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Primordial non-Gaussianity from multifield DBI Galileons

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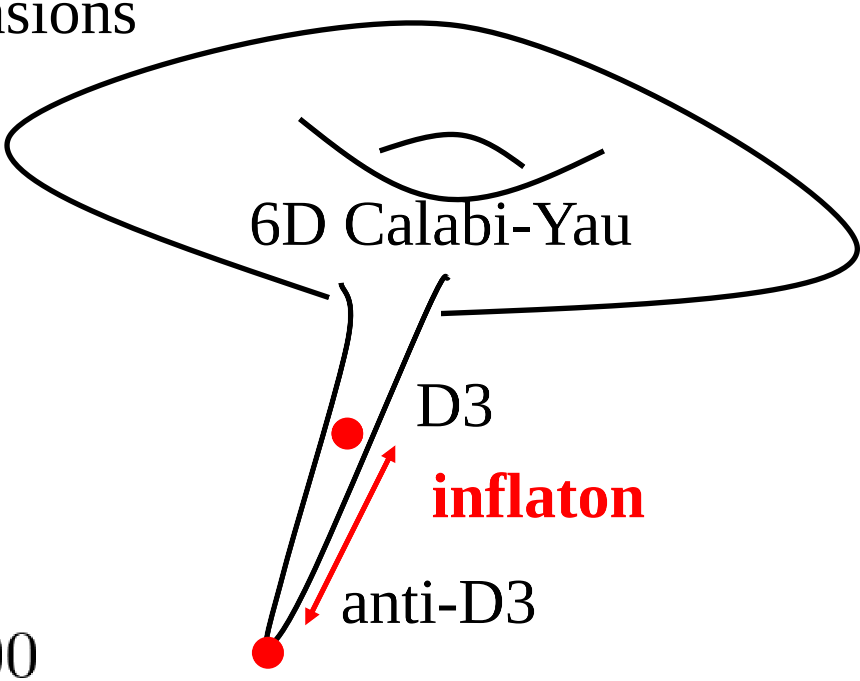
DBI inflation

- A natural inflation model based on string theory which can generate large equilateral NG Silverstein, Tong `04
- Inflation is identified as a radial position modulus of a probe D3 brane in extra-dimensions

small sound speed enhances NG

$$f_{NL}^{equil} = -\frac{35}{108}\frac{1}{c_s^2}$$



- Single field model in string theory is severely constrained

$$r = 16 c_s \epsilon < 10^{-7}$$
$$1 - n_s \sim 4\epsilon \sim 0.04$$
$$\Rightarrow |f_{NL}^{equil}| > 300$$

Lidsey, Huston `07
Kobayashi, Mukohyama, Kinoshita `07

Multifield DBI inflation

- DBI inflation is naturally a multifield model

  (Angular directional coordinates) Easson, Gregory, Tasinato, Zavala `07

- Multifield effect can ease the problem

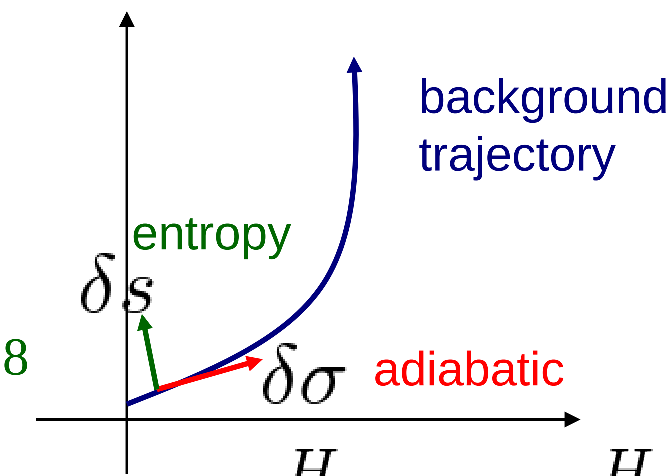
$$\zeta = \zeta_* + T_{\sigma s} S_*$$

same momentum dependence

$$f_{\rm NL}^{equil} \sim \frac{\epsilon^2}{r^2} \frac{1}{(1+T_{\sigma s}^2)^3}$$

Langlois, Renaux-Petel, Steer, Tanaka `08

Arroja, Mizuno, Koyama `08



$$\zeta = \frac{H}{\dot\sigma}\delta\sigma, \quad S = \frac{H}{\dot\sigma}\delta s$$

- Trispectrum

momentum dependence is different from single field model

Arroja, Mizuno, Koyama, Tanaka `09

$$\tau_{\rm NL}^{equil} \sim T_{\sigma s}^2 (f_{\rm NL}^{equil})^2$$

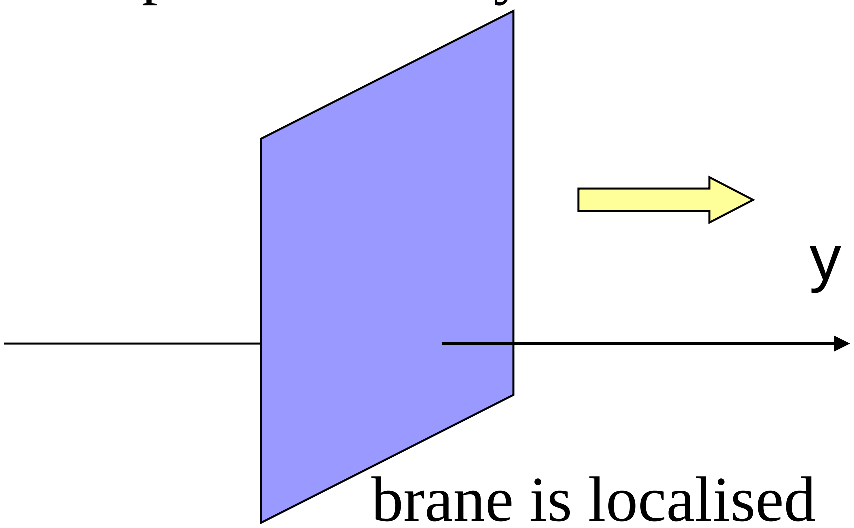
easier to detect if there is large transfer

Mizuno, Arroja, Koyama, Tanaka `09

DBI Galileon

- Natural unification of DBI and Galileon
- A probe brane dynamics in 5D

de Rham, Tolley \`10



brane is localised at $y = \pi(x^\mu)$

Induced metric of the brane

$$q_{\mu\nu} = g_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$$

Extrinsic curvature of the brane

$$K_{\mu\nu} = -\gamma \partial_\mu \partial_\nu \pi$$

Lorentz factor $\gamma = \frac{1}{c_s} = \frac{1}{\sqrt{1+(\partial\pi)^2}}$

Requiring E.O.Ms are kept to be second order

$$\Rightarrow \quad \mathcal{L} = \sqrt{-q}\left(-\lambda - M_5^3 K + \frac{M^2}{2} R - \beta \frac{M^4}{M_5^3} \mathcal{K}_{\rm GB}\right)$$

DBI — Relativistic extension of Galileon

Non-Gaussianity from single field DBI Galileon

- Simplest extension of single field DBI inflation Mizuno, Koyama `10

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{pl}^2 R + 2P(\phi,X) - 2G(\phi,X)\Box\phi] \qquad X = -(1/2)(\partial\phi)^2$$

with $P(\phi,X) = -f(\phi)^{-1}\sqrt{1-2Xf(\phi)} + f(\phi)^{-1} + V(\phi)$, $G(\phi,X) = \dfrac{g(\phi)X}{1-2Xf(\phi)}$

standard DBI (L_2) (L_3)

$$\Rightarrow \quad f_{\rm NL}^{equil} = -\frac{5}{324c_s^2}\frac{(21+546b_D+3776b_D^2+6048b_D^3)}{(1+4b_D)(1+12b_D)^2} \qquad b_D \equiv \frac{gH\dot\phi}{\sqrt{1-2fX}^3}$$

$$f_{\rm NL}^{equil} \simeq -20\frac{(1-n_s)^2}{r^2} \longleftarrow$$ almost independent of b_D

- Related works for NG from single field Galileon models

Kobayashi, Yamaguchi, Yokoyama `11 general $P(X,\phi) - G(X,\phi)\Box\phi$

Burrage et al `10 all terms satisfying exact Galileon symmetry

De Felice, Tsujikawa `11 further generalised model

1. Motivations

2. Model and background

3. Linear perturbations

4. Non-Gaussianity

5. Conclusions

DBI Galileon with higher co-dimensions

- Induced metric

$$q_{\mu\nu} = g_{\mu\nu} + f(\phi^I) G_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J \ , \quad I = 1, \ldots, N$$

↓

L 2

If we require the action to be scalar with respect to I, only tension and scalar curvature can be allowed!!

$$\mathcal{L} = \sqrt{-q}\left(-\lambda - M_5^3 K + \frac{M^2}{2} R - \beta \frac{M^4}{M_5^3} \mathcal{K}_{\rm GB}\right)$$

L 3 L 4 L 5

- Furthermore, it was shown that for even codimensions with $n > 3$ the only possible term is the induced gravity term (R)

Hinterbichler, Trodden, Wesley \`10

cf. Matching condition for a brane of arbitrary codimensions

Charmousis, Zegers \`05

Model

Renaux-Petel, Mizuno, Koyama `11

- Embedding a brane in 10-dimensions

$$ds^2 = \underline{h^{-1/2}}(y^K)\, g_{\mu\nu} dx^\mu dx^\nu + \underline{h^{1/2}}(y^K)\, G_{IJ}(y^K)\, dy^I dy^J$$

warp factor

$\mu = 0, \ldots 3$ and $I = 1, \ldots, 6$

- Induced metric on the brane

$$\gamma_{\mu\nu} = h^{-1/2} \tilde{g}_{\mu\nu}, \quad \tilde{g}_{\mu\nu} \equiv g_{\mu\nu} + f\, G_{IJ} \partial_\mu \phi^I \partial_\nu \phi^J$$

with $f = \dfrac{h}{T_3}, \quad \phi^I = \sqrt{T_3} y^I,$

- Action

$$S = \int d^4x \left[\frac{M_p^2}{2} \sqrt{-g} R[g] + \underline{\frac{M^2}{2} \sqrt{-\gamma} R[\gamma]} + \sqrt{-g} \mathcal{L}_{multiDBI} \right]$$

with $\mathcal{L}_{multiDBI} = -\dfrac{1}{f(\phi^I)} \left(\sqrt{\mathcal{D}} - 1 \right) - V(\phi^I)$

$$\mathcal{D} \equiv \det(\delta^\mu_\nu + f\, G_{IJ} g^{\mu\rho} \partial_\rho \phi^I \partial_\nu \phi^J)$$

Equations of motion

$$M_P^2 G^{\mu\nu}[g] + M^2 \frac{\sqrt{\mathcal{D}}}{h^{3/2}} G^{\mu\nu}[\gamma] = -\frac{1}{f}\sqrt{\mathcal{D}}\tilde{g}^{\mu\nu} - g^{\mu\nu}\left(V - \frac{1}{f}\right)$$

Including ϕ^I dependence

$$\left(\frac{M^2}{h^{3/2}} G^{\mu\nu}[\gamma] + \frac{1}{f}\tilde{g}^{\mu\nu}\right)\left(\delta^I_J + 2fG_{JK}X^{IK}_{\tilde{g}}\right)\left(\Pi^J_{\mu\nu} + \frac{f^{,J}}{4f}\frac{\tilde{g}_{\mu\nu}}{f}\right) - \frac{G^{IJ}}{f\sqrt{\mathcal{D}}}\left(V_{,J} + \frac{f_{,J}}{f^2}\right) = 0$$

$$X^{IJ} \equiv -\frac{1}{2} g^{\mu\nu}\partial_\mu \phi^I \partial_\mu \phi^J$$

$$\mathcal{D} = 1 - 2fX^I_I + 4f^2 X^{[I}_I X^{J]}_J - 8f^3 X^{[I}_I X^J_J X^{K]}_K + 16 f^4 X^{[I}_I X^J_J X^K_K X^{L]}_L$$

$$X^{IJ}_{\tilde{g}} \equiv -\tfrac{1}{2}\tilde{g}^{\mu\nu}\partial_\nu \phi^I \partial_\mu \phi^J$$

$$\Pi^I_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi^I + \hat{\Gamma}^I_{AB} \nabla_\mu \phi^A \nabla_\nu \phi^B$$

Christoffel symbol associated with $\hat{G}_{IJ} \equiv fG_{IJ}$

....

Cosmological background

- Friedmann equation

$$3H^2\left(M_P^2+\frac{\tilde{M}^2}{c_{\mathcal{D}}^3}\right)=V+\frac{1}{f}\left(\frac{1}{c_{\mathcal{D}}^2}-1\right)$$

$$\tilde{M}^2\equiv\frac{M^2}{\sqrt{h}}$$

$$c_{\mathcal{D}}^2\equiv 1-f\dot{\sigma}^2 \qquad \dot{\sigma}\equiv\sqrt{G_{IJ}\dot{\phi}^I\dot{\phi}^J}$$

$$-\dot{H}\left(M_P^2+\frac{\tilde{M}^2}{c_{\mathcal{D}}}\right)=\frac{\dot{\sigma}^2}{2c_{\mathcal{D}}}-\frac{\tilde{M}^2}{c_{\mathcal{D}}}\left[\frac{3}{2}\left(\frac{1}{c_{\mathcal{D}}^2}-1\right)+\frac{\dot{c}_{\mathcal{D}}}{Hc_{\mathcal{D}}}\right]$$

- In the relativistic regime ($c_{\mathcal{D}}^2\ll 1$), the induced gravity gives a term that breaks the null energy condition
- Condition for inflation in the relativistic regime

$$c_{\mathcal{D}}fV\gg 1\,,\ \tilde{M}^2\ll c_{\mathcal{D}}^3M_P^2$$

$$\Rightarrow\quad \epsilon\equiv-\frac{\dot{H}}{H^2}\simeq\frac{3}{2c_{\mathcal{D}}fV}(1-3\alpha)\ll 1 \qquad \text{with}\quad \alpha\equiv\frac{fH^2\tilde{M}^2}{c_{\mathcal{D}}^2}$$

Linear perturbations

- Second-order action

$$c_{\mathcal{D}}^2 \equiv 1 - f\dot{\sigma}^2 \qquad \alpha \equiv \frac{fH^2\tilde{M}^2}{c_{\mathcal{D}}^2}$$

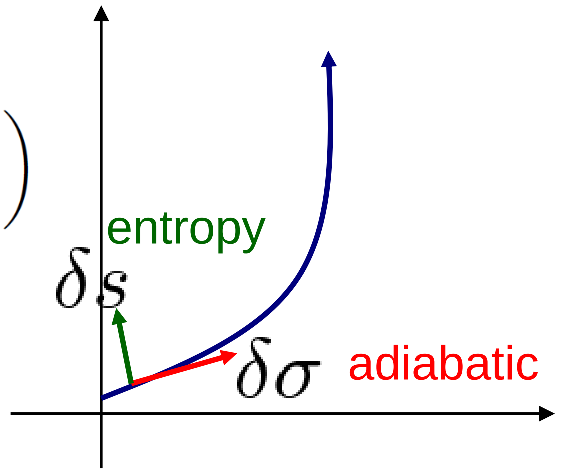
$$S_{(2)} = \frac{1}{2}\int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left(\frac{\dot{Q}_\sigma^2}{c_{\mathcal{D}}^3}\left(1-3\alpha(3-2c_{\mathcal{D}}^2)\right) - \frac{(\partial Q_\sigma)^2}{c_{\mathcal{D}}a^2}\left(1-\alpha(5-2c_{\mathcal{D}}^2)\right) + \frac{1-3\alpha}{c_{\mathcal{D}}}\left(\dot{Q}_s^2 - c_{\mathcal{D}}^2\frac{(\partial Q_s)^2}{a^2}\right) + \ldots\right),$$

- No ghost instabilities

$$\alpha < \frac{1}{9-6c_{\mathcal{D}}^2} \quad \Leftrightarrow \quad \rho < \frac{1}{fc_{\mathcal{D}}(3-2c_{\mathcal{D}}^2)}\left(1+\frac{c_{\mathcal{D}}^2M_P^2}{\tilde{M}^2}\right)$$

- Sound speeds

$$c_\sigma^2 = \frac{1-5\alpha}{1-9\alpha}c_{\mathcal{D}}^2 \qquad c_s^2 = c_{\mathcal{D}}^2$$



Entropic sound speed is smaller than adiabatic one (they become same in the DBI inflation $\alpha = 0$)

Non-Gaussianity

- Third-order action $c_{\mathcal{D}}^2 \equiv 1 - f\dot{\sigma}^2 \qquad \alpha \equiv \frac{fH^2\tilde{M}^2}{c_{\mathcal{D}}^2} \qquad \lambda \equiv \frac{c_s}{c_\sigma} \simeq \sqrt{\frac{1-3\alpha(3-2c_{\mathcal{D}}^2)}{1-\alpha(5-2c_{\mathcal{D}}^2)}}$

$$S_{(3)}^{\rm eff} = \int {\rm d}t\,{\rm d}^3x\, a^3 \frac{f\dot{\sigma}}{2c_{\mathcal{D}}^5}\Big[A_{\dot{Q}_\sigma^3}\dot{Q}_\sigma^3 - c_{\mathcal{D}}^2 A_{\dot{Q}_\sigma(\partial Q_\sigma)^2}\dot{Q}_\sigma\frac{(\partial Q_\sigma)^2}{a^2}$$
$$+c_{\mathcal{D}}^2\dot{Q}_\sigma\dot{Q}_s^2 + c_{\mathcal{D}}^4\dot{Q}_\sigma\frac{(\partial Q_s)^2}{a^2} - 2c_{\mathcal{D}}^4(1-2\alpha)\frac{\partial Q_\sigma \partial Q_s}{a^2}\dot{Q}_s$$
$$-\frac{3}{2}\alpha c_{\mathcal{D}}^4\left(\frac{1}{\lambda^2}-1\right)\dot{Q}_s^2\frac{\partial^2 Q_\sigma}{Ha^2} + \frac{\alpha}{2}c_{\mathcal{D}}^6\left(\frac{1}{\lambda^2}-1\right)\frac{(\partial Q_s)^2}{a^2}\frac{\partial^2 Q_\sigma}{Ha^2}\Big]$$

with $A_{\dot{Q}_\sigma^3} = 1 - 3\alpha\left(5 - 2c_{\mathcal{D}}^2 - 4\lambda^2 + \lambda^4\right) \qquad A_{\dot{Q}_\sigma(\partial Q_\sigma)^2} = 1 - \alpha\left(9 - 2c_{\mathcal{D}}^2 - 3\lambda^2\right)$

- Bispectrum $\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^7\delta(\sum_{i=1}^{3}\boldsymbol{k}_i)\mathcal{P}_\zeta^2\frac{S(k_1,k_2,k_3)}{(k_1k_2k_3)^2}$

$$S = -\frac{1}{2}\left(\frac{1}{c_{\mathcal{D}}^2}-1\right)\frac{1}{1-3\alpha(3-2c_{\mathcal{D}}^2)}\frac{1}{(1+T_{\sigma s}^2)^2}\left(S^{(\sigma\sigma\sigma)} + T_{\sigma s}^2 S^{(\sigma ss)}\right)$$

adiabatic ($S^{(\sigma\sigma\sigma)}$)   entropy induced ($T_{\sigma s}^2 S^{(\sigma ss)}$)

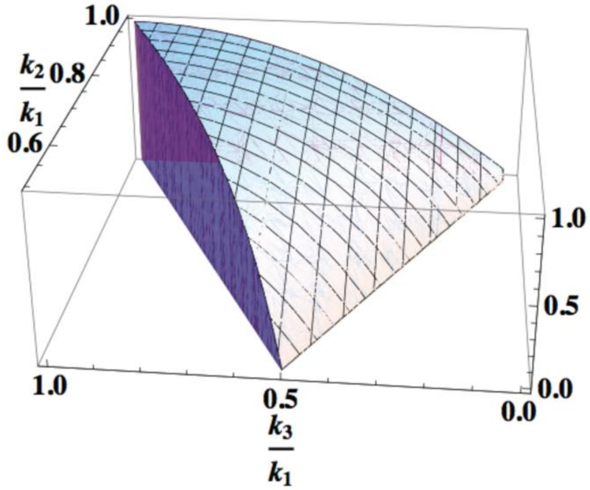
$S^{(\sigma\sigma\sigma)} = S^{(\sigma ss)}$ for standard DBI inflation $\alpha = 0$

Momentum dependence of Bispectrum

- For the fast analysis of NG from the bispectrum, separable theoretical templates are used.

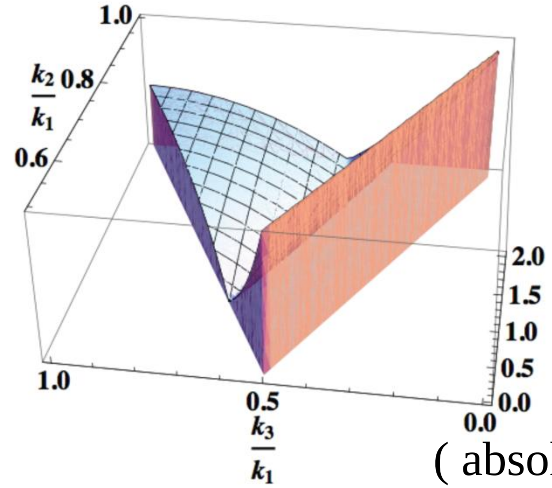
- Equilateral shape

$$S_{\rm eq} = -\left(\frac{k_1^2}{k_2 k_3} + 2\,{\rm perm.}\right) + \left(\frac{k_1}{k_2} + 5\,{\rm perm.}\right) - 2$$



- Orthogonal shape

$$S_{\rm orth} = -3\left(\frac{k_1^2}{k_2 k_3} + 2\,{\rm perm.}\right) + 3\left(\frac{k_1}{k_2} + 5\,{\rm perm.}\right) - 8$$



(absolute value)

- Constraints

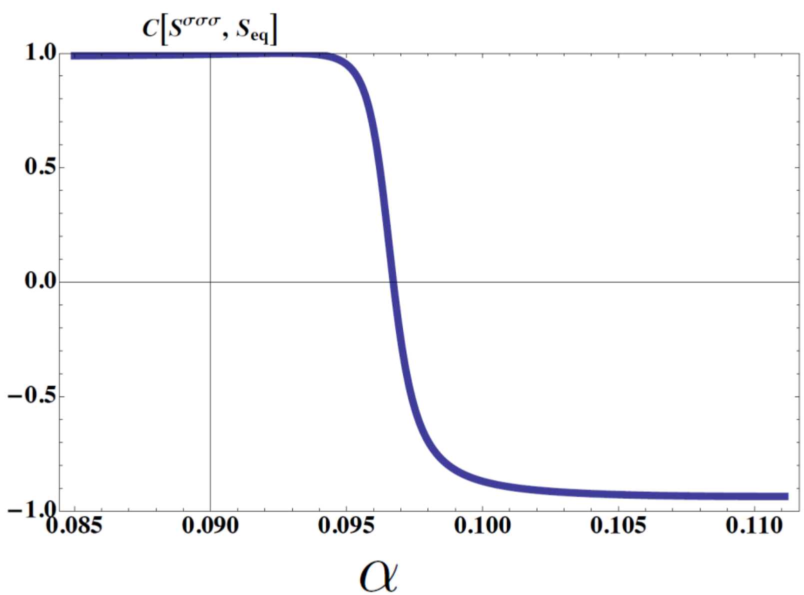
Fergusson, Shellard `10

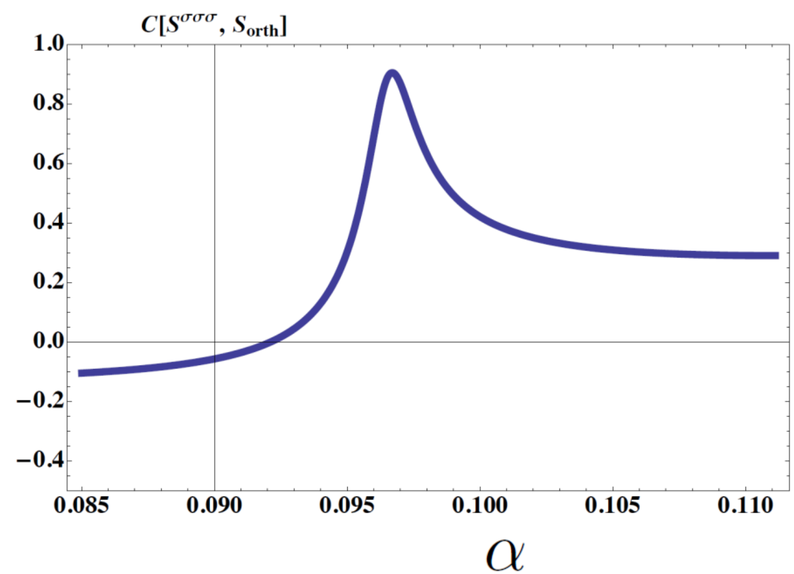
$$f_{\rm NL}^{eq} = 143.5 \pm 151.2 \;\text{and}\; f_{\rm NL}^{orth} = -79.4 \pm 133.3 \;(68\%\,{\rm C.L})$$

Correlation of the adiabatic shape

(relativistic limit $c_{\mathcal{D}}^2 \ll 1$)

- vs equilateral template
- vs orthogonal template



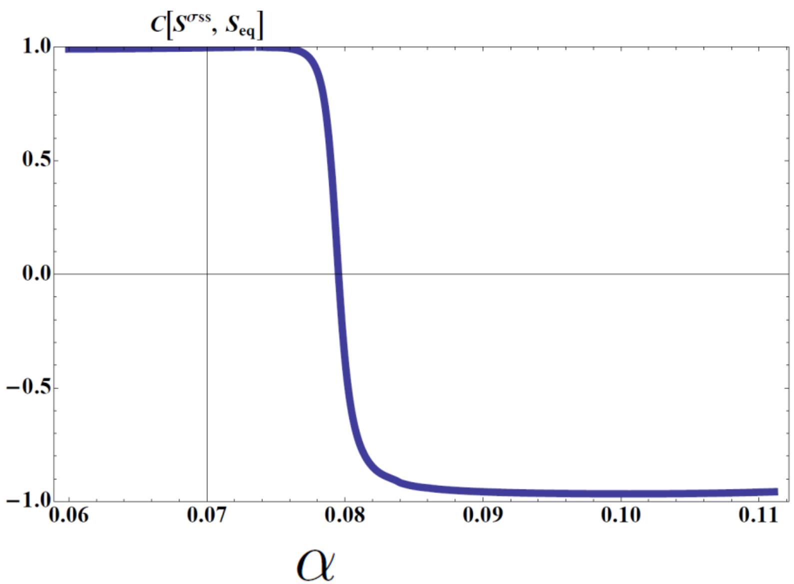


⇒ Mostly it gives equilateral NG but for a narrow region of α , it gives orthogonal type NG !

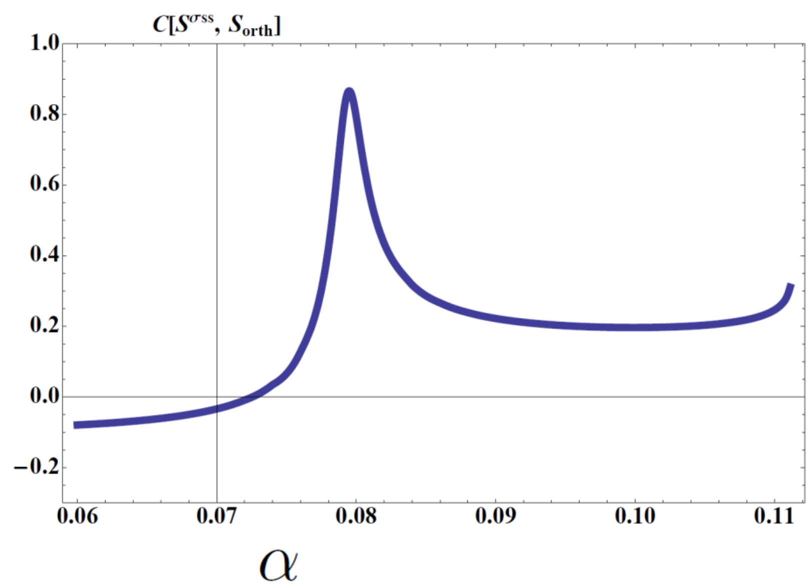
Correlation of the entropic-induced shape

(relativistic limit $c_{\mathcal{D}}^2 \ll 1$)

- vs equilateral template



- vs orthogonal template

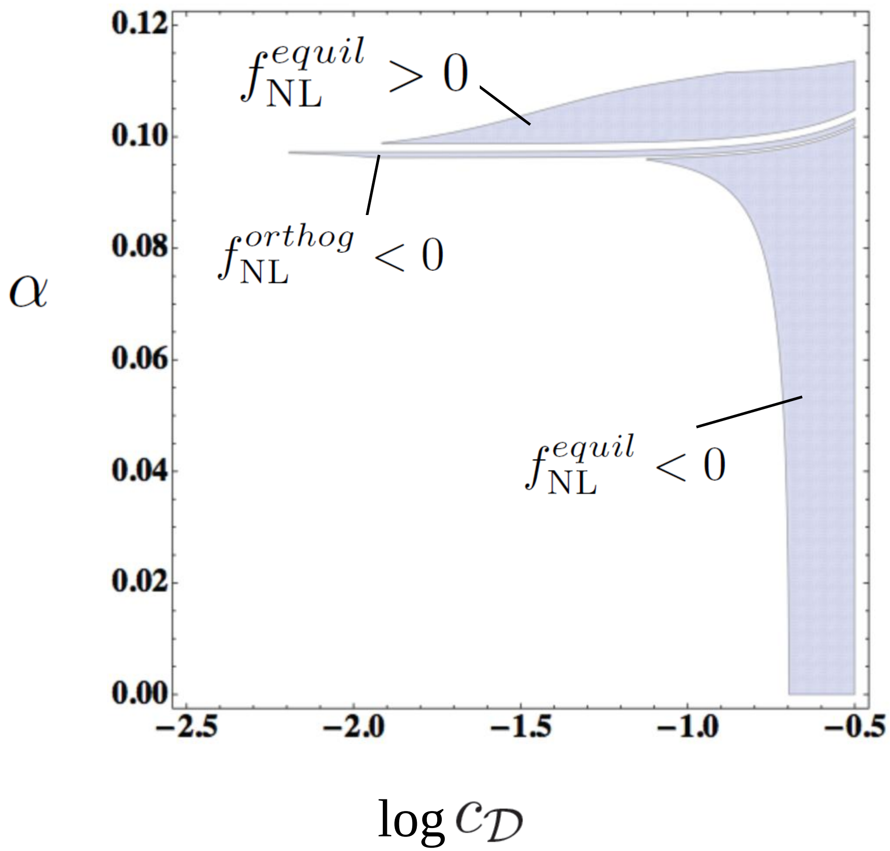


⇨ Qualitatively similar to the adiabatic shape , but value of α which gives orthogonal type NG is different

Observational constraint

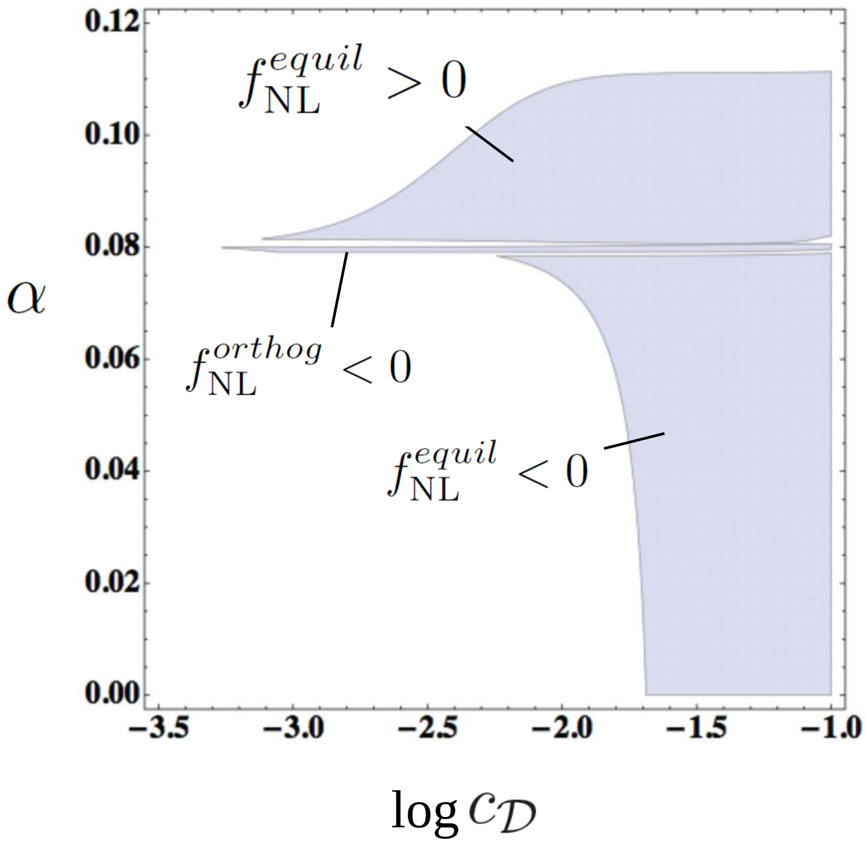
- No entropic transfer

$T_{\sigma s} = 0$



- Large entropic transfer

$T_{\sigma s} = 10$



Conclusion

- Multifield DBI Galileon

  a probe brane action with an induced gravity

- Relativistic inflation can be realised if $c_{\mathcal{D}} f V \gg 1\,, \;\; \tilde{M}^2 \ll c_{\mathcal{D}}^3 M_P^2$

- Cosmological perturbations are controlled by

  two parameters $c_{\mathcal{D}}^2 \equiv 1 - f\dot{\sigma}^2 \quad \alpha \equiv \dfrac{f H^2 \tilde{M}^2}{c_{\mathcal{D}}^2}$

  ghost condition is specified

  sound speeds c_σ and c_s are different

  constraints on these parameters for given $T_{\sigma s}$

  concrete example to give orthogonal type NG and positive $f_{\rm NL}^{equil}$

Discussions

- Entropy transfer $T_{\sigma s}$

  Concrete model ($f(\phi^I)$, $V(\phi^I)$)

- New shape from $S_{\dot{Q}_s^2 \partial^2 Q_\sigma}$

  general multifield model

- general cosmological background

$$S_{(2)} = \frac{1}{2} \int \mathrm{d}\tau \, \mathrm{d}^3x \Big\{ v_\sigma'^2 + v_s'^2 - 2\xi v_\sigma' v_s - c_\sigma^2 (\partial v_\sigma)^2 - c_s^2 (\partial v_s)^2 - \underline{2 c_{\sigma s}^2 \partial v_\sigma \partial v_s}$$
$$+ \frac{z''}{z} v_\sigma^2 + \left(\frac{w''}{w} - a^2 \mu_s^2 \right) v_s^2 + 2 \frac{z'}{z} \xi v_\sigma v_s \Big\}$$

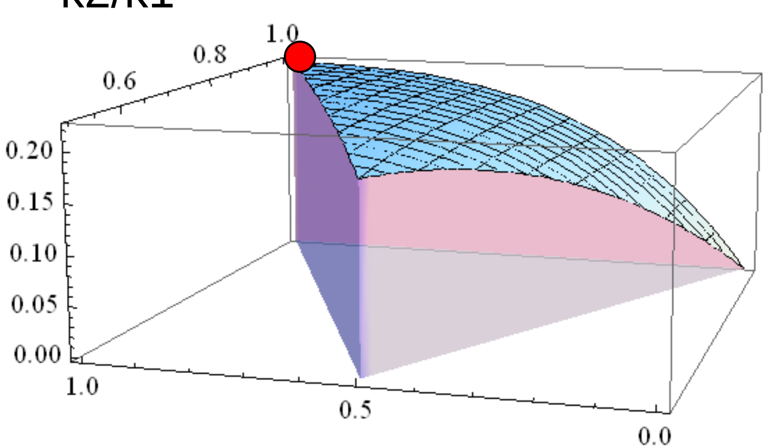
$$c_{\sigma s}^2 = \frac{c_{\mathcal{D}}^3 (1-\kappa)(1-c_{\mathcal{D}}^2)}{(\kappa(1-3\alpha)^3 \left((1-9\alpha)\kappa + 6\alpha c_{\mathcal{D}}^2\right))^{1/2}} \frac{2 M_P^2 H V_s}{\dot{\sigma}^3} \qquad \kappa \equiv \frac{M_P^2 + \frac{\tilde{M}^2}{c_{\mathcal{D}}}}{M_P^2 + \frac{\tilde{M}^2}{c_{\mathcal{D}}^3}}$$

- Trispectrum

Multi-field non-Gaussianity (new-effect)

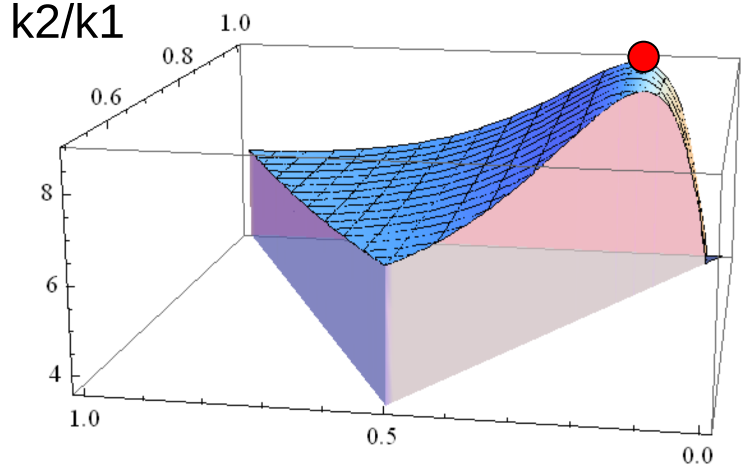
- Bispectrum shape generated by $\dot{Q}_s^2 \partial^2 Q_\sigma$

$$\lambda \equiv \frac{c_s}{c_\sigma} = 1.0$$



``Equilateral”

$$\lambda \equiv \frac{c_s}{c_\sigma} = 0.1$$



``local”

location of the maximum changes from Equilateral to local !!

Concrete form of $R[\gamma]$

$$\gamma_{\mu\nu} = h^{-1/2}\tilde{g}_{\mu\nu} \qquad \tilde{g}_{\mu\nu} \equiv g_{\mu\nu} + f\, G_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J$$

$$\begin{aligned}
R[\tilde{g}] =\ & A^2 R[g] - 2A\mathcal{A}_{IJ}(\partial\phi^I \cdot R[g] \cdot \partial\phi^J) + \mathcal{A}_{IJ}\mathcal{A}_{ML}\nabla^\lambda\phi^I\nabla^\nu\phi^J\nabla^\mu\phi^M\nabla^\kappa\phi^L R_{\lambda\mu\nu\kappa}[g] \\
& + A^2\, H_{AB}\left([\Pi^A][\Pi^B] - [\Pi^A \cdot \Pi^B]\right) \\
& - 2A\,\mathcal{A}_{CD}\,H_{AB}\left([\Pi^A](\partial\phi^C \cdot \Pi^B \cdot \partial\phi^D) - (\partial\phi^C \cdot \Pi^A \cdot \Pi^B \cdot \partial\phi^D)\right) \\
& + (\mathcal{A}_{IJ}\mathcal{A}_{KL} - \mathcal{A}_{IL}\mathcal{A}_{KJ})H_{BE}\,(\partial\phi^I \cdot \Pi^B \cdot \partial\phi^J)(\partial\phi^K \cdot \Pi^E \cdot \partial\phi^L) \\
& + 4(AX^{LJ} + 2X^{LM}X^{JN}\mathcal{A}_{MN})(AX^{IK} + 2X^{IC}X^{KD}\mathcal{A}_{CD})\hat{R}_{LIJK}\,. \qquad \text{(A.13)}
\end{aligned}$$

$$A \equiv \frac{1}{\mathcal{D}}\left(1 - 2fX_I^I + 4f^2X_I^{[I}X_J^{J]} - 8f^3X_I^{[I}X_J^JX_K^{K]}\right)$$

$$\mathcal{A}_{IJ} \equiv \frac{f}{\mathcal{D}}\left(\left(1 - 2fX_A^A + 4f^2X_B^{[B}X_C^{C]}\right)G_{IJ} + 2f\left(1 - 2fX_K^K\right)X_{IJ} + 4f^2X_{IK}X_J^K\right)$$

$$\Pi^I_{\mu\nu} = \nabla_\mu\nabla_\nu\phi^I + \hat{\Gamma}^I_{AB}\nabla_\mu\phi^A\nabla_\nu\phi^B$$