Topics in Eternal Inflation

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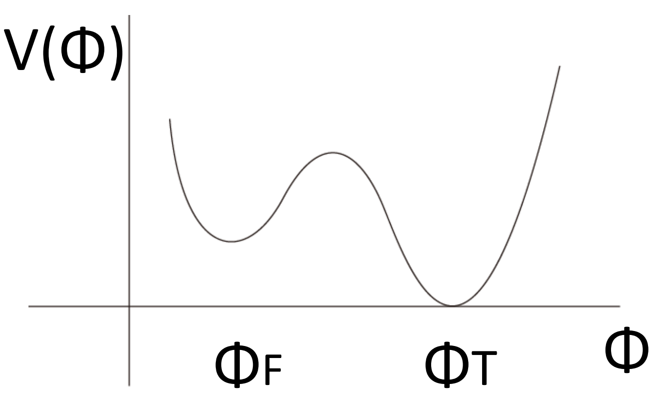
Introduction

- There is evidence that string theory has many vacua that have positive vacuum energy (Landscape of vacua).

- At the level of low-energy effective theory, described by gravity coupled to field theory whose potential has local minima. Consider a simple model:



Basic picture

- False vacuum is inflating, and decays through bubble nucleation.
- Bubble nucleation: mechanism for the creation of our universe (Natural realization of “open inflation”)
- Eternal inflation generically occurs: If the nucleation rate is small, false vacuum does not disappear, and produce infinitely many bubbles. (Concept of multiverse)
- Bubble collisions inevitably occur.

Topics of this talk

- Fluctuations in the universe inside the bubble
  - What are the signatures of the false vacuum?

    Freivogel, YS, Susskind, Yeh, hep-th/0606204;

  See also:

    Garriga, Montes, Sasaki, Tanaka, '97, '98;

    Yamauchi, Linde, Naruko, Sasaki, Tanaka, arXiv:1105.2674.

- Effect of bubble collisions
  - Non-trivial topology, "Phases" of eternal inflation

  Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, arXiv:0807.1947;

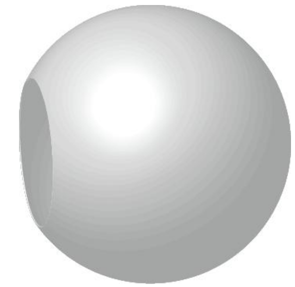
  YS, Shenker, Susskind, arXiv:1003.1347.

- Holographic dual description
  - Theoretical foundation for the landscape

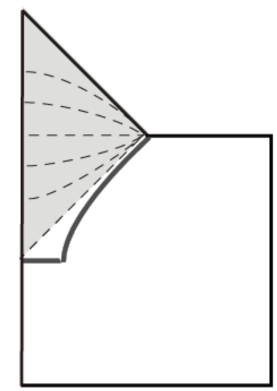
    Freivogel, YS, Susskind, Yeh, hep-th/0606204;

    YS, Susskind, arXiv:0908.3844.

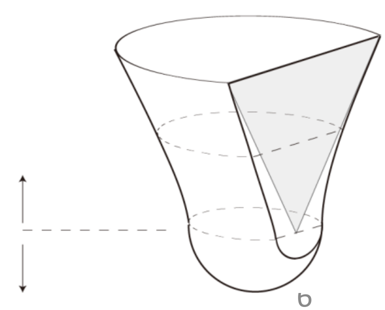
Bubble nucleation in de Sitter

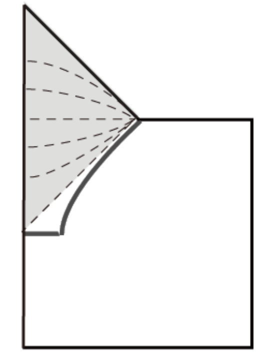


- Described by Coleman-De Luccia instanton.
  - Euclidean geometry: interpolates between true vacuum (flat disc) and
    false vacuum (de Sitter: S^4 in Euclidean)
  - Rotationally symmetric: SO(4)



- Evolution of the bubble: analytic continuation
  - Symmetry: SO(3,1)
  - Domain wall constant accelerates.
  - Open FRW universe inside the bubble.
    Constant time slice: H^3.
    "Big bang" is just a coordinate singularity.





- Bubbles are nucleated with the rate

$$\Gamma \sim e^{-(S_{\rm cl} - S_{\rm deSitter})}$$

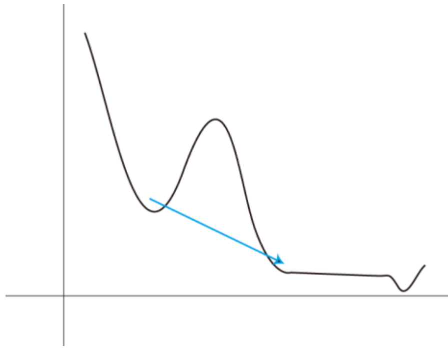
per unit time, unit volume.

- Many bubbles are nucleated in the de Sitter region.
  - One bubble does not fill the whole space.
  - If $\Gamma \ll H^4$, eternal inflation occurs.
  - Bubble collisions are inevitable ; Infinite 4-volume of false vacuum in the past light cone.

- (For the moment, let us ignore bubble collisions.)

Perturbative fluctuations

Universe inside the bubble



- Universe is curvature dominated (i.e. flat spacetime) just after tunneling.
- Slow-roll inflation ($H_I \ll H_A$) after tunneling:
  At time $t \sim H_I^{-1}$, vacuum energy of SR inflation takes over.

$$\left(\frac{\dot{a}}{a}\right)^2 = H_I^2 + \frac{1}{a^2}$$

- Tunneling gives the initial condition for SR inflation (choice of vacuum for the fluctuations).
- Constraint on curvature: $\Omega_0 > 0.98 \quad \Rightarrow \quad 7H_0^{-1} < R_{\rm curv}$
- Surface of last scattering: $R_{\rm l.s.} = \int_{z=1100}^{z=0} \frac{dt}{a} \sim 0.5 R_{\rm curv}$

Functions on hyperboloid

- Harmonics:

$$\nabla_H^2 f_{klm}(R,\Omega) = -(k^2+1) f_{klm}(R,\Omega)$$
$$e.g. \quad f_{k00}(R,\Omega) = N\frac{\sin kR}{\sinh R}$$

- Normalizable mode: continuous spectrum (real k)
  decays as exp(-R) at large R.
- Non-normalizable (supercurvature mode):
  e.g. k=i: constant, or arbitrary function finite as R -> ∞.

- Time-dep. of each mode during curvature domination is

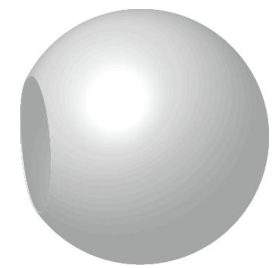
$$\ddot{\phi}_k + \frac{3}{t}\dot{\phi}_k - (k^2+1)\phi_k = 0, \qquad \phi_k \sim t^{-1\pm ik}$$

Calculation of correlation function

- Euclidean (Hartle-Hawking) prescription:
  Compute the correlator in Euclidean space, and analytically continue.

$$ds_E^2 = a^2(X)\left(dX^2 + d\theta^2 + \sin^2\theta d\Omega_2^2\right) \qquad (-\infty \le X \le \infty)$$

$$a(X) = \tilde{H}_A^{-1} e^X \quad (\text{flat}), \qquad a(X) = \frac{H_A^{-1}}{\cosh X} \quad (\text{de Sitter})$$

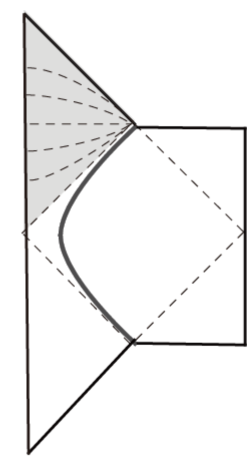


- Analytic continuation to FRW:

$$X \to T + \frac{\pi}{2} i, \quad \theta \to iR$$

$$ds^2 = a^2(T)(-dT^2 + dR^2 + \sinh^2 R d\Omega_2^2)$$

$$\text{Early time: } a(T) \sim \tilde{H}_A e^T$$



- (Analytic cont. to the “center” region):

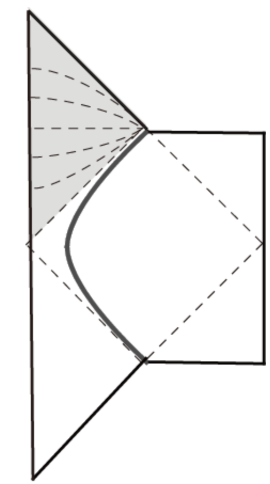
$$\theta \to \frac{\pi}{2} + i\tau, \quad ds^2 = a^2(X)\left(dX^2 - d\tau^2 + \cosh^2\tau d\Omega_2^2\right)$$

Euclidean correlator

- e.o.m. for a minimally coupled scalar:

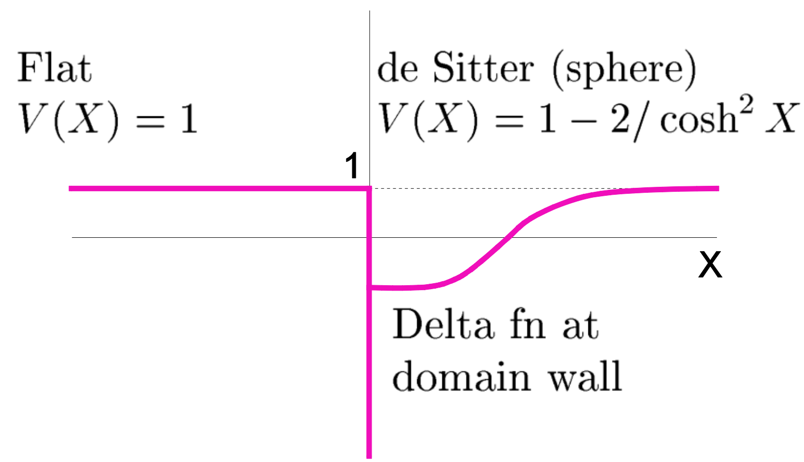
$$\left[-\partial_X^2 + \frac{a''}{a} - \nabla_S^2 + m^2 a^2\right](a\phi) = 0$$

- Calculation of the correlator is essentially a 1-dimensional scattering problem (c.f. Garriga, Montes, Sasaki, Tanaka, '98):



$$\left[-\partial_X^2 + \frac{a''}{a} + m^2 a^2\right] u_k(X) = (k^2+1) u_k(X)$$

$V(X) = a''/a$ (in the thin-wall limit)



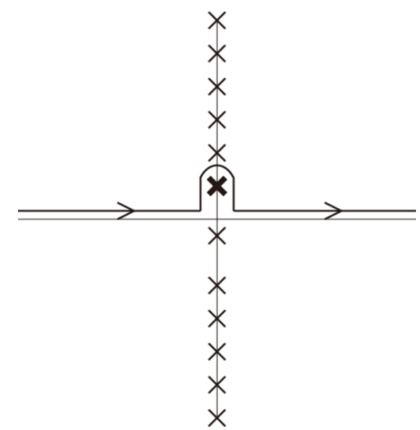
- Using the solutions to the 1D scattering,

$$\langle\phi(X,\theta)\phi(X',0)\rangle = \frac{1}{a(X)a(X')}\int dk u_k(X)u_k^*(X')G_k(\theta) + \text{(bound state)}$$

where $\left[\nabla_S^2 + (k^2+1)\right]G_k(\theta) = \delta(\theta)/\sin\theta, \quad G_k(\theta) = \frac{\sinh k(\pi-\theta)}{\sinh k\pi \sin\theta}$

- Correlator can be written in terms of reflection coeff.

$$\langle\phi(X,\theta)\phi(X',0)\rangle = \tilde{H}_A^2 e^{-(X+X')}\int_{C_1} dk\Big(e^{ik(X-X')} + \mathcal{R}(k)e^{-ik(X+X')}\Big)\frac{\sinh k(\pi-\theta)}{\sinh k\pi \sin\theta}$$



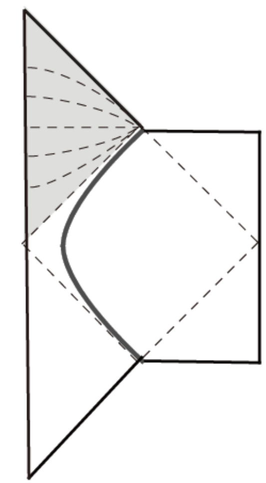
- Bound state exists when mass in the false vacuum is small compared to H_A.
  - Taken into account by deforming the contour.
  - Mode localized near the domain wall.

Correlator in open FRW

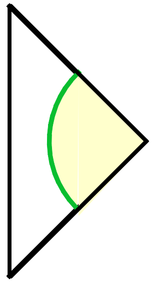
- At early time (curvature dominated era),

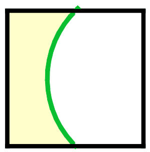
$$\langle\phi(T,R)\phi(T',0)\rangle = \tilde{H}_A^2 e^{-(T+T')}\int_{C_1} dk\Big(e^{ik(T-T')}\cosh k\pi + \mathcal{R}(k)e^{-ik(T+T')}\Big)\frac{\sin kR}{\sinh k\pi \sinh R}$$

(R: Geodesic distance on H^3)



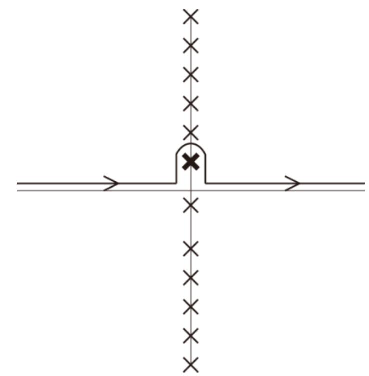
- 1st term: "Flat space piece" (Minkowski correlator written in hyperbolic slicing: Initial condition usually assumed for inflation)
- 2nd term: Effect of the ancestor vacuum.





Time-dependence: In terms of scaling dimensions

- The 1st term gives the scale invariant spectrum with square amplitude H_I^2
- The 2nd term has different time-dependence:



$$H_A^2 e^{-(T+T')} e^{-ik(T+T')}$$

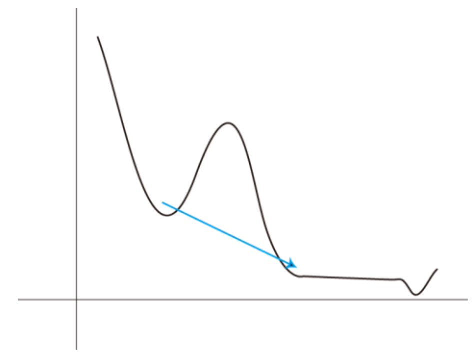
- The k-integral is given as a sum over poles.
- At the onset of SR inflation $T \sim \log(H_A/H_I)$, $H_I^2 \left(\frac{H_I}{H_A}\right)^{2b}$

- The term with the smallest b decays the least.
- Super-curvature mode (b<0) is most important, if exists.

Characteristic features of the spectrum
(see also Yamauchi et al, 2011)

- The effect of the false vacuum (if there is any) is exponentially peaked at low l (since $f_{(-ib,l,m)} \sim R^l$):

$$C_l^{(\text{ancestor})} \sim \left(\frac{R}{2}\right)^{2l} \quad (\text{ when } R < 1)$$



- For scalar (inflaton), there is small effect, at least in one field model: Mass in false vacuum is large, and there is no supercurvature mode.

- For extra fields (isocurvature perturbations), the effect could be large: There could be field with small mass in the false vacuum.

Tensor modes

- Generically, small effect on temperature fluctuations

$$\frac{\delta \mathrm{T}(\Omega)}{\mathrm{T}} = \int dT \partial_T h_{RR}(T, R(T), \Omega)$$

There is no supercurvature mode, leading dimension is

$$b \sim \alpha \equiv (r_c H_A)^2$$

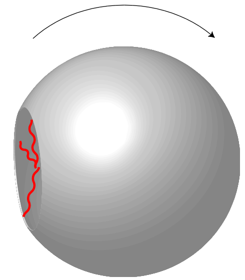
When $\alpha \ll 1$, effect could be large,

$$\langle h_{RR} h_{RR} \rangle \sim \frac{H_I^2}{\alpha^2} \left(\frac{H_I}{H_A} \right)^{2b}$$

- The effect in the B-mode polarization will be small (since the above mode is “parity-even”).

Three-point functions
(work in progress, w/ Daniel Park)

- Tree diagram for minimally coupled scalar
- Integrate the vertex over the whole Euclidean space:



$$\int dX_0 a^4(X_0) \int d^3\Omega_0 \prod_{I=1}^{3} \langle \phi(X_I, \Omega_I)\phi(X_0, \Omega_0)\rangle$$

- Where

$$\langle \phi(X_I, \Omega_I)\phi(X_0, \Omega_0)\rangle$$
$$= \frac{1}{a(X_I)a(X_0)} \int dk (e^{ik(X_I - X_0)} + \mathcal{R}(k) e^{-ik(X_I + X_0)}) \frac{\sinh k(\pi - \theta_{I0})}{\sinh k\pi \sin\theta_{I0}} \quad (X_0 < X_{\rm DW})$$
$$= \frac{1}{a(X_I)a(X_0)} \int dk \mathcal{T}(k) e^{ik(X_I - X_0)} \frac{\sinh k(\pi - \theta_{I0})}{\sinh k\pi \sin\theta_{I0}} \quad (X_{\rm DW} < X_0)$$

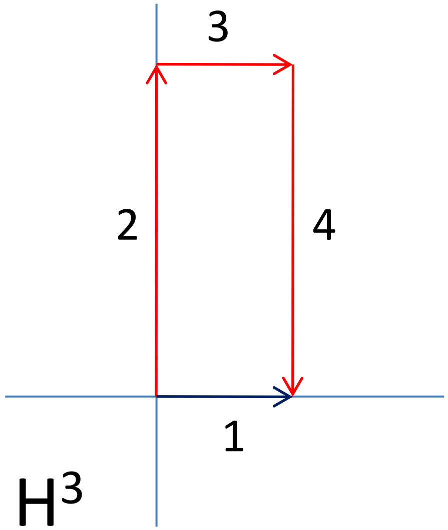
(the $H_A \to 0$ limit for simplicity)

- First do the X integral:

$$C_{k_1k_2k_3}(X_1,X_2,X_3;X_{\rm DW}) = \int_{-\infty}^{X_{\rm DW}} dX_0 a(X_0) \prod_{I=1}^{3} e^{-ik_IX_0}(e^{ik_IX_I} + \mathcal{R}(k_I)e^{-ik_IX_I}) + \int_{X_{\rm DW}}^{\infty} dX_0 a(X_0) \prod_{I=1}^{3} e^{-ik_IX_0}\mathcal{T}(k_I)e^{ik_IX_I}$$

- Analytically continue external points: $\theta_I \to iR_I,\ X_I \to T_I + \frac{\pi}{2}i$
- Then, deform contour for θ_0 integral:

$$\begin{aligned} 1 &: \ \text{real } \theta, \quad 0 \le \theta_0 \le \pi \\ 2 &: \ \theta_0 \to iR_0, \quad 0 \le R_0 \le \infty \\ 3 &: \ \theta_0 \to \tilde{\theta}_0 + i\infty, \quad \text{Can be neglected} \\ 4 &: \ \theta_0 \to \pi + iR_0, \quad 0 \le R_0 \le \infty \end{aligned}$$



- We turn integral over S^3 to integral over H^3

- Three-point function in open universe:

$$\langle\phi(T_1,R_1,\omega_1)\phi(T_2,R_2,\omega_2)\phi(T_3,R_3,\omega_3)\rangle$$
$$=\prod_{I=1}^{3}\left(\frac{1}{a(T_I)}\int dk_I\,\frac{1}{\sinh k_I\pi}\right)\tilde{C}_{k_1,k_2,k_3}(T_1,T_2,T_3;X_{\rm DW})$$
$$\times\int_0^\infty dR_0\sinh^2 R_0\int d^2\omega_0\,\frac{\sin k_1R_{10}}{\sinh R_{10}}\,\frac{\sin k_2R_{20}}{\sinh R_{20}}\,\frac{\sin k_3R_{30}}{\sinh R_{30}}$$

Bubble Collisions:
Phases of eternal inflation,
Non-trivial topology

Outline

- There are three phases of eternal inflation, depending on the nucleation rate.

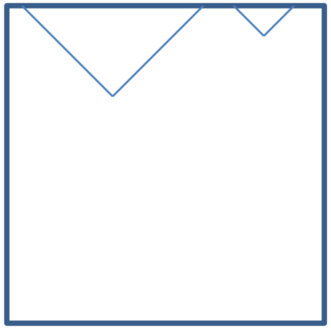
- Phases are characterized by the existence of percolating structures (lines, sheets) of bubbles in global de Sitter. (First proposed by Winitzki, ’01)

- The geometry of the true vacuum region is qualitatively different in each phase.

Distribution of bubbles: view from future infinity

- Consider conformal future of de Sitter.
  (future infinity in comoving coordinates)

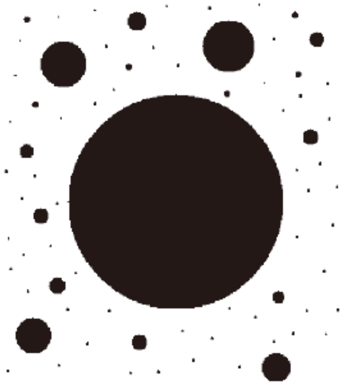
$$ds^2 \sim \frac{-d\eta^2 + d\vec{x}^2}{H^2\eta^2} \qquad (-\infty < \eta < 0)$$



- A bubble: represented as a sphere cut out from de Sitter.

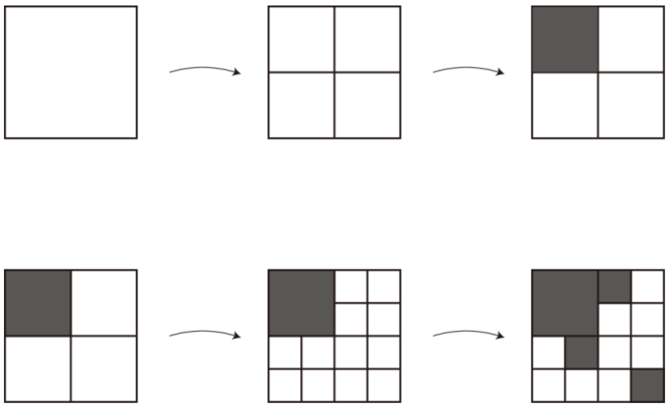
- "Scale invariant" distribution of bubbles

  Bubbles nucleated earlier:
  appear larger: radius $\sim H^{-3}|\eta|^3$
  rarer: volume of nucleation sites $\sim |\eta|^{-3}$



Model for eternal inflation

- Mandelbrot model (Fractal percolation)
  - Start from a white cell.
    (White: inflating, Black: non-inflating
    Cell: One horizon volume)
  - Divide the cell into cells
    with half its linear size.
    (The space grows by a factor of 2.
    Time step: $\Delta t = H^{-1} \ln 2$)
  - Paint each cell in black with probability P.
    (P ~ $\Gamma V_{\rm hor} \Delta t$ = nucleation rate per horizon volume: constant)
  - Subdivide the surviving (white) cells, and paint cells
    in black w/ probability P. Repeat this infinite times.



Picture of the 2D version

Mandelbrot model defines a fractal

- If P > 1 - $(1/2)^3$ =7/8, the whole space turns black, since (the rate of turning black) > (the rate of branching). (No eternal inflation)

- If P < 7/8, white region is a fractal. Non-zero fractal dimension d_F (rate of growth of the cells):

$$N_{\rm cells} = 2^{n d_F}, \quad d_F = 3 - |\log(1-P)|/\log 2$$
$$(n : \# \text{ of steps})$$

  Physical volume of de Sitter region grows. (Eternal inflation)

- Fractals in eternal inflation: c.f. Vilenkin, Winitzki, …

Three phases of eternal inflation

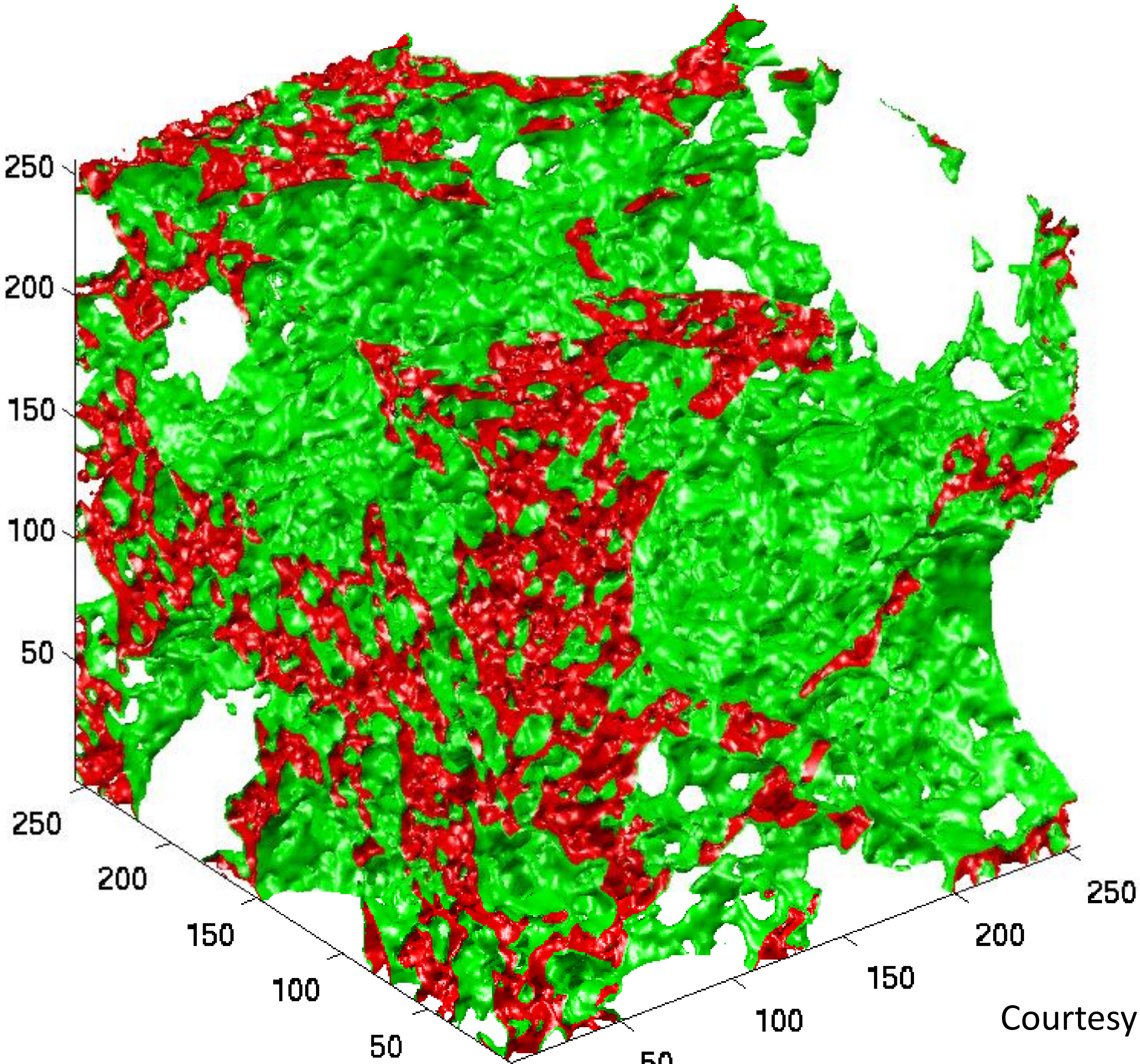
From the result on the 3D Mandelbrot model

[Chayes et al, Probability Theory and Related Fields 90 (1991) 291]

In order of increasing P (or Γ), there are

(white = inflating, black = non-inflating)

- Black island phase: Black regions form isolated clusters;
  ・ percolating white sheets.
- Tubular phase: Both regions form tubular network;
  ・ percolating black and white lines.
- White island phase: White regions are isolated;
  ・ percolating black sheets.



Courtesy of Vitaly Vanchurin

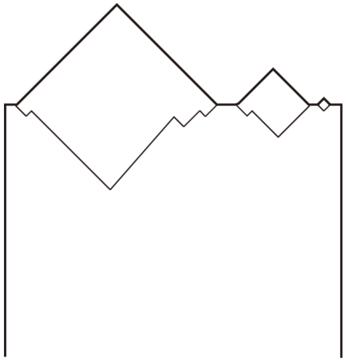
Geometry of the true vacuum region

- Mandelbrot model: the picture of the de Sitter side. (de Sitter region outside the light cone of the nucleation site is not affected by the bubble.)

- To find the spacetime in the non-inflating region inside (the cluster of) bubbles, we need to understand the dynamics of bubble collisions.

- In the following, we study this using the intuition gained from simple examples of bubble collisions.

Black island phase (isolated cluster of bubbles)

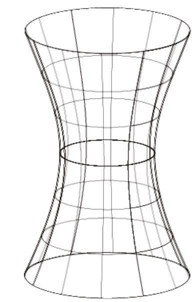


Small deformations of open FRW universe.

- Basic fact: A collision of two bubbles (of the same vacuum) does not destroy the bubble
  [c.f. Bousso, Freivogel, Yang, '07]

  – Residual symmetry SO(2,1): spatial slice has H^2 factor
  – Negative curvature makes the space expand.
  – Spatial geometry approaches smooth H^3 at late time.

Collision of two bubbles

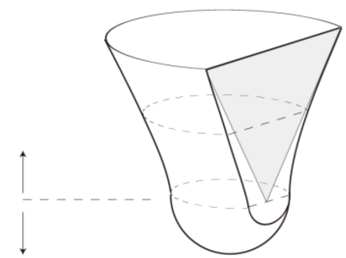


- De Sitter space: hyperboloid in $R^{4,1}$

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2$$

- One bubble: plane at $X_4 = \text{const.} = \sqrt{\ell^2 - r_0^2}$
- Second bubble: plane at $X_3 = \text{const.} = \sqrt{\ell^2 - r_0^2}$

  Residual sym: SO(2,1)



- Parametrization of de Sitter w/ manifest H^2 factor :

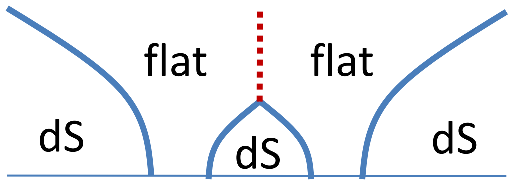
$$ds^2 = -f^{-1}(t)dt^2 + f(t)dz^2 + t^2 dH_2^2$$
$$f(t) = 1 + t^2/\ell^2, \quad (0 \le z \le 2\pi\ell)$$

$$\left(X_a = tH_a \ (a = 0,1,2), \quad X_3 = \sqrt{t^2+\ell^2}\cos(z/\ell), \quad X_4 = \sqrt{t^2+\ell^2}\sin(z/\ell)\right)$$

- Parametrization of flat space:

$$ds^2 = -dt^2 + dz^2 + t^2 dH_2^2$$

- Profile of domain wall in (t, z) space (H^2 is attached)

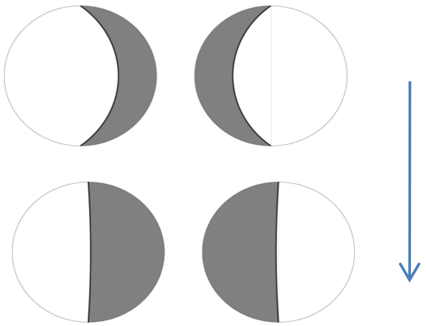


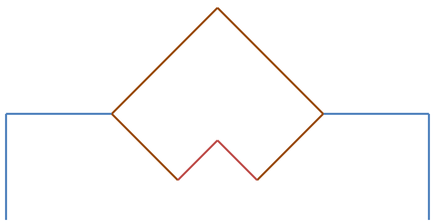
$$ds_{\rm DW}^2 = -d\tau^2 + R^2(\tau) dH_2^2 \qquad (R(\tau) = t(\tau))$$

- Energy on the domain wall decays at late time

$$\rho = \rho_0/t^2 \quad \text{(for dust wall)}$$

- The spatial geometry approach smooth H^3





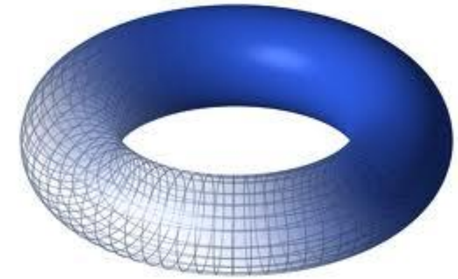
Tubular phase

- True vacuum region form tubular network.
- In the late time limit, negatively curved space which has a boundary with non-trivial topology:

$$ds^2 = -dt^2 + t^2 ds^2_{H/\Gamma}$$

  - $ds^2_{H/\Gamma}$: H^3 modded out by discrete elements of isometry
  - Boundary genus = # of elements

Example: toroidal boundary

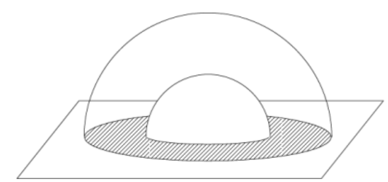


- Space: bulk of a torus
- Infinite distance to get to the boundary
- Non-contractible circle in the bulk has finite length.

- Can be constructed by identification of H^3 by

$$ds^2_{H^3} = \frac{dx^2 + dy^2 + dz^2}{z^2}$$

$$(x, y, z) \sim \lambda(x, y, z)$$

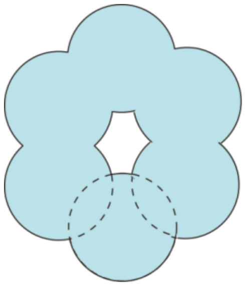


- There are geodesics can come back to the original point (generically once; comes in from boundary and goes out to boundary).

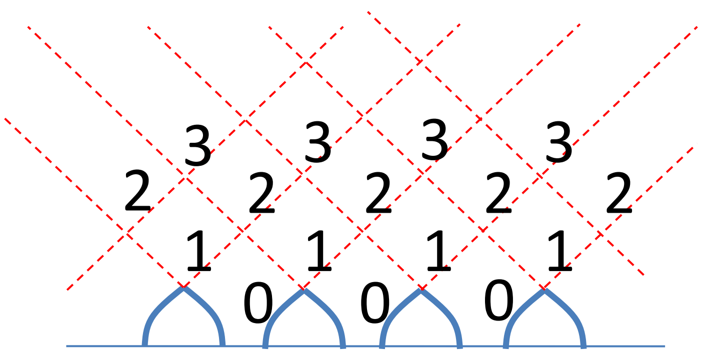
- Simpler example: true vacuum with toroidal boundary

  [Bousso, Freivogel, YS, Shenker, Susskind, Yang, Yeh, '08]

  - Ring-like initial configuration of bubbles (with the hole larger than horizon size)
  - Solve a sequence of junction conditions



$$ds^2 = -f(t)dt^2 + f^{-1}(t)dz^2 + t^2 dH_2^2$$
$$f(t) = 1 + t^2/\ell^2 \quad \text{(de Sitter)}$$
$$f(t) = 1 - t_n/t \quad (\text{in region } n;\ t_n\text{: const.})$$

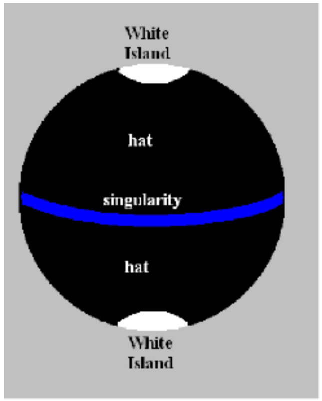


  - Approaches flat spacetime at late time. (Negatively curved spatial slice with toroidal boundary)

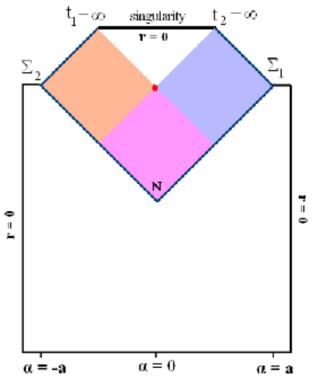
White island phase (isolated inflating region)

An observer in the black region is "surrounded" by the white region (contrary to the intuition from Mandelbrot model).

- Simple case: two white islands (with S^2 symmetry)
  [Kodama et al '82, BFSSSYY '08]



Global slicing (S^3) of de Sitter

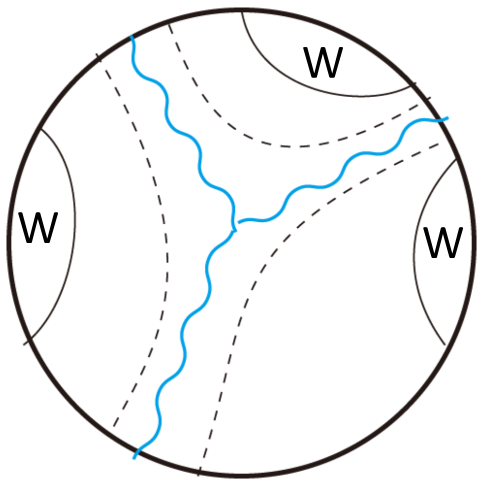


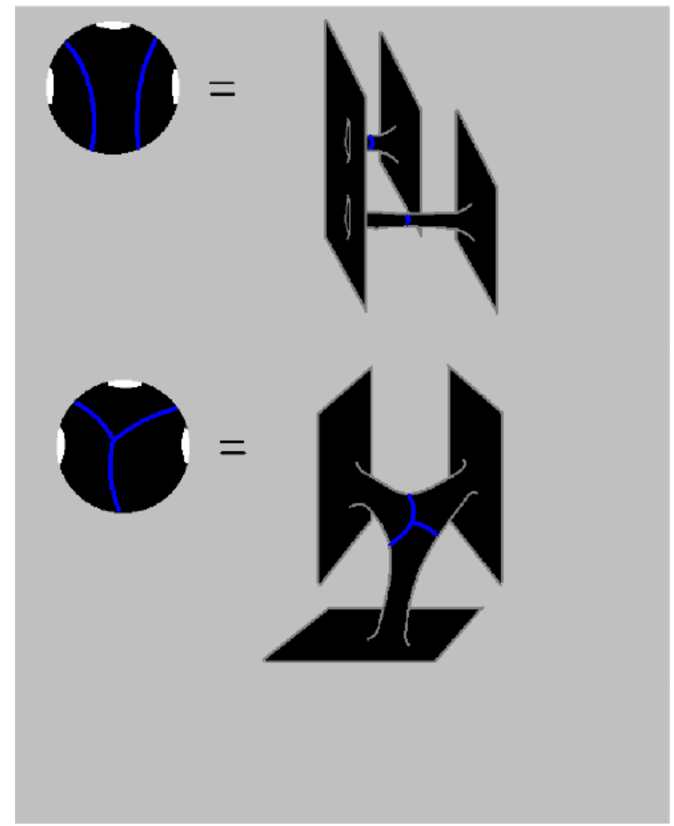
Penrose diagram

  - An observer can see only one boundary; the other boundary is behind the black hole horizon. [c.f. "non-traversability of a wormhole", "topological censorship"]

- In the white island phase, a white region will split.
  - Late time geometry for the three white island case:

[Kodama et al ’82]



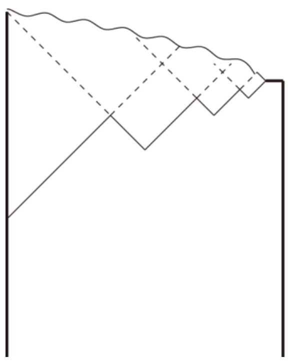


  - Singularity and horizons will form so that the boundaries are causally disconnected from each other.

- From the Mandelbrot model: A single white island is of order Hubble size (due to frequent bubble nucleation)

- The boundary moves away from a given observer, but its area remains finite. (Effectively a closed universe)
- Black hole in the bulk.

- This universe will eventually collapse.
  - Simpler model: Shells of bubbles constantly colliding to a given bubble
  - Any given observer in the true vacuum will end up at singularity.



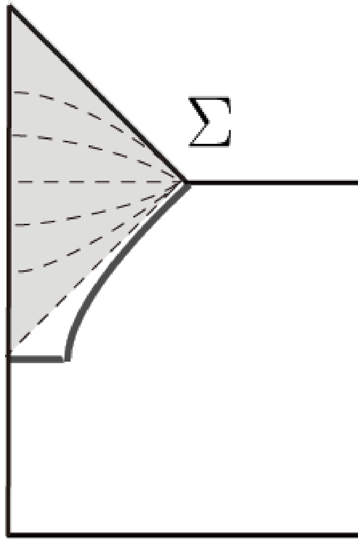
Summary of this part

Three phases of eternal inflation and their cosmology:

- Black island phase:
  Small deformation of an open FRW
- Tubular phase:
  Negatively curved space with an infinite genus boundary
- White island:
  Observer sees one boundary and one or more black hole horizons (behind which there are other boundaries).

Holographic duality

Proposal: FRW/CFT duality



The open FRW created by bubble nucleation is described by a conformal field theory on S^2 (at the boundary of H^3)

- SO(3,1): conformal sym in 2D (as in AdS/CFT).
- The dual has 2 less dimensions than the bulk.
  - The dual theory contains gravity (Liouville field). (It is a 2D gravity coupled to matter with c>25).
  - Liouville field plays the role of time.
  - "Census taker" sees more and more stuff at late time: Corresponds to finer and finer cutoff.

Near boundary behavior of correlators

- Two-point function: sum of terms with definite dimensions

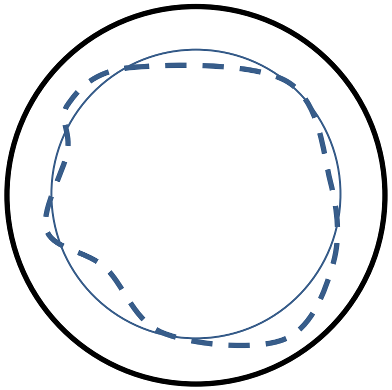
$$\langle\phi(T,R)\phi(T',0)\rangle = \sum_{\Delta} G^{(1)}_{\Delta} e^{-\Delta(R_1+R_2)} e^{-\Delta(T_1+T_2)} (1-\cos\Omega)^{-\Delta} + \sum_{\Delta'=2}^{\infty} G^{(2)}_{\Delta'} e^{-\Delta'(R_1+R_2)} e^{(\Delta'-2)(T_1+T_2)} (1-\cos\Omega)^{-\Delta'}$$

(Ω: angular separation on S^2)

Δ = 2,3,4,…; and 0, 1+α ($0<\alpha<1$) for massless fields).

- One bulk field corresponds to a tower of CFT operators.
  – Operators which scale like $e^{-\Delta(T+R)}$:
  "RG-invariant" operators (defined at the UV scale)
  – Operators which scale like $e^{(\Delta-2)T-\Delta R}$:
  "RG-covariant" operators (defined at reference scale)

Graviton correlator



- Supercurvature mode with $k = i \ \ (\Delta = 0)$
  Pure gauge in the bulk; has physical effect on the boundary ("How the boundary is embedded in the bulk").

- From Euclidean prescription, we get correlator of 2D curvature

$$\langle R^{(2)} R^{(2)} \rangle = \frac{1}{(1 - \cos \Omega)^2}$$

This remains finite as $R \to \infty$:
Gravity is not decoupled at the boundary.

- Dimension 2 piece of graviton is transverse (conserved)-traceless in 2D.
  - Identified as energy-momentum tensor of 2D CFT
  - Evidence for the existence of local 2D theory.

- 3-point function $\langle h\phi\phi\rangle$ will tell us about operator product expansion.
  - Normalization of energy momentum tensor will be fixed, and central charge will be found
  - From scaling argument, it is of order de Sitter entropy.

Conclusions

- We have studied
  - Fluctuations in the universe in a bubble
  - Phases of eternal inflation; non-trivial topology
  - Holographic duality

- There are many issues that are not well understood:
  - Multi-field tunneling
  - Observational consequences of the phases
  - Holographic duality (does dS/CFT make sense?)
  - Measure problem