Higher Derivative Corrections to Holographic Entanglement Entropy for AdS Soliton

Noriaki OGAWA
(IPMU)

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Collaboration with: Tadashi Takayanagi (IPMU)

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Plan to Talk

- Holographic Entanglement Entropy (HEE) and AdS Soliton Geometries [Review]

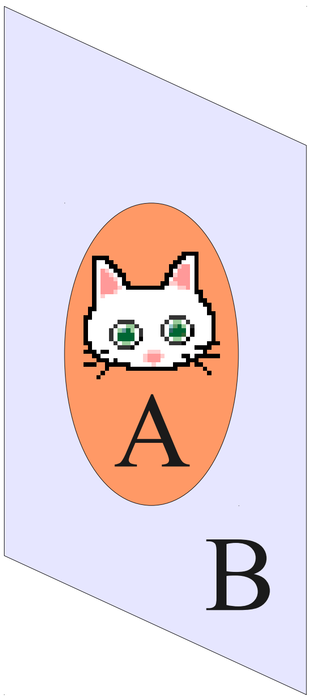
- Higher Derivative Corrections [Our work]
  - String/M theories
  - Gauss-Bonnet Gravity → phase transition structure

Holographic Entanglement Entropy

And

AdS Soliton Geometries

Quantum Entanglement and Entanglement Entropy

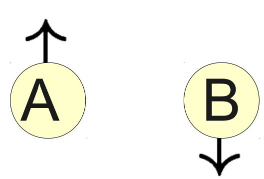


Only subsystem A is visible to observer
→ He/She observes an "entropy"

$$S_A = -\mathrm{Tr}\,\rho_A \log \rho_A \qquad (\rho_A = \mathrm{Tr}_B\,\rho_{total})$$

This quantity measures the degree of
"quantum entanglement" between A and B
→ called "Entanglement Entropy" (EE)

ex.) 2-spins system

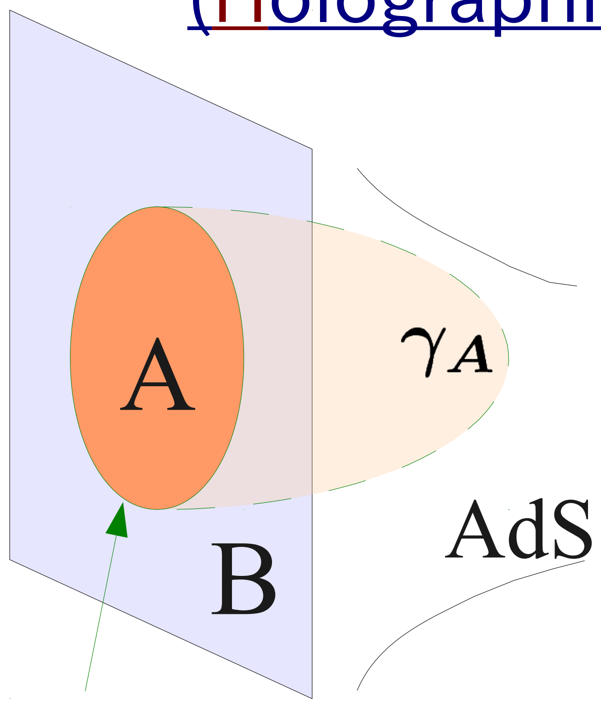


$$|\psi>= \frac{1}{\sqrt{2}}(|\uparrow>_A \otimes |\downarrow>_B + |\downarrow>_A \otimes |\uparrow>_B) \quad \Rightarrow \quad S_A = \log 2$$

$$|\psi>= \frac{1}{2}(|\uparrow>_A + |\downarrow>_A) \otimes (|\uparrow>_B + |\downarrow>_B) \quad \Rightarrow \quad S_A = 0$$

Entanglement Entropy and AdS/CFT

(Holographic Entanglement Entropy, HEE)

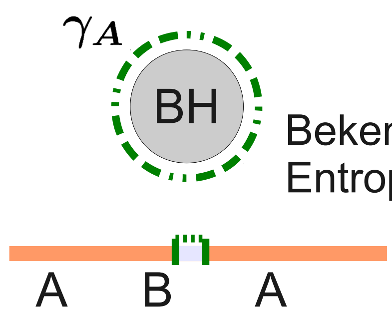


conical deficit 2π(1-n)

$$S_A = \frac{\min(\text{Area}(\gamma_A))}{4G}$$

[Ryu-Takayanagi, 2006]

Consistent with BH entropy:



Bekenstein-Hawking Entropy

If $K_{\gamma_A} = 0,$

binormal tensor to γ_A

$$\mathcal{R}_{\mu\nu\rho\sigma} = \mathcal{R}^{(0)}_{\mu\nu\rho\sigma} + 2\pi(1-n)(N_{\mu\rho}N_{\nu\sigma} - N_{\mu\sigma}N_{\nu\rho})\delta(\gamma_A)$$

$$\mathcal{R}_{\mu\nu} = \mathcal{R}^{(0)}_{\mu\nu} + 2\pi(1-n)N_{\mu\nu}\delta(\gamma_A)$$

$$\mathcal{R} = \mathcal{R}^{(0)} + 4\pi(1-n)\delta(\gamma_A)$$

[Fursaev-Solodukhin, 1995]

⇒ Einstein-Hilbert action $I = -\frac{1}{16\pi G}\int\sqrt{-g}\mathcal{R}$ leads to the formula above.

AdS Soliton Geometry

AdS soliton: double-Wick-rotated AdS black hole

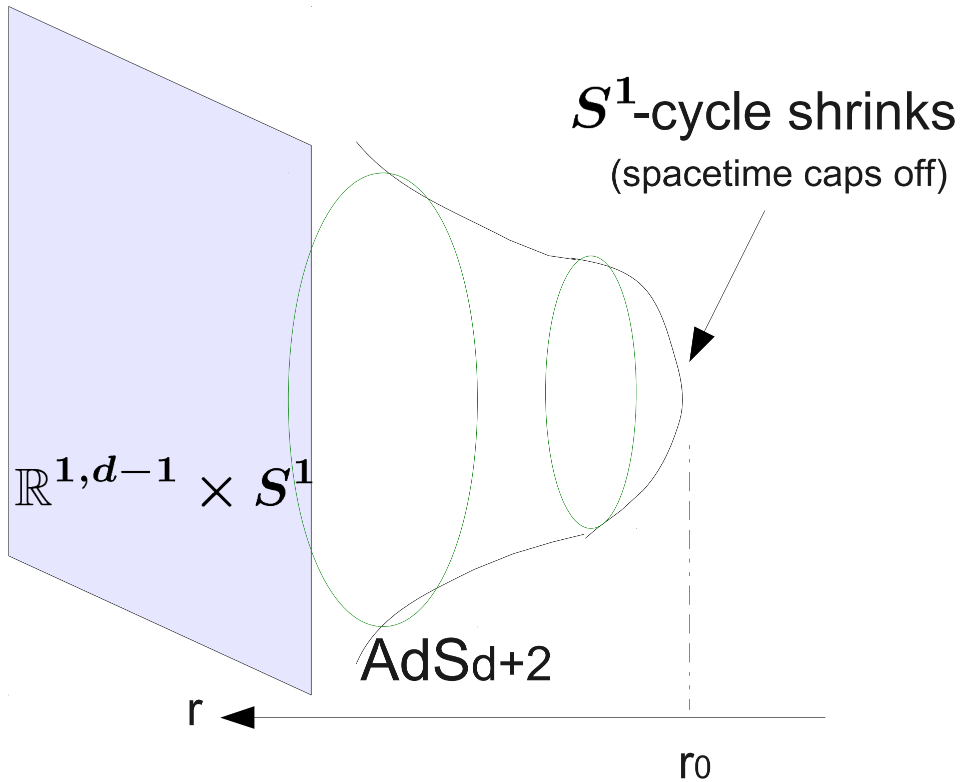
BH: time direction is compactified
Soliton: one of space direction is compactified

Dual to CFT on S^1-compactified spacetime

System is characterized by the scale of KK radius R.

E >> 1/R → (d+1)-dim like

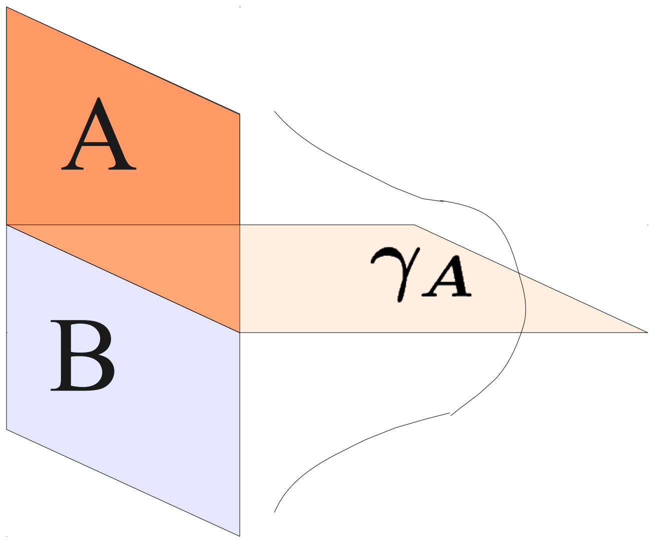
E << 1/R → d-dim like



AdS5 soliton (in Einstein gravity)

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx^2 + dy^2) + \frac{L^2}{r^2}\frac{dr^2}{1-(r_0/r)^4} + \frac{r^2}{L^2}[1-(r_0/r)^4]d\theta^2 \qquad R = L^2/2r_0$$

Half Space Subsystem in AdS Soliton



γ_A is obviously like this, from symmetry.

$$K_{\gamma_A} = 0$$

$$\mathcal{R}_{\mu\nu\rho\sigma} = \mathcal{R}^{(0)}_{\mu\nu\rho\sigma} + 2\pi(1-n)(N_{\mu\rho}N_{\nu\sigma} - N_{\mu\sigma}N_{\nu\rho})\delta(\gamma_A)$$
$$\mathcal{R}_{\mu\nu} = \mathcal{R}^{(0)}_{\mu\nu} + 2\pi(1-n)N_{\mu\nu}\delta(\gamma_A)$$
$$\mathcal{R} = \mathcal{R}^{(0)} + 4\pi(1-n)\delta(\gamma_A)$$

is applicable

$$S_A = \frac{\partial}{\partial n}(I_n - n \cdot I_1)$$

is calculable for any action *I*.

Higher Derivative Corrections

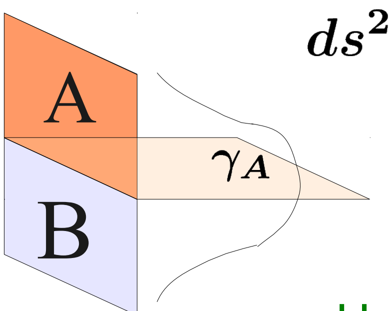
Systems We Investigate

- AdS5 soliton in IIB SUGRA with R^4 term
  - String (α') correction
  - compactified N=4 SYM with finite (large) λ
- AdS4/7 solitons in 11D SUGRA with R^4 term
  - Quantum ($l_p^{(11)}$) correction
  - compactified M2/M5, finite (large) N
- AdS4/5 solitons in Gauss-Bonnet Gravity
  - Toy model
  - Correction to the HEE formula has recently proposed

AdS5 Soliton in IIB

Action: $$I_{\mathrm{IIB}} = -\frac{1}{16\pi G_N}\int d^{10}x\,(\mathcal{R} + \gamma e^{-\frac{3}{2}\phi}\underbrace{W}_{\sim \mathcal{R}^4} + \ldots)$$ $\left(\gamma = \frac{\zeta(3)}{8}\alpha'^3\right)$

AdS$_5$-soliton $\times S^5$ $\Leftrightarrow$ $\mathcal{N}=4$ Yang-Mills on $\mathbb{R}^{1,2}\times S^1$



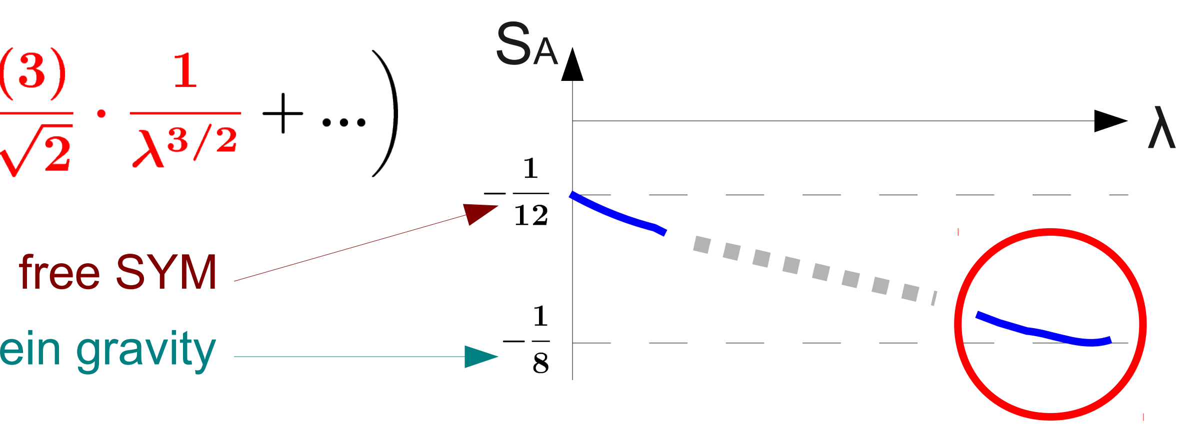
$$ds^2 = ds^2_{(0)} + \gamma \cdot ds^2_{(1)} + \ldots$$

$$ds^2_{(0)} = \frac{r^2}{L^2}(dt_E^2 + dx^2 + dy^2) + \frac{L^2}{r^2}\frac{dr^2}{1-(r_0/r)^4} + \frac{r^2}{L^2}[1-(r_0/r)^4]d\theta^2 + L^2 d\Omega_5^2$$

Using replica method holographically, with

$$\mathcal{R}_{\mu\nu\rho\sigma} = \mathcal{R}^{(0)}_{\mu\nu\rho\sigma} + 2\pi(1-n)(N_{\mu\rho}N_{\nu\sigma} - N_{\mu\sigma}N_{\nu\rho})\delta(\gamma_A)$$
$$\mathcal{R}_{\mu\nu} = \mathcal{R}^{(0)}_{\mu\nu} + 2\pi(1-n)N_{\mu\nu}\delta(\gamma_A)$$
$$\mathcal{R} = \mathcal{R}^{(0)} + 4\pi(1-n)\delta(\gamma_A)$$

$$S_A = (\mathrm{div}) + \frac{N^2 V}{R}\left(-\frac{1}{8} + \frac{\zeta(3)}{8\sqrt{2}}\cdot\frac{1}{\lambda^{3/2}} + \ldots\right)$$



M2/M5 Solitons

Action: $$I_{\mathrm{M}} = -\frac{1}{16\pi G_N}\int d^{11}x(\mathcal{R}+\gamma \underbrace{W}_{\sim \mathcal{R}^4}+\ldots)$$ $$\left(\gamma = \frac{16\pi^{8/3}}{3}G_N^{2/3}\right)$$

AdS4 (M2) soliton: $$ds^2 = ds^2_{(0)} + \gamma \cdot ds^2_{(1)} + \ldots$$

$$ds^2_{(0)} = \frac{r^2}{L^2}(-dt^2+dx^2)+\frac{L^2}{r^2}\frac{dr^2}{1-(r_0/r)^3}+\frac{r^2}{L^2}[1-(r_0/r)^3]d\theta^2+4L^2d\Omega_7^2$$

$$S_A = (\mathrm{div}) - \frac{2\sqrt{2}\pi}{9}N^{3/2} + \frac{2^{43/6}\pi^{19/3}}{9}N^{1/2}+\ldots$$

AdS7 (M5) soliton: $$ds^2 = ds^2_{(0)} + \gamma \cdot ds^2_{(1)} + \ldots$$

$$ds^2_{(0)} = \frac{r^2}{L^2}(-dt^2+dx_1^2+\ldots+dx_4^2)+\frac{L^2}{r^2}\frac{dr^2}{1-(r_0/r)^6}+\frac{r^2}{L^2}[1-(r_0/r)^6]d\theta^2+\frac{L^2}{4}d\Omega_4^2$$

$$S_A = (\mathrm{div}) - \frac{2}{3^5\pi}\frac{N^3V_3}{R^3} + 0\cdot N^1+\ldots$$

AdS4/5 Solitons in Gauss-Bonnet

Einstein-Gauss-Bonnet Gravity:

One of the simplest toy-models of higher derivertive gravities

$$I_{\mathrm{EGB}} = -\frac{1}{16\pi G_N}\int d^D x \left(-2\Lambda + \mathcal{R} + \frac{\eta L^2}{2}\mathcal{L}_{GB}\right)$$

$$\mathcal{L}_{GB} = \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2$$

⇒ Exact AdS soliton (BH) solutions are known. [Myers-Simon, 1988]

4D case $ds^2 = ds^2_{(0)}$ (← 4D GB term is topological)

$$S_A = (\mathrm{div}) + \frac{\pi L^2}{G_N}\left(-\frac{1}{3} + \eta\right)$$

5D case $ds^2 = ds^2(\eta)$ (← nontrivial exact solution)

$$S_A = (\mathrm{div}) + \frac{\pi V L^3 \sqrt{f_\infty(\eta)}}{16 G_N R}(-1 + 8\eta)$$

$$f_\infty(\eta) = \frac{2}{1+\sqrt{1-4\eta}}$$

HEE formula for Gauss-Bonnet

$$I_{\mathrm{EGB}} = -\frac{1}{16\pi G_N}\int d^D x\left(-2\Lambda + \mathcal{R} + \frac{\eta L^2}{2}\mathcal{L}_{GB}\right)$$

$$\mathcal{L}_{GB} = \mathcal{R}_{\mu\nu\rho\sigma}\mathcal{R}^{\mu\nu\rho\sigma} - 4\mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu} + \mathcal{R}^2$$

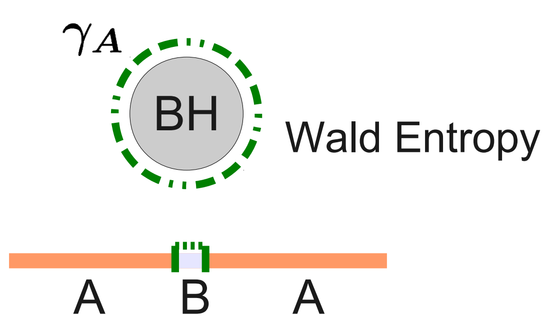
Recent conjecture by [Hung-Myers-Smolkin, 1001.5813]
[de Boer-Kulaxizi-Parnachev, 1001.5781]

$$S_A = \frac{1}{4G}\int_{\gamma_A} d^{D-2}x\sqrt{h}(1 + \eta L^2\mathcal{R}_h)$$

induced metric on γ_A

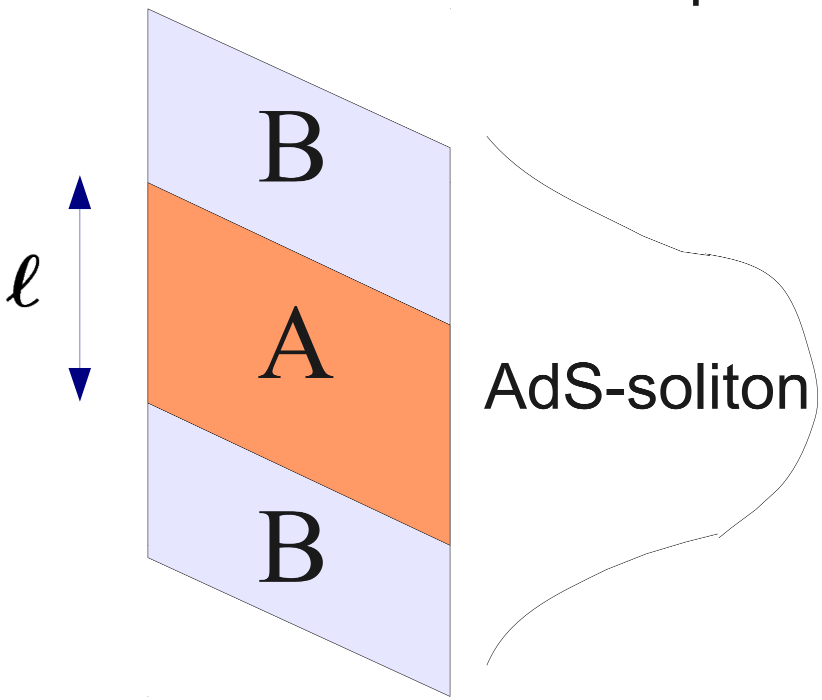
- Consistent with Wald entropy.
- Choosed the terms vanishing on horizon.

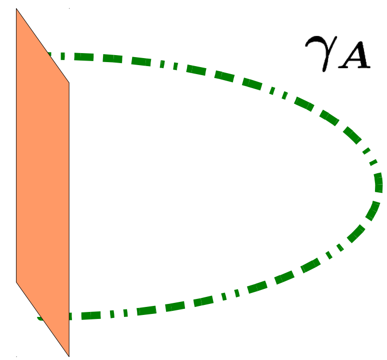
Expected to be applicable
to any curved surfaces γ_A



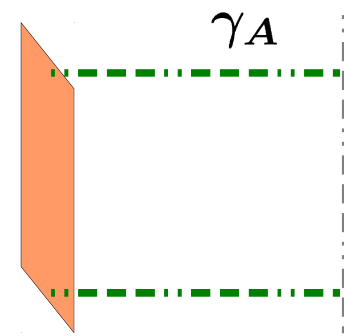
Confinement/Deconfinement Phase Structure

AdS-Soliton and Strip Subsystem





"Nontrivial" surface



"Trivial" surface

Depend on ℓ

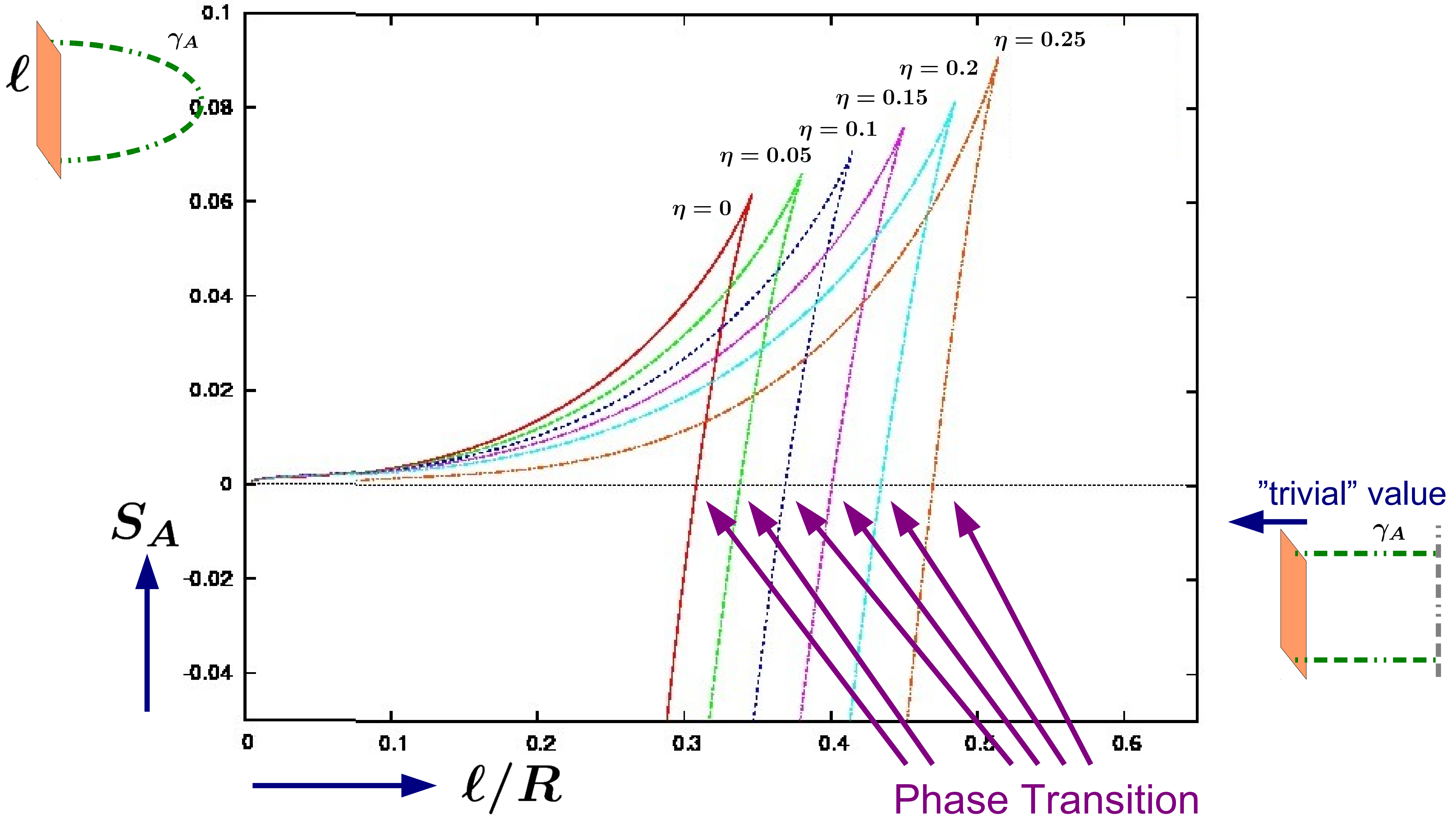
Deconfinement phase

Independent from ℓ

Confinement phase

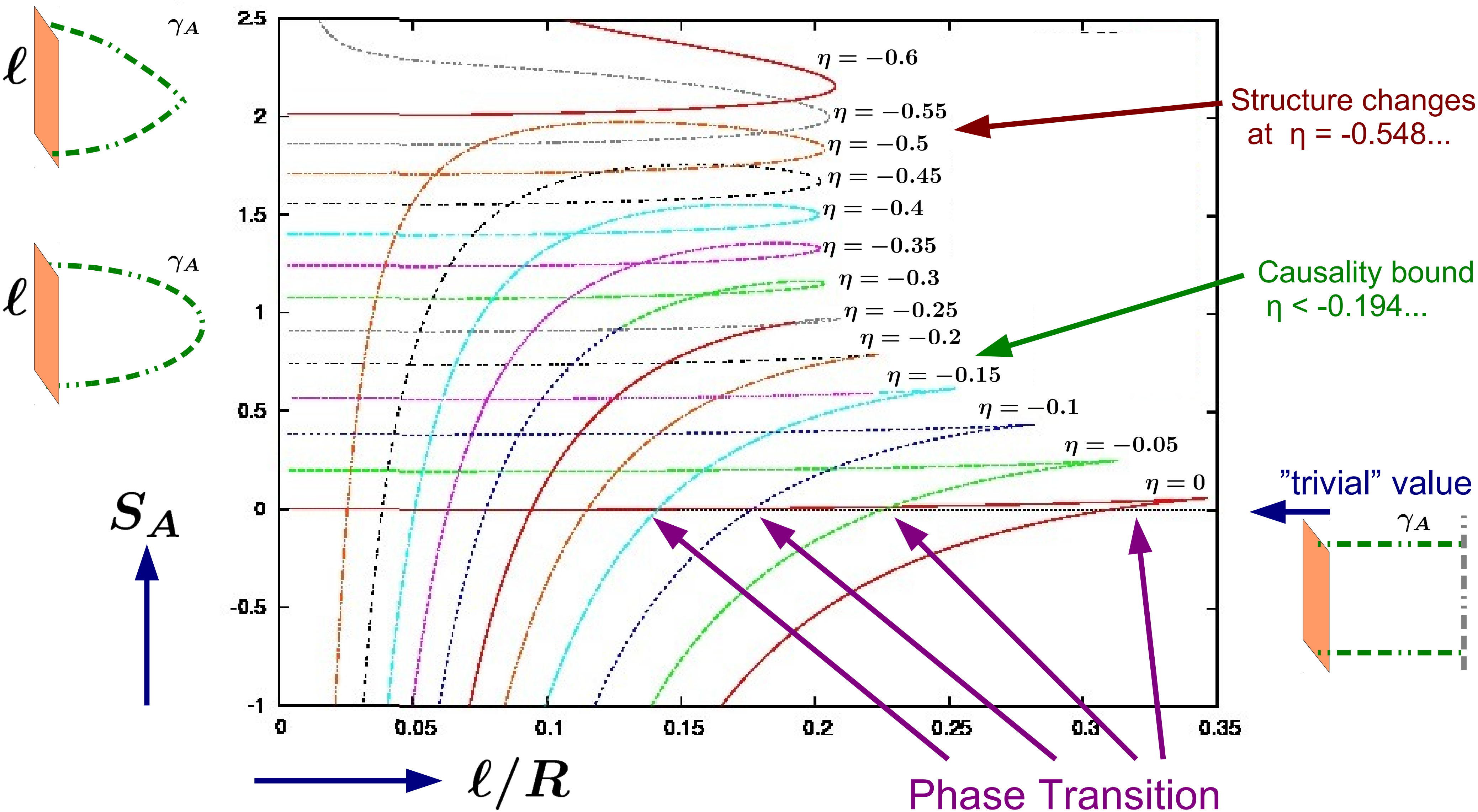
Numerical Results for 5D GB (1)

Plots of ”nontrivial” phase for various η >= 0



Numerical Results for 5D GB (2)

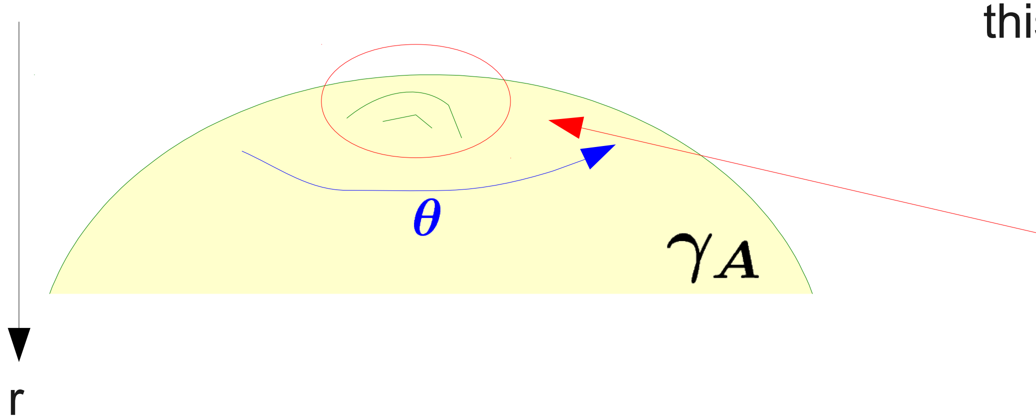
Plots of "nontrivial" phase for various η <= 0



Instability for $\eta > 0$

$$S_A = \frac{1}{4G}\int_{\gamma_A} d^3x\sqrt{h}(1+\eta L^2\mathcal{R}_h)$$

On dimensional reduction along θ,
this term becomes genus # of 2-dim sheet.



By adding small handles,
we can make S_A arbitrary small !

This implies inconsistency of the theory.
→ Higher order (R^3 or more) terms are essential.

Summary

- Corrections to S_A for A=Half-Space were computed for D3/M2/M5 solitons.

  (1/ λ , 1/N corrections.)

- For 5D GB, we investigated the dependence of the phase structure on the coupling η .

- Future work:

  extension of
  $$\mathcal{R}_{\mu\nu\rho\sigma} = \mathcal{R}^{(0)}_{\mu\nu\rho\sigma} + 2\pi(1-n)(N_{\mu\rho}N_{\nu\sigma} - N_{\mu\sigma}N_{\nu\rho})\delta(\gamma_A)$$
  $$\mathcal{R}_{\mu\nu} = \mathcal{R}^{(0)}_{\mu\nu} + 2\pi(1-n)N_{\mu\nu}\delta(\gamma_A)$$
  $$\mathcal{R} = \mathcal{R}^{(0)} + 4\pi(1-n)\delta(\gamma_A)$$
  to general γ_A

  → stringy corrections for various S_A may become caliculable.

Thank you

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