The Lagrangian resummation theory and observables in the large-scale structure

Takahiko Matsubara

(Nagoya U.)

@Fuji Calm

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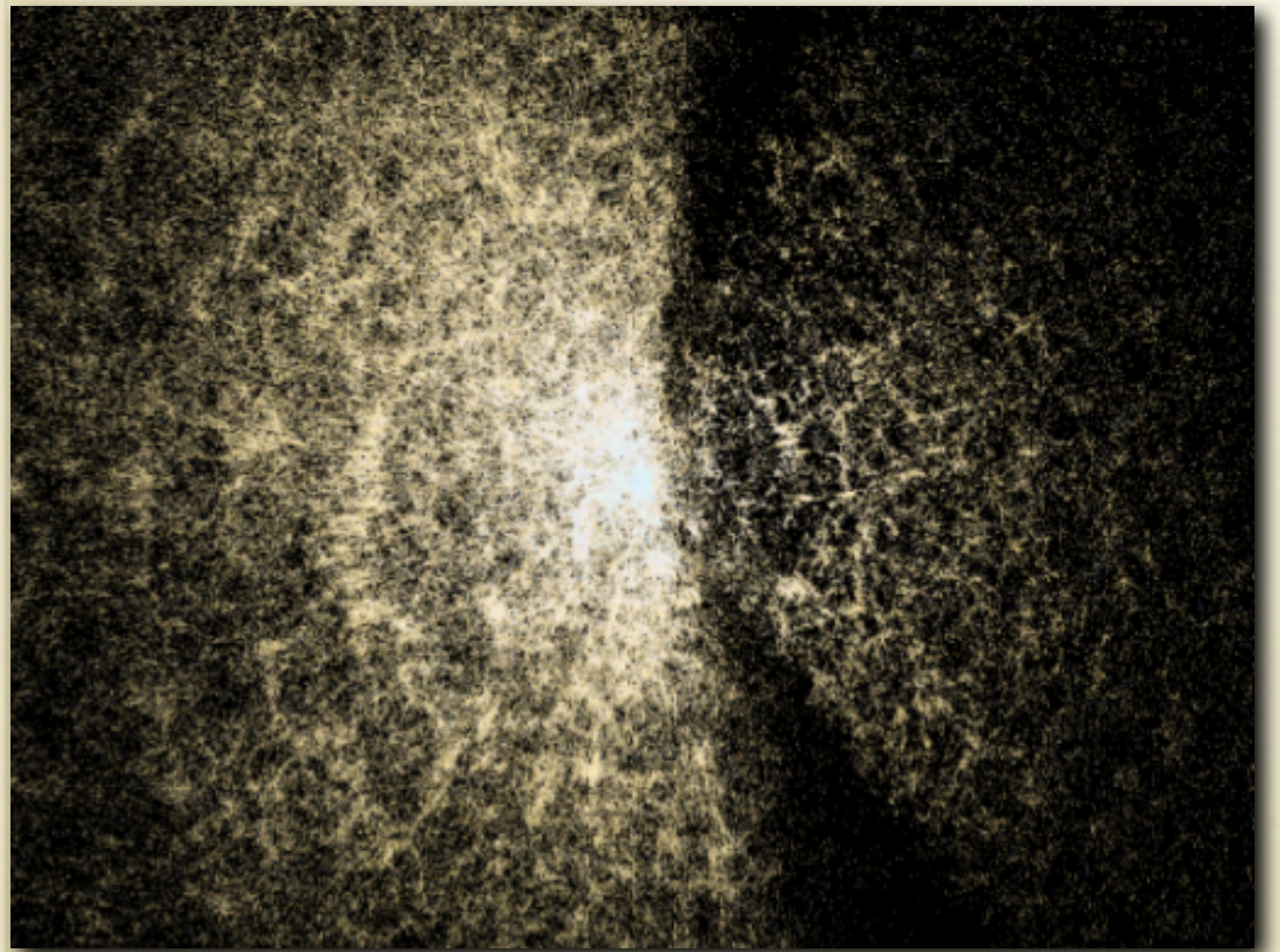
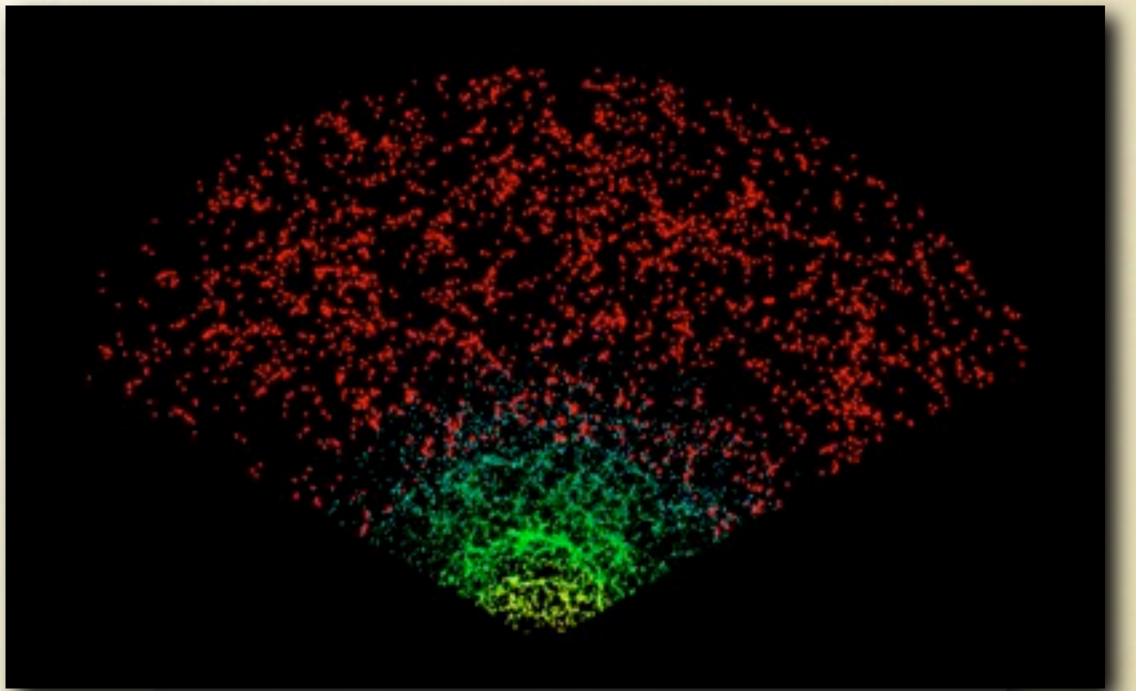
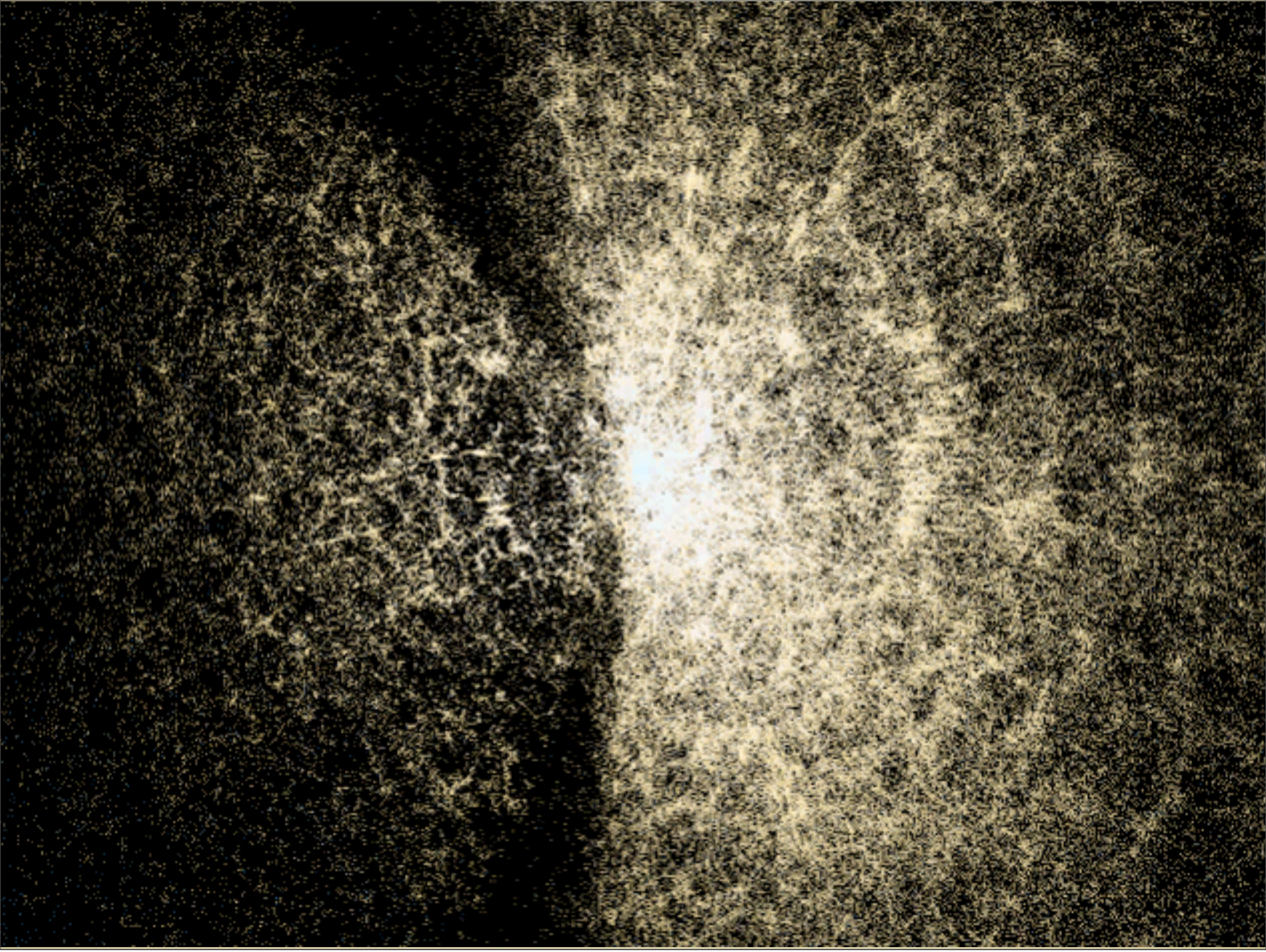
Galaxy redshift survey

- Sloan Digital Sky Survey (SDSS): The largest redshift survey ever made







Dynamics of large-scale structure

- Spatial distritubution of dark matter

  - density fluctuations $\delta(\boldsymbol{x}, t) \equiv \dfrac{\varrho(\boldsymbol{x}, t) - \bar{\varrho}(t)}{\bar{\varrho}(t)}$

  - Evolution equations of collisionless matter in expanding universe

$$\text{Continuity:} \quad \frac{\partial \delta}{\partial t} + \frac{1}{a} \boldsymbol{\nabla} \cdot [(1+\delta)\boldsymbol{v}] = 0$$

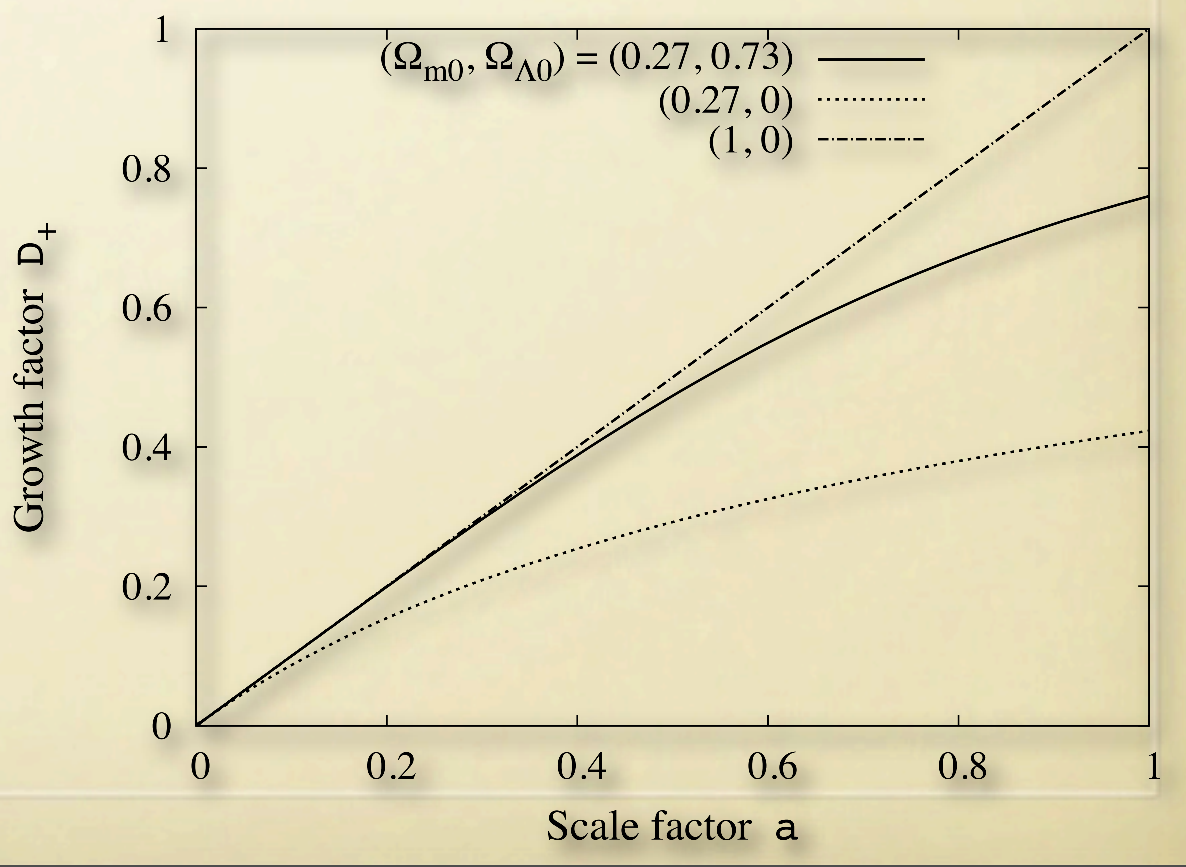
$$\text{Euler:} \quad \frac{\partial \boldsymbol{v}}{\partial t} + \frac{\dot{a}}{a}\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{v} = -\frac{1}{a}\boldsymbol{\nabla}\Phi$$

$$\text{Poisson:} \quad \triangle\Phi = 4\pi G a^2 \bar{\varrho} \delta$$

Linear theory

- Applicability of the linear theory
  - Density fluctuations are small enough
    - high-redshift universe
    - large scales
- Linear evolutions
  - Evolutions of each Fourier mode are INDEPENDENT

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} - 4\pi G\bar{\varrho}\delta = 0$$

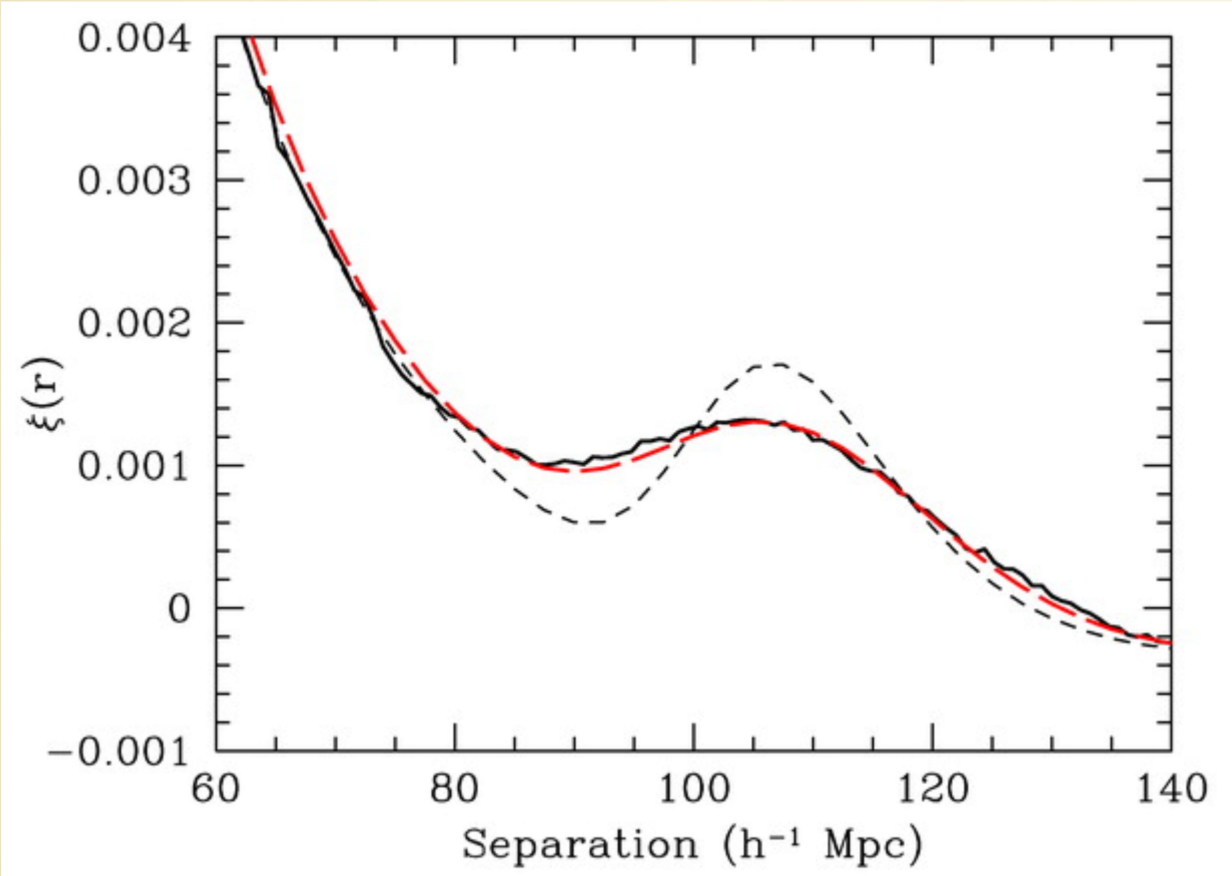


Importance of nonlinearity

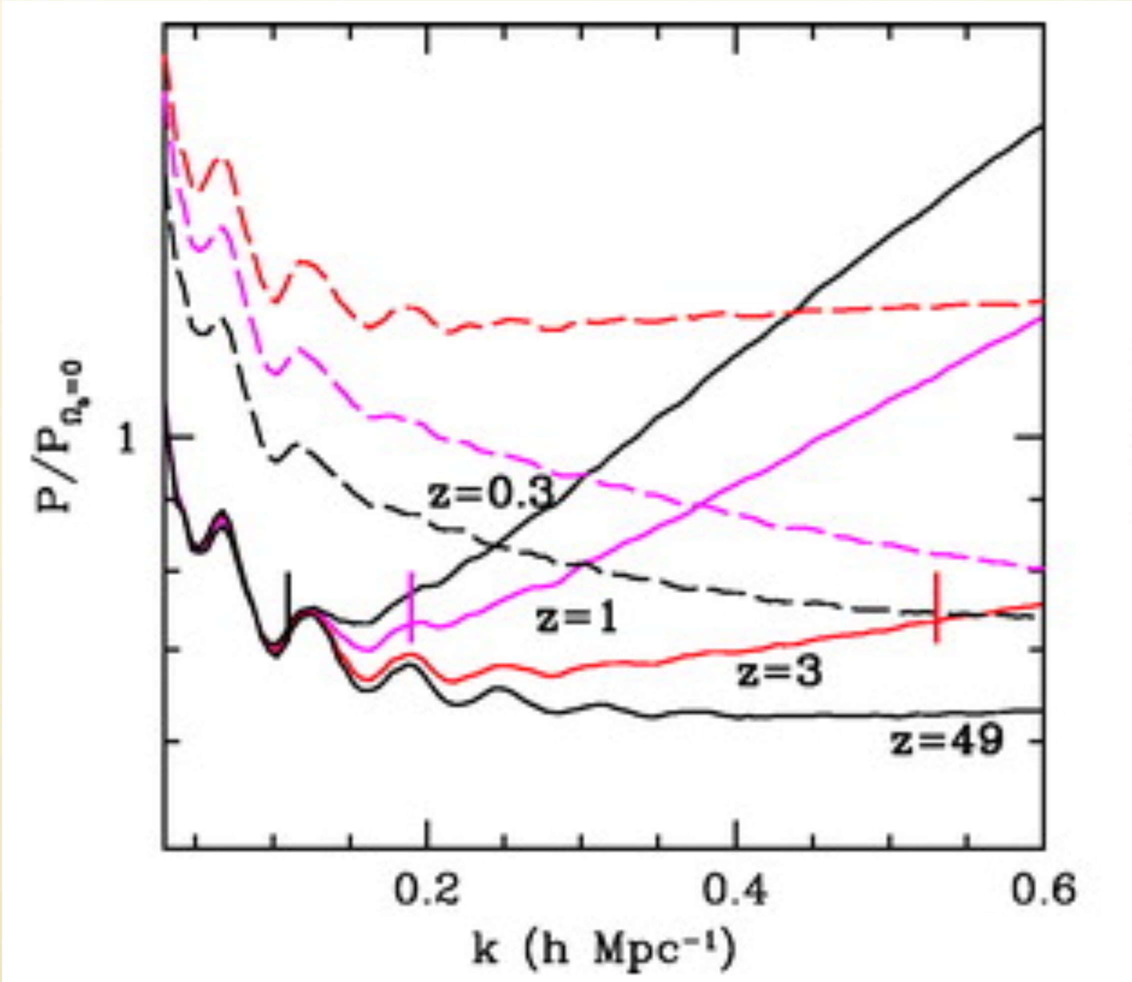
- Precision cosmology: cosmology is now sufficiently quantitative
    - Nature of dark energy
    - Primordial fluctuations (e.g., non-Gaussianity, etc.)
- Nonlinear dynamics on large scales
    - Some time ago, large-scale dynamics are considered to be completely described by linear theory
    - However, the nonlinear dynamics plays an important role on large scales in the era of precision cosmology

Nonlinear dynamics and BAO

- BAO (Baryon Acoustic Oscillations) is a standard ruler to probe the nature of dark energy
- Effects of nonlinear dynamics on BAO
    - linear theory is insufficient to measure BAO scales



Eisenstein, Seo & White (2005)



Eisenstein & Seo (2005)

The nonlinear perturbation theory

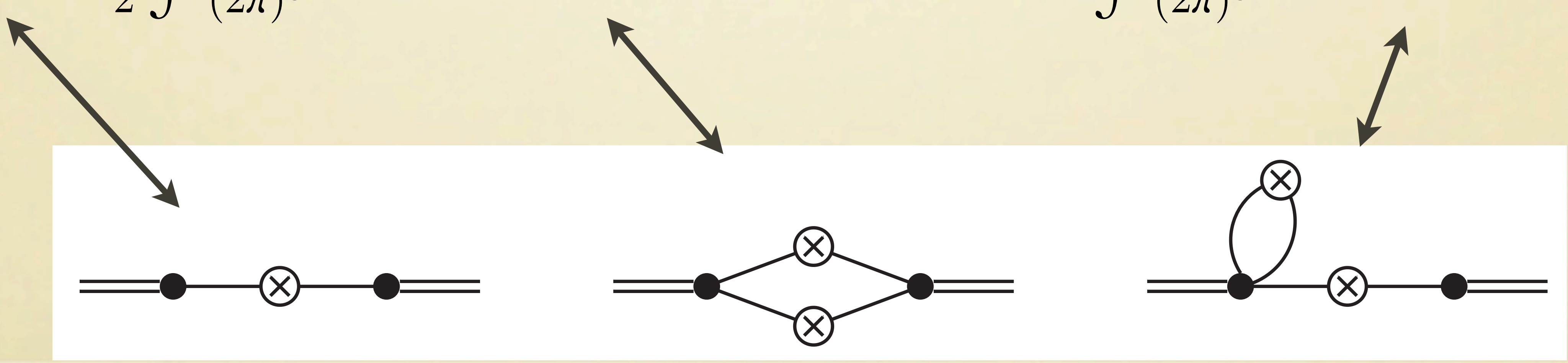
Juszkiewicz (1981), Vishniac (1983), Goroff et al. (1986), Makino et al. (1992), ...

- The nonlinear perturbation theory
  - Evolutions of density fluctuations are solved by perturbative expansions of initial (linear) density field

$$\delta_{\rm m}(\boldsymbol{k}) = \delta_{\rm L}(\boldsymbol{k}) + \frac{1}{2!}\int \frac{d^3k'}{(2\pi)^3} F_2(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}')\delta_{\rm L}(\boldsymbol{k}')\delta_{\rm L}(\boldsymbol{k}-\boldsymbol{k}') + \cdots$$

  - Ex.: the power spectrum $\langle \delta_{\rm m}(\boldsymbol{k})\delta_{\rm m}(\boldsymbol{k}')\rangle = (2\pi)^3\delta^3_{\rm D}(\boldsymbol{k}+\boldsymbol{k}')P_{\rm m}(k)$

$$P_{\rm m}(k) = P_{\rm L}(k) + \frac{1}{2}\int \frac{d^3k'}{(2\pi)^3}[F_2(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}')]^2 P_{\rm L}(k')P_{\rm L}(|\boldsymbol{k}-\boldsymbol{k}'|) + P_{\rm L}(k)\int \frac{d^3k'}{(2\pi)^3} F_3(\boldsymbol{k}', -\boldsymbol{k}', \boldsymbol{k})P_{\rm L}(k') + \cdots$$

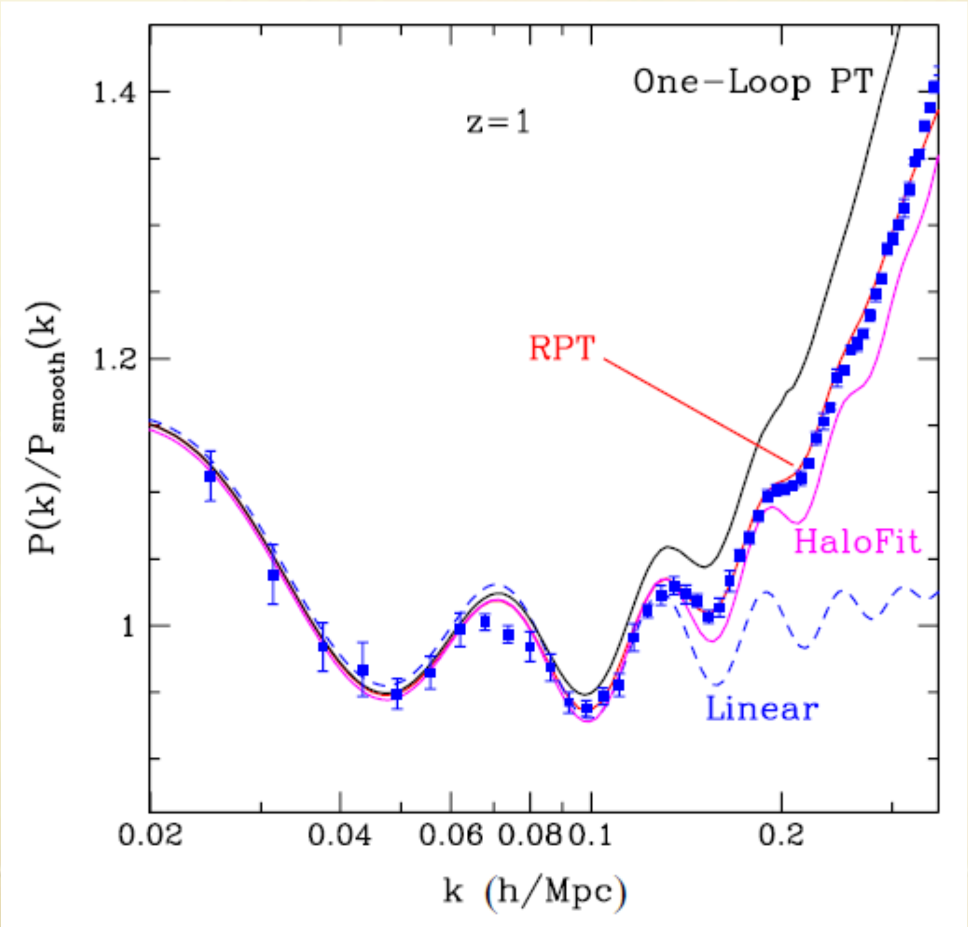


Renormalized perturbation theory etc.

- **Standard perturbation theory (SPT) is still insufficient to describe the redshift range [z ~ 0-3] of interest for BAO**
- **Improving SPT**

Crocce & Scoccimarro (2006), Valageas (2007), Matarrese & Pietroni (2007), Taruya & Hiramatsu (2007), TM (2008),...

  - **Recently, new resummation methods are introduced**
  - **Infinite series of higher-order perturbations are partially resummed (the way of resummation is not unique)**

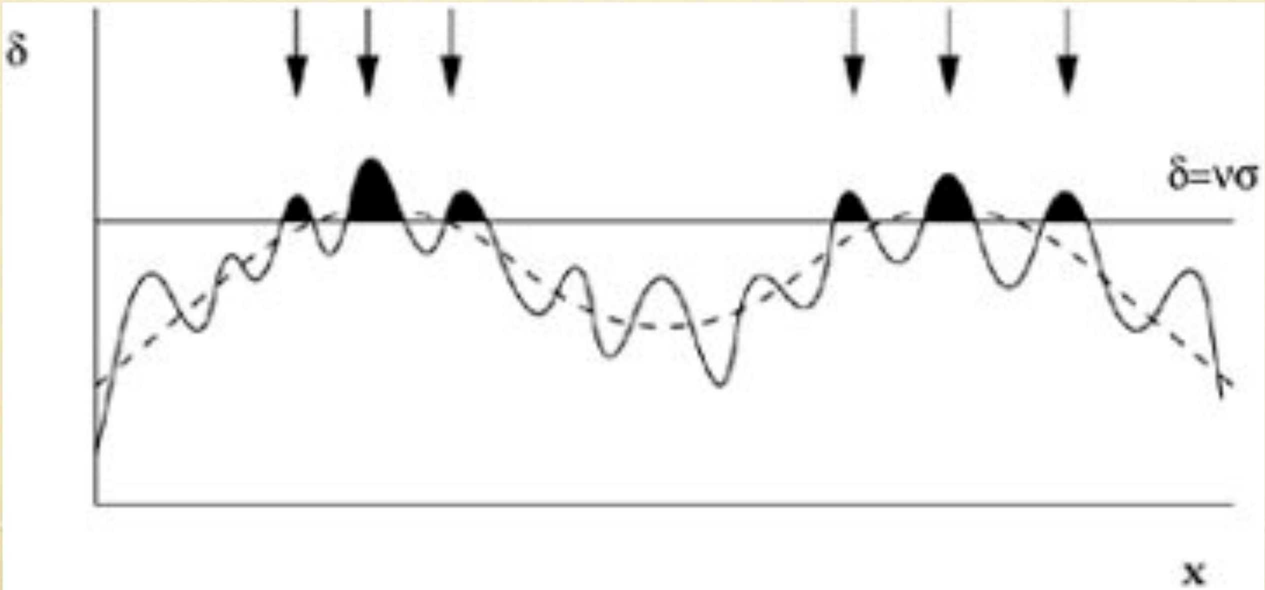


“Renormalized perturbation theory”
Crocce & Scoccimarro (2006)

- However, even if the nonlinear dynamics are solved, there are other issues in comparing theory and observations

Issues of bias

- Distributions of dark matter $\neq$ distributions of galaxies
- galaxy formation is a complicated nonlinear process: difficult
- On large scales, an empirical model "Halo Bias" turns out to be quite useful, and accurately reproduces results of N-body simulations.
    - Halo bias (Mo & White 1996)
        - Based on the Extended Press-Schechter theory
        - Conditional mass function $n(M|\delta_0)$ is predicted in a region where large-scale linear density fluctuation has a particular value δ_0

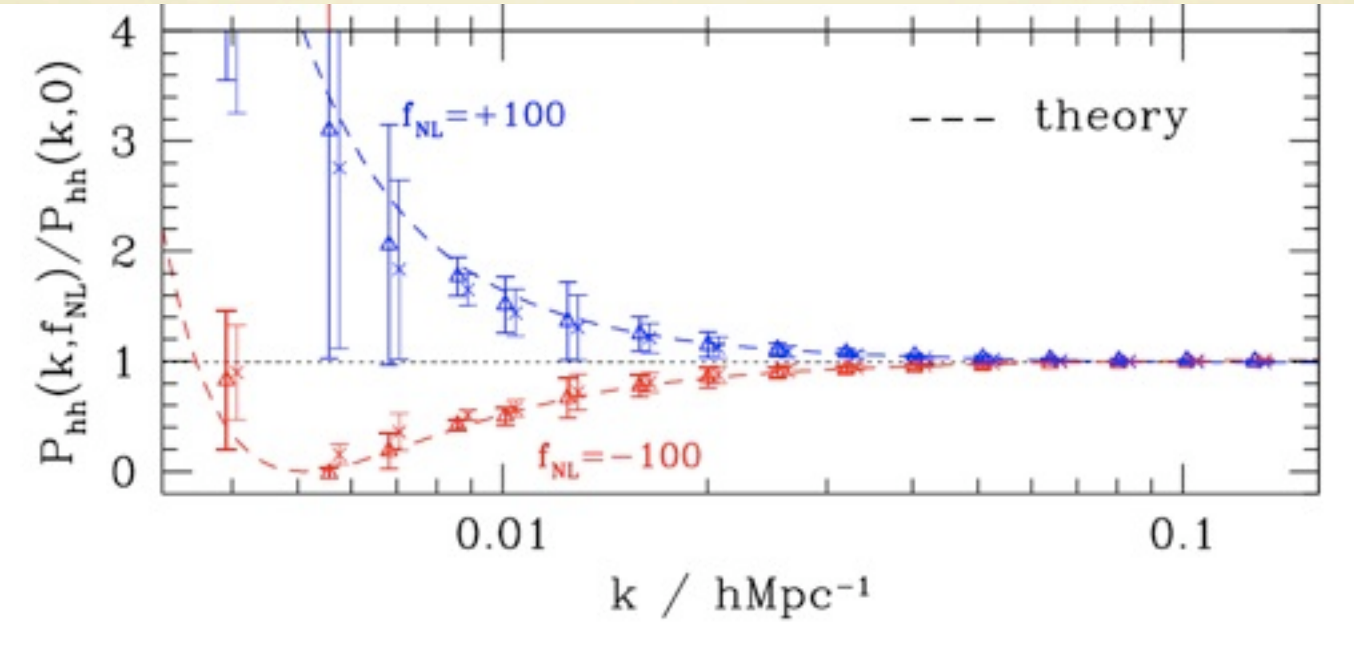


Primordial non-Gaussianity and bias in the large-scale structure

- Primordial non-Gaussianity in the initial density fluctuations
  - Discriminate many possible inflationary theories (or other theories of the early universe)
- Primordial non-Gaussianity and scale-dependent bias
  - Large-scale halo bias becomes scale-dependent in the presence of local-type nG

$$\Phi(\mathbf{r}) = \Phi_L(\mathbf{r}) + f_{\rm NL}\left(\Phi_L^2(\mathbf{r}) - \langle\Phi_L^2(\mathbf{r})\rangle\right)$$

$$\Delta b(M,k) = 3 f_{\rm NL}(b-1)\delta_c \frac{\Omega_m}{k^2 T(k) D(z)}\left(\frac{H_0}{c}\right)^2$$

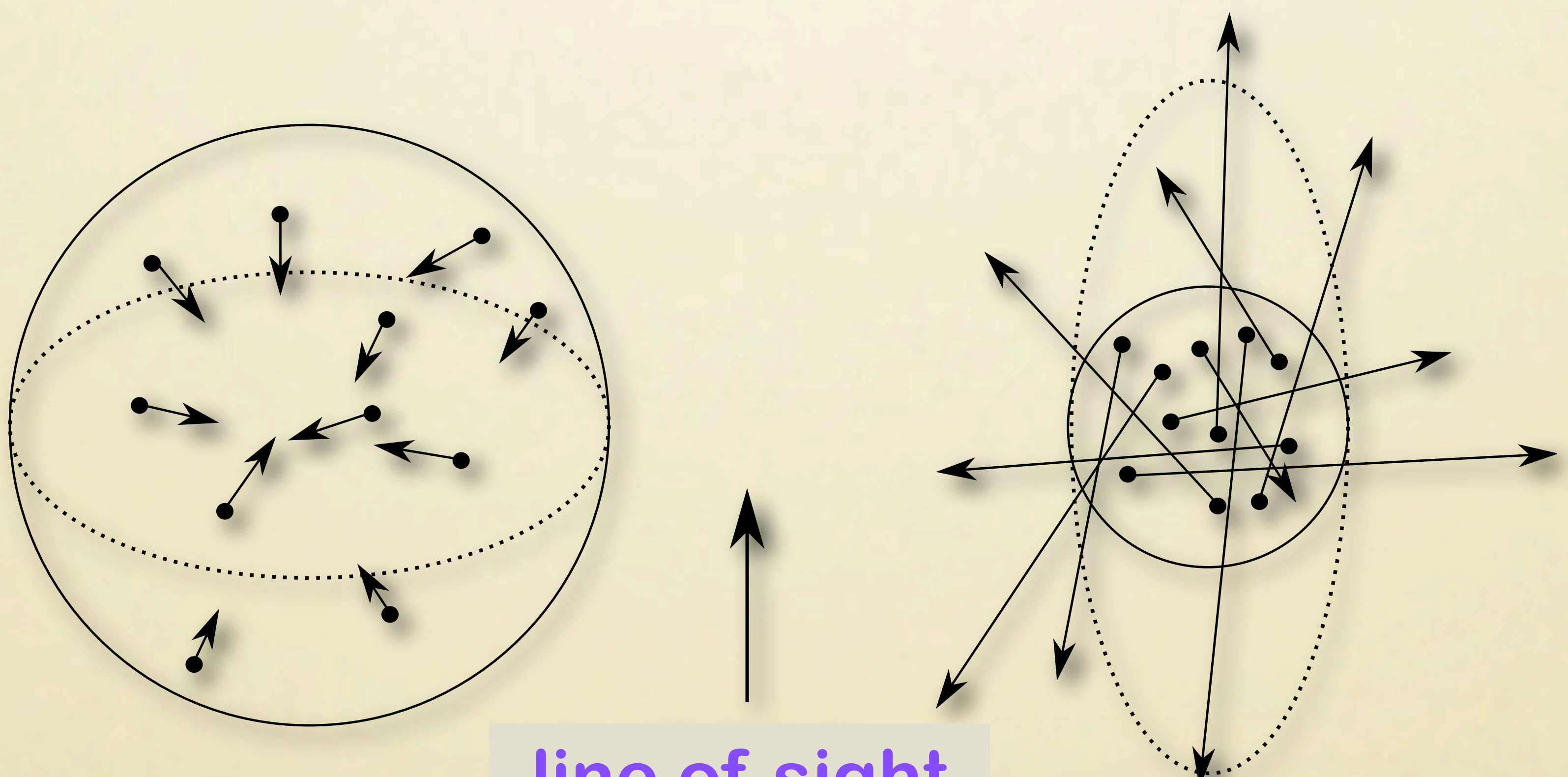


Redshift-space distortions

- observed distributions of galaxies ≠ real distributions
  - radial distances are measured by redshifts
  - peculiar velocities distort the clustering pattern along lines of sight

Large scales:
“Kaiser effect”

small scales:
“Fingers-of-God” effect

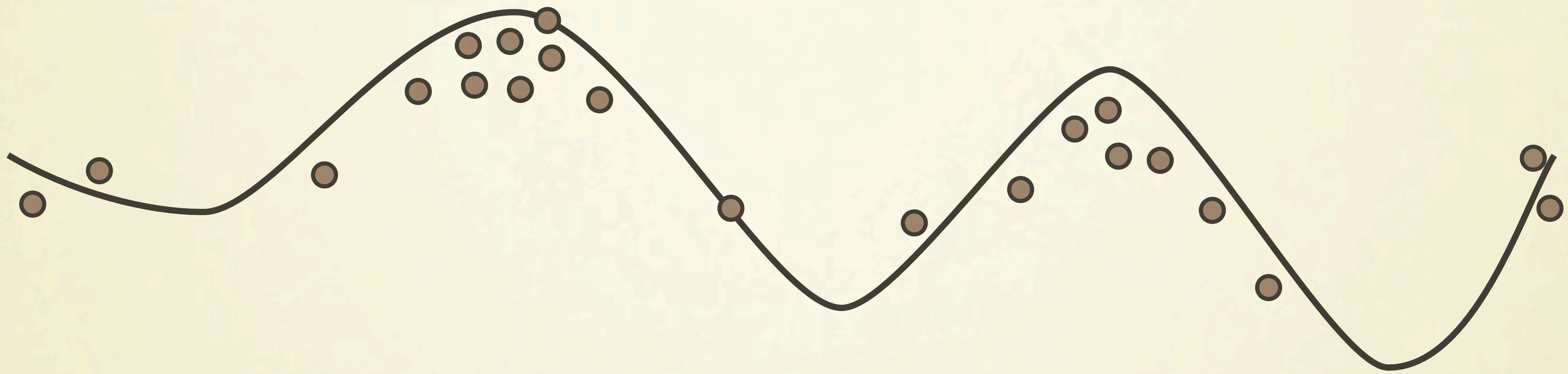


line of sight

Integrated perturbation theory

- Traditional nonlinear perturbation theories
  - nonlinear evolutions of “dark matter” in “real space”
  - Do not correspond to observed distributions
    - what we observe: nonlinear evol. of “galaxy distributions” in “redshift space”
- Integrated perturbation theory (TM 2011):
  - Extension of the standard perturbation theory to include nonlinear bias and redshift-space distortions
  - Model-independent formulation

Bias between mass and objects



- Mass density ≠ number density (in general)
- Densities of both mass and astronomical objects are determined by initial density field
  - There should be a relation

$$\delta_{\mathrm{m}}(\boldsymbol{x}) = \frac{\rho_{\mathrm{m}}(\boldsymbol{x})}{\bar{\rho}_{\mathrm{m}}} - 1 \quad \Leftrightarrow \quad \delta_{\mathrm{X}}(\boldsymbol{x}) = \frac{\rho_{\mathrm{x}}(\boldsymbol{x})}{\bar{\rho}_{\mathrm{x}}} - 1$$

Eulerian Local Bias model

- **Local bias: a simple model usually adopted in the nonlinear perturbation theory**
  - The number density is assumed to be locally determined by (smoothed) mass density

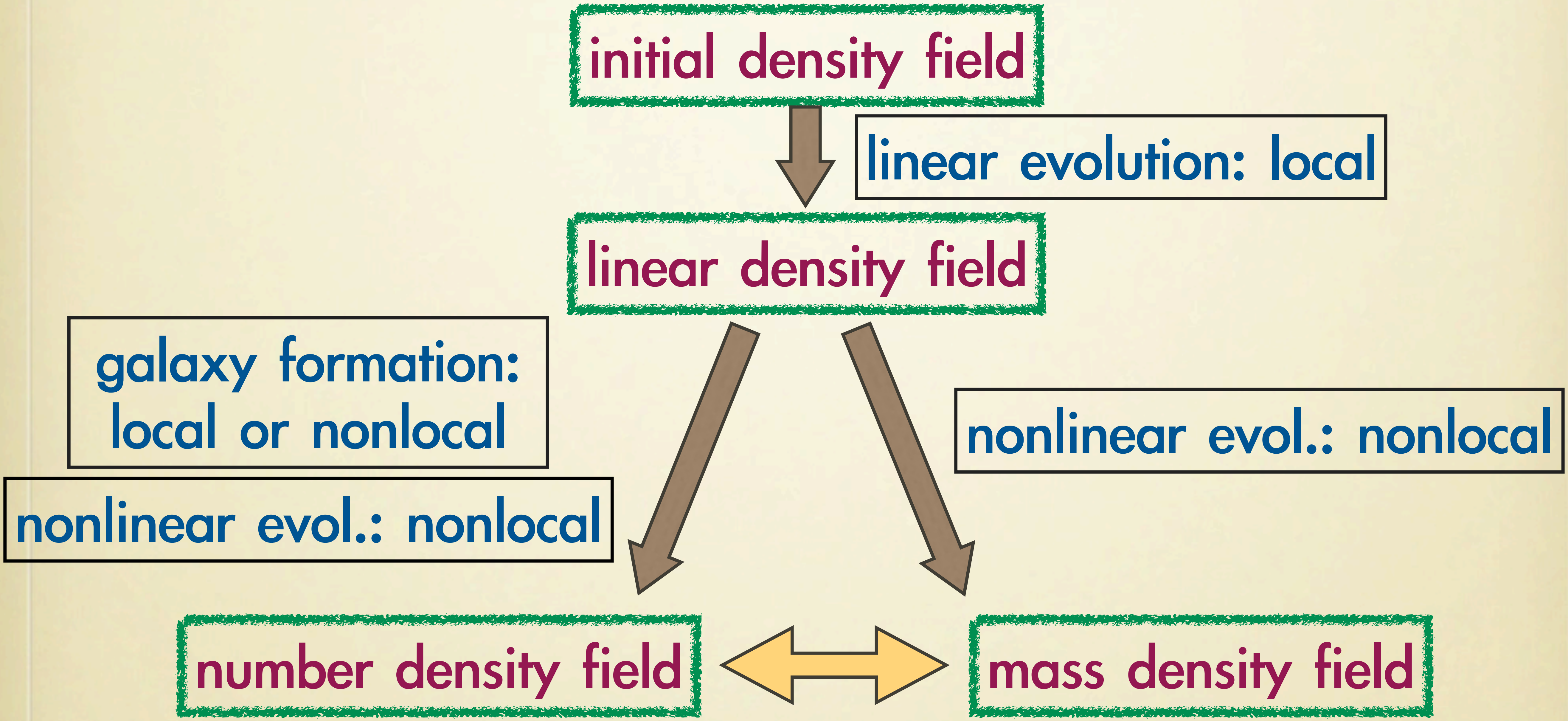
$$\delta_{\mathrm{X}}(\boldsymbol{x}) = F_{\mathrm{X}}(\delta_{\mathrm{L}}(\boldsymbol{x}))$$

  - Apply a Taylor expansion

$$\delta_{\mathrm{X}}(\boldsymbol{x}) = b_0 + b_1\delta_{\mathrm{m}}(\boldsymbol{x}) + \frac{b_2}{2!}\delta_{\mathrm{m}}^2(\boldsymbol{x}) + \cdots$$

  - Phenomenological model, just for simplicity, but divergences in loop corrections

Eulerian Local bias is not physical



Nonlocal relation in general
Local only in linear regime & local galaxy formation

Nonlocal Bias

- "Functional" instead of function

$$\delta_{\mathrm{m}} = \mathcal{F}_{\mathrm{m}}[\delta_{\mathrm{L}}], \quad \delta_{\mathrm{X}} = \mathcal{F}_{\mathrm{X}}[\delta_{\mathrm{L}}]$$

- For a single streaming fluid (quasi-nonlinear)

$$\delta_{\mathrm{L}} = \mathcal{F}_{\mathrm{m}}^{-1}[\delta_{\mathrm{m}}] \quad \Rightarrow \quad \delta_{\mathrm{X}} = \mathcal{F}_{\mathrm{X}}[\mathcal{F}_{\mathrm{m}}^{-1}[\delta_{\mathrm{m}}]]$$

- Taylor expansion of the functional

$$\delta_{\mathrm{X}}(\boldsymbol{x}) = \int d^3x_1 b_1(\boldsymbol{x} - \boldsymbol{x}_1)\delta_{\mathrm{m}}(\boldsymbol{x}_1) + \frac{1}{2!}\int d^3x_1 d^3x_2 b_2(\boldsymbol{x} - \boldsymbol{x}_1, \boldsymbol{x} - \boldsymbol{x}_2)\delta_{\mathrm{m}}(\boldsymbol{x}_1)\delta_{\mathrm{m}}(\boldsymbol{x}_2) + \cdots$$

Perturbation theory with nonlocal bias

- Perturbative expansions in Fourier space:
  - nonlocal bias:

$$\delta_X(\boldsymbol{k}) = b_1(\boldsymbol{k})\delta_m(\boldsymbol{k}) + \frac{1}{2!}\int \frac{d^3k'}{(2\pi)^3} b_2(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}')\delta_m(\boldsymbol{k}')\delta_m(\boldsymbol{k}-\boldsymbol{k}') + \cdots$$

  - nonlinear dynamics

$$\delta_m(\boldsymbol{k}) = \delta_L(\boldsymbol{k}) + \frac{1}{2!}\int \frac{d^3k'}{(2\pi)^3} F_2(\boldsymbol{k}', \boldsymbol{k}-\boldsymbol{k}')\delta_L(\boldsymbol{k}')\delta_L(\boldsymbol{k}-\boldsymbol{k}') + \cdots$$

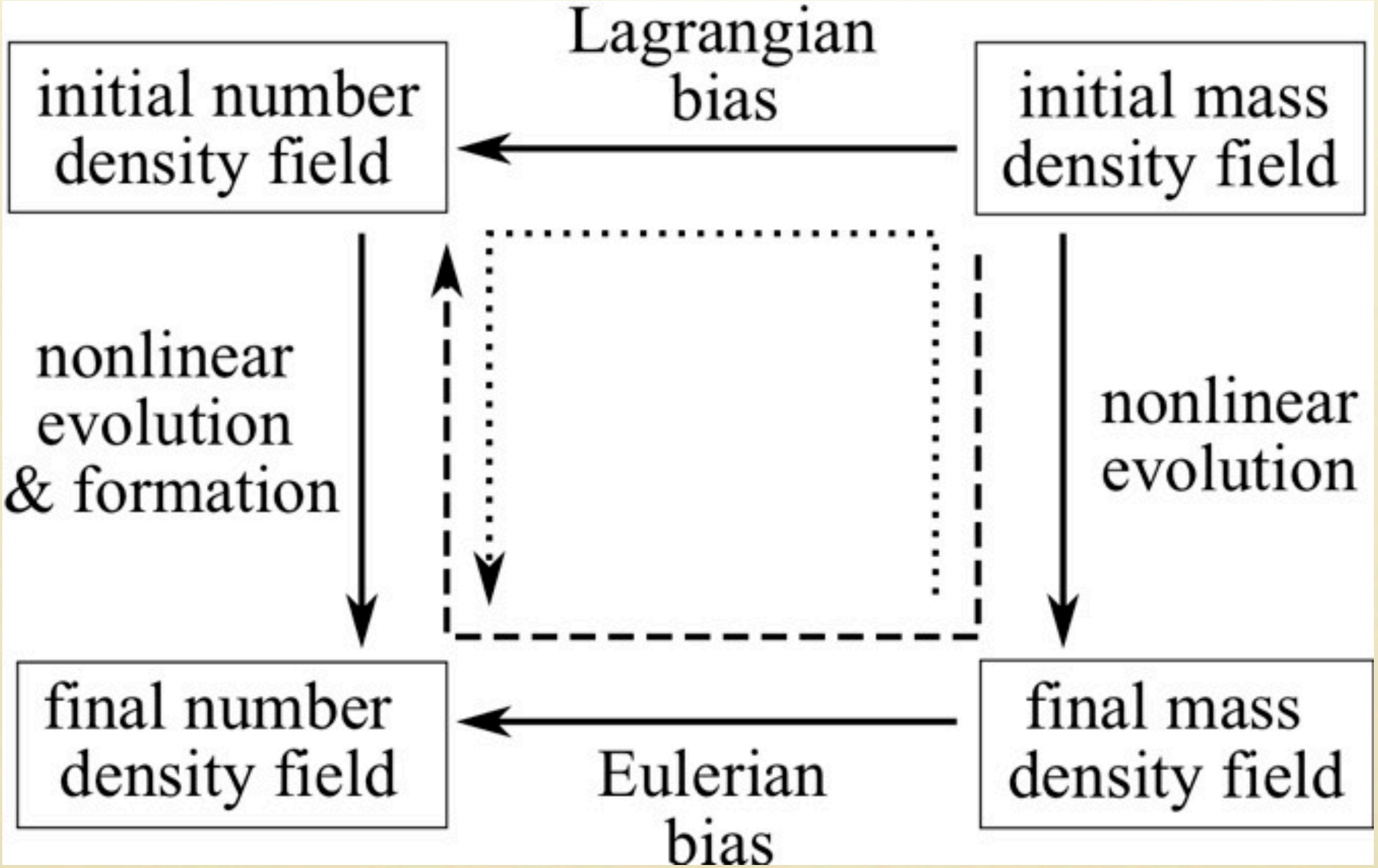
  - It is straightforward to calculate observables, such as power spectrum, bispectrum, etc.

The problem with Eulerian bias

- No physical model of Eulerian nonlocal bias !!
- Physical models of bias known so far is provided in Lagrangian space
  - e.g., Halo bias model, Peak bias model,...
  - In those models, conditions of galaxy formation are imposed on initial (Lagrangian) density field
- What is the relation between Eulerian bias and Lagrangian bias?

Eulerian local bias

- The Eulerian local bias in nonlinear perturbation theory is dynamically inconsistent



Eulerian and Lagrangian bias

- Equivalence of Eulerian and Lagrangian nonlocal bias
  - Nonlocal Eulerian and Lagrangian biases are equivalent. Only representations are different
  - The relations can be explicitly derived in perturbation theory:

$$b_1(\boldsymbol{k}) = b_1^{\mathrm{L}}(\boldsymbol{k}) + 1,$$

$$b_2(\boldsymbol{k}_1, \boldsymbol{k}_2) = b_2^{\mathrm{L}}(\boldsymbol{k}_1, \boldsymbol{k}_2) - b_1^{\mathrm{L}}(\boldsymbol{k}_1 + \boldsymbol{k}_2) F_2(\boldsymbol{k}_1, \boldsymbol{k}_2) + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_1^2}\right) b_1^{\mathrm{L}}(\boldsymbol{k}_2) + \left(1 + \frac{\boldsymbol{k}_1 \cdot \boldsymbol{k}_2}{k_2^2}\right) b_1^{\mathrm{L}}(\boldsymbol{k}_1),$$

etc.

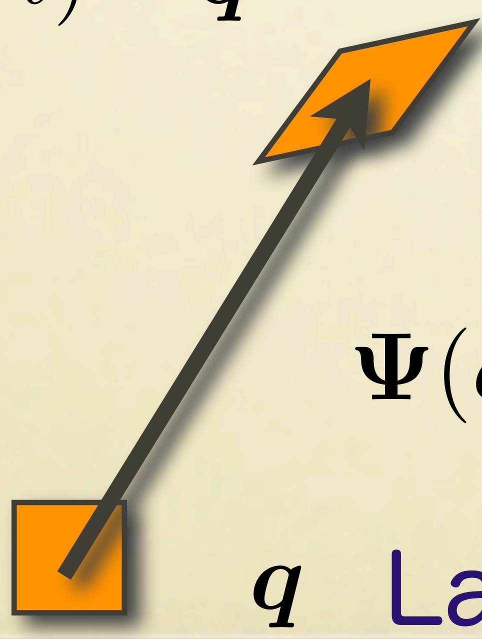
local biases are incompatible! At least one must be nonlocal

Lagrangian perturbation theory

- Lagrangian perturbation theory
  - suitable for handling Lagrangian bias
- Fundamental variables in Lagrangian picture
  - Displacement field

Buchert (1989)

$$\boldsymbol{\Psi}(\boldsymbol{q},t) = \boldsymbol{x}(\boldsymbol{q},t) - \boldsymbol{q}$$



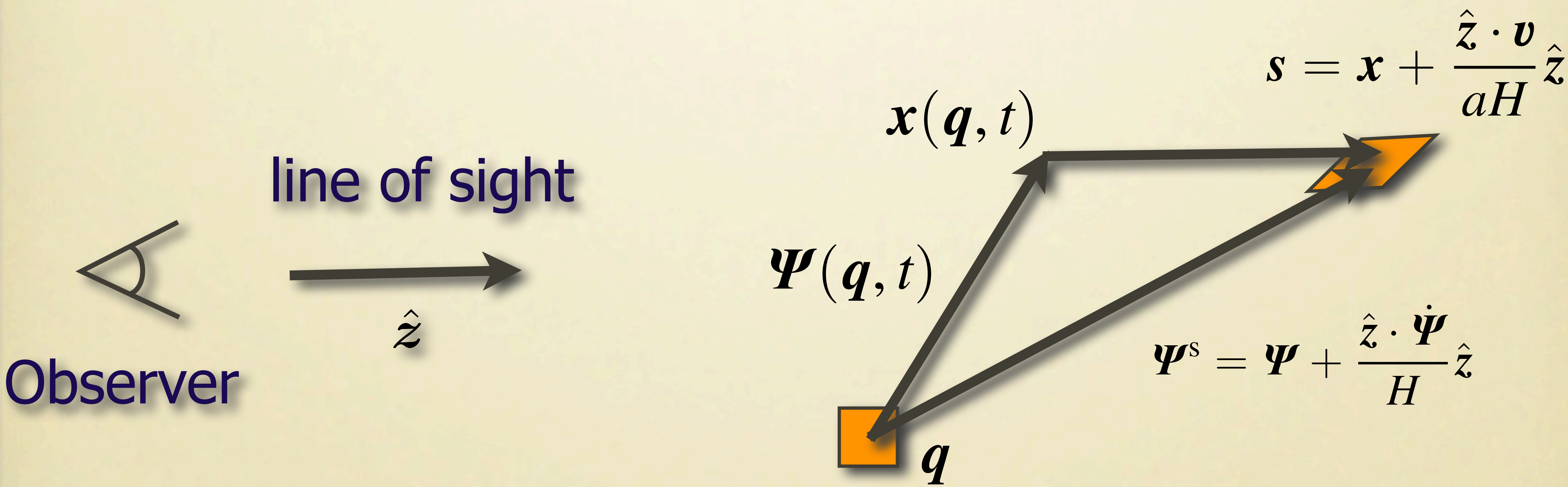
$\boldsymbol{x}(\boldsymbol{q},t)$ Eulerian (final) position

$\boldsymbol{\Psi}(\boldsymbol{q},t)$ Displacement vector

$\boldsymbol{q}$ Lagrangian (initial) position

Redshift-space distortions

- Redshift-space distortions are easily incorporated to Lagrangian perturbation theory
  - Mapping from real space to redshift space is exactly linear in Lagrangian variables   c.f.) nonlinear in Eulerian
  - Mapping of the displacement field



Lagrangian perturbation theory with Lagrangian (nonlocal) bias

- The relation between Eulerian density fluctuations and Lagrangian variables

$$1+\delta_{\mathrm{X}}(\boldsymbol{x}) = \int d^3q \left[1+\delta^{\mathrm{L}}_{\mathrm{X}}(\boldsymbol{q})\right] \delta^3_{\mathrm{D}}[\boldsymbol{x}-\boldsymbol{q}-\boldsymbol{\Psi}(\boldsymbol{q})]$$

Eulerian density field

Biased field in Lagrangian space

displacement (& redshift distortions)

- Perturbative expansion in Fourier space

Kernel of the Lagrangian bias

$$\delta^{\mathrm{L}}_{\mathrm{X}}(\boldsymbol{k}) = \sum_{n=1}^{\infty} \frac{1}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta^3_{\mathrm{D}}(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k}) b^{\mathrm{L}}_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) \delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta_{\mathrm{L}}(\boldsymbol{k}_n)$$

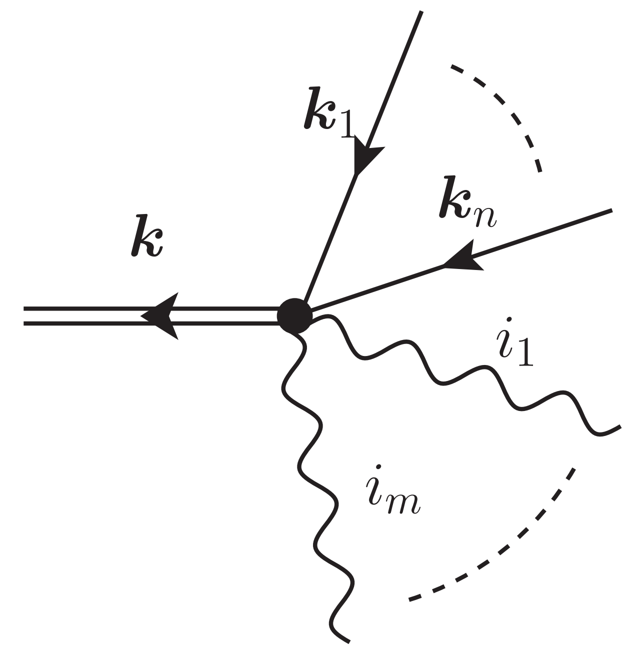
$$\tilde{\boldsymbol{\Psi}}(\boldsymbol{k}) = \sum_{n=1}^{\infty} \frac{i}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta^3_{\mathrm{D}}(\boldsymbol{k}_{1\cdots n}-\boldsymbol{k}) \boldsymbol{L}_n(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) \delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta_{\mathrm{L}}(\boldsymbol{k}_n)$$

Kernel of the displacement field (& redshift distortions)

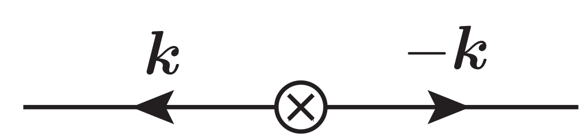
$$\boldsymbol{k}_{1\cdots n} \equiv \boldsymbol{k}_1 + \cdots + \boldsymbol{k}_n$$

Diagrammatics

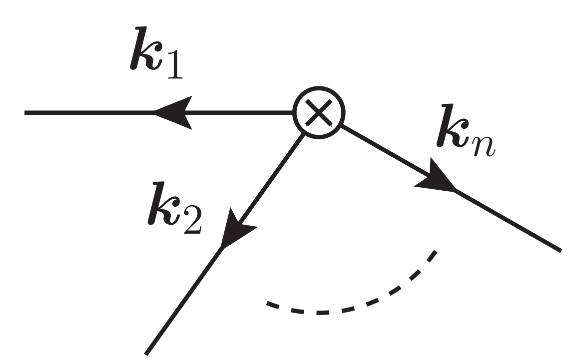
- Introducing diagrammatic rules is useful



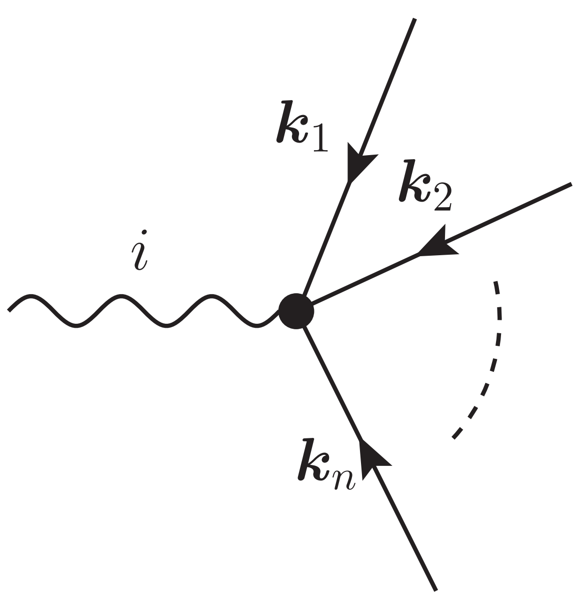
$$\Leftrightarrow \quad b_n^{\mathrm{L}}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) k_{i_1} \cdots k_{i_m}$$



$$\Leftrightarrow \quad P_{\mathrm{L}}(k)$$



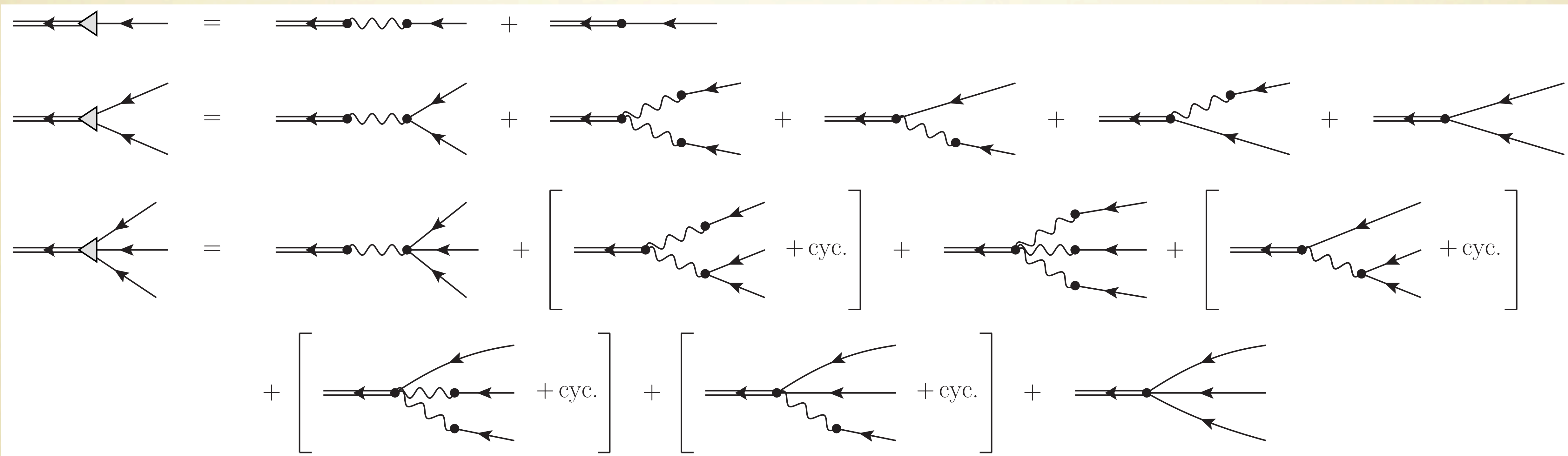
$$\Leftrightarrow \quad P_{\mathrm{L}}^{(n)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n)$$



$$\Leftrightarrow \quad L_{n,i}(\boldsymbol{k}_1, \boldsymbol{k}_2, \ldots, \boldsymbol{k}_n)$$

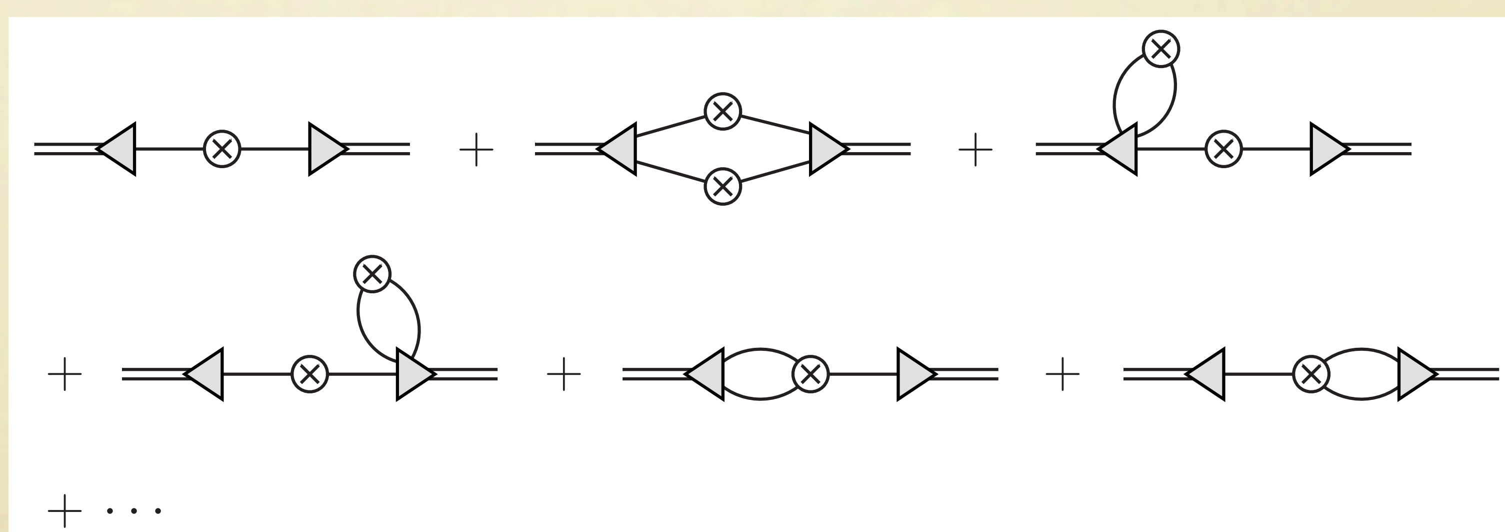
Diagrammatics

- Shrunk vertices



- Example: power spectrum

$$P_{\mathrm{X}}(k) =$$



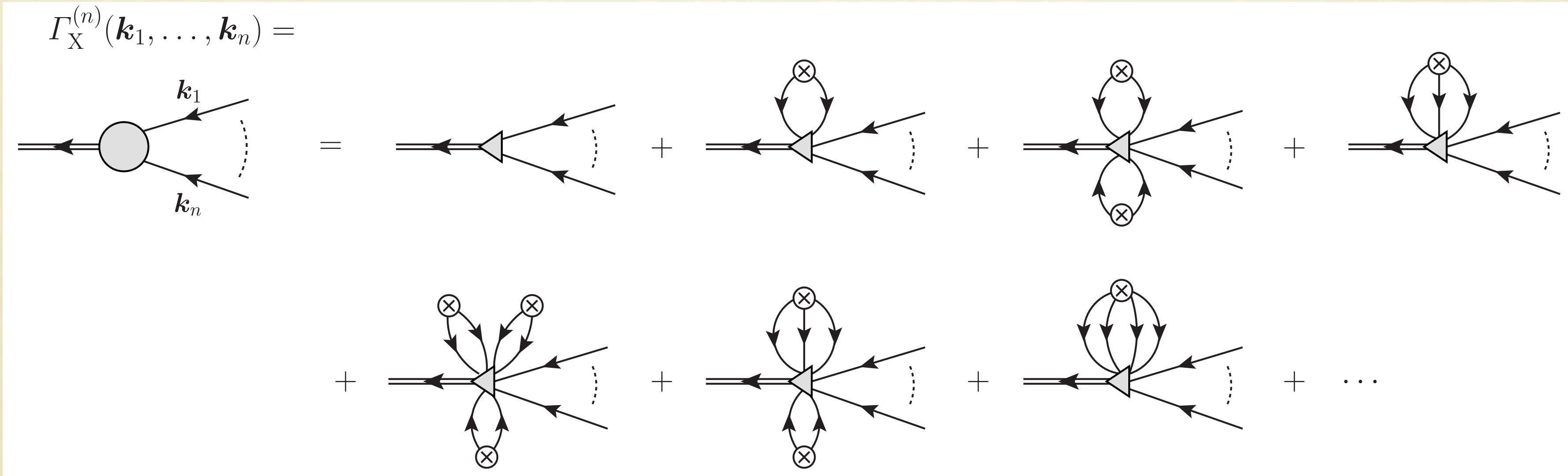
Multi-point propagator

- **Multi-point propagator**
  - Responses to the nonlinear density field from initial density fluc.
  - Central role in renormalized perturbation theory

Crocce & Scoccimarro (2006), Bernardeau et al. (2008)

  - Define corresponding quantity in Lagrangian perturbation theory and Lagrangian bias

$$\left\langle \frac{\delta^n \delta_{\rm X}(\boldsymbol{k})}{\delta\delta_{\rm L}(\boldsymbol{k}_1)\cdots\delta\delta_{\rm L}(\boldsymbol{k}_n)} \right\rangle = (2\pi)^{3-3n}\delta_{\rm D}^3(\boldsymbol{k}-\boldsymbol{k}_{1\cdots n})\Gamma_{\rm X}^{(n)}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n)$$



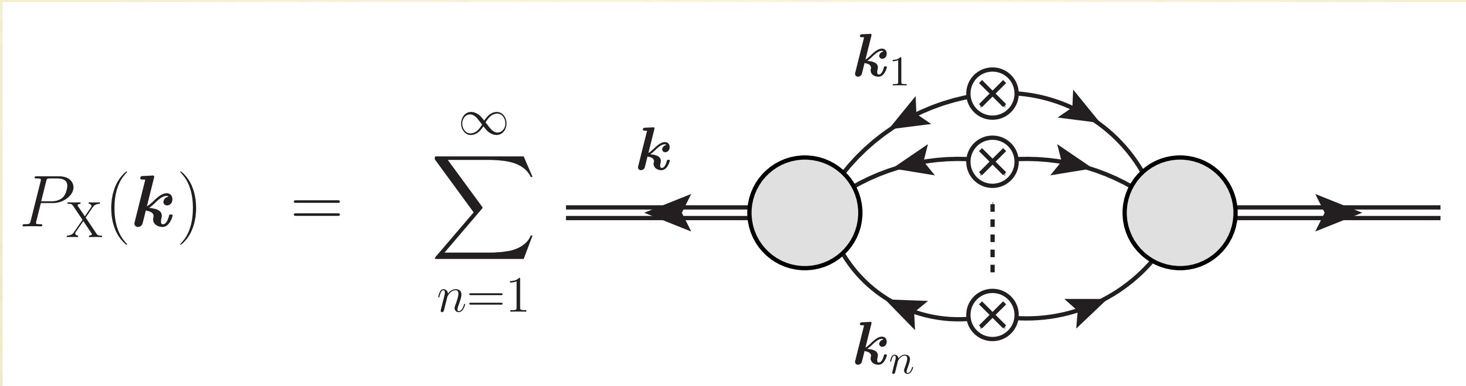
  - Renormalization of external vertices

Multi-point propagator

- Example: nonlinear power spectrum in terms of multi-point propagator

Bernardeau et al. (2008)

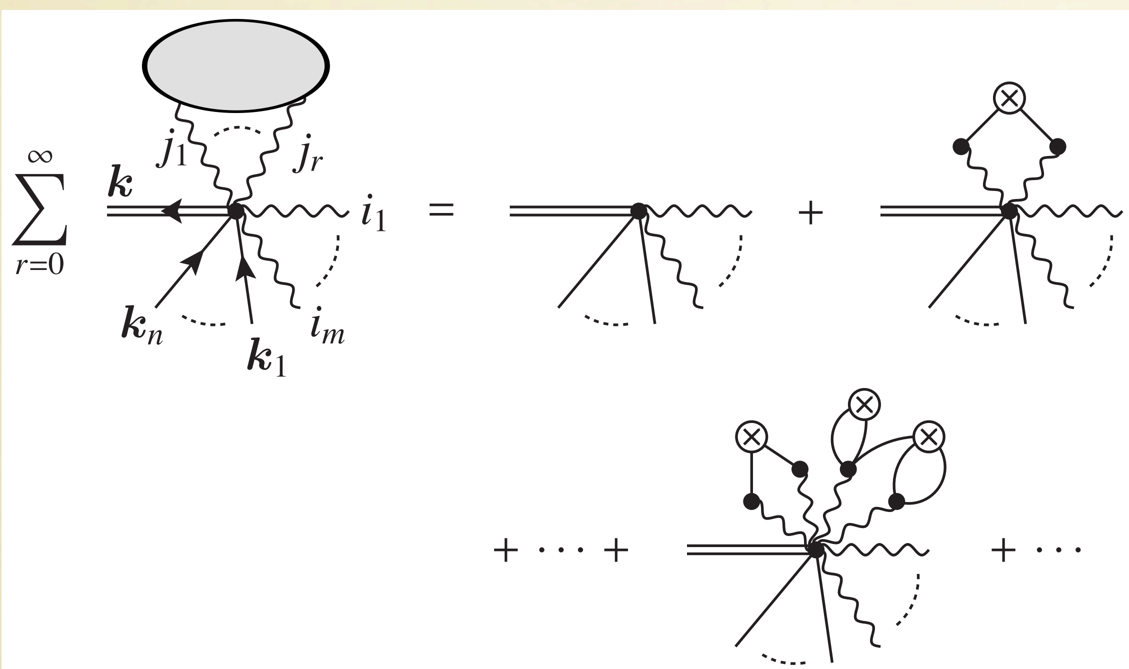
$$P_X(\boldsymbol{k}) = \sum_{n=0}^{\infty} \frac{1}{n!} \int \frac{d^3k_1}{(2\pi)^3} \cdots \frac{d^3k_n}{(2\pi)^3} (2\pi)^3 \delta_D^3(\boldsymbol{k} - \boldsymbol{k}_{1\cdots n})$$
$$\times \left| \Gamma_X^{(n)}(\boldsymbol{k}_1, \ldots, \boldsymbol{k}_n) \right|^2 P_L(k_1) \cdots P_L(k_n)$$

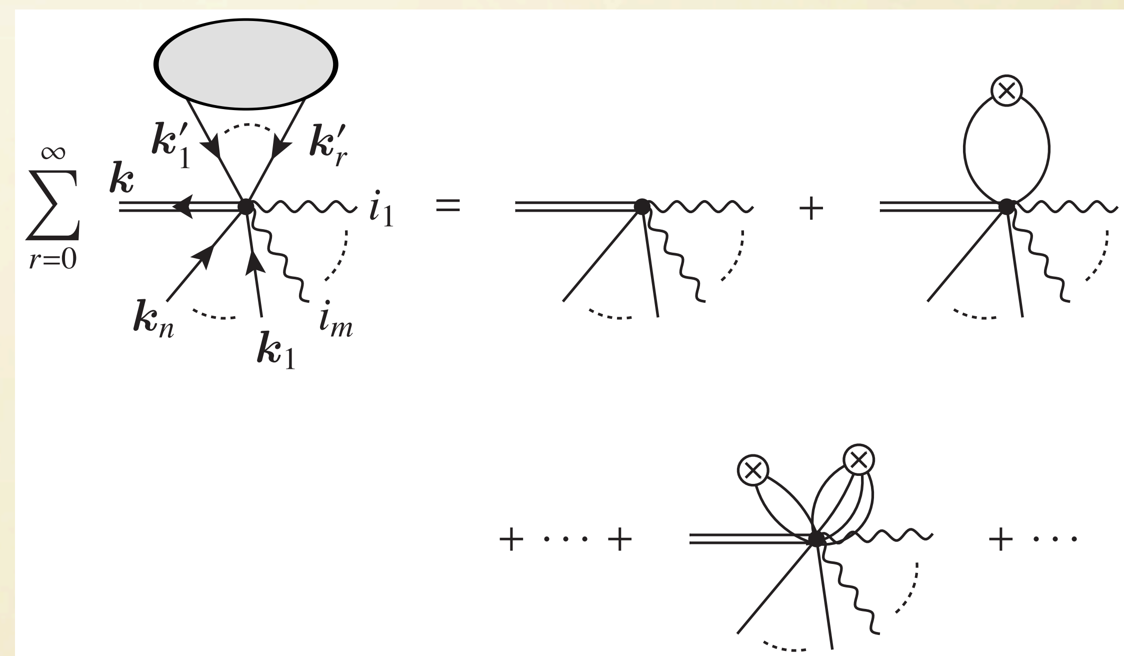


- No way of obtaining exact multi-point propagator
  - In renormalized perturbation theory, large-k limit and one-loop approximation are interpolated by hand

Multi-point propagator

- **Partial renormalization**
  - **Infinite series are partially resummed in Lagrangian bias + Lagrangian perturbation theory**





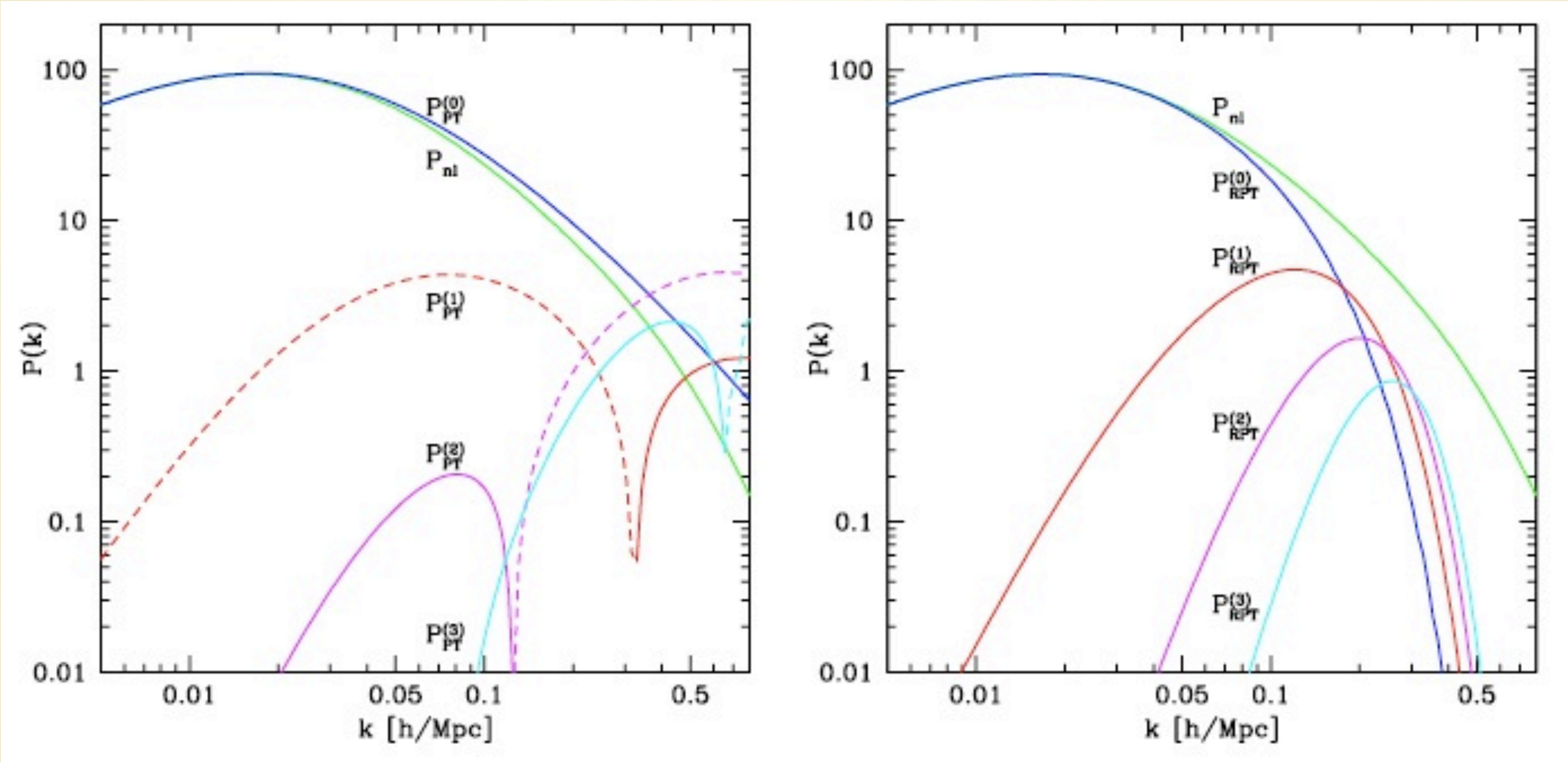
$$\Pi(\boldsymbol{k}) = \left\langle e^{-i\boldsymbol{k}\cdot\boldsymbol{\Psi}} \right\rangle = \exp\left[\sum_{n=1}^{\infty} \frac{(-i)^n}{n!} \langle (\boldsymbol{k}\cdot\boldsymbol{\Psi})^n \rangle_{\mathrm{c}} \right]$$

$$b_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n} \int \frac{d^3k'}{(2\pi)^3} \left. \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right|_{\delta_{\mathrm{L}}=0}$$

$$\Rightarrow\ c_n^{\mathrm{L}}(\boldsymbol{k}_1,\ldots,\boldsymbol{k}_n) = (2\pi)^{3n} \int \frac{d^3k'}{(2\pi)^3} \left\langle \frac{\delta^n \delta_{\mathrm{X}}^{\mathrm{L}}(\boldsymbol{k}')}{\delta\delta_{\mathrm{L}}(\boldsymbol{k}_1)\cdots\delta\delta_{\mathrm{L}}(\boldsymbol{k}_n)} \right\rangle$$

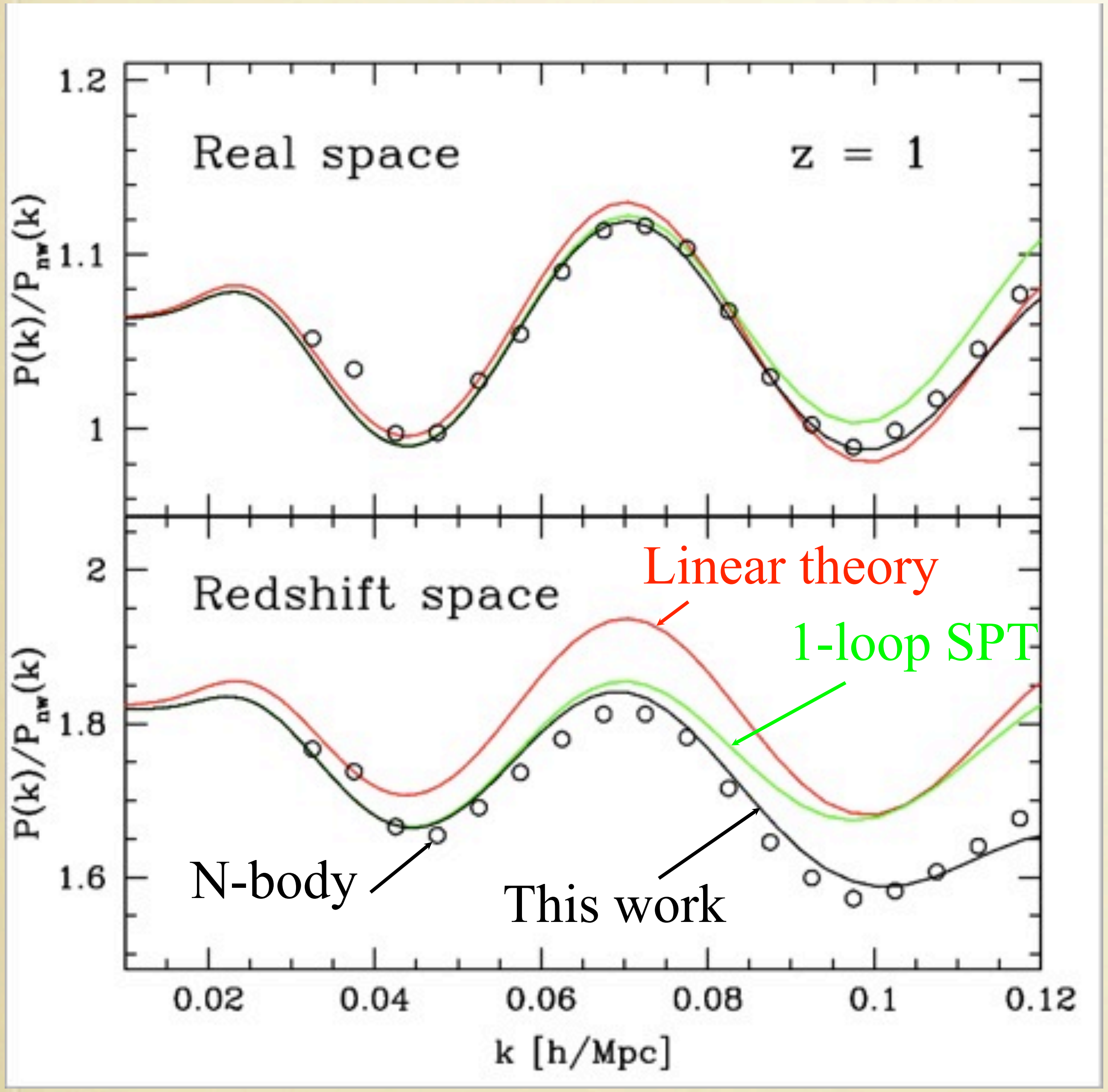
Positiveness

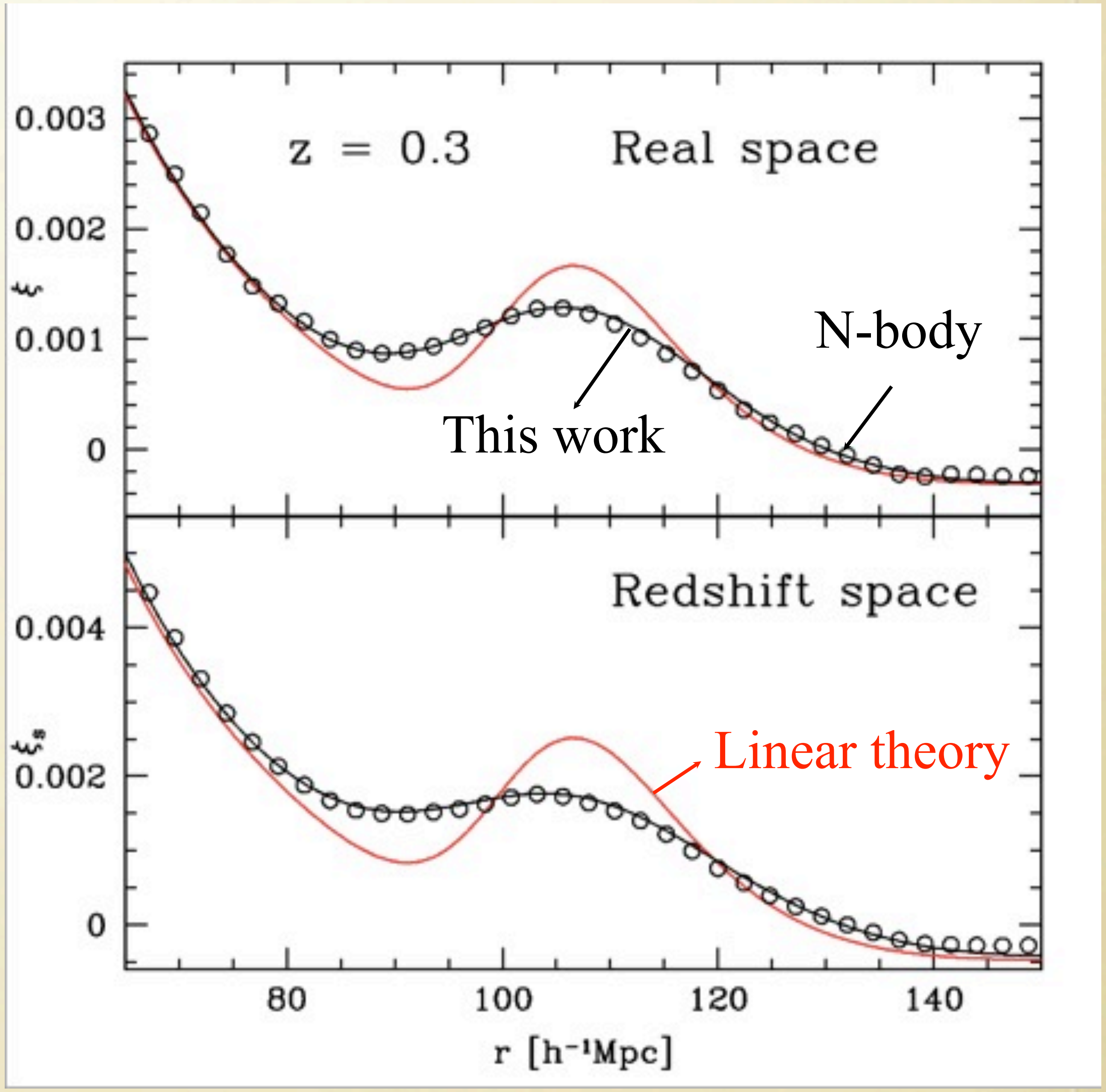
- Each term in the resummed series is positive and add constructively (common feature with RPT)



Crocce & Scoccimarro (2006)

Application: Baryon Acoustic Oscillations

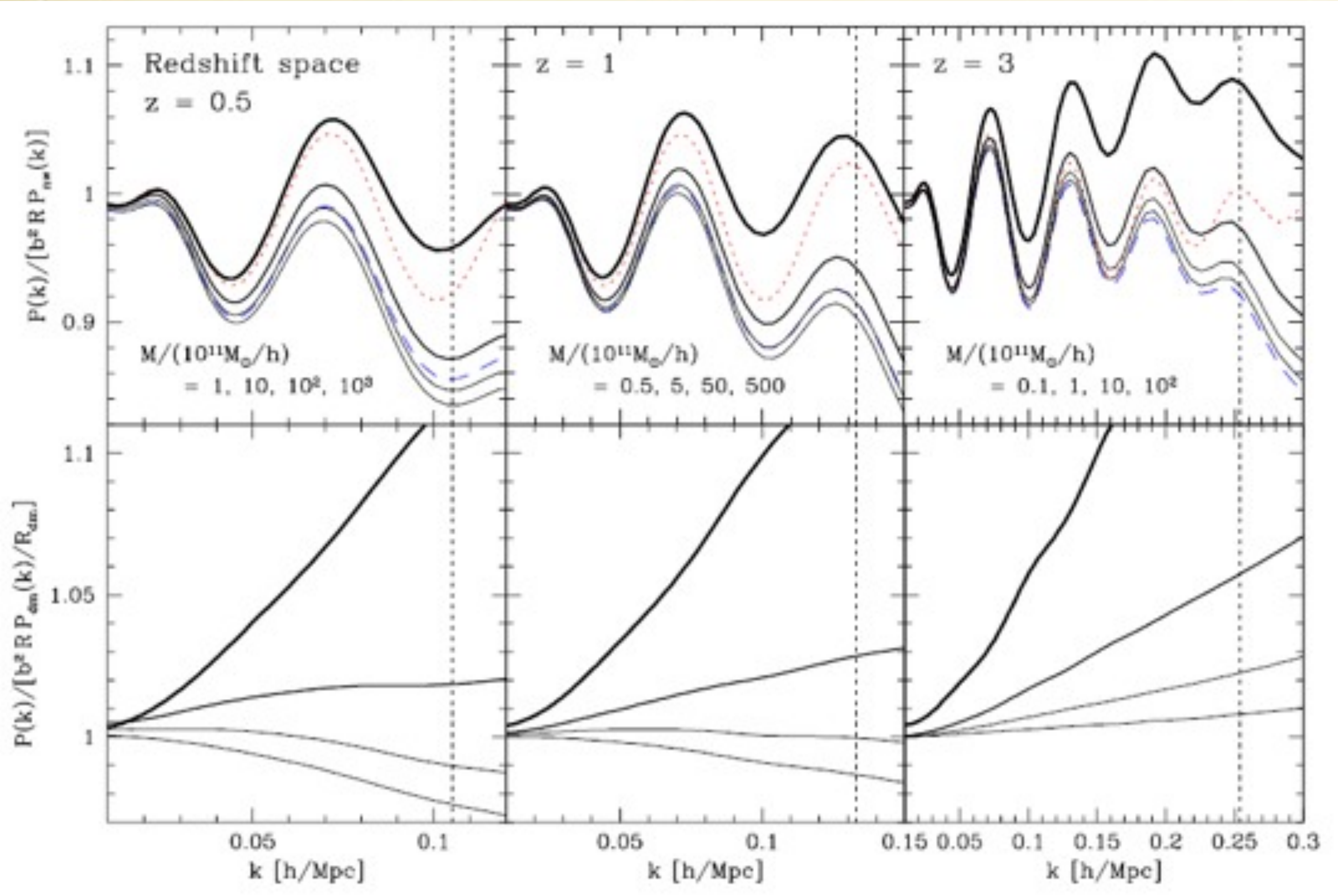


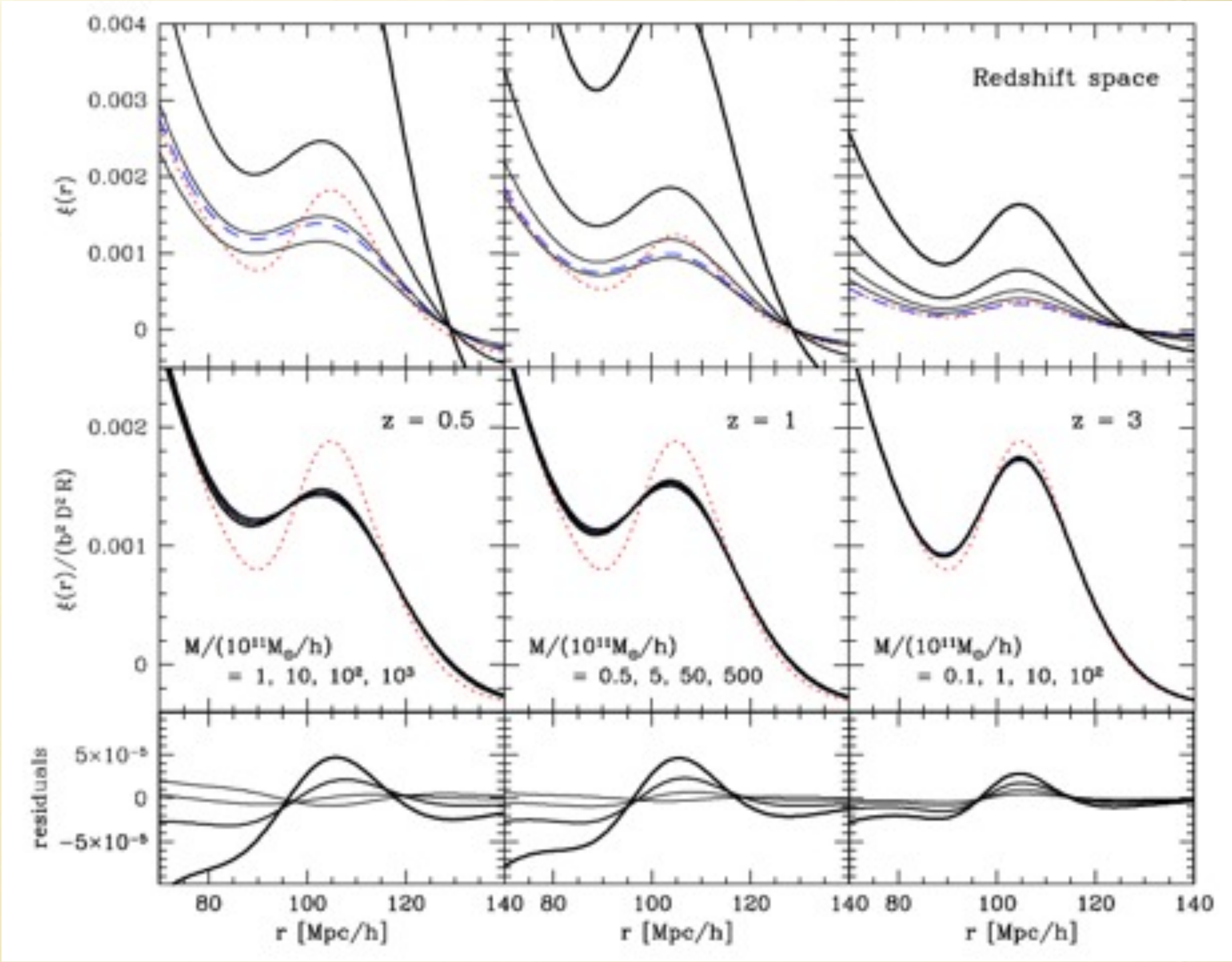


TM (2008)

Application:
Effects of halo bias on BAO

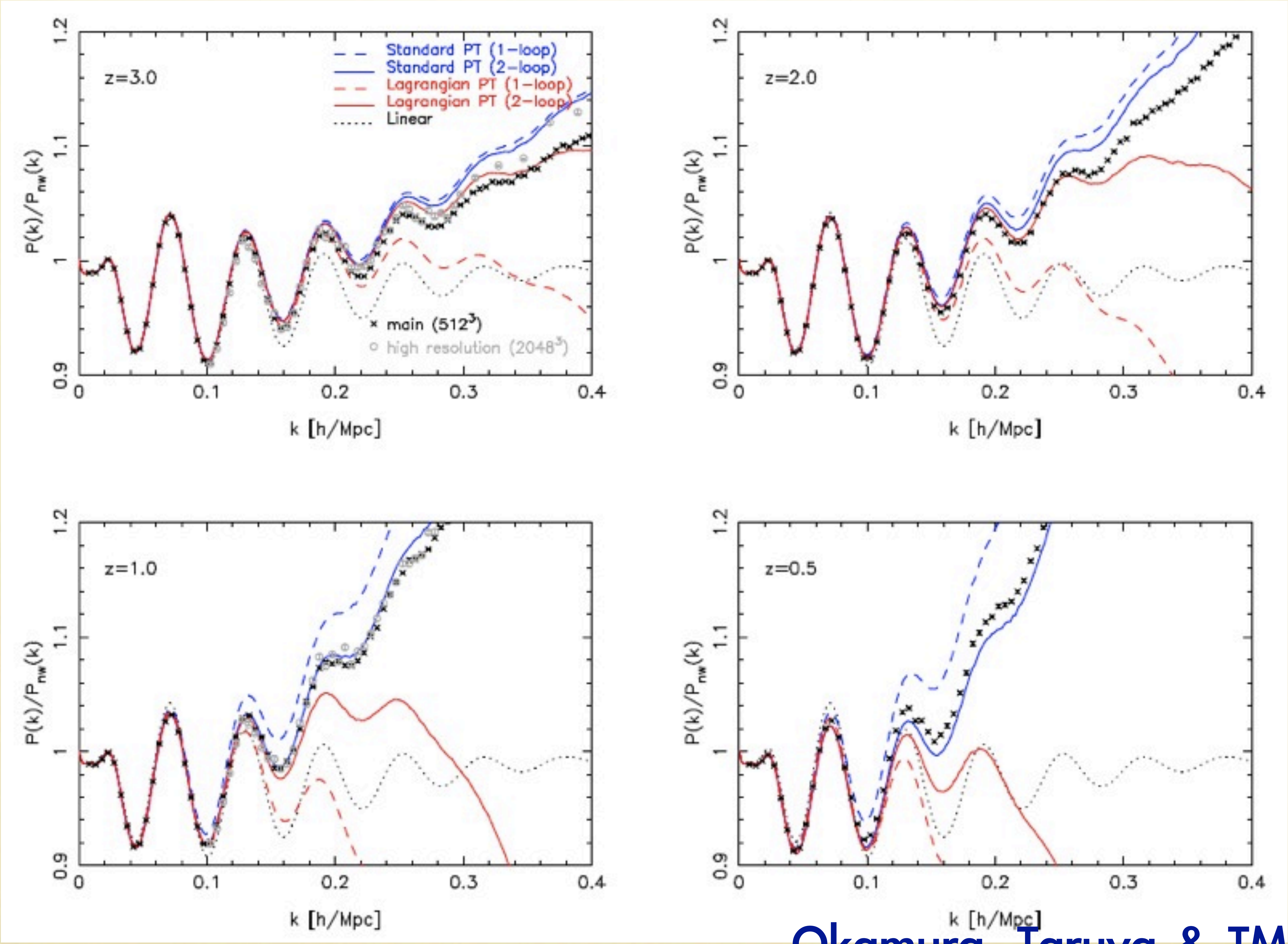
- Apply halo bias (local Lagrangian bias)
- redshift-space distortions also included





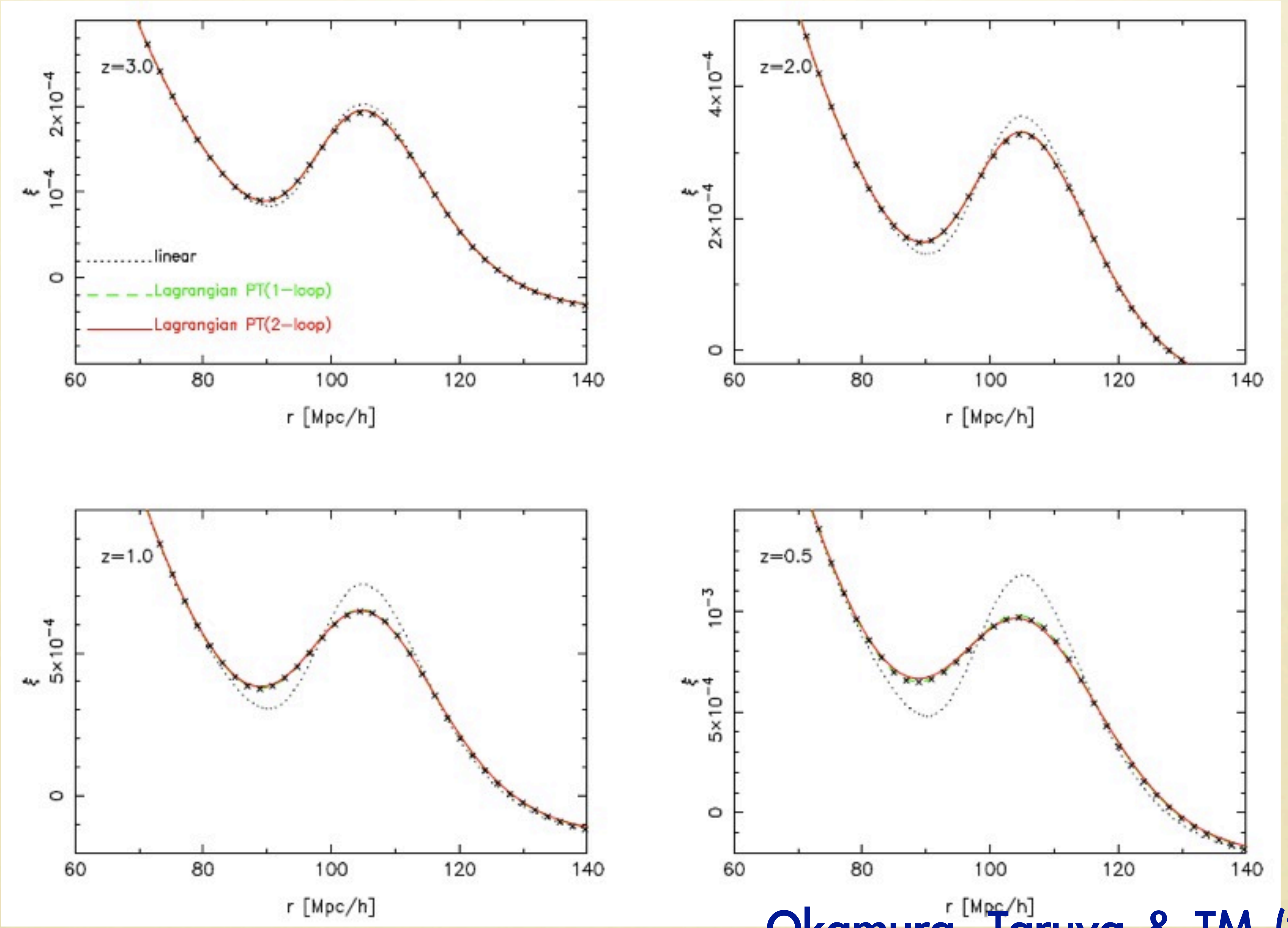
TM (2008)

2-loop corrections



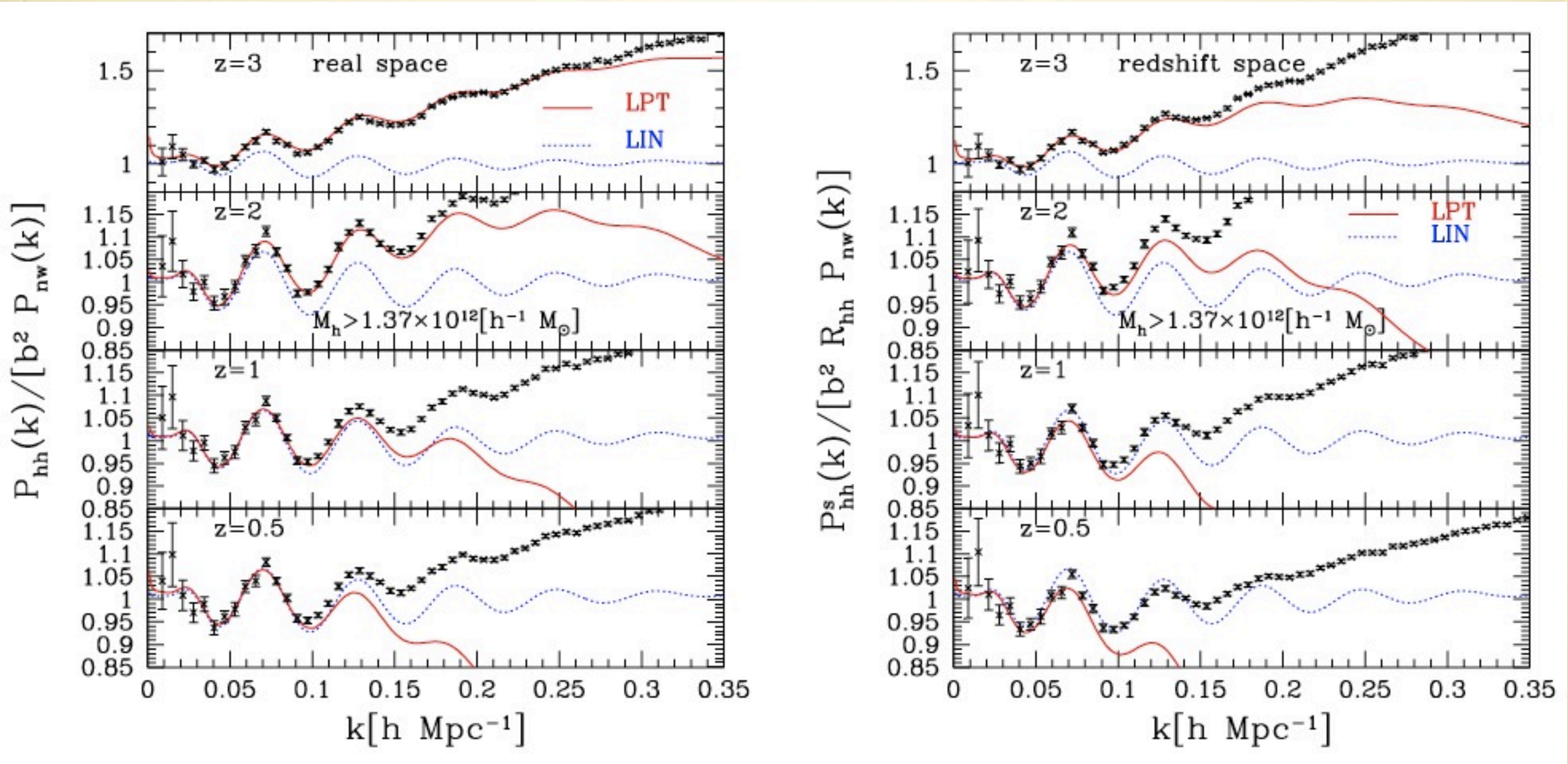
Okamura, Taruya & TM (2011)

2-loop corrections



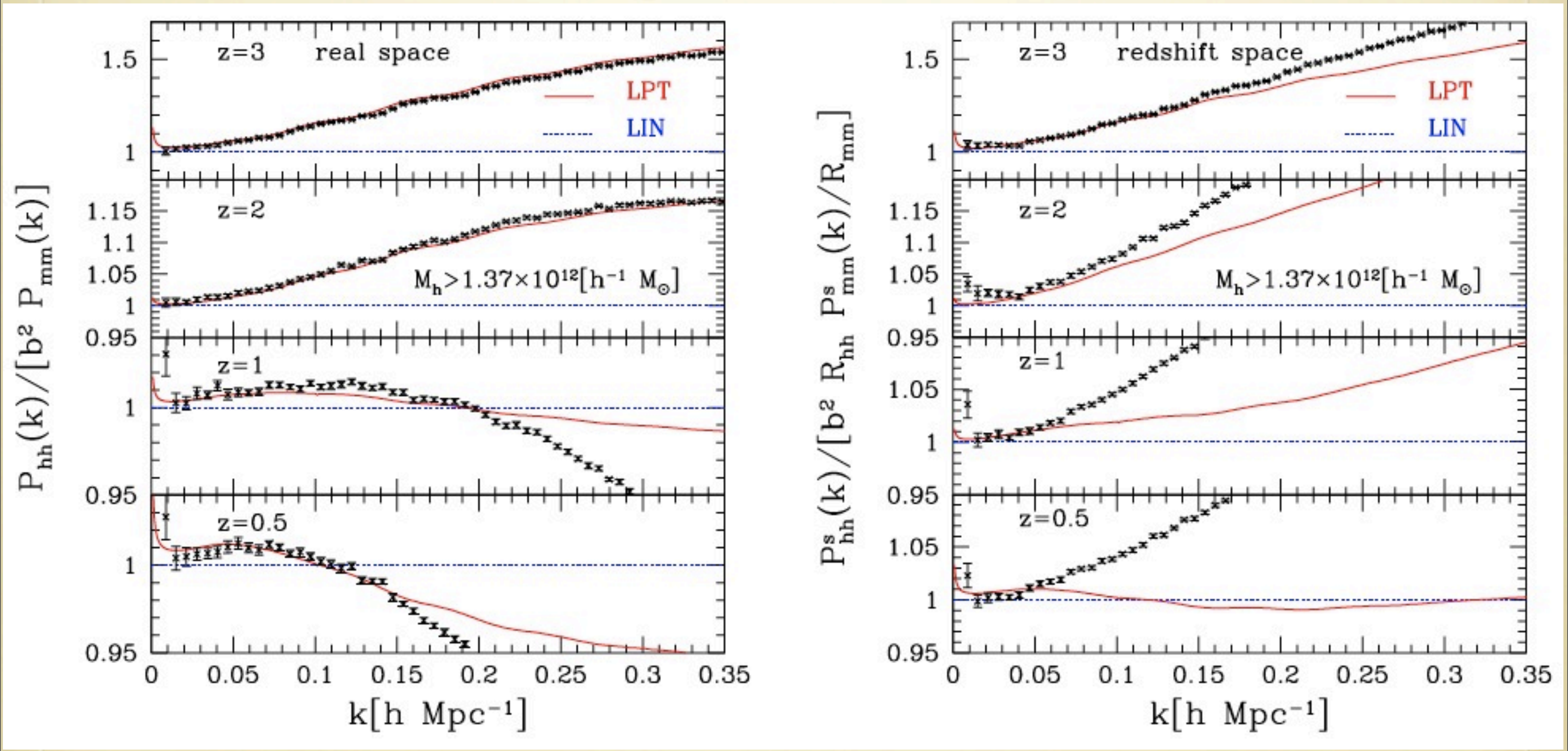
Okamura, Taruya & TM (2011)

Halo clustering: Comparison with N-body simulations



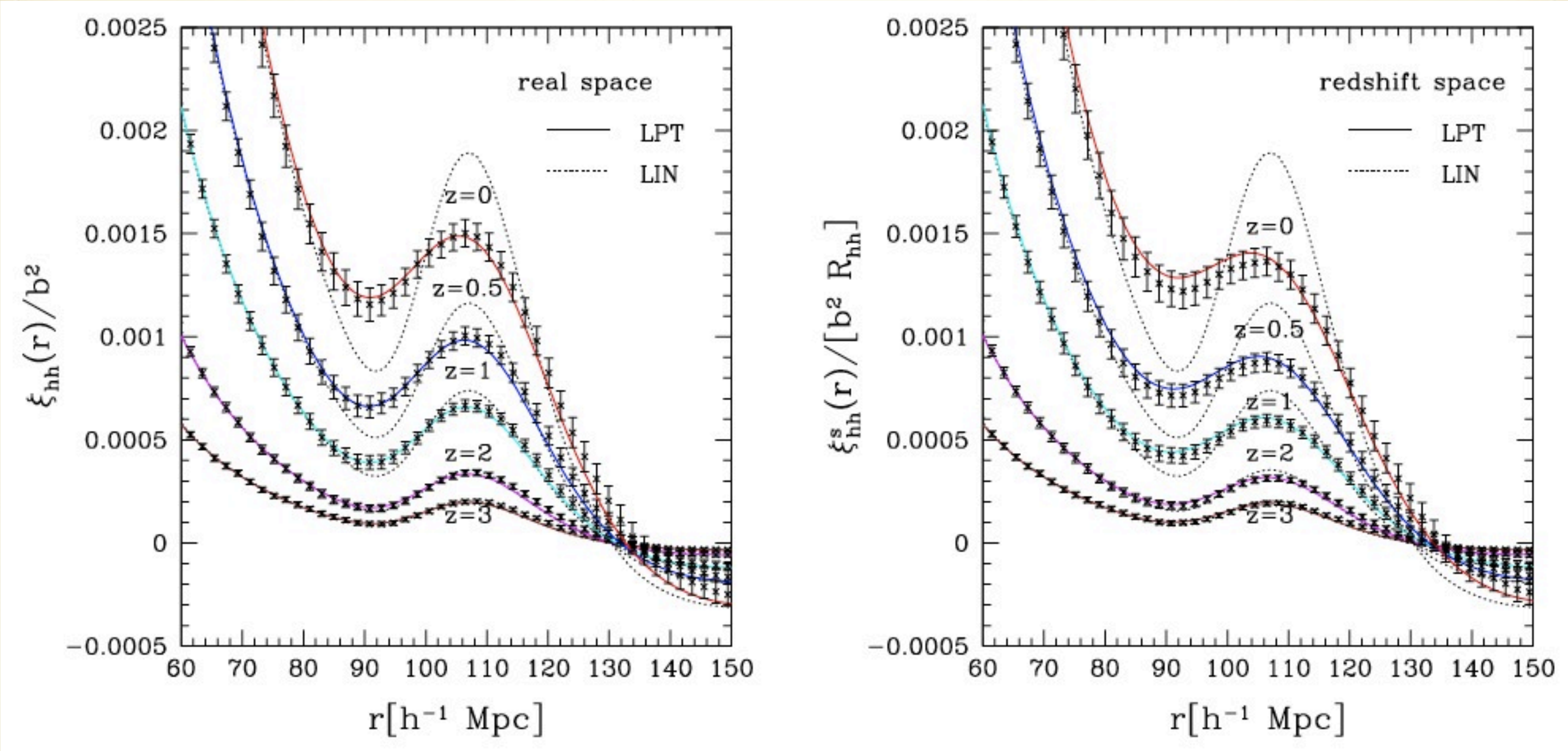
Sato & TM (2011)

Halo clustering: Comparison with N-body simulations



Sato & TM (2011)

Halo clustering: Comparison with N-body simulations

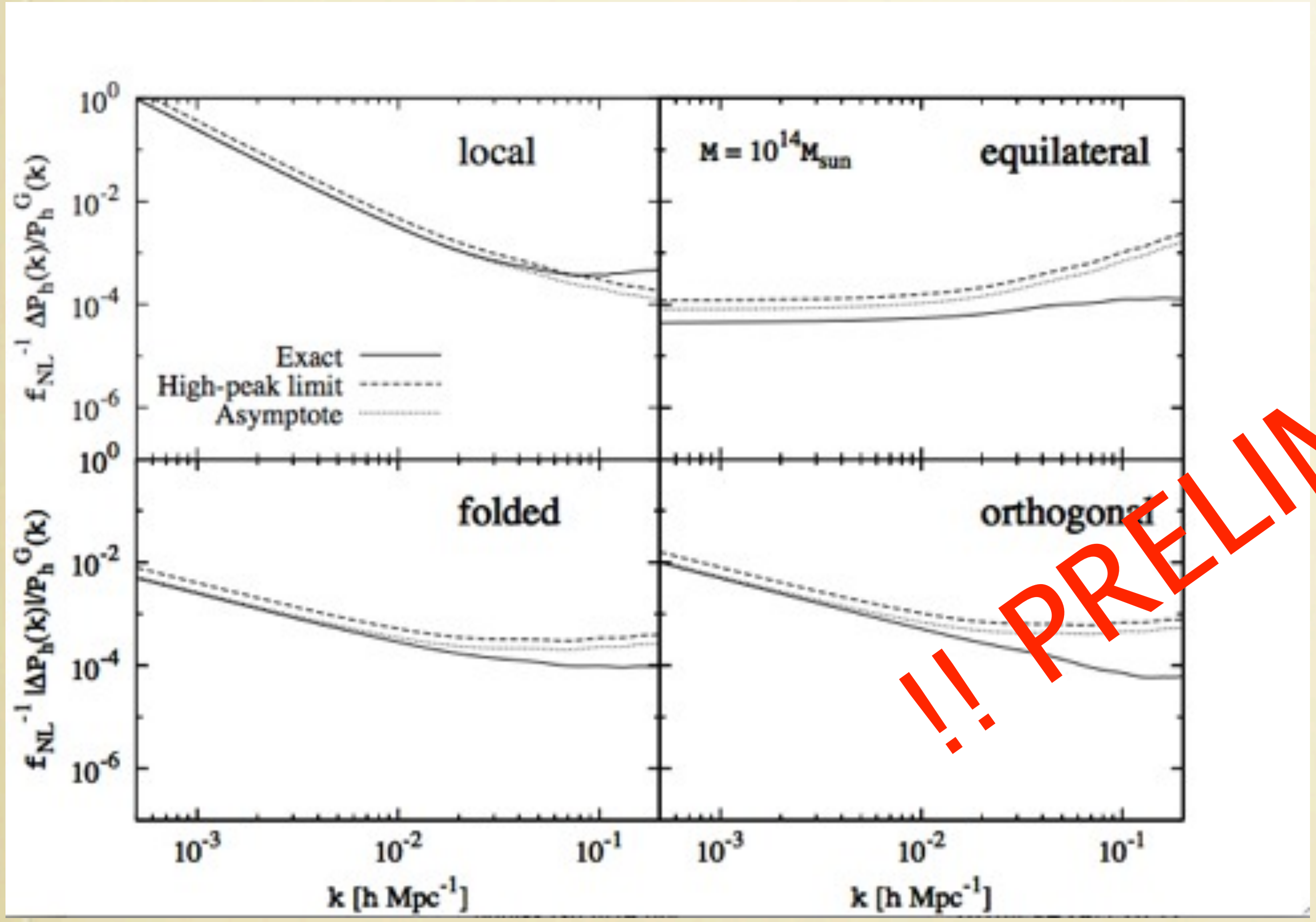


Sato & TM (2011)

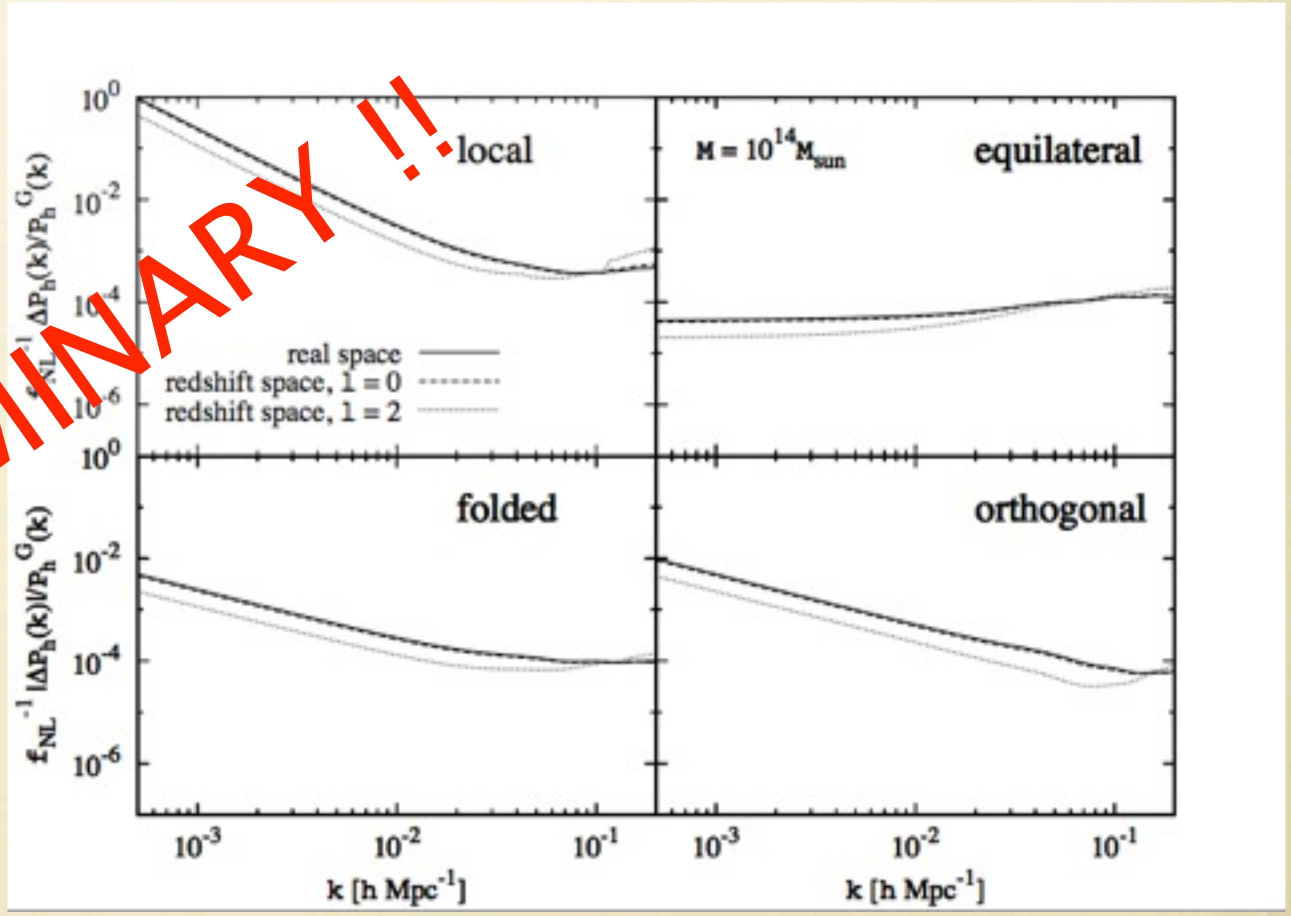
Application: Scale-dependent bias and prim.nG

- Previous methods are not accurate enough

real space: comparison with simple formula



redshift space



$$\Delta b^{\rm loc.}(k) = \frac{2 f_{\rm NL}(b-1)\delta_{\rm c}}{D(z)M(k)}$$ is accurate only in a high-peak limit

Summary

- Nonlinear perturbation theory is attracting renewed interests in the context of precision cosmology
- Traditional perturbation theory does not predict observables
- Integrated perturbation theory
  - A consistent formulation of perturbation theory to include bias and redshift-space distortions
- Lagrangian resummation theory
  - a simple resummation technique
  - bias and redshift-space distortions are included
  - Nicely reproduce the results of expensive N-body sims