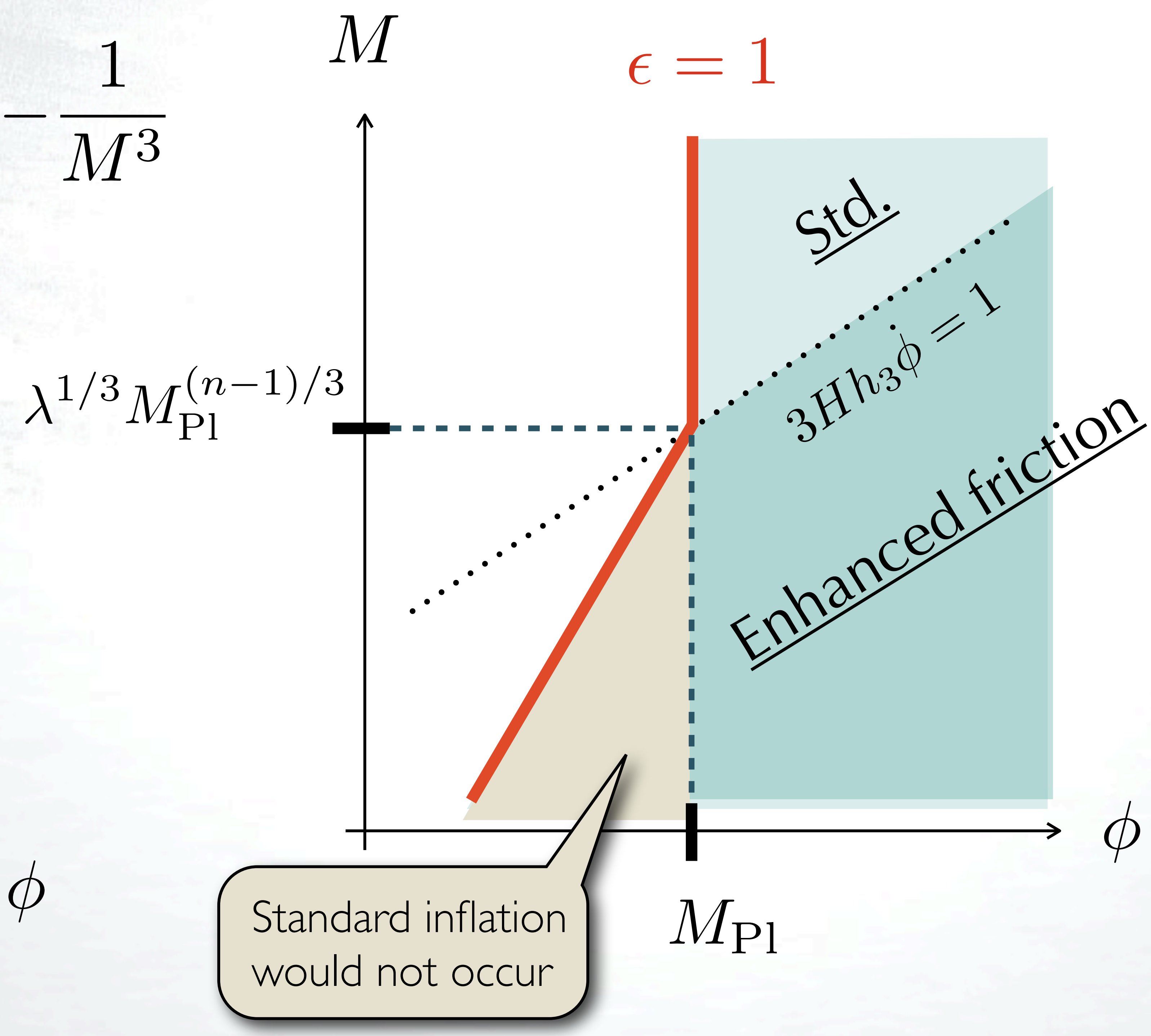
Even when $Hh_3\dot{\phi} \gg 1$
we still have $V \gg |h_3 X \Box \phi|$

Kamada, TK, Yamaguchi, Yokoyama (2011)

Chaotic G-inflation

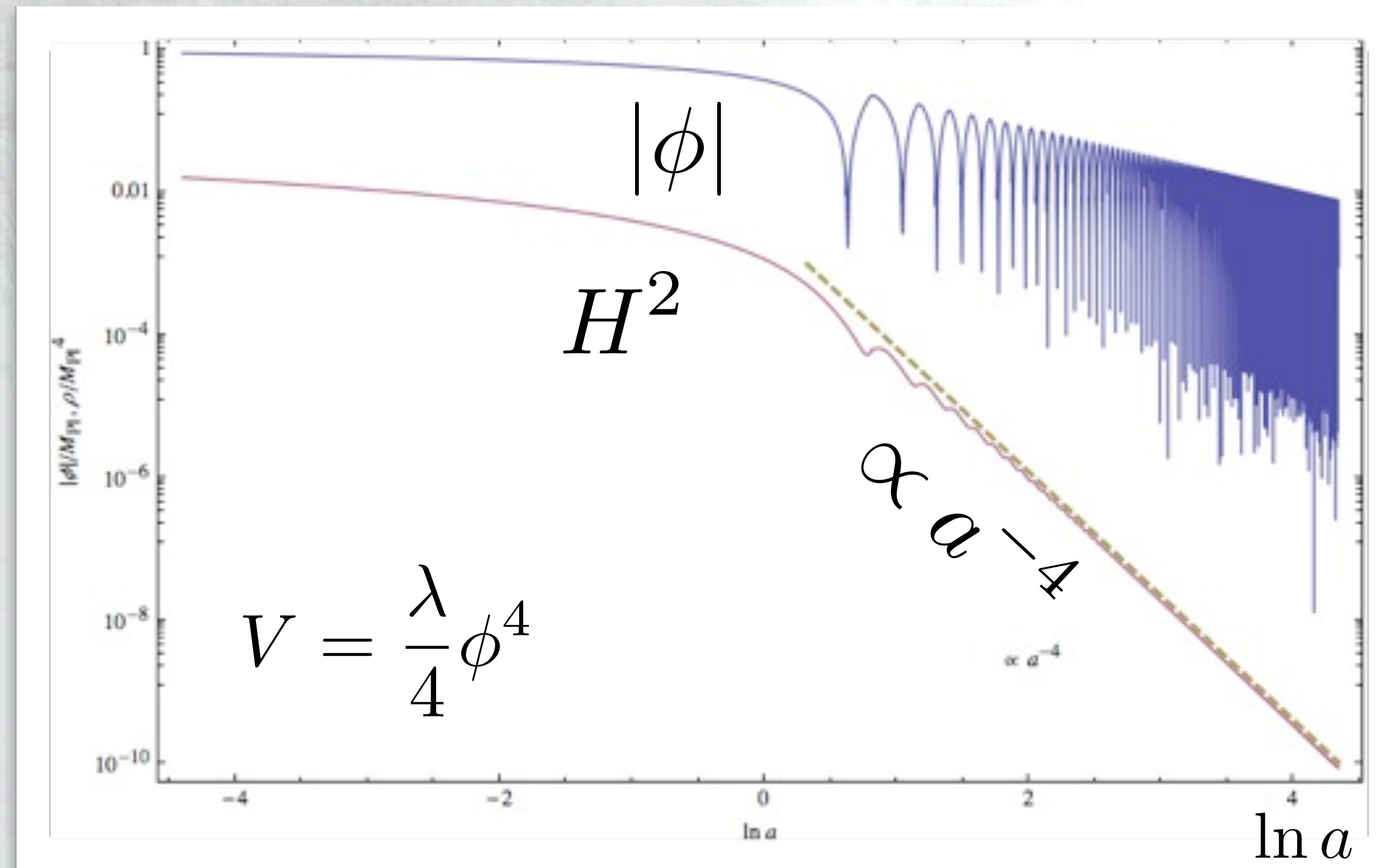
$$V = \frac{\lambda}{n}\phi^n \qquad h_3 = -\frac{1}{M^3}$$





Reheating

Non-trivial in general



But... arrange so that

$$K \simeq X - V(\phi)$$

$$G_i = g_i(\phi) + h_i(\phi)X + \cdots$$

$\simeq$ const

$\simeq 0$

around the minimum of V,

then reheating will proceed in an usual way

TK, Yamaguchi, Yokoyama (2010, 2011)

2. Kinetically driven model

Shift symmetry: $\phi \to \phi + c$ ⟶ $K = K(X),\ G_i = G_i(X)$

Scalar-field EOM: $\dot{J} + 3HJ = 0$ ⟶ $J(\dot\phi, H) \propto a^{-3} \to 0$

$$J = \dot\phi K_X + 6HXG_{3X} + 6H^2\dot\phi\left(G_{4X} + 2XG_{4XX}\right) + 2H^3X\left(3G_{5X} + 2XG_{5XX}\right)$$

de Sitter attractor

$$\dot\phi = \text{const.}$$
$$H = \text{const.}$$

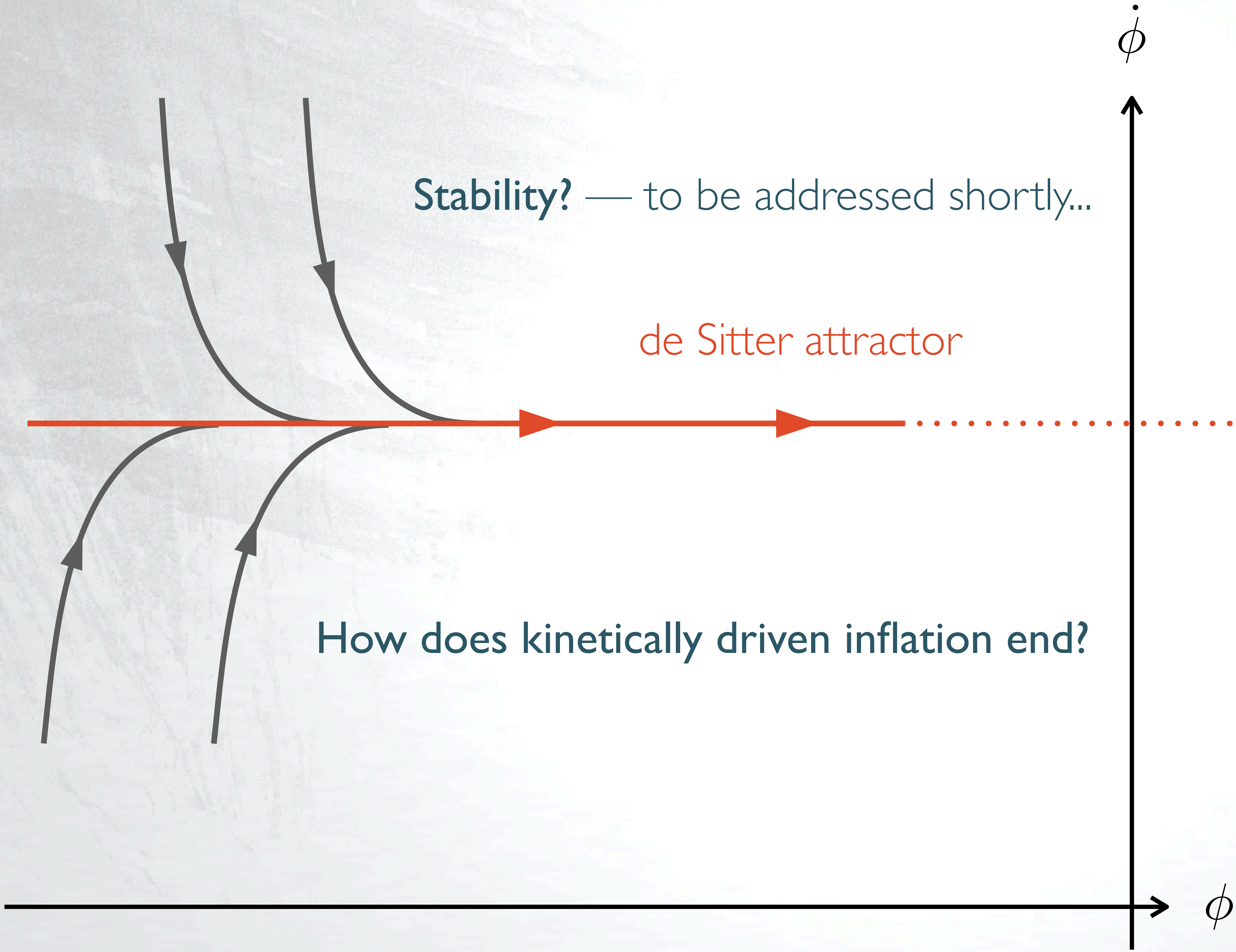
satisfying

$$J(\dot\phi, H) = 0$$
$$F(\dot\phi, H) = 0$$

"Friedmann equation"

$$\sum_{i=2}^{5}\mathcal{E}_i = 0 = \dot\phi J - \underbrace{\left[K + 6H^2(G_4 - 2XG_{4X}) - 4H^3X\dot\phi G_{5X}\right]}$$

$$=: F(\dot\phi, H) \to 0$$

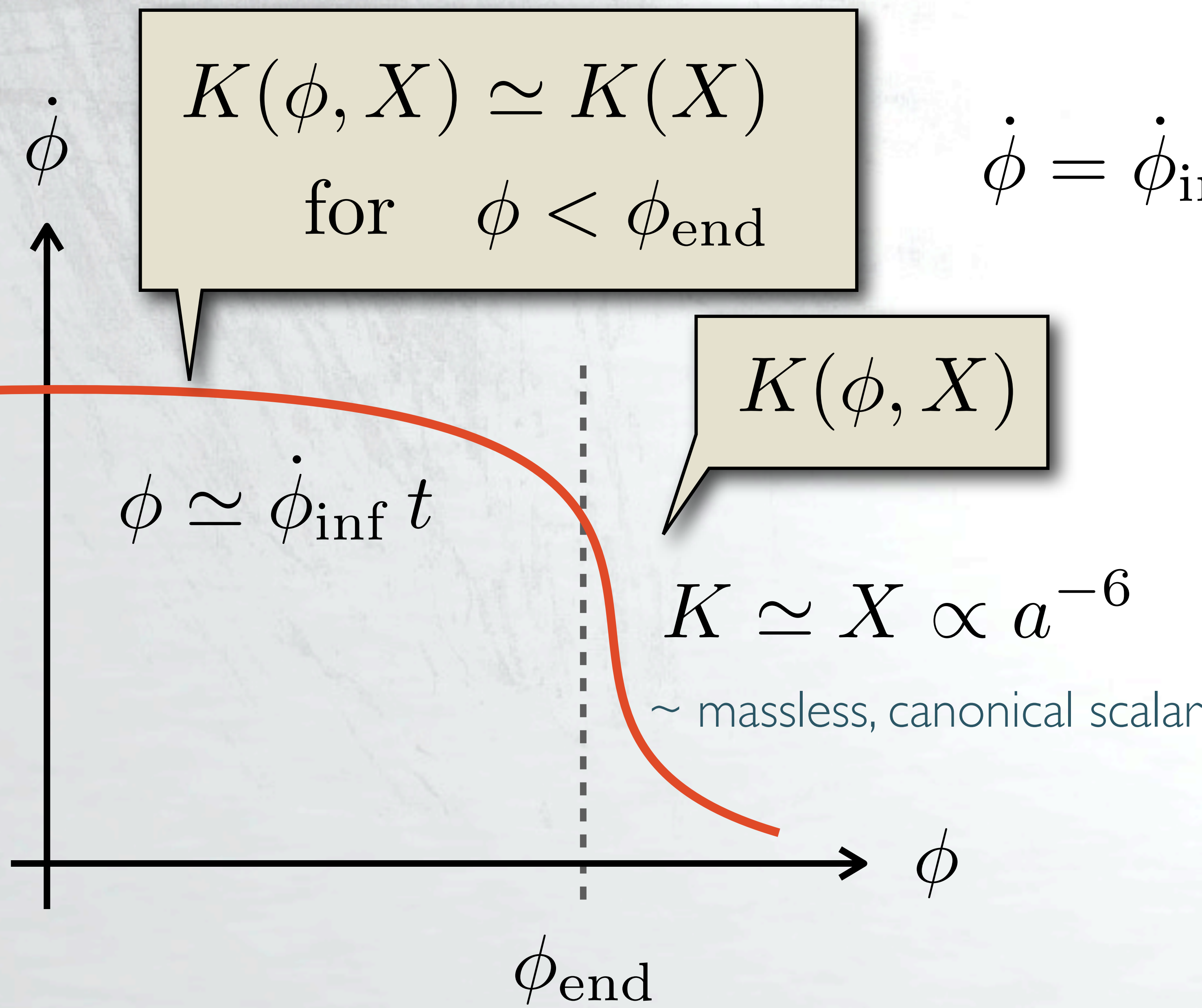

$\dot{\phi}$
Stability? — to be addressed shortly...
de Sitter attractor
How does kinetically driven inflation end?
ϕ

Exit from kinetically driven G-inflation

Shift symmetry $\phi \to \phi + c$ must be broken in order to end inflation

The situation is essentially the same as k-inflation

$\dot{\phi} = \dot{\phi}_{\rm inf}$ is no longer a solution for $\phi > \phi_{\rm end}$



Reheating through gravitational particle production   Ford (1987)

- GW spectrum is enhanced at high frequencies
- Everything in the world, including what you don't want, will be produced

2-2

ζ

h_{ij}

Perturbations

TK, Yamaguchi, Yokoyama (2011)

Tensor perturbation

$$g_{ij} = a^2 \left(\delta_{ij} + h_{ij}\right) \longrightarrow$$

Substitute this to the action and expand to second order

General quadratic action for tensor perturbations:

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla}h_{ij})^2\right]$$

$$\mathcal{F}_T := 2\left[G_4 - X\left(\ddot{\phi}G_{5X} + G_{5\phi}\right)\right]$$

$$\mathcal{G}_T := 2\left[G_4 - 2XG_{4X} - X\left(H\dot{\phi}G_{5X} - G_{5\phi}\right)\right]$$

$$S_T^{(2)} = \frac{1}{8}\int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2}(\vec{\nabla} h_{ij})^2\right]$$

Propagation speed: $c_T^2 := \mathcal{F}_T/\mathcal{G}_T$ → $c_T^2 \neq 1$ in general

Stability: $\mathcal{F}_T > 0$ – avoid gradient instabilities

$\mathcal{G}_T > 0$ – avoid ghost instabilities

Normalized mode: $z\, h_{ij} = \frac{\sqrt{\pi}}{2}\sqrt{-y} H_\nu^{(1)}(-ky)\mathrm{e}_{ij}$

$$z := \frac{a}{2}(\mathcal{F}_T \mathcal{G}_T)^{1/4} \qquad \mathrm{d}y := \frac{c_T}{a}\mathrm{d}t \qquad \nu := \frac{3 - \epsilon - 2s_T + f_T}{2(1 - \epsilon - s_T)}$$

Constant (slow) **variation parameters**

$$\epsilon = \mathrm{const}, \quad s_T := \frac{\dot{c}_T}{H c_T} = \mathrm{const}, \quad f_T := \frac{\dot{\mathcal{F}}_T}{H \mathcal{F}_T} = \mathrm{const}$$

Tensor power spectrum

- $$\mathcal{P}_T = 2^{2\nu} \left| \frac{\Gamma(\nu)}{\Gamma(3/2)} \right|^2 (1-\epsilon-s) \frac{\mathcal{G}_T^{1/2}}{\mathcal{F}_T^{3/2}} \frac{H^2}{4\pi^2} \Bigg|_{-ky=1}$$

evaluated at sound horizon crossing

- $$n_T = 3 - 2\nu$$

$n_T > 0$ ⟷ $2\epsilon + s_T + f_T < 0$

In principle, this is possible without causing instabilities both in scalar and tensor modes

TK, Yamaguchi, Yokoyama (2011)

Curvature perturbation

Unitary gauge: $\delta\phi(t,\mathbf{x})=0$

$$ds^2=-N^2dt^2+\gamma_{ij}\left(dx^i+N^idt\right)\left(dx^j+N^jdt\right)$$

$$N=1+\alpha,\quad N_i=\partial_i\beta,\quad \gamma_{ij}=a^2(t)e^{2\zeta}\delta_{ij}$$

$$\longrightarrow\quad S_S^{(2)}=\int dtd^3xa^3\Bigg[-3\mathcal{G}_T\dot{\zeta}^2+\frac{\mathcal{F}_T}{a^2}(\vec{\nabla}\zeta)^2+\Sigma\alpha^2$$

defined in the next slide...

$$-2\Theta\alpha\frac{\vec{\nabla}^2}{a^2}\beta+2\mathcal{G}_T\dot{\zeta}\frac{\vec{\nabla}^2}{a^2}\beta+6\Theta\alpha\dot{\zeta}-2\mathcal{G}_T\alpha\frac{\vec{\nabla}^2}{a^2}\zeta\Bigg]$$

Get quadratic action for ζ

✔ $\delta\alpha$ → $\Sigma\alpha-\Theta\frac{\vec{\nabla}^2}{a^2}\beta+3\Theta\dot{\zeta}-\mathcal{G}_T\frac{\vec{\nabla}^2}{a^2}\zeta=0$

Hamiltonian constraint

✔ $\delta\beta$ → $\Theta\alpha-\mathcal{G}_T\dot{\zeta}=0$

Momentum constraint

$$
\begin{aligned}
\Sigma \quad := \quad & XK_X + 2X^2K_{XX} + 12H\dot{\phi}XG_{3X} \\
& +6H\dot{\phi}X^2G_{3XX} - 2XG_{3\phi} - 2X^2G_{3\phi X} - 6H^2G_4 \\
& +6\Big[H^2\left(7XG_{4X} + 16X^2G_{4XX} + 4X^3G_{4XXX}\right) \\
& -H\dot{\phi}\left(G_{4\phi} + 5XG_{4\phi X} + 2X^2G_{4\phi XX}\right)\Big] \\
& +30H^3\dot{\phi}XG_{5X} + 26H^3\dot{\phi}X^2G_{5XX} \\
& +4H^3\dot{\phi}X^3G_{5XXX} - 6H^2X\big(6G_{5\phi} \\
& +9XG_{5\phi X} + 2X^2G_{5\phi XX}\big)
\end{aligned}
$$

$$
\begin{aligned}
\Theta \quad := \quad & -\dot{\phi}XG_{3X} + 2HG_4 - 8HXG_{4X} \\
& -8HX^2G_{4XX} + \dot{\phi}G_{4\phi} + 2X\dot{\phi}G_{4\phi X} \\
& -H^2\dot{\phi}\left(5XG_{5X} + 2X^2G_{5XX}\right) \\
& +2HX\left(3G_{5\phi} + 2XG_{5\phi X}\right)
\end{aligned}
$$

Compact expressions

$$\Sigma = X \sum_{i=2}^{5} \frac{\partial \mathcal{E}_i}{\partial X} + \frac{1}{2} H \sum_{i=2}^{5} \frac{\partial \mathcal{E}_i}{\partial H}$$

$$\Theta = -\frac{1}{6} \sum_{i=2}^{5} \frac{\partial \mathcal{E}_i}{\partial H}$$

General quadratic action for curvature perturbation:

$$S_S^{(2)} = \int \mathrm{d}t\mathrm{d}^3x\, a^3 \left[\mathcal{G}_S\dot{\zeta}^2 - \frac{\mathcal{F}_S}{a^2}(\vec{\nabla}\zeta)^2\right]$$

$$\mathcal{F}_S := \frac{1}{a}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a}{\Theta}\mathcal{G}_T^2\right) - \mathcal{F}_T$$

$$\mathcal{G}_S := \frac{\Sigma}{\Theta^2}\mathcal{G}_T^2 + 3\mathcal{G}_T$$

Sound speed: $c_s^2 = \mathcal{F}_S/\mathcal{G}_S$

Stability: $\mathcal{F}_S > 0, \quad \mathcal{G}_S > 0$

k-inflation

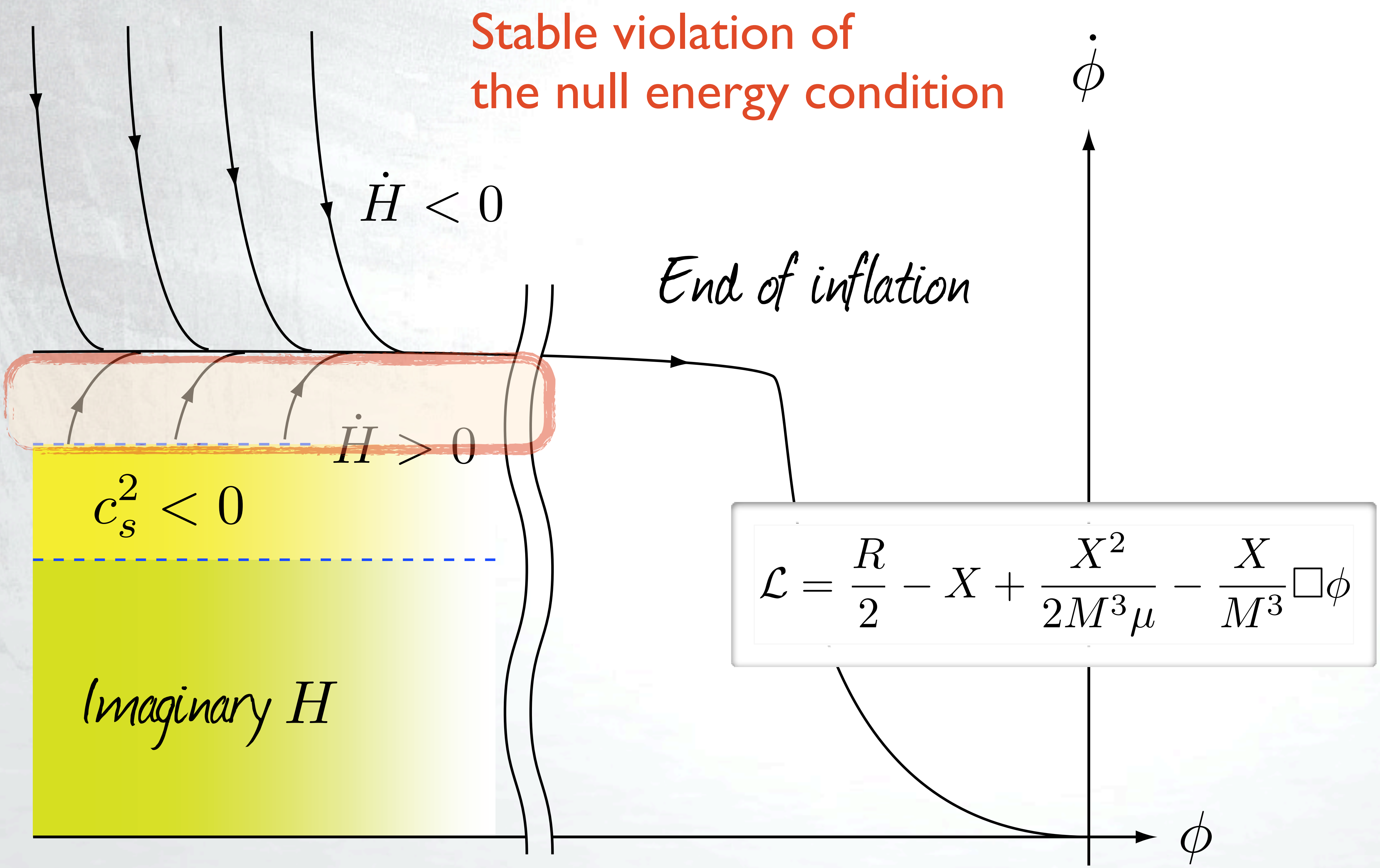
$\dot{H} > 0 \longleftrightarrow$ unstable

$$\mathcal{L} = \frac{M_{\mathrm{Pl}}^2}{2}R + K(\phi, X) \longrightarrow \mathcal{F}_S = M_{\mathrm{Pl}}^2\epsilon$$

Garriga, Mukhanov (1999)

In more general cases, the sign of $\dot{H}$ and the stability criteria are not correlated

Phase space of kinetically driven G-inflation



Normalized mode: $$z\,\zeta = \frac{\sqrt{\pi}}{2}\sqrt{-y}H_{\nu}^{(1)}(-ky)$$

$$z := \sqrt{2}a\left(\mathcal{F}_S\mathcal{G}_S\right)^{1/4}$$

$$\mathrm{d}y := \frac{c_s}{a}\mathrm{d}t$$

$$\nu := \frac{3-\epsilon-2s+f_S}{2(1-\epsilon-s)}$$

Constant (slow) variation parameters

$$\epsilon = \text{const}, \quad s := \frac{\dot{c}_s}{Hc_s} = \text{const}, \quad f_S := \frac{\dot{\mathcal{F}}_S}{H\mathcal{F}_S} = \text{const}$$

Power spectrum

- $$\mathcal{P}_\zeta = 2^{2\nu-4}\left|\frac{\Gamma(\nu)}{\Gamma(3/2)}\right|^2 (1-\epsilon-s)\,\frac{\mathcal{G}_S^{1/2}}{\mathcal{F}_S^{3/2}}\frac{H^2}{4\pi^2}\bigg|_{-ky=1}$$
- $$n_s - 1 = 3 - 2\nu$$

Approximately scale-invariant if $\nu \simeq \frac{3}{2}$

Consistency relation

$$r = 16 \frac{\mathcal{F}_S}{\mathcal{F}_T} \frac{c_s}{c_T}$$

($r = \mathcal{P}_T / \mathcal{P}_\zeta$, Slow-variation parameters << 1)

Canonical inflation $r = 16\epsilon = -8 n_T$

k-inflation $r = 16\epsilon c_s = -8 n_T c_s$

Consistency relation in **potential-driven** G^2

$$K(\phi, X) = -V(\phi) + \mathcal{K}(\phi)X + \cdots$$
$$G_i(\phi, X) = g_i(\phi) + h_i(\phi)X + \cdots$$

$$\mathcal{F}_S \simeq \frac{X}{H^2}\left(\mathcal{K} + 6H^2 h_4\right) + \frac{4\dot{\phi}X}{H}\left(h_3 + H^2 h_5\right)$$

$$\rightarrow \quad \mathcal{G}_S \simeq \frac{X}{H^2}\left(\mathcal{K} + 6H^2 h_4\right) + \frac{6\dot{\phi}X}{H}\left(h_3 + H^2 h_5\right)$$

$$\mathcal{F}_T \simeq \mathcal{G}_T \simeq 2g_4 \quad \rightarrow \quad c_T^2 \simeq 1$$

Usual consistency relation

$$c_s^2 \simeq 1$$
$$r \simeq -8n_T$$

New consistency relation

$$c_s^2 \simeq \frac{2}{3}$$
$$r \simeq -\frac{32\sqrt{6}}{9} n_T$$

2-3

Non-Gaussianity

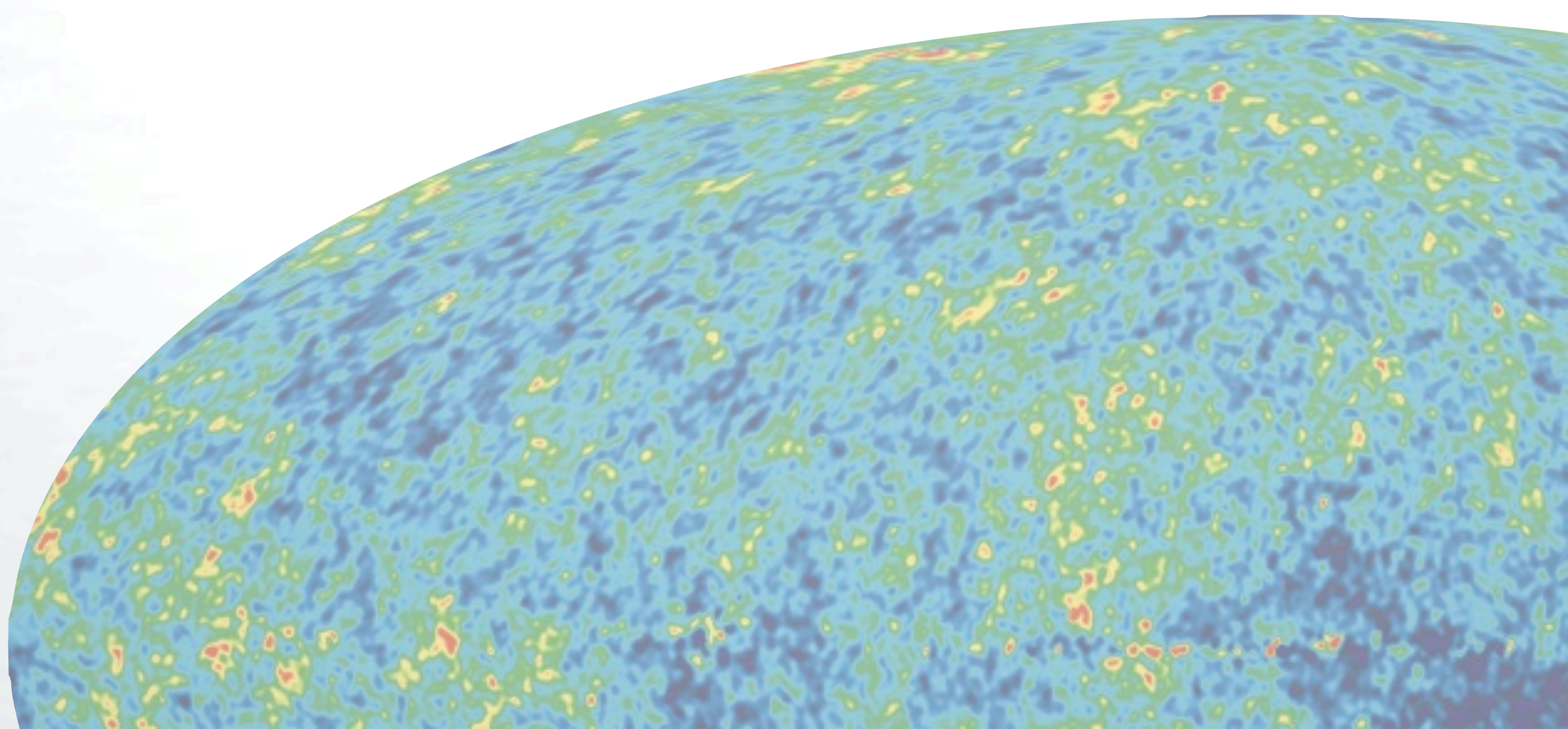
f_{NL}

Let's focus on the minimal example

$$\mathcal{L} = \frac{R}{2} + K(\phi, X) - G(\phi, X)\Box\phi,$$

and evaluate $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle$ $M_{\rm Pl} = 1$

TK, Yamaguchi, Yokoyama (2011)



Key quantities

Recall the quadratic action ——

$$S = \int \mathrm{d}t\mathrm{d}^3x\, a^3\, \sigma \left[\frac{1}{c_s^2}\dot{\zeta}^2 - \frac{1}{a^2}(\partial\zeta)^2\right]$$

$$\sigma := \mathcal{F}_S$$

$$c_s^2 := \frac{\mathcal{F}_S}{\mathcal{G}_S}$$ Sound speed

reduces to $\epsilon \ (= -\dot{H}/H^2)$

in standard (k-)inflation (G=0)

$\sigma \neq \epsilon$, not necessarily slow-roll

suppressed in (kinetically driven) G^2

Tensor-to-scalar ratio

$$r = 16\sigma c_s$$

Cubic action

$$S_3 = \int dt d^3x\, a^3 \left[\frac{\mathcal{C}_1}{H}\dot\zeta^3 + \mathcal{C}_2 \zeta\dot\zeta^2 + \frac{\mathcal{C}_3}{a^4H^2}\partial^2\zeta(\partial\zeta)^2 + \frac{\mathcal{C}_4}{a^2H^2}\dot\zeta^2\partial^2\zeta + \mathcal{C}_5 H\zeta^2\dot\zeta \right.$$

$$\left. + \frac{\mathcal{C}_6}{a^4H}\partial^2\zeta(\partial\zeta\cdot\partial\chi) + \frac{\mathcal{C}_7}{a^4}\partial^2\zeta(\partial\chi)^2 + \frac{\mathcal{C}_8}{a^2}\zeta(\partial\zeta)^2 + \frac{\mathcal{C}_9}{a^2}\dot\zeta(\partial\zeta\cdot\partial\chi) + \frac{2}{a^3}f(\zeta)\,\frac{\delta L}{\delta\zeta}\Big|_1 \right]$$

$$\chi := \frac{a^2\,\sigma}{c_s^2}H^{-2}\dot\zeta$$

at most four derivatives

$$\mathcal{C}_1 = -\frac{\sigma}{\Theta c_s^2}\left(1+2\frac{\mathcal{I}}{\mathcal{G}}\right) - 2\dot\phi X\left(G_X + XG_{XX}\right)\frac{H\sigma}{c_s^2\Theta^2} + \frac{H^2\sigma}{c_s^4\Theta^2},$$

$$\mathcal{C}_2 = \frac{\sigma}{c_s^2}\left[3 - \frac{H^2}{c_s^2\Theta^2}\left(3+\epsilon+\frac{2\dot\Theta}{H\Theta}\right)\right],$$

$$\mathcal{C}_3 = -\frac{H^2\dot\phi XG_X}{\Theta^3},$$

$$\mathcal{C}_4 = \frac{2H^2\dot\phi X\left(G_X+XG_{XX}\right)}{\Theta^3},$$

$$\mathcal{C}_5 = \frac{\sigma}{2c_s H}\frac{d}{dt}\left(\frac{H^2\delta}{c_s^2\Theta^2}\right),$$

$$\mathcal{C}_6 = \frac{2H\dot\phi XG_X}{\Theta^2},$$

$$\mathcal{C}_7 = \frac{\sigma}{4} - \frac{\dot\phi XG_X}{\Theta},$$

$$\mathcal{C}_8 = -\sigma\frac{H}{\Theta^2}\frac{\sigma}{c_s^2}\left(1-\epsilon-2s-\frac{2\dot\Theta}{H\Theta}\right),$$

$$\mathcal{C}_9 = \frac{\sigma}{c_s^2}\left(-\frac{2H}{\Theta} + \frac{\sigma}{2}\right),$$

$$\mathcal{I} := XK_{XX} + \frac{2X^2}{3}K_{XXX} + H\dot\phi G_X + 6X^2G_X^2 \ldots H\dot\phi XG_{XX} + 6X^3G_XG_{XX} + 2H\dot\phi X^2G_{XXX} \ldots \frac{X}{3}\left(2G_{\phi X} + XG_{\phi XX}\right)$$

σ instead of ϵ

No new shapes beyond k-in...

Creminelli *et al.* (2011); Reneux-Petel (2011)

Evaluating Non-Gaussianity

in-in formalism

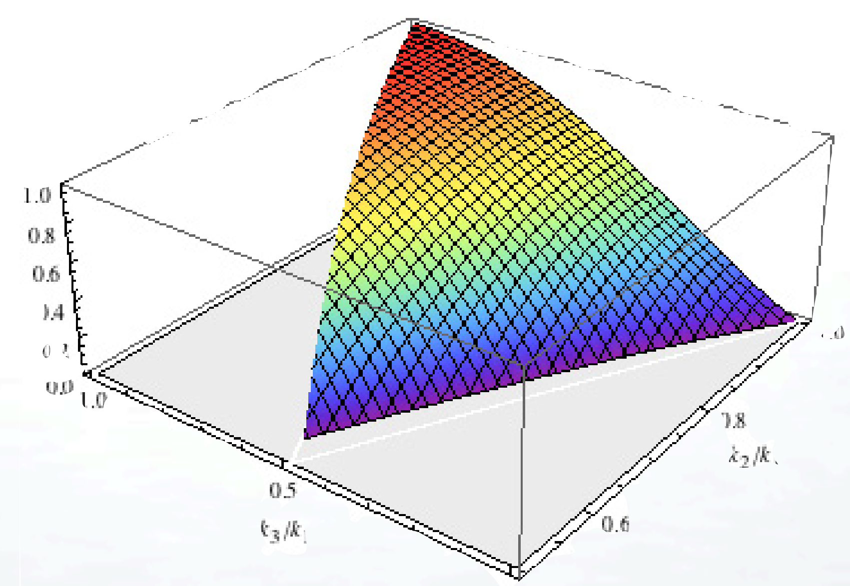
$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = -i\int_{t_0}^{t} \mathrm{d}t' \langle [\zeta(\mathbf{k}_1,t)\zeta(\mathbf{k}_2,t)\zeta(\mathbf{k}_3,t), H_{\rm int}(t')]\rangle$$

$$H_{\rm int}(t) = -\int \mathrm{d}^3x\, a^3 \left[\frac{\mathcal{C}_1}{H}\dot{\zeta}^3 + \mathcal{C}_2\zeta\dot{\zeta}^2 + \cdots\right]$$

$$\longrightarrow \quad \langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^7\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\mathcal{P}_\zeta^2 \frac{\mathcal{A}}{k_1^3k_2^3k_3^3}$$

A peaks at $k_1=k_2=k_3$ (except in the case of fine-tuned parameters)

$$f_{\rm NL}^{\rm equi} = 30\frac{\mathcal{A}_{k_1=k_2=k_3}}{(k_1+k_2+k_3)^3}$$

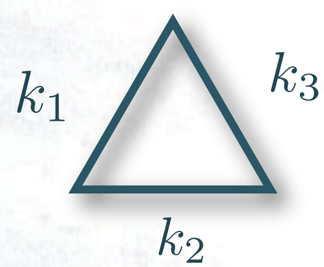


Size of Non-Gaussianity

$$f_{\rm NL}^{\rm equi} = \mathcal{O}\left(\frac{\tilde{\sigma}^2}{c_s^2}\right) + \mathcal{O}\left(\tilde{\sigma}^2 \frac{XG_{XX}}{G_X}\right) + \mathcal{O}\left(\tilde{\sigma}\frac{\mathcal{I}}{\mathcal{G}_S}\right), \quad \tilde{\sigma} := \max\{1, \sigma\}$$

$$\mathcal{I} := XK_{XX} + \frac{2X^2}{3}K_{XXX} + H\dot{\phi}G_X + \cdots$$

C.f. $r = 16\sigma c_s$



<u>k-inflation</u> $\sigma = \epsilon$

$$f_{\rm NL} \sim \frac{1}{c_s^2} \qquad r = 16\epsilon c_s$$

Large $f_{\rm NL}$ (r) → Small r ($f_{\rm NL}$)

<u>G-inflation</u> $\sigma \neq \epsilon$

Both $f_{\rm NL}$ and r can be large

Say, $f_{\rm NL} = 210$ with $r = 0.17$ is possible in kinetically driven models

The most general case

$$G_4 = G_4(\phi, X), \quad G_5 = G_5(\phi, X)$$

$$S_{(3)}[\zeta] = \int d\eta d^3x a^2 \Bigg\{ \frac{\Lambda_1}{\mathcal{H}}\zeta'^3 + \Lambda_2 \zeta'^2 \zeta + \Lambda_3 \zeta \left(\partial_i \zeta\right)^2 + \frac{\Lambda_4}{\mathcal{H}^2}\zeta'^2 \partial^2 \zeta + \Lambda_5 \zeta' \partial_i \zeta \partial^i \psi + \Lambda_6 \partial^2 \zeta \left(\partial_i \psi\right)^2$$
$$+ \frac{\Lambda_7}{\mathcal{H}^2}\left[\partial^2 \zeta \left(\partial_i \zeta\right)^2 - \zeta \partial_i \partial_j \left(\partial^i \zeta \partial^j \zeta\right)\right] + \frac{\Lambda_8}{\mathcal{H}}\left[\partial^2 \zeta \partial_i \zeta \partial^i \psi - \zeta \partial_i \partial_j \left(\partial^i \zeta \partial^j \psi\right)\right] + F\left(\zeta\right) \frac{\delta \mathcal{L}_2}{\delta \zeta}\bigg|_1 \Bigg\},$$

Gao & Steer 1107.2642

See also De Felice & Tsujikawa 1107.3917

No new operators beyond k-inflation Reneux-Petel (2011)

More complicated expressions for coefficients...

Summary of Part 2

The generalized Galileon offers a framework to study
the most general single-field inflation model

We now have **the most general quadratic (and cubic) actions** for curvature and tensor perturbations, which can be used to **determine stability** and **compute 2-(and 3-) point functions** of **all** the single-field inflation models

Non-Gaussianity: No new shapes beyond k-inflation, but large r and large f_{NL} are compatible in more general models than k-inflation

Talk Plan

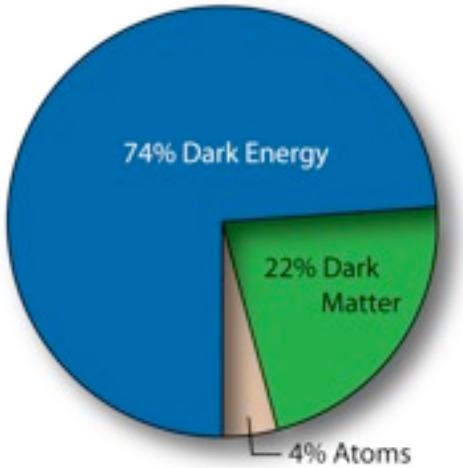
1. Introduction to the *Galileon*
2. G-inflation – Inflation driven by the Galileon field –
3. Galileon models of dark energy

3

Galileon models of dark energy

Many **dynamical dark energy** models and **modified gravity** models are described in a generic, unified manner by

$$\sqrt{-g}\sum_{i=2}^{5}\mathcal{L}_i + \sqrt{-g}\mathcal{L}_{\rm m}$$



Most general scalar-tensor theory
= generalized Galileon

Ordinary matter:
dark matter, photons, baryons, ...

* *f*(*R*) gravity is also in this class

Assume matter is coupled to $g_{\mu\nu}$, and not directly to ϕ

If you want to consider matter coupled to $\tilde{g}_{\mu\nu} = A(\phi)g_{\mu\nu}$,
a conformal transformation brings your theory to the above form

Solar-system constraints on scalar-mediated force

Severe constraint on **Brans-Dicke theory** (prototype example of ST theories)

$$\mathcal{L} = \phi R - \frac{\omega}{\phi}(\partial\phi)^2 + \mathcal{L}_{\rm m}$$

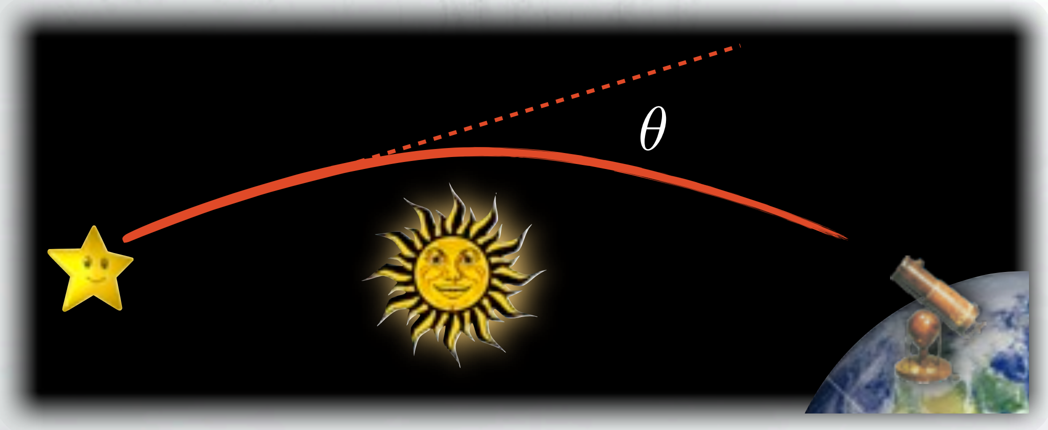
Gravitational field around a point mass

$$T_{00} = M\delta^{(3)}(\mathbf{x})$$

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + (1-2\Psi)\mathrm{d}\mathbf{x}^2$$

Cf. General Relativity $\gamma = 1$

$$\gamma = \frac{\Psi}{\Phi} = \frac{1+\omega}{2+\omega}$$



$$\theta = 1.75''(1+\gamma)/2$$

Light bending $|\gamma - 1| < 10^{-4}$

Will gr-qc/0103036

Screening mechanisms

Scalar d.o.f. must be screened somehow in the vicinity of matter

(1) Scalar d.o.f. is effectively massive
in the vicinity of matter
— not fluctuate

Mota, Barrow (2004)
Khoury, Weltman (2004)

Chameleon mechanism

$$\sum_{i=2}^{5} \mathcal{L}_i \supset -V(\phi)$$

Potential term at work

(2) Scalar d.o.f. is effectively weakly coupled
in the vicinity of matter
— fluctuate, but do not care

Vainshtein (1992)

Vainshtein mechanism

$$\sum_{i=2}^{5} \mathcal{L}_i \supset (\partial\phi)^2 \Box\phi, \cdots$$

Non-linear derivative interaction
at work

Improving BD theory

Add Galileon-type interaction to BD theory

$$\mathcal{L} = \phi R - \frac{\omega}{\phi}(\partial\phi)^2 + \underline{f(\phi)(\partial\phi)^2\Box\phi} + \mathcal{L}_{\rm m}$$

$$G_3 = 2f(\phi)X$$

Schematically,

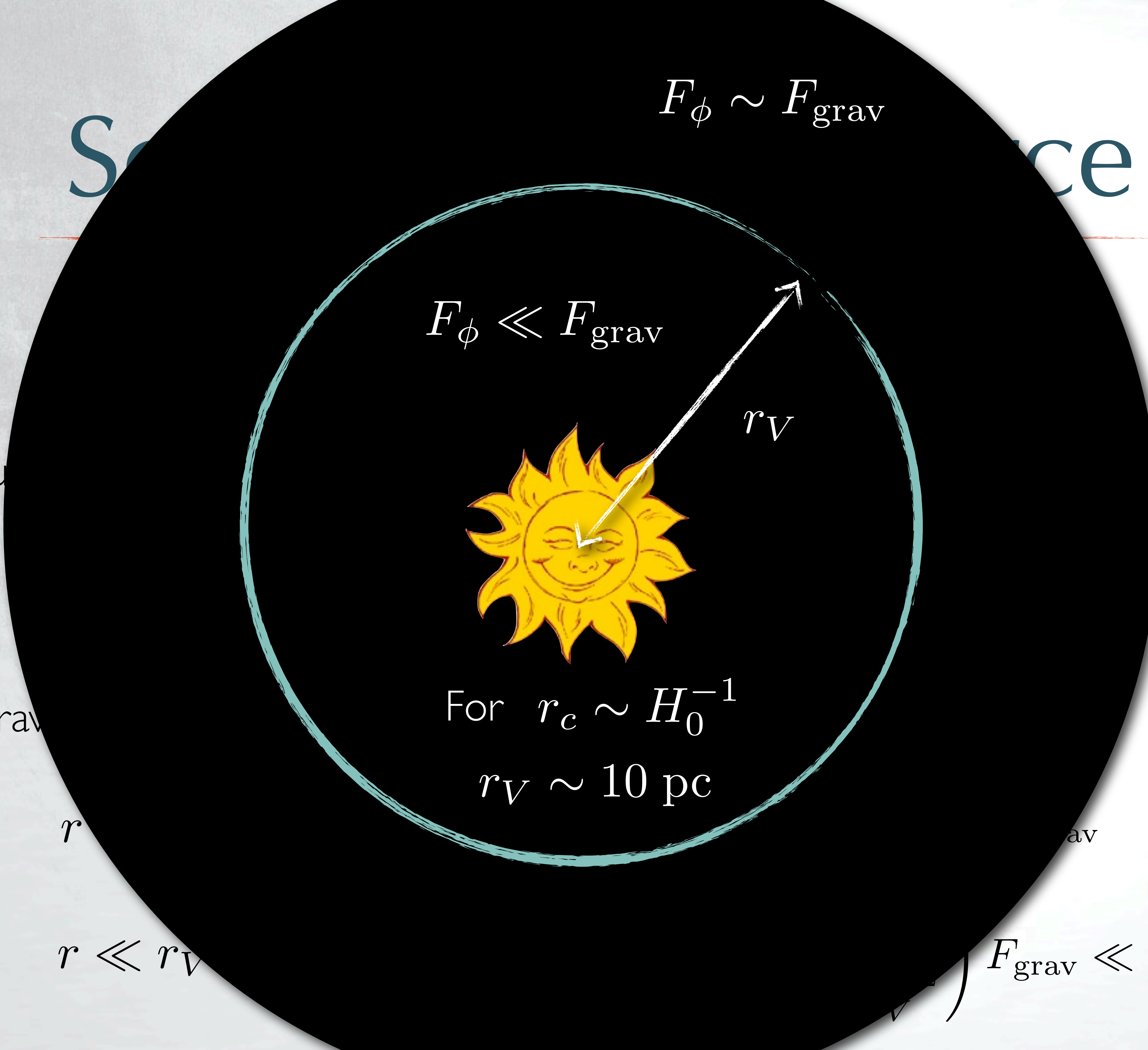
$$\Box\phi \sim \phi R \sim \rho \quad \longrightarrow \quad G_3\Box\phi \propto \rho(\partial\phi)^2$$

Large kinetic term at high densities

$$\omega \ll \omega_{\rm eff} \propto \rho$$

S ce

Arou

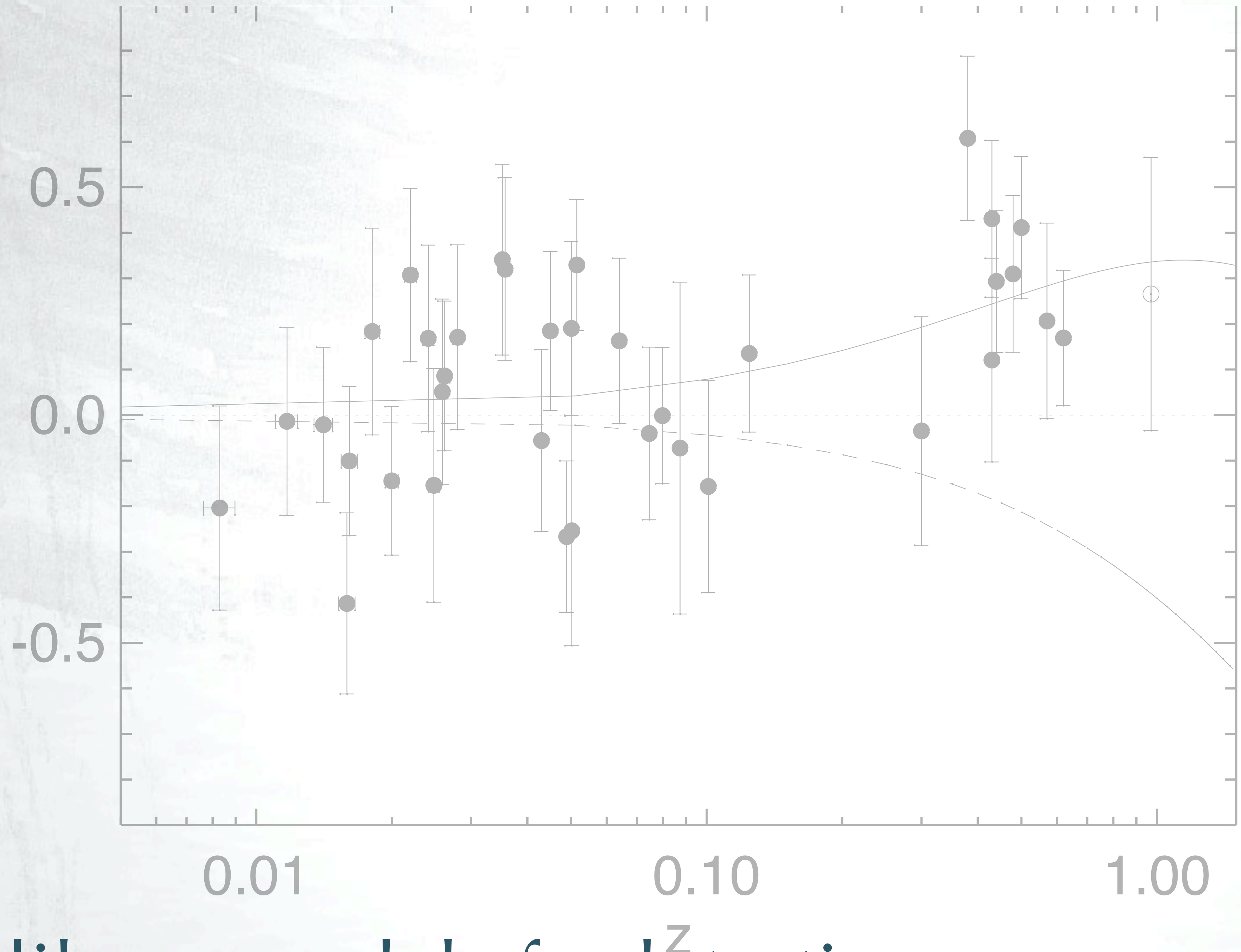


Cf. grav

r av

$r \ll r_V$ $) \; F_{\rm grav} \ll F_{\rm grav}$

3-1



Some Galileon models for late-time acceleration

Chow, Khoury (2009); Silva, Koyama (2009); TK, Tashiro, Suzuki (2010); TK (2011)

Cosmology of BD + $\mathcal{L}_3$

Two-parameter model BD + $\mathcal{L}_3 + \mathcal{L}_{\rm m}$

ω α

$$G_3 = 2f(\phi)X, \quad f(\phi) = \frac{r_c^2}{\phi^2}\left(\frac{2\phi}{M_{\rm Pl}^2}\right)^\alpha \propto \phi^{\alpha-2}$$

Early-time behavior $H \gg r_c^{-1}$

$3H^2 \simeq 8\pi G\rho$ Standard cosmology is *not* destroyed

Late-time behavior $H \lesssim r_c^{-1}$

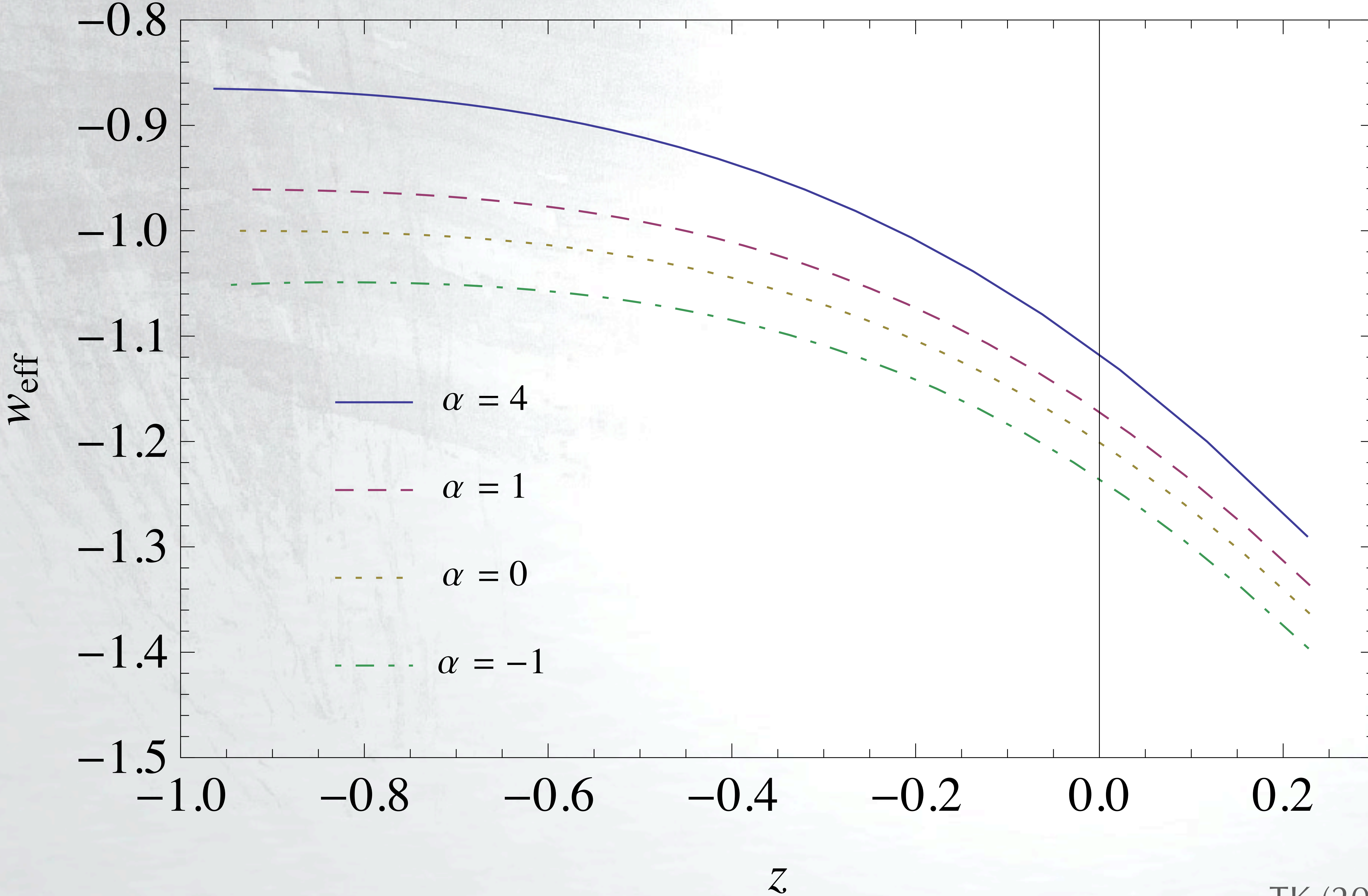
$$1 + w_{\rm eff} \to \frac{\alpha}{3}\left[\frac{-2+\alpha\pm\sqrt{-6\omega-(2-\alpha)(4+\alpha)}}{\omega+2-\alpha}\right]$$

> 0 for $\alpha > 0$

$= 0$ for $\alpha = 0$

< 0 for $\alpha < 0$

$$1 + w_{\rm eff} = -\frac{\dot{\rho}_{\rm eff}}{3H\rho_{\rm eff}} \qquad \omega = -500$$



TK (2011)

Shift-symmetric scalar

$K = K(X),\ G_i = G_i(X)$ — Recall *kinetically driven G-inflation*

"Friedmann equation" $\sum^{5} \mathcal{E}_i + \rho = 0$

can mimic phenomenological models of modified Friedmann equation

$$\tilde{F}(H) = \rho$$

scalar EOM $J(\dot\phi, H) \propto a$

Along the attractor,

$$F(\dot\phi, H) = \rho$$
$$J(\dot\phi, H) = 0$$

→ de Sitter

Recent applications

"Kinetic gravity braiding"

Deffayet *et al.* (2010)
Kimura, Yamamoto (2011)

$G_3 \propto X^n$

"Purely kinetic coupled gravity"

Gubitosi, Linder (2011)

$G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \longleftarrow G_4 \propto X$

Kimura, Yamamoto (2011)

$$K = -X, \quad G_3 \propto X^{n}$$

Reproduce the phenomenological model of Dvali and Turner (2003)

$n = 1$ – covariant Galileon

$n = \infty$ – LCDM

$$\alpha = -\frac{2}{2n-1}$$

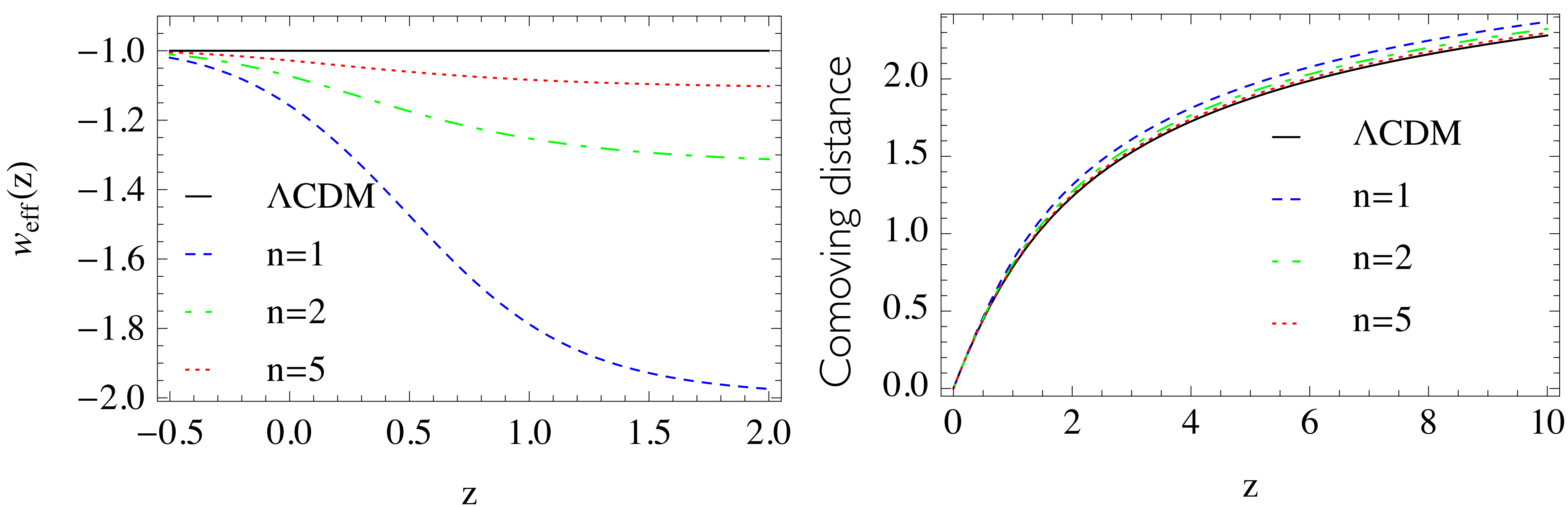
Friedmann equation

$$3M_{\rm Pl}^2 H^2 - \left(X + \dot{\phi} J\right) = \rho$$

Along the attractor,

$$J = -\dot{\phi} + 6HXG_{3X} = 0$$

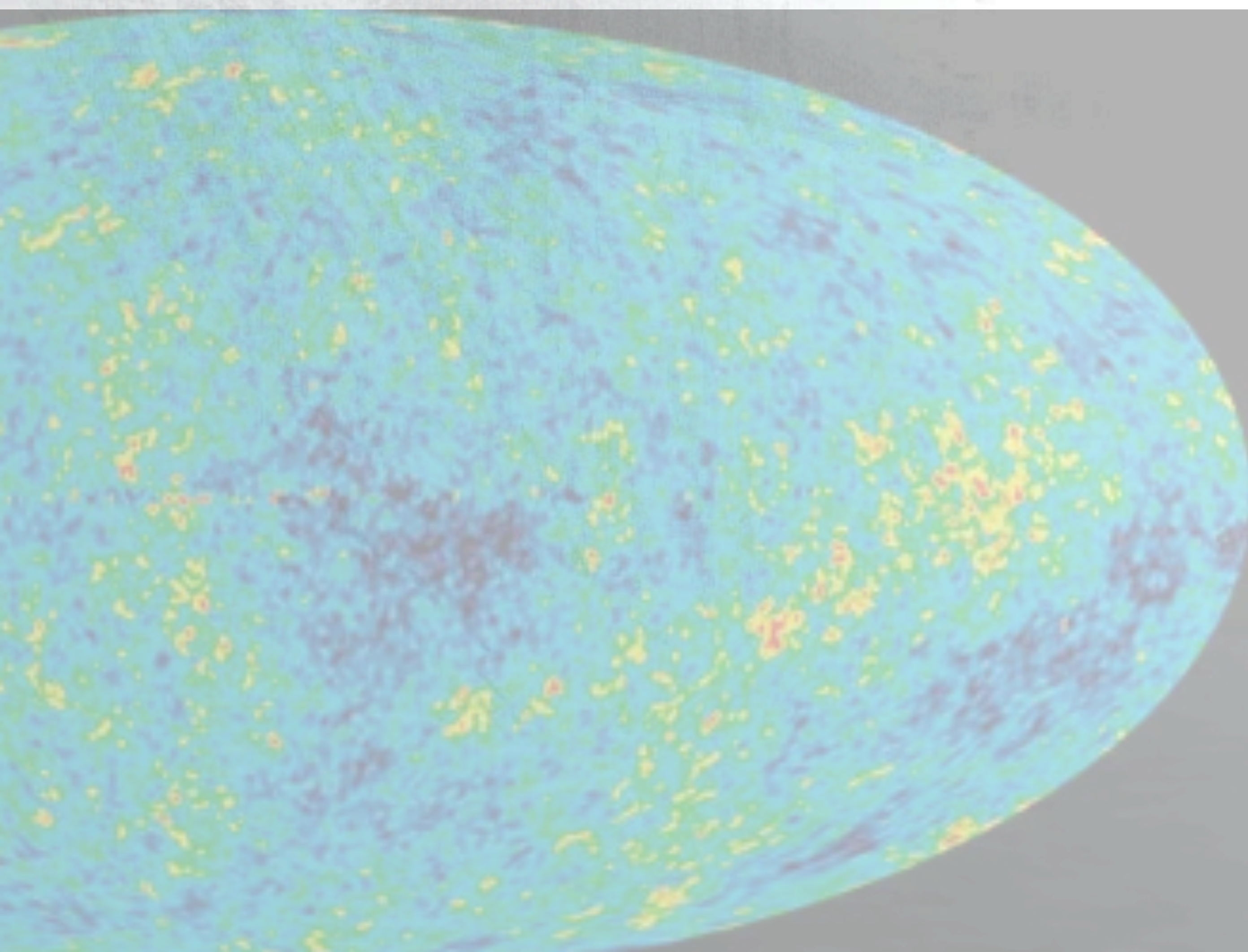
$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{m0}}{a^3} + (1-\Omega_{m0})\left(\frac{H}{H_0}\right)^{\alpha}$$

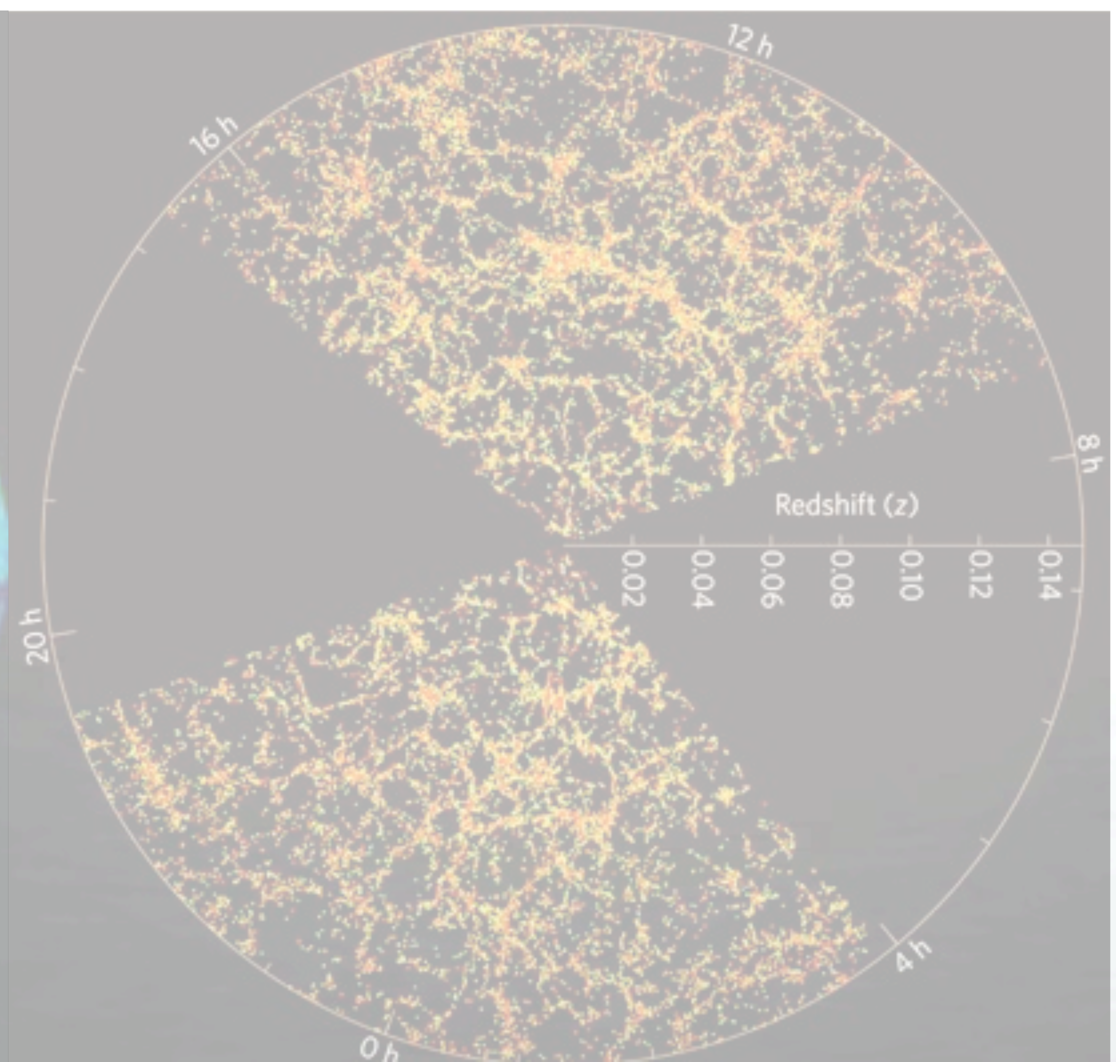


3-2

Density perturbations

The aim: test/distinguish different models of dark energy and modified gravity





Density perturbations in the most general ST theory

TK, to appear

For *all* the ST theories with second-order field equations, the **sub-horizon evolution of the density perturbation** is described by

$$\ddot{\delta} + 2H\dot{\delta} = \frac{\xi(t;k)}{2}\rho_{\rm m}\delta$$

$4\pi G_{\rm eff}$

This can be shown by using the **quasi-static approximation**

$$\partial_t \sim H \ll \frac{\partial_i}{a}$$

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(t)(1-2\Psi)\mathrm{d}\mathbf{x}^2, \quad Q = H\frac{\delta\phi}{\dot{\phi}}$$

$$m^2 := -K_{\phi\phi}$$

ij equation

$$\mathcal{F}_T\Psi - \mathcal{G}_T\Phi = \left(\frac{\dot{\mathcal{G}}_T}{H} + \mathcal{G}_T - \mathcal{F}_T\right)Q$$

Cf. GR $\Psi - \Phi = 0$

00 equation

$$\mathcal{G}_T\frac{\nabla^2}{a^2}\Psi + \left(\mathcal{G}_T - \frac{\Theta}{H}\right)\frac{\nabla^2}{a^2}Q \simeq \frac{1}{2}\rho\delta$$

Scalar-field equation

$$\left[\frac{\dot{\Theta}}{H^2} + \frac{\Theta}{H} + \mathcal{F}_T - 2\mathcal{G}_T - 2\frac{\dot{\mathcal{G}}_T}{H} - \frac{\mathcal{E}+\mathcal{P}}{2H^2}\right]\frac{\nabla^2}{a^2}Q + m^2\frac{X}{H^2}Q$$
$$-\left(\frac{\dot{\mathcal{G}}_T}{H} + \mathcal{G}_T - \mathcal{F}_T\right)\frac{\nabla^2}{a^2}\Psi - \left(\mathcal{G}_T - \frac{\Theta}{H}\right)\frac{\nabla^2}{a^2}\Phi \simeq 0$$

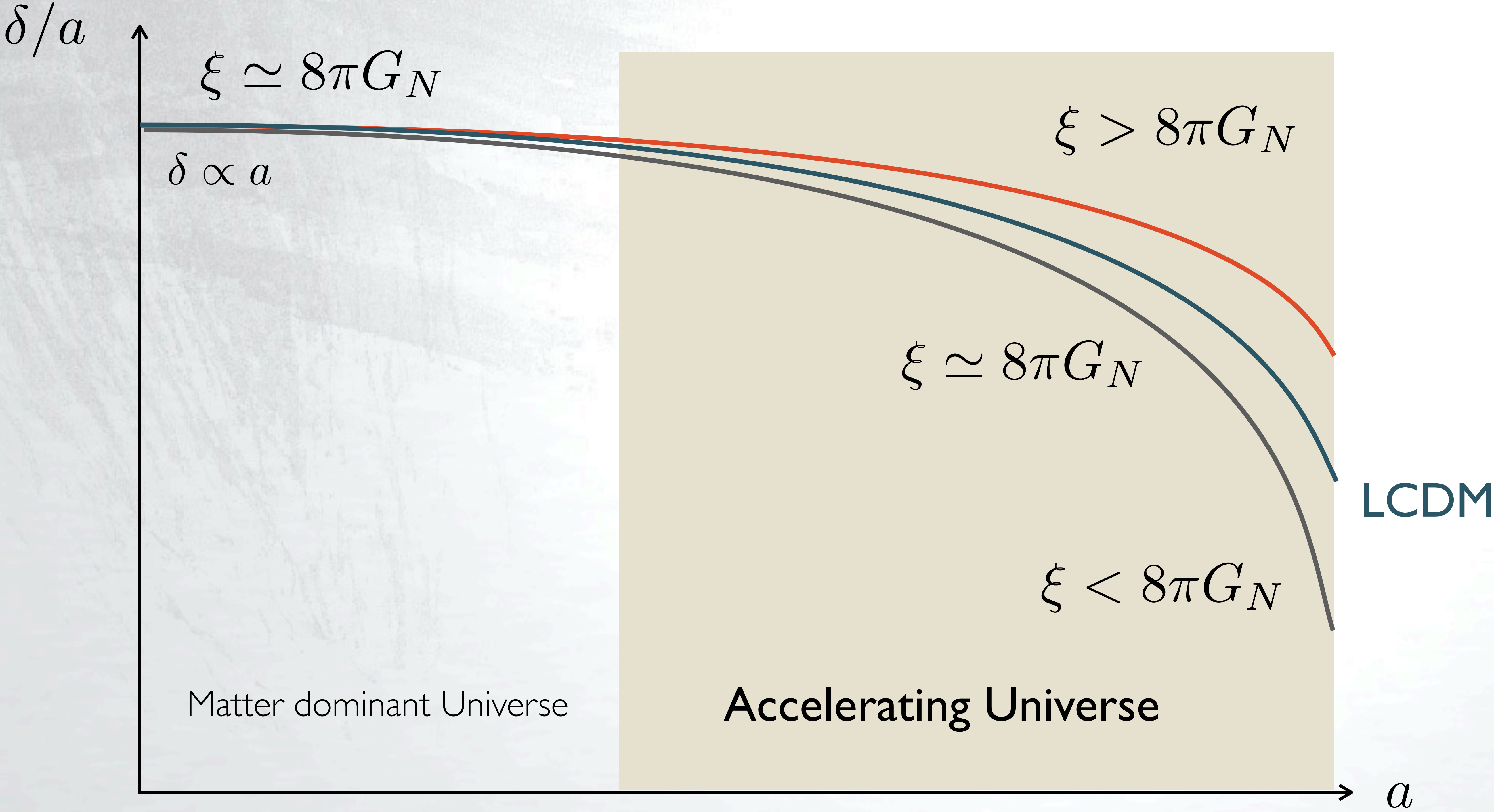
$$\frac{\nabla^2}{a^2}\Phi = \frac{\xi}{2}\delta\rho$$

$$\nabla_\mu T_\nu^{\mu(\mathrm{m})} = 0$$

Evolution equation for δ

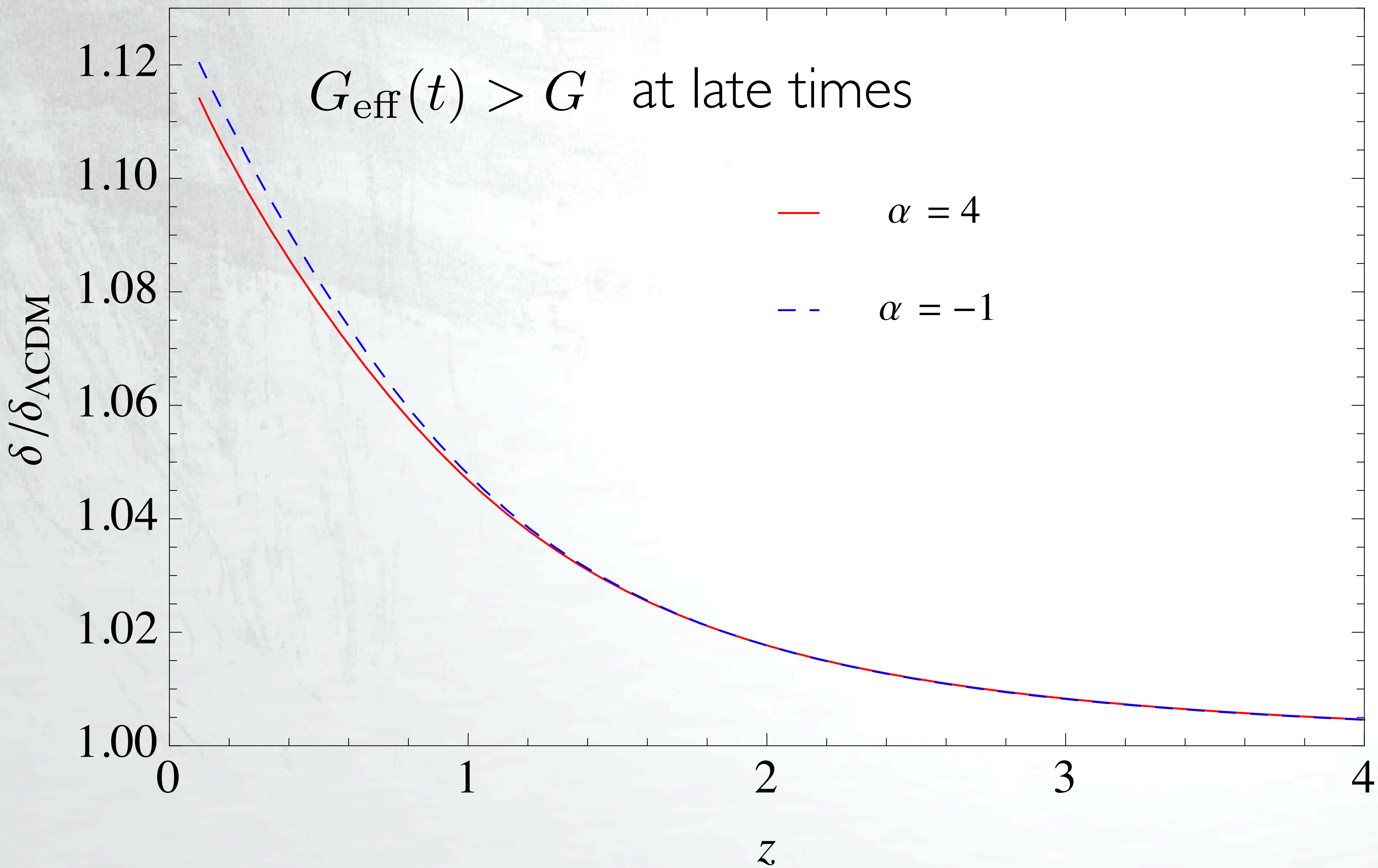
$$\xi := \frac{\left(\dot{\Theta} + H\Theta\right)\mathcal{F}_S + \left(\dot{\mathcal{G}}_T - \dot{\Theta}\mathcal{G}_T/\Theta\right)^2 + \mathcal{F}_T\left[(\mathcal{E}+\mathcal{P})/2 + Xa^2m^2/k^2\right]}{\Theta^2\mathcal{F}_S + \mathcal{G}_T^2\left[(\mathcal{E}+\mathcal{P})/2 + Xa^2m^2/k^2\right]}$$

Schematically...



BD + $\mathcal{L}_3$ model

$\omega = -500$



Characterizing growth history

Growth factor: $g = \delta / a$

Growth rate: $f = \dfrac{\mathrm{d} \ln \delta}{\mathrm{d} \ln a}$

Useful discriminant for distinguishing different models

Growth index:

$$f = [\Omega_{\mathrm{m}}(a)]^{\gamma}, \quad \Omega_{\mathrm{m}}(a) = \frac{\rho_{\mathrm{m}}}{3 M_{\mathrm{Pl}}^2 H^2}$$

Wang, Steinhardt (1998)
Linder (2005)
Linder, Cahn (2007)

LCDM $\gamma \simeq 0.55$

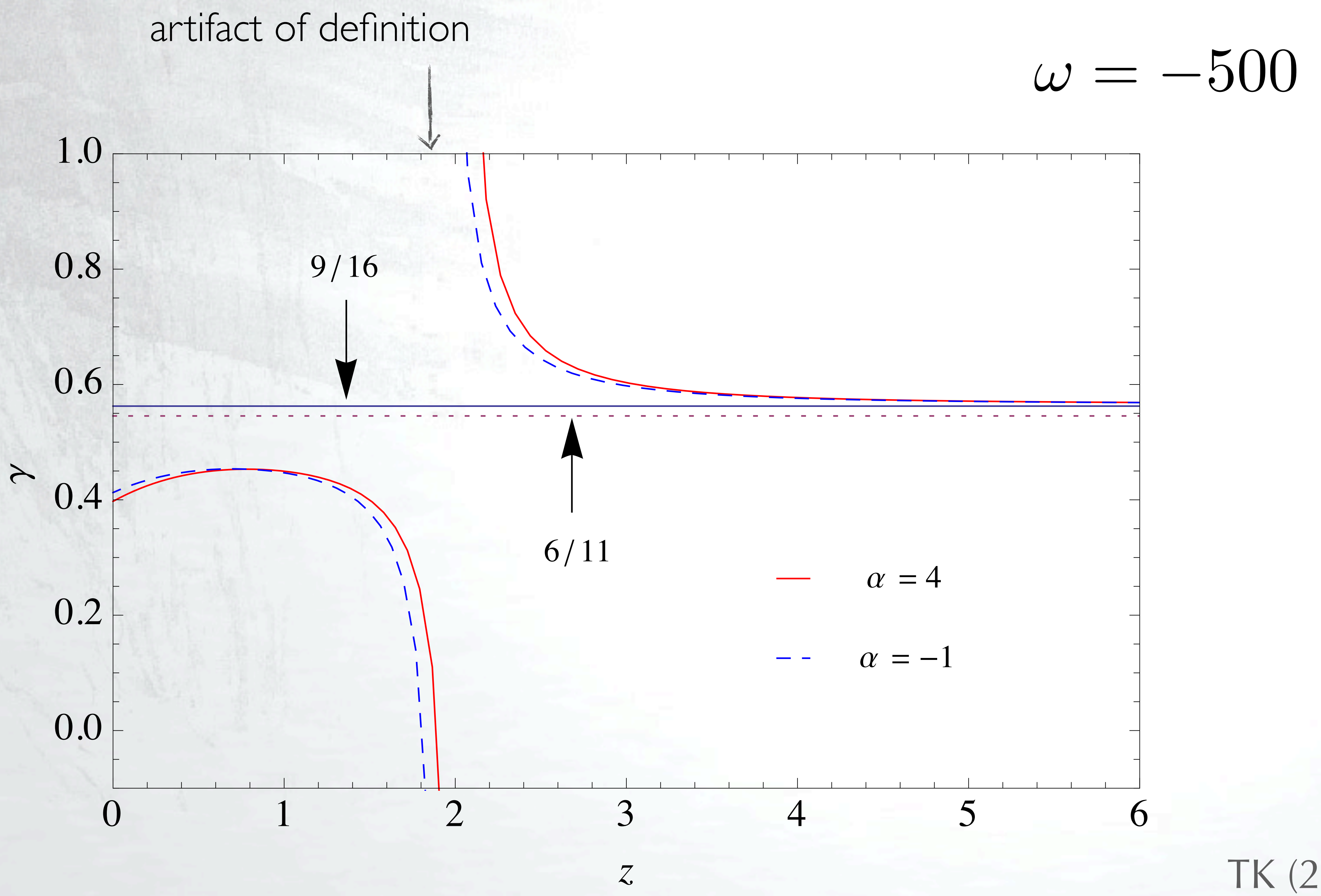
DGP $\gamma \simeq 0.69$

approximately constant

f(*R*) γ small

Gannouji, Moraes, Polarski (2009); Tsujikawa, Gannouji, Moraes, Polarski (2009); Narikawa, Yamamoto (2009); Motohashi, Starobinsky, Yokoyama (2010); ...

BD + $\mathcal{L}_3$ model



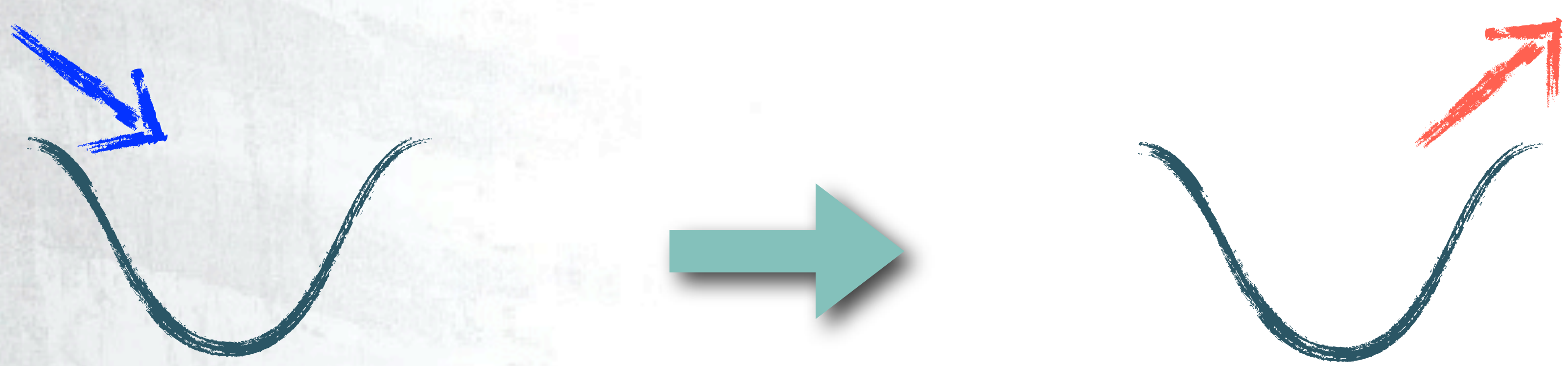
TK (2011)

Integrated Sachs-Wolfe effect

$$\dot{\Phi} + \dot{\Psi} = 0$$

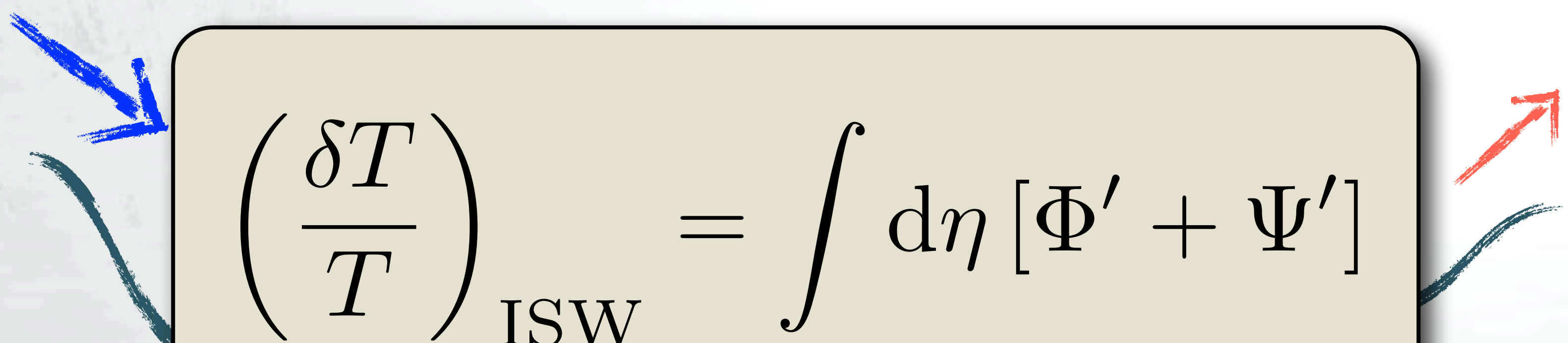
CMB photon

Blue shift = Red shift



$$\dot{\Phi} + \dot{\Psi} \neq 0$$

Blue shift ≠ Red shift



$$\left(\frac{\delta T}{T}\right)_{\rm ISW} = \int \mathrm{d}\eta \, [\Phi' + \Psi']$$

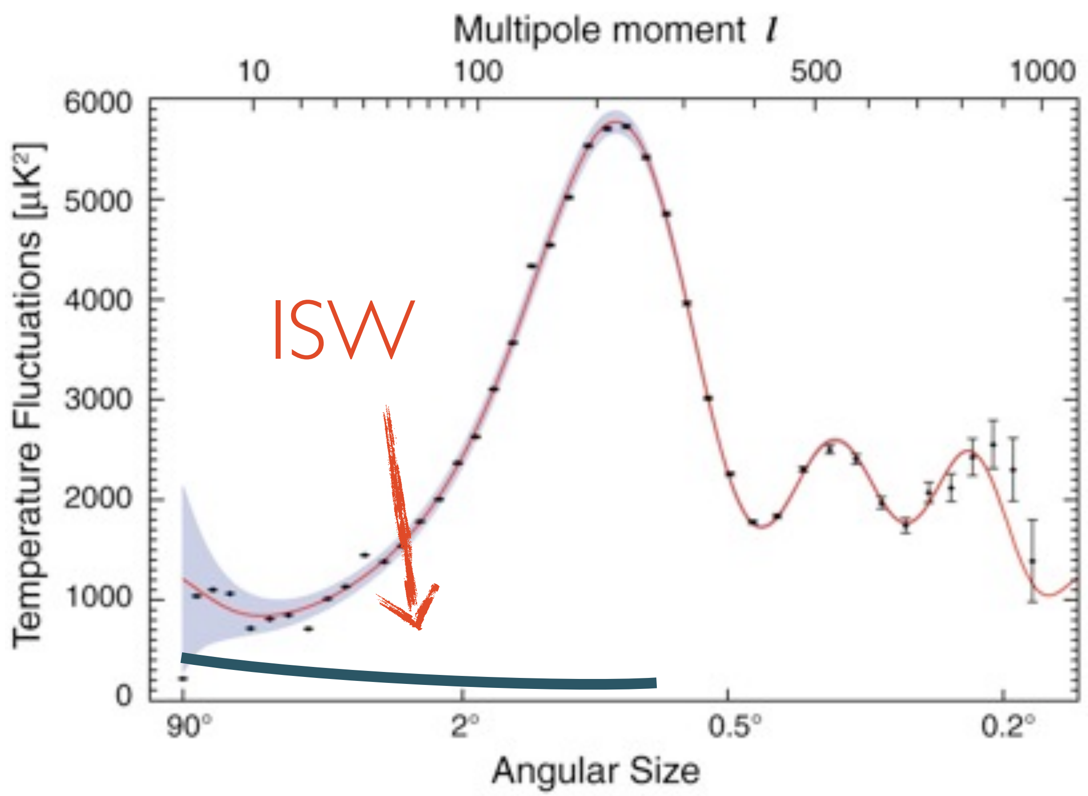
$$\frac{\delta T}{T}$$

$\dot{\Phi} + \dot{\Psi} \neq 0$ in the accelerating Universe

Late ISW — powerful dark energy probe

Difficult to measure ISW because:

- ✓ SW >> ISW
- ✓ Cosmic variance



This problem can be evaded:

- ✓ ISW is correlated with matter density through potential
- ✓ Primary CMB is generated long before and is not correlated

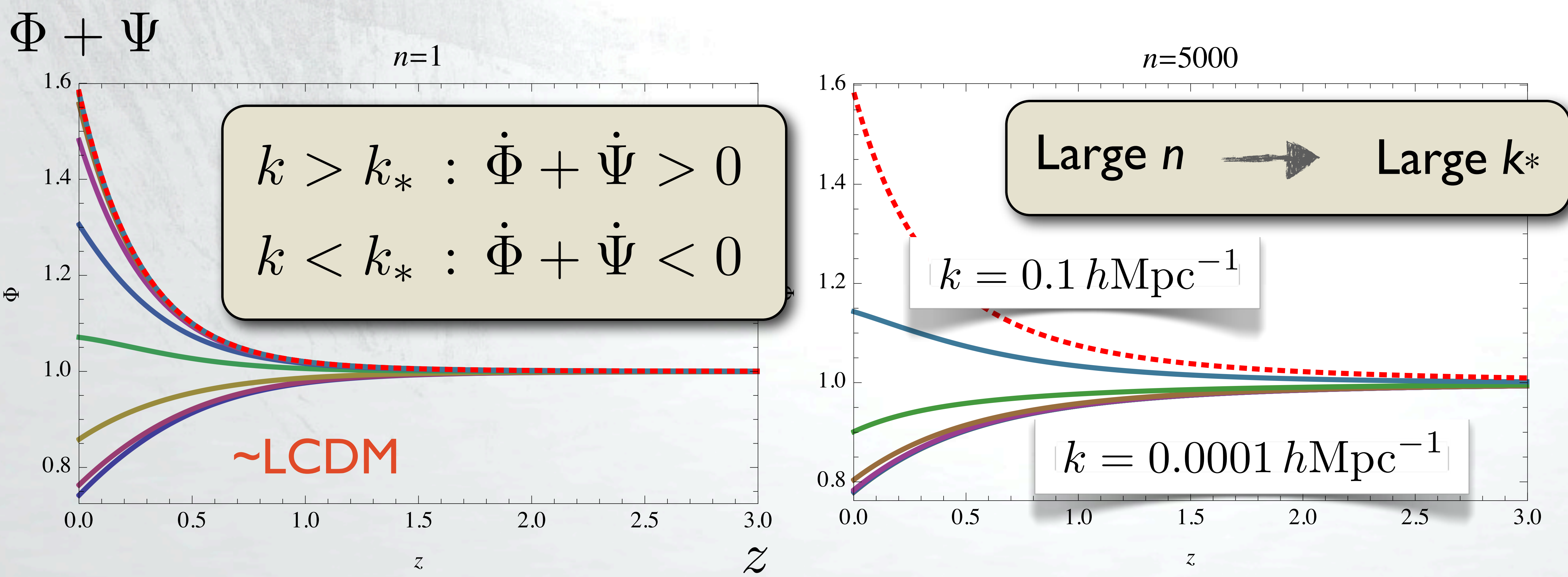
→ **CMB-galaxy cross-correlation** Crittenden, Turok (1995) w/ Rampei Kimura, Kazuhiro Yamamoto (Hiroshima), to appear

ISW in KGB

Kimura-Yamamoto model:

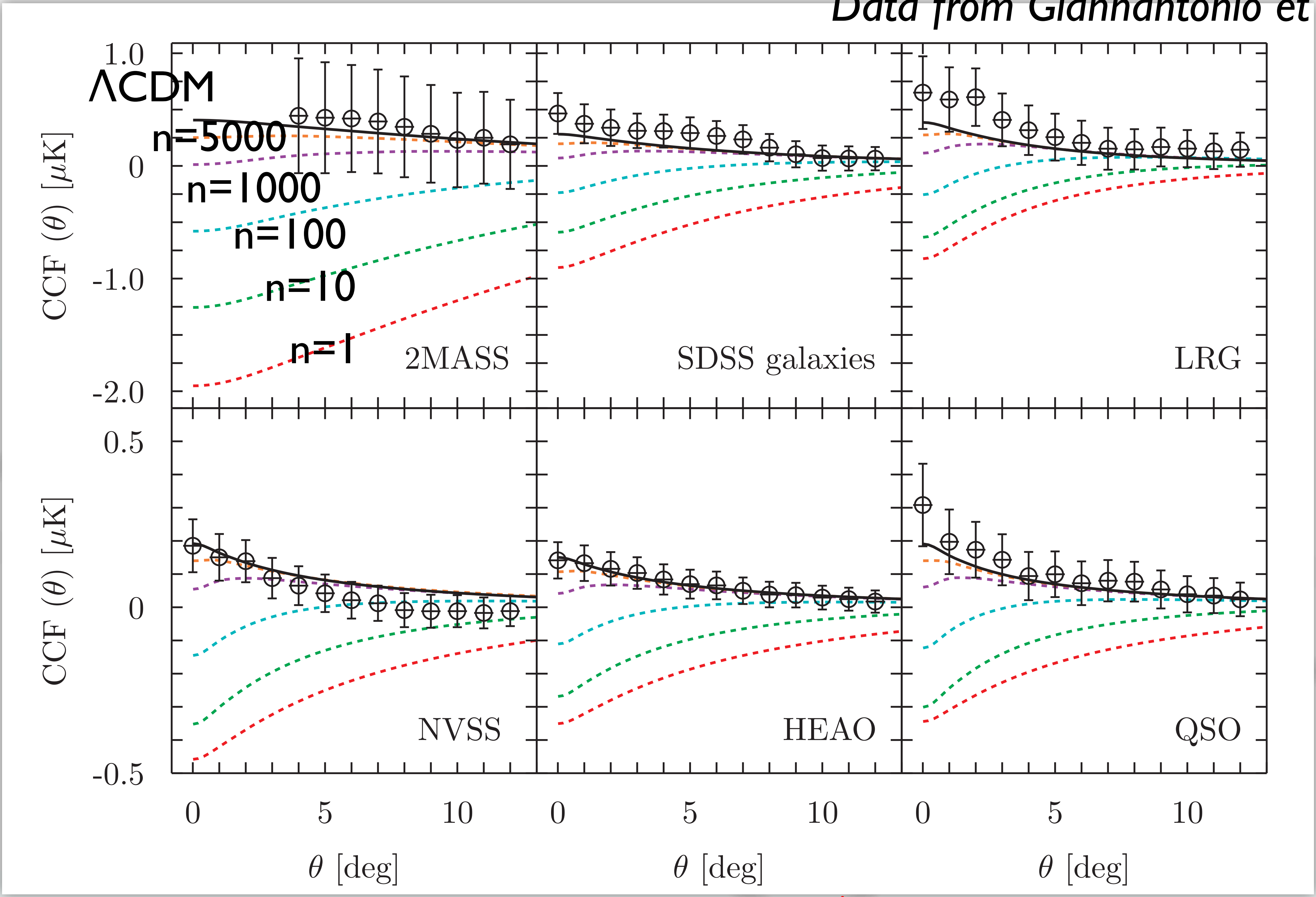
$$\mathcal{L} = \frac{R}{2} - X - G(X)\Box\phi, \quad G \propto X^n$$

Quasi-static approximation is insufficient; need full perturbation analysis



Galaxy-LSS Cross-correlation

Data from Giannantonio et al. '08



$n \gtrsim 10^4$ *(95% C.L.)*

Summary of Part 3

Dark energy and modified gravity models with a single scalar degree of freedom (in addition to metric) are described by the generalized Galileon

Need screening mechanism:
Chameleon / Vainshtein

Implications for cosmological observations are interesting

Growth of matter perturbations / ISW / ...

Conclusion

The Galileon extends far beyond a specific scalar-field theory

The generalized Galileon is **the most general** scalar-tensor theory with second-order field equations (equivalent to Horndeski's theory)

The generalized Galileon is a useful framework to study inflation and dark energy models in a generic/unified/systematic way

New models and new scenarios, as well as all the previous examples proposed so far in the single-field context

Large GWs / large non-Gaussianity / $\dot{H} > 0$ ……