Summer Institute 2011 @ Fujiyoshida August 5, 2011

ASPECTS OF D-BRANE INFLATION IN STRING COSMOLOGY Takeshi Kobayashi (RESCEU, Tokyo U.)

TODAY'S PLAN

Cosmic Inflation and String Theory

- D-Brane Inflation and its Difficulties arXiv:0708.4285 with Shunichiro Kinoshita and Shinji Mukohyama
- Inflation from Rapid-Rolling D-Branes arXiv:0810.0810 with Shinji Mukohyama arXiv:0905.1752 with Shinji Mukohyama and Brian A. Powell
- Curvatons in Warped Throats arXiv:0905.2835 with Shinji Mukohyama arXiv:1107.6011 with Masahiro Kawasaki and Fuminobu Takahashi

COSMIC INFLATION



image: NASA/WMAP Science Team

COSMIC INFLATION



homogeneous and isotropicflat

• without unwanted relics

•tiny inhomogeneities

image: NASA/WMAP Science Team

TOWARDS MICROSCOPIC REALIZATION

Inflaton : a scalar field which acts like vacuum energy



FUNDAMENTAL QUESTIONS

- What is the inflaton?
- What microphysics governed the inflationary universe?

• UV-sensitivity of inflationary cosmology

How valid is the effective field theory description?

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- What is the inflaton?
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How valid is the effective field theory description?

String Theory may help!

STRING COSMOLOGY

Good things for Cosmology

allows description of the universe at high energy
provides new ideas, new ingredients, and new ways of thinking about the early universe

Good things for String Theory

 may provide tests of string theory through cosmological observables

STRINGTHEORY

• quantum gravity candidate based on closed / open strings



STRINGTHEORY

quantum gravity candidate based on closed / open strings





open strings end on Dp-branes, i.e. physical objects with p spatial dimensions

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quantum gravity candidate based on closed / open strings





- open strings end on Dp-branes, i.e. physical objects with p spatial dimensions
- string theory predicts 10 (or 11) spacetime dimensions
 → the extra 6 (or 7) dimensions need to be compactified

COMPACTIFICATION OF TYPE IIB STRING THEORY

Klebanov, Strassler '00 Gidding, Kachru, Polchinski '02 Kachru, Kallosh, Linde, Trivedi '03

warped compactification to 4-dim. dS with all moduli fixed via fluxes, brane sources, and nonperturbative effects

extra 6-dim. space

R-R flux

NS-NS flux

warped throat region

 $ds^{2} = h(y)^{2} g^{(4)}_{\mu\nu} dx^{\mu} dx^{\nu} + h(y)^{-2} g^{(6)}_{mn} dy^{m} dy^{n}$

D-BRANE INFLATION IN A WARPED THROAT

Kachru, Kallosh, Linde, Maldacena, McAllister, Trivedi '03

A brane-antibrane pair drives inflation.



D-BRANE INFLATION : A CLOSER LOOK

inflaton ϕ : radial position of the D3



D3

D3

D3-D3 annihilation

INFLATON POTENTIAL

 ϕ : (normalized) D3 position h_0 : warp factor at the throat tip

$$V(\phi) = 2h_0^4 T_3 \left(1 - \frac{\mu^4}{\phi^4}\right) + H^2 \phi^2 + \cdots$$

D3-tension

Coulomb interaction

From moduli stabilization



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D3-tension from mo

Coulomb interaction

from moduli stabilization

$$H^2 \simeq \frac{V}{3M_p^2} \longrightarrow \eta = M_p^2 \frac{V''}{V} \simeq \frac{2}{3}$$
 η-problem

slow-roll inflation

(unless delicate fine-tuning)

Baumann et al. '06 - '10

Inflation from D-branes moving with relativistic velocities.

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However...

Throat too short for a relativistically moving D3-brane to drive sufficient inflation.

Baumann, McAllister '06 Lidsey, Huston '07

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Not only for D3-branes,

but also for higher-dimensional wrapped branes (D5 and D7).

TK, Mukohyama, Kinoshita '07

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Relaxation of constraints by multi-field/Galileon extensions? (talks by Tsutomu & Shuntaro yesterday)

SUMMARY SO FAR FOR D-BRANE INFLATION

Slow-Roll Inflation

• suffers from the η -problem (i.e. Hubble size mass)

DBI Inflation (relativistic limit)

suffers from geometrical constraints

INFLATION FROM RAPID-ROLLING D-BRANES

Kofman, Mukohyama '07 TK, Mukohyama '08 TK, Mukohyama, Powell '09

• An inflationary attractor solution DOES exist even for $\eta \sim \mathcal{O}(1)$.

• This corresponds to D-branes moving rapidly, but non-relativistically.

• The inflaton is quickly decelerating / accelerating. (+---- slow-roll inf.)

RAPID-ROLL INFLATION

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi - V(\phi)$$
$$\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta \equiv M_p^2\frac{V''}{V}$$

Hubble eq.
$$3M_p^2H^2 = \frac{\phi^2}{2} + V$$

EOM $\ddot{\phi} + 3H\dot{\phi} = -V'$

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Hubble eq.
$$3M_p^2 H^2 = \frac{\dot{\phi}^2}{2} + V \longrightarrow 3M_p^2 H^2 \simeq V$$

EOM $\ddot{\phi} + 3H\dot{\phi} = -V' \longrightarrow cH\dot{\phi} \simeq -V'$

where $c = \frac{3 + \sqrt{9 - 12\eta}}{2}$

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$$\text{slow-roll recovered}$$

$$\text{upon } \eta \to 0$$

EOM
$$\ddot{\phi} + 3H\dot{\phi} = -V' \longrightarrow cH\dot{\phi} \simeq -V'$$

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INFLATION FROM RAPID-ROLLING D3-BRANE

$$\frac{\mathcal{L}}{\sqrt{-g}} = -\frac{1}{2}g^{\mu\nu}\partial_{\mu}\phi\partial_{\nu}\phi - V_0\left\{1 + \mathcal{O}(1) \times \frac{\phi^2}{M_p^2} - \frac{\mu^4}{\phi^4}\right\}$$

$$V(\phi)$$

$$(\phi)$$

$$(\phi)$$

$$(\phi)$$

 ϕ

•

inflationary attractor

$$\frac{\dot{\phi}}{H} \simeq -\frac{6\eta}{3+\sqrt{9-12\eta}}\phi$$

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 \bigcirc

inflationary attractor

$$\frac{\dot{\phi}}{H} \simeq -\frac{6\eta}{3+\sqrt{9-12\eta}}\phi$$





strong warping gives sufficient inflationary period

 $11 \quad 3 \pm \sqrt{9} = 121$

 ϕ/M_p

INFLATION FROM RAPID-ROLLING D3-BRANE

- D3-D3 system in a warped geometry can give rise to rapid-roll inflation
- no need to cancel the Hubble size mass
 - \rightarrow remedy to the **n**-problem
- sufficient inflationary expansion can be obtained in a single throat
 - geometrical constraints circumvented

- However, the inflaton itself cannot produce scale-invariant density perturbations.
 - → perturbations need to be generated by something else

Angular directions of warped throats can generate curvature perturbations



TK, Mukohyama '10 and work in progress

•as multi-field inflation

• light fields modulating rapid-roll inflation can generate scale-invariant perturbations

after inflation (as curvatons) TK, Mukohyama '09

•D-branes oscillating in warped throats can be curvatons

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CURVATON MECHANISM

Linde, Mukhanov '97 Enqvist, Sloth '01 Lyth, Wands '01 Moroi, Takahashi '01



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CURVATON MECHANISM

Linde, Mukhanov '97

Enqvist, Sloth '01





ANGULAR POTENTIAL

- isometry breaking bulk effects
- moduli stabilizing non-perturbative effects

Standard Model brane (anti-D3)

+ warping

small mass to the angular degrees of freedom of the SM brane II CURVATON **o**

> eventually decays into other open string modes (reheating)

ACTION

$$S = -T_3 \int d^4 \xi \sqrt{-\det G_{\mu\nu}} \left(1 - \bar{\Psi} i \not\!\!\!D\Psi\right) - T_3 \int C_4$$

$$\sim \int d^4x \sqrt{-g^{(4)}} \left[-(\partial \sigma)^2 + \bar{\psi} i \not\!\!D \psi -(m_{\text{bulk}}^2 + m_{\text{np}}^2) \sigma^2 + \frac{g_s M^{1/2} \alpha'^{3/2}}{h_0^3} \frac{m_{\text{bulk}}^2 m_{\text{np}}^2}{m_{\text{bulk}}^2 + m_{\text{np}}^2} \sigma \bar{\psi} i \not\!\!D \psi + \cdots \right]$$

$$m_{\text{bulk}}^2 = \frac{h_0^{\Delta - 2}}{g_s M \alpha'} \quad \text{: bulk effects}$$
$$m_{\text{np}}^2 = \frac{h_0^{\lambda - 2}}{g_s M \alpha'} \quad \text{: nonperturbative effects}$$

 h_0 : warp factor at the tip

$$\left(ds^{2} = h^{2}g^{(4)}_{\mu\nu}dx^{\mu}dx^{\nu} + h^{-2}g^{(6)}_{mn}dx^{m}dx^{n}\right)$$





Here we have considered quadratic curvaton potentials, but density perturbations can be quite different for non-quadratic potentials.

Kawasaki, TK, Takahashi '11

related works by Enqvist, Takahashi '08 Kawasaki, Nakayama, Takahashi '08

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Curvatons with Non-Quadratic Potentials



Curvatons with Non-Quadratic Potentials



Curvatons with Non-Quadratic Potentials



Additional contributions to the density perturbations!

Density Perturbations

$$\mathcal{P}_{\zeta} = \left(\frac{\partial \mathcal{N}}{\partial \sigma_*} \frac{H_*}{2\pi}\right)^2$$

$$\frac{\partial \mathcal{N}}{\partial \sigma_*} = \frac{r}{4+3r} \left(1 - X(\sigma_{\rm osc})\right)^{-1} \left\{ \frac{V'(\sigma_{\rm osc})}{V(\sigma_{\rm osc})} - \frac{3X(\sigma_{\rm osc})}{\sigma_{\rm osc}} \right\} \frac{V'(\sigma_{\rm osc})}{V'(\sigma_*)}$$

 $r \equiv rac{
ho_{\sigma}}{
ho_{r}} @ ext{curvaton decay}$

* : @ horizon exit

osc: @ onset of curvaton oscillation

 $X(\sigma_{\rm osc}) \equiv \frac{1}{2(c-3)} \left(\frac{\sigma_{\rm osc} V''(\sigma_{\rm osc})}{V'(\sigma_{\rm osc})} - 1 \right)$

: effects due to non-uniform onset of oscillation

Non-Gaussianity f_{NL} also modified.

towards the hilltop:

strong enhancement of linear-order density pert. with mild increase of f_{NL}

f_{NL} = O(10) even for a dominant curvaton





Additional features can show up on the spectral tilt when the inflationary background is given by rapid-roll inflation.

 $rac{\delta
ho}{
ho} \propto H_{
m inf}$

SPECTRALTILT IN RAPID-ROLL INFLATION

The Hubble parameter in rapid-roll inflation does NOT possess a hierarchy amongst its higher-order time derivatives:

$$\frac{1}{H}\frac{d}{dt}\ln H \bigg| \sim \left| \left(\frac{1}{H}\frac{d}{dt}\right)^2 \ln H \right| \sim \left| \left(\frac{1}{H}\frac{d}{dt}\right)^3 \ln H \right| \sim \cdots$$

cf. SLOW-ROLL INFLATION: $\left|\frac{1}{H}\frac{d}{dt}\ln H\right| \gg \left|\left(\frac{1}{H}\frac{d}{dt}\right)^{2}\ln H\right| \gg \left|\left(\frac{1}{H}\frac{d}{dt}\right)^{3}\ln H\right| \gg \cdots$

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The resulting curvature perturbation spectrum can have large running (and its running, and so on).

$$\frac{d}{d\ln k}\ln\left(\frac{\delta\rho}{\rho}\right) \quad , \quad \frac{d^2}{d(\ln k)^2}\ln\left(\frac{\delta\rho}{\rho}\right) \quad , \quad \frac{d^3}{d(\ln k)^3}\ln\left(\frac{\delta\rho}{\rho}\right) \quad , \quad \cdots$$

CONSTRAINTS ON RAPID-ROLL INFLATION

TK, Mukohyama, Powell '09

When
$$P(k) \propto H^2$$
,

$$P(k) = \frac{P(k_0)}{1+A} \left[1 + A \left(\frac{k}{k_0} \right)^{-E} \right]$$

$$n_s(k_0) - 1 = -\frac{AB}{1+A}$$

$$\frac{dn_s}{d\ln k}(k_0) = \frac{AB^2}{(1+A)^2}$$



CONSTRAINTS ON RAPID-ROLL INFLATION

TK, Mukohyama, Powell '09



distinguishing features especially at small scales

CMB & LSS data.

SUMMARY

- Slow-roll and relativistic limits of D-brane inflation suffer from the η-problem and geometrical constraints, respectively.
- Rapid-rolling D-branes exhibit stable inflationary attractors, as well as provide sufficient inflationary expansions.
- Rapid-roll inflationary backgrounds give rise to density perturbation spectra with running spectral index.
- Angular oscillations of D-branes at throat tips can source the primordial density perturbations through the curvaton mechanism.
- Various roles in inflationary cosmology can be shared by the many degrees of freedom that show up in string theory.

BACKUP SLIDES

INFLATON KINETIC TERM

throat geometry

D3

 ρ

$$ds^{2} = h(\rho)^{2} g^{(4)}_{\mu\nu} dx^{\mu} dx^{\nu} + h(\rho)^{-2} (d\rho^{2} + \rho^{2} d\Sigma_{X_{5}}^{2})$$

DBI action
$$S_{\text{DBI}} = -T_3 \int d^4x \sqrt{-\det(G_{\mu\nu})}$$

 $= -T_3 \int d^4x \sqrt{-g^{(4)}} h^4 \sqrt{1 + h^{-4}g^{(4)\alpha\beta}\partial_{\alpha}\rho\partial_{\beta}\rho}$
nflaton
 $d\phi = \sqrt{T_3} d\rho$
 $= \int d^4x \sqrt{-g^{(4)}} \left(-T_3 h^4 - \frac{1}{2}g^{(4)\alpha\beta}\partial_{\alpha}\phi\partial_{\beta}\phi \right)$

ALTERNATIVE APPROACH: DBI INFLATION Silverstein, Tong '04 Inflation driven by D-branes moving with relativistic velocities.

$$ds^{2} = h(\rho)^{2} g^{(4)}_{\mu\nu} dx^{\mu} dx^{\nu} + h(\rho)^{-2} (d\rho^{2} + \rho^{2} d\Sigma_{X_{5}}^{2})$$

$$S_{\text{DBI}} = -T_3 \int d^4 x \sqrt{-\det(G_{\mu\nu})}$$
$$= -T_3 \int d^4 x \sqrt{-g^{(4)}} h(\rho)^4 \sqrt{1 - \frac{\dot{\rho}^2}{h(\rho)^4}}$$

The warping of the throat enforces the D-brane to slow down, regardless of the potential.

 $\frac{\dot{\rho}^2}{h(\rho)^4} < 1$

DBI INFLATION

$$\frac{\mathcal{L}}{\sqrt{-g^{(4)}}} = -T(\phi)\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}} + T(\phi) - V(\phi)$$

$$T(\phi) \equiv T_3 h(\phi)^4$$

• inflation can occur for some $T(\phi)$, $V(\phi)$ \rightarrow a new stringy inflation mechanism

• a remedy to the η -problem (no need for slow-roll)

produces large non-Gaussianity

→ can be tested in future experiments

NEW DIFFICULTIES: GEOMETRICAL CONSTRAINTS ON FIELD RANGE

Baumann, McAllister '06 Lidsey, Huston '07

• DBI inflation generates (too) large non-Gaussianity. In order to suppress non-Gaussianity down to a level consistent with WMAP data, the inflaton need to travel at least some field range $\Delta \phi$.

• On the other hand, the inflaton field range is geometrically restricted by the throat length, which is restricted by the Planck mass.

$$M_p^2 = \frac{2V_6}{(2\pi)^7 g_s^2 \alpha'^4} > \frac{2\text{Vol}(X_5)(\Delta\phi)^6}{(2\pi)^7 g_s^2 \alpha'^4 h_*^4 T_3^3}$$

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CONTRADICTION!

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Throat too short for a relativistically moving D3-brane to drive sufficient inflation.

WRAPPED BRANES

D5,D7

TK, Mukohyama, Kinoshita '07 Becker, Leblond, Shandera '07

D3-brane $d\phi \equiv T_3^{1/2} d\rho$

D3

ρ

D(3+2n)-brane $d\phi \equiv T_{3+2n}^{1/2} \left\{ \int d^{2n} \xi \sqrt{\det(G_{kl} - B_{kl})} \right\}^{1/2} d\rho$

 \sim volume of the wrapped cycle

effective field range increased for wrapped branes Geometrical constraint for DBI inflation can be solved!

BACKGROUND CHARGE

A long throat is sufficient in any case.

→ Large flux number required.

tadpole-cancellation condition

$$N = \frac{\chi}{24}$$

 χ : Euler number of a Calabi-Yau fourfold

The large flux number required exceeds the largest known Euler number for a CY 4-fold $\chi = 1820448$ Klemm, Lian, Roan, Yau 1998

SLOW-ROLL AND RAPID-ROLL INFLATIONS

$$V(\phi) = V_0 \left(1 + \mu \frac{\phi^2}{M_p^2} \right)$$

$$p = \frac{\dot{\phi}}{V_0^{1/2}} \qquad q = \frac{\phi}{M_p}$$



 $\mu = 1/200$

COSMOLOGICAL CONSTRAINTS ON RAPID-ROLL

