Inflation in modified gravitational theories

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Inflationary models

Up to now many inflationary models have been proposed. The conventional inflation is driven by a field potential with the Lagrangian

\[ P = X - V(\phi) \]

where \( X = -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2 \)

Meanwhile there are other types of single-field models such as

- K-inflation: the Lagrangian includes non-linear terms in \( X \).
  \[ P = P(\phi, X) \]  
  Examples: ghost condensate, DBI

- f(R) gravity: a simple example is \( f(R) = R + R^2/(6M^2) \) \[\text{[Starobinsky, 1980]}\]
  f(R) gravity is equivalent to (generalized) Brans-Dicke theory with \( \omega_{BD} = 0 \)

- Non-minimal coupling models: a scalar field couples to the Ricci scalar.
  \[ F(\phi) R \]  
  Example: Higgs inflation

- Galileon (G) inflation: the Lagrangian is constructed to satisfy the symmetry.
  \[ \partial_\mu \phi \rightarrow \partial_\mu \phi + b_\mu \]  
  (in the limit of flat space-time)
The most general single-field scalar-tensor theories having second-order equations of motion:

\[ S = \int d^4 x \sqrt{-g} \left[ \frac{M_{\text{Pl}}^2}{2} R + P(\phi, X) - G_3(\phi, X) \Box \phi + \mathcal{L}_4 + \mathcal{L}_5 \right]. \]

\[ \mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right] \]

\[ \mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} (\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5,X} \left[ (\Box \phi)^3 - 3(\Box \phi) (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi) (\nabla^\alpha \nabla_\beta \phi) (\nabla^\beta \nabla_\mu \phi) \right] \]

This action covers most of the single-field scalar field models of inflation proposed in literature.

- K-inflation
- Non-minimal coupling models
  \[ G_4 = F(\phi) \quad \rightarrow \quad \text{Scalar-tensor theories (including f(R) gravity), Higgs inflation} \]
  \[ G_5 = F(\phi) \quad \rightarrow \quad \text{Field-derivative coupling models (‘New Higgs inflation’)} \]
- Galileon inflation
  \[ P = X - c \phi, \quad G_3 \propto X, \quad G_4 \propto X^2, \quad G_5 \propto X^2 \]

Even the Gauss-Bonnet coupling \( \xi(\phi)G \) can be recovered for a particular choice of \( P, G_3, G_4, G_5 \). (Kobayashi, Yamaguchi, Yokoyama, arXiv: 1105.5723)
Discrimination between single-field inflationary models

One can classify a host of inflationary models observationally from

1. The spectral index $n_R$ of scalar curvature perturbations
2. The tensor-to-scalar ratio $r$

For the most general scalar-tensor theories, these observables are evaluated by Kobayashi, Yamaguchi, Yokoyama (arXiv: 1105.5723).

See also Naruko and Sasaki (CQG, 2011), De Felice and S.T. (JCAP, 2011), Gao (2011)

In those theories the scalar propagation speed $C_s$ is in general different from 1.

In k-inflation the scalar non-Gaussianities are large for $C_s^2 \ll 1$

How about the scalar non-Gaussianities in the most general single-field scalar-tensor theories?

The spectrum of curvature perturbations

We consider scalar metric perturbations $\alpha, \psi, \mathcal{R}$ with the ADM metric

$$ds^2 = -[(1 + \alpha)^2 - a(t)^{-2} e^{-2\mathcal{R}} (\partial \psi)^2] dt^2 + 2\partial_i \psi dt dx^i + a(t)^2 e^{2\mathcal{R}} dx^2$$

We choose the uniform field gauge: $\delta \phi = 0$

Using the momentum and Hamiltonian constraints, the second-order action for perturbations reduces to

$$S_2 = \int dt d^3 x a^3 Q \left[ \dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2} (\partial \mathcal{R})^2 \right]$$

where

$$Q = \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}, \quad c_s^2 = \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1w_1w_2 - 2w_1^2w_2)}{w_1(4w_1w_3 + 9w_2^2)}$$

$$w_1 = M_{pl}^2 F - 4XG_{4,X} - 2H X \dot{\phi}G_{5,X} + 2XG_{5,\phi}$$

$$w_2 = 2M_{pl}^2 H F - 2X \dot{\phi}G_{3,X} - 16H(XG_{4,X} + X^2G_{4,XX}) + 2\dot{\phi}(G_{4,\phi} + 2XG_{4,\phi X}) - 2H^2 \dot{\phi}(5XG_{5,X} + 2X^2G_{5,XX}) + 4HX(3G_{5,\phi} + 2XG_{5,\phi X})$$

$$w_3 = -9M_{pl}^2 H^2 F + 3(XP_{,X} + 2X^2P_{,XX}) + 18H\dot{\phi}(2XG_{3,X} + X^2G_{3,XX}) - 6X(G_{3,\phi} + XG_{3,\phi X}) + 18H^2 (7XG_{4,X} + 16X^2G_{4,XX} + 4X^3G_{4,XXX}) - 18H\dot{\phi}(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2G_{4,\phi XX}) + 6H^3 \dot{\phi}(15XG_{5,X} + 13X^2G_{5,XX} + 2X^3G_{5,XXX}) - 18H^2 X(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2G_{5,\phi XX})$$

$$w_4 = M_{pl}^2 F - 2XG_{5,\phi} - 2XG_{5,X} \ddot{\phi}$$

The scalar power spectrum is

$$\mathcal{P}_\mathcal{R} = \frac{H^2}{8\pi^2 Q c_s^3}$$

Kobayashi, Yamaguchi, Yokoyama (arXiv: 1105.5723)
De Felice and S. T. (arXiv: 1107.3917)
The spectrum of tensor perturbations

The second-order action for tensor perturbations is

\[ S = \sum_{\lambda} \int dt d^3 x \ a^3 Q_T \left[ \dot{h}_\lambda^2 - \frac{c_T^2}{a^2} (\partial h_\lambda)^2 \right] \]

\[ \lambda \text{ corresponds to polarization modes} \]

where

\[ Q_T = \frac{w_1}{4} = \frac{1}{4} M_{\text{pl}}^2 F [1 + \mathcal{O}(\epsilon)] \]

\[ c_T^2 = w_4 / w_1 = 1 + \mathcal{O}(\epsilon) \]

\[ \text{where } F = 1 + 2 G_4 / M_{\text{pl}}^2 \text{ and } \epsilon = -\dot{H} / H^2 \ll 1. \]

This term comes from the nonminimal coupling in \( \mathcal{L}_4 \)

\[ \mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} \left[ (\Box \phi)^2 - (\nabla_\mu \nabla_\nu \phi) (\nabla^\mu \nabla^\nu \phi) \right] \]

The no-ghost condition is satisfied for \( F > 0 \).

The tensor power spectrum is

\[ P_T = \frac{H^2}{2 \pi^2 Q_T c_T^3} \approx \frac{2 H^2}{\pi^2 M_{\text{pl}}^2 F} \]

The tensor-to-scalar ratio is

\[ r = \frac{P_T}{P_{\mathcal{R}}} \approx 16 c_s \epsilon_s \]

where \( \epsilon_s = \frac{Q c_s^2}{M_{\text{pl}}^2 F} = \mathcal{O}(\epsilon) \)
Scalar non-Gaussianities

For the ADM metric with scalar metric perturbations the third-order perturbed action is given by (obtained after many integrations by parts)

\[ S_3 = \int dt \, \mathcal{L}_3 \]

where

\[
\mathcal{L}_3 = \int d^3x \left\{ a^3 c_1 M_{pl}^2 \mathcal{R} \mathcal{R}^2 + a c_2 M_{pl}^2 \mathcal{R} (\partial \mathcal{R})^2 + a^3 c_3 M_{pl} \dot{\mathcal{R}}^3 + a^3 (c_4 + M_{pl}^2) \partial \mathcal{R} (\partial_i \mathcal{R})(\partial_i \mathcal{X}) + a^3 (c_5 + M_{pl}^2) \partial^2 \mathcal{R} (\partial_i \mathcal{X})^2 \\
+ a c_6 \mathcal{R}^2 \partial \mathcal{R} + c_7 \left[ \partial^2 \mathcal{R} (\partial \mathcal{R})^2 - \mathcal{R} \partial_i \partial_j (\partial_i \mathcal{R})(\partial_j \mathcal{R}) \right] / a + a (c_8 + M_{pl}) \left[ \partial^2 \mathcal{R} \partial_i \mathcal{R} \partial_i \mathcal{X} - \mathcal{R} \partial_i \partial_j (\partial_i \mathcal{R})(\partial_j \mathcal{X}) \right]
\]

\[ \left. \mathcal{F}_1 \right|_{\delta \mathcal{R}} = -2 \left[ \frac{d}{dt} (a^3 Q \dot{\mathcal{R}}) - a Q c_2 \partial^2 \mathcal{R} \right] \]

This vanishes at first order in \( \mathcal{R} \).

The vacuum expectation value of \( \mathcal{R} \) for the three-point operator at the conformal time \( \tau = \tau_f \) is

\[
\langle \mathcal{R}(k_1) \mathcal{R}(k_2) \mathcal{R}(k_3) \rangle = -i \int_{\tau_i}^{\tau_f} d\tau \, a \langle 0 | [\mathcal{R}(\tau_f, k_1) \mathcal{R}(\tau_f, k_2) \mathcal{R}(\tau_f, k_3), \mathcal{H}_{\text{int}}(\tau)] | 0 \rangle
\]

where \( \mathcal{H}_{\text{int}} = -\mathcal{L}_3 \). 

We find that the three-point correlation function is given by

\[
\langle R(k_1)R(k_2)R(k_3) \rangle = (2\pi)^7 \delta^{(3)}(k_1 + k_2 + k_3) \langle P_R \rangle^2 \frac{A_R}{\prod_{i=1}^3 k_i^3}
\]

where

\[
A_R = \frac{M_{pl}^2}{Q} \left\{ \frac{1}{4} \left( \frac{2}{K} \sum_{i>j} k_i^2 k_j^2 - \frac{1}{K^2} \sum_{i \neq j} k_i^2 k_j^3 \right) C_1 + \frac{1}{4c_s^2} \left( \frac{1}{2} \sum_{i} k_i^3 + \frac{2}{K} \sum_{i>j} k_i^2 k_j^2 - \frac{1}{K^2} \sum_{i \neq j} k_i^2 k_j^3 \right) C_2 
\]

\[+ \frac{3}{2} \frac{H}{M_{pl}} \frac{(k_1 k_2 k_3)^2}{K^3} C_3 + \frac{1}{8} \frac{Q}{M_{pl}^2} \left( \sum_{i} k_i^3 - \frac{1}{2} \sum_{i \neq j} k_i k_j^2 - \frac{2}{K^2} \sum_{i \neq j} k_i^2 k_j^3 \right) C_4 \]

\[+ \frac{1}{4} \left( \frac{Q}{M_{pl}^2} \right)^2 \frac{1}{K^2} \left[ \sum_{i} k_i^5 + \frac{1}{2} \sum_{i \neq j} k_i k_j^4 - \frac{3}{2} \sum_{i \neq j} k_i^2 k_j^3 - k_1 k_2 k_3 \sum_{i} k_i k_j \right] C_5 + \frac{3}{c_s^2} \left( \frac{H}{M_{pl}} \right)^2 \frac{(k_1 k_2 k_3)^2}{K^3} C_6 \]

\[+ \frac{1}{2c_s^4} \left( \frac{H}{M_{pl}} \right)^2 \frac{1}{K} \left( 1 + \frac{1}{K^2} \sum_{i>j} k_i k_j^2 + \frac{3k_1 k_2 k_3}{K^3} \right) \left[ \frac{3}{4} \sum_{i} k_i^4 - \frac{3}{2} \sum_{i>j} k_i^2 k_j^2 \right] C_7 \]

\[+ \frac{1}{8c_s^2 M_{pl} M_{pl}^2 K^2} \left[ \frac{3}{2} k_1 k_2 k_3 \sum_{i} k_i^2 - \frac{5}{2} k_1 k_2 k_3 K^2 - 6 \sum_{i \neq j} k_i^2 k_j^3 - \sum_{i} k_i^5 + \frac{7}{2} K \sum_{i} k_i^4 \right] C_8 \}

The terms \( C_i \) (\( i = 1, \cdots, 5 \)) appear in k-inflation (Seery and Lidsey, Chen et al), which can be well approximated as an equilateral estimator.

The shape of the non-Gaussianities can be also approximated as an equilateral one even in the presence of \( C_6 \) (Mizuno and Koyama) and \( C_7, C_8 \) (De Felice and S.T.).
The nonlinear parameter

We define the nonlinear parameter $f_{NL}$, as

$$f_{NL} = \frac{10}{3} \frac{A_R}{\sum_{i=1}^{3} k_i^3}$$

For the equilateral configuration ($k_1 = k_2 = k_3$) one has

$$f_{NL}^{\text{equil}} = \frac{40}{9} \frac{M_{pl}^2}{Q} \left[ \frac{1}{12} c_1 + \frac{17}{96 c_s^2} c_2 + \frac{1}{72 M_{pl} c_3} - \frac{1}{24} \frac{Q}{M_{pl}^2} c_4 - \frac{1}{24} \left( \frac{Q}{M_{pl}^2} \right)^2 c_5 + \frac{1}{36 c_s^2} \left( \frac{H}{M_{pl}} \right)^2 c_6 - \frac{13}{96 c_s^4} \left( \frac{H}{M_{pl}} \right)^2 c_7 - \frac{17}{192 c_s^2 M_{pl} Q}{M_{pl}^2 c_8} \right]$$

Under the expansion of the slow-variation parameters it follows that

$$f_{NL}^{\text{equil}} = \frac{85}{324} \left( 1 - \frac{1}{c_s^2} \right) - \frac{10}{81} \lambda + \frac{55}{81} s + \frac{1}{12} \frac{\eta_s}{c_s^2} + \frac{5}{54} c_s^2 + \frac{85}{81} \frac{s}{c_s^2} + \frac{20}{81} \frac{1 + \lambda_3 x}{\epsilon_s} + \frac{65}{162 c_s^2} \frac{\delta_{G3X}}{c_s^2}$$

$$+ \left( \frac{80}{81} \frac{3 + 2 \lambda_4 x}{\epsilon_s} + \frac{65}{27 c_s^2} \frac{\delta_{G4XX}}{c_s^2} \right) \epsilon_s + \left( \frac{20}{81} + \frac{65}{162 c_s^2} \right) \frac{\delta_{G5X}}{c_s^2} + \left( \frac{20}{81} + \frac{65}{162 c_s^2} \right) \frac{\delta_{G5XX}}{c_s^2}$$

where

$$\eta_s = \frac{\dot{c}_s}{H c_s}, \quad s = \frac{\dot{c}_s}{H c_s}, \quad \delta_{G3X} = \frac{G_{3,X} \dot{\phi} X}{M_{pl}^2 H F}, \quad \delta_{G4XX} = \frac{G_{4,XX} X^2}{M_{pl}^2 F}, \quad \delta_{G5X} = \frac{G_{5,X} H \dot{\phi} X}{M_{pl}^2 F}, \quad \delta_{G5XX} = \frac{G_{5,XX} H \dot{\phi} X}{M_{pl}^2 F}$$

$$\lambda_3 = \frac{X G_{3,XX}}{G_{3,XX}}, \quad \lambda_4 = \frac{X G_{4,XX}}{G_{4,XX}}, \quad \lambda_5 = \frac{X G_{5,XX}}{G_{5,XX}}, \quad \Sigma = \frac{w_1 (4 w_1 w_3 + 9 w_2)}{12 M_{pl}^4}.$$ 

$$\lambda = (F^2/3)[3 X^2 P_{,XX} + 2 X^3 P_{,XXX} + 3 H \dot{\phi} (X G_{3,X} + 5 X^2 G_{3,XX} + 2 X^3 G_{3,XXX}) - 2 (2 X^2 G_{3,\phi,X} + X^3 G_{3,\phi,XX})$$

$$+ 6 H^2 (9 X^2 G_{4,XX} + 16 X^3 G_{4,XXX} + 4 X^4 G_{4,XXXX}) - 3 H \dot{\phi} (3 X G_{4,\phi,X} + 12 X^2 G_{4,\phi,XX} + 4 X^3 G_{4,\phi,XXX})$$

$$+ H^3 \dot{\phi} (3 X G_{5,X} + 27 X^2 G_{5,XX} + 24 X^3 G_{5,XXX} + 4 X^4 G_{5,XXXX})$$

$$- 6 H^2 (6 X^2 G_{5,\phi,X} + 9 X^3 G_{5,\phi,XX} + 2 X^4 G_{5,\phi,XXX})]$$

Standard inflation: $P = X - V(\phi), c_s^2 = 1, \lambda = 0 \quad \Rightarrow \quad f_{NL}^{\text{equil}} = \frac{55}{36} \epsilon_s + \frac{5}{12} \eta_s$
The scalar propagation speed

If $c_s^2 \ll 1$, the large non-linear parameter $|f_{\text{NL}}|^\text{equil} \gg 1$ can be realized. Expansion in terms of slow-variation parameters gives

$$c_s^2 \approx \frac{\delta_{PX} + 4\delta_{G3X} - 2\delta_{G3\phi} + 6\delta_{G4X} + 20\delta_{G4XX} + 4\delta_{G5X} + 4\delta_{G5XX} - 6\delta_{G5\phi}}{\delta_{PX}(1 + 2\lambda_{PX}) + 6\delta_{G3X}(1 + \lambda_{3X}) - 2\delta_{G3\phi} + 6\delta_{G4X} + 24\delta_{G4XX}(2 + \lambda_{4X}) + 6\delta_{G5X} + 2\delta_{G5XX}(7 + 2\lambda_{5X}) - 6\delta_{G5\phi}}$$

where

$$\delta_{PX} = \frac{P_{,X}X}{M_{\text{pl}}^2 H^2 F}, \quad \lambda_{PX} = \frac{XP_{,XX}}{P_{,X}}, \quad \delta_{G3\phi} = \frac{G_{3,\phi}X}{M_{\text{pl}}^2 H^2 F}, \quad \delta_{G4X} = \frac{G_{4,X}X}{M_{\text{pl}}^2 F}, \quad \delta_{G5\phi} = \frac{G_{5,\phi}X}{M_{\text{pl}}^2 F}.$$

If either of the following conditions is satisfied, it is possible to realize the large non-Gaussianities:

$$\lambda_{PX} \gg 1, \quad \lambda_{3X} \gg 1, \quad \lambda_{4X} \gg 1, \quad \lambda_{5X} \gg 1.$$

In k-inflation one has

$$c_s^2 = \frac{1}{1 + 2\lambda_{PX}} = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}} \quad \epsilon = \delta_{PX} = \frac{P_{,X}X}{3M_{\text{pl}}^2 H^2}$$

Since k-inflation occurs around $P_{,X} \approx 0$, it follows that $c_s^2 \ll 1$.

If $P$ is function of $X$ only, a de Sitter solution with $P_{,X} = 0$ is problematic because the power spectrum $\mathcal{P}_\mathcal{R}$ diverges.

We require the $\phi$-dependence in $P$ [like DBI inflation] or additional terms $G_i$ ($i = 3, 4, 5$) [like G-inflation].

$$|f_{\text{NL}}|^\text{equil} \gg 1$$
K-inflation + Galileon terms

- The ghost condensate model plus the Galileon $G_3$ term

$$P = -X + X^2 / (2M^4), \quad G_3 = \mu X / M^4$$  
(Kobayashi, Yamaguchi, Yokoyama, PRL, 2010)

There is a de Sitter solution with $1 - x \simeq \sqrt{3} \mu / M_{\text{pl}}$ for $x = X / M^4$ close to 1.

$$f_{\text{NL}}^{\text{equil}} \simeq 5 / [6(1 - x)] \simeq 4.62r^{-2/3}$$  
(Mizuno and Koyama, PRD, 2010)

$r$ is the tensor-to-scalar ratio.

- Let us consider the Galileon $G_4$ term

$$P = -X + X^2 / (2M^4), \quad G_4 = \mu X^2 / M^7$$

For $x = X / M^4$ close to 1 there is a de Sitter solution characterized by

$$H^2 = \frac{M^3}{36\mu} \frac{1 - x}{x}, \quad \frac{\mu M}{M_{\text{pl}}^2} = \frac{1 - x}{6x^2(3 - 2x)} \quad \text{and} \quad c_s^2 = \frac{2(1 - x)}{9}$$

The inflationary observables are

$$P_R \simeq \frac{3\sqrt{2}}{256\pi^2} \left( \frac{M}{M_{\text{pl}}} \right)^4 \frac{1}{(1 - x)^{3/2}}, \quad r \simeq \frac{128\sqrt{2}}{9} (1 - x)^{3/2}, \quad f_{\text{NL}}^{\text{equil}} \simeq 1.28r^{-2/3}$$

(DeFelice and S.T., arXiv: 1107.3917)

For $r = 0.01$, $f_{\text{NL}}^{\text{equil}} = 9.4$

- Let us consider the Galileon $G_5$ term

$$P = -X + X^2 / (2M^4), \quad G_5 = \mu X^2 / M^{10}$$

$$f_{\text{NL}}^{\text{equil}} \simeq 0.17r^{-2/3}$$
Our formula of $f_{\text{NL}}^{\text{equil}}$ can be applied to most of single-field inflation models proposed so far.

Let us consider nonminimal coupling models.

(i) $G_4 = F(\phi) \quad \Rightarrow \quad \mathcal{L}_4 = F(\phi)R$ including scalar-tensor gravity and $f(R)$ gravity

$\delta_{G4X} = \delta_{G4XX} = 0 \quad \Rightarrow \quad c_s^2 = 1/(1 + 2\lambda_{P X})$

If $P$ does not have non-linear terms in $X$ we have $c_s^2 = 1$ and

$f_{\text{NL}}^\text{equil} = \mathcal{O}(\epsilon_s, \eta_s)$

This is the case for Higgs inflation ($P = X - V(\phi)$) and Brans-Dicke theories ($P = \omega_{BD}X/\phi - V(\phi)$).

(ii) $G_5 = F(\phi) \quad \Rightarrow \quad \mathcal{L}_5 = F(\phi)G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi)$

New Higgs inflation (with the coupling $G_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi$) corresponds to the choice $F(\phi) \propto \phi$.

$\delta_{G5X} = \delta_{G5XX} = 0 \quad \Rightarrow \quad c_s^2 = \frac{\delta_{P X} - 6\delta_{G5\phi}}{\delta_{P X}(1 + 2\lambda_{P X}) - 6\delta_{G5\phi}}$

If $P$ does not have non-linear terms in $X$ we have $c_s^2 = 1$ and

$f_{\text{NL}}^\text{equil} = \mathcal{O}(\epsilon_s, \eta_s)$

The above two types of nonminimal couplings themselves do not give rise to large equilateral non-Gaussianities.
Potential-driven G-inflation

Let us consider the following model

\[ P = X - V(\phi), \quad G_3 = -\frac{1}{M^{4n-1}} e^{\mu \phi/M_{pl}} X^n \]

The Galileon coupling corresponds to \( \mu = 0 \) and \( n = 1 \).

In this case the scalar propagation speed squared is

\[ c_s^2 \simeq \frac{\delta_{PX} + 4\delta_{G3X}}{\delta_{PX} + 6n\delta_{G3X}} \]

\[ \Rightarrow \quad c_s^2 \simeq \frac{2}{3n} \] for \( \delta_{G3X} \gg \delta_{PX} \).

For the specific theories with \( n \gg 1 \) one has \( c_s^2 \ll 1 \) and

\[ f_{NL}^{\text{equil}} \simeq -\frac{865}{3888} n \]

\[ \Rightarrow \quad |f_{NL}^{\text{equil}}| \gg 1 \]

In the following let us consider the theories with \( n = 1 \) (in which case the non-Gaussianities cannot be large).

For the potential \( V(\phi) = V_0(\phi/M_{pl})^p \) the spectral index and the tensor-to-scalar ratio in the regime \( \delta_{G3X} \gg \delta_{PX} \) are

\[ n_s \simeq 1 - \frac{3(p+1)}{(p+3)N + p} \left[ 1 - \frac{2(p-1)}{3(p+1)(p+5)} \mu x \right] \]

\[ N \text{ is the number of e-foldings from the end of inflation.} \]

For \( N = 55 \), in the limit where \( \mu \to 0 \), \( n_s = 0.9614 \) and \( r = 0.1791 \) for \( p = 4 \).

The quartic potential can be saved (Kamada et al, PRD, 2010).
Observational constraints on G-inflation with the quartic potential

\[ V(\phi) = V_0\left(\phi/M_{\text{pl}}\right)^4 \quad \text{and} \quad G_3 = -\frac{1}{M^3}e^{\mu\phi/M_{\text{pl}}}X \]

The left figure corresponds to \( \mu = 0 \) and \( \mu = 1 \) where \( B = [V_0/(M^3 M_{\text{pl}})]^{1/4} \)

The quartic potential is saved by the Galileon-like corrections.

Kamada, Kobayashi, Yamaguchi, Yokoyama, PRD, 2010 \((\mu = 0)\)

De Felice, S.T., Elliston, Tavakol, JCAP to appear \((\mu \neq 0)\)

For \( \mu > 0 \) the intermediate values of \( B \) are within 1\( \sigma \) observational bound.

\[ B \to 0 \text{ corresponds to } \frac{\delta_{PX}}{\delta_{G3X}} \gg 1. \quad \text{(standard inflation)} \]

\[ B \to \infty \text{ corresponds to } \frac{\delta_{PX}}{\delta_{G3X}} \ll 1. \quad \text{(G-inflation)} \]
Gauss-Bonnet couplings

Our general action covers the Gauss-Bonnet coupling $-\xi(\phi)G$ by choosing

\[
\begin{align*}
P &= -8\xi^{(4)}(\phi)X^2(3 - \ln X), \\
G_3 &= -4\xi^{(3)}(\phi)X(7 - 3\ln X), \\
G_4 &= -4\xi^{(2)}(\phi)X(2 - \ln X), \\
G_5 &= 4\xi^{(1)}(\phi)\ln X
\end{align*}
\]

(Kobayashi, Yamaguchi, Yokoyama, arXiv: 1105.5723)

where $\xi^{(n)}(\phi) = \partial^n \xi(\phi)/\partial \phi^n$

For the potential-driven inflation (i.e. in the presence of the term $P = X - V(\phi)$) one has

\[
c_s^2 = 1 - 64\delta_\xi^2(6\delta_\xi + \delta_X)/\delta_X
\]

very close to $c_s^2 = 1$

where $\delta_X = X/(M_{\text{pl}}^2 H^2)$ and $\delta_\xi = H\dot{\xi}/M_{\text{pl}}^2$.

The nonlinear parameter is estimated as

\[
f_{\text{NL}}^{\text{equl}} = \frac{55}{36}\delta_X + \frac{5}{12}\eta_X + \frac{275}{81}\delta_\xi \left(4\epsilon + 2\eta_\xi - \eta_X\right)
\]

small

where $\eta_\xi = \dot{\delta}_\xi/(H\delta_\xi)$ and $\eta_X = \dot{\delta}_X/(H\delta_X)$.

This result matches with that derived explicitly in the presence of the Gauss-Bonnet coupling (De Felice and S.T., JCAP, 2011).
Observational constraints on the Gauss-Bonnet coupling

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M_{\text{pl}}^2}{2} R + X - V(\phi) - \xi(\phi)G \right] \]

where \( V(\phi) = m^2 \phi^2 / 2 \), \( \xi(\phi) = \xi_0 e^{\mu \phi / M_{\text{pl}}} \)

From the observational data of the scalar spectral index and the tensor-to-scalar ratio the GB Coupling is constrained by varying two parameters:

\[ \epsilon_s(= \delta_X), \quad r_\xi = \delta_\xi / \delta_X \]

The CMB likelihood analysis shows that the Gauss-Bonnet contribution needs to be suppressed:

\[ r_\xi \equiv \delta_\xi / \delta_X < 0.1 \quad (95\% \text{ CL}) \]

CMB likelihood analysis by CAMB

De Felice, S.T., Elliston, Tavakol, JCAP to appear (2011)
Conclusions

We have evaluated the primordial non-Gaussianities for the very general single-field models characterized by

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R + P(\phi, X) - G_3(\phi, X) \Box \phi + \mathcal{L}_4 + \mathcal{L}_5 \right].$$

We found that the equilateral nonlinear parameter is given by

$$f_{\text{NL}}^{\text{equil}} = \frac{85}{324} \left( 1 - \frac{1}{c_s^2} \right) - \frac{10 \lambda}{81 \Sigma} + \frac{55}{36} \epsilon_s + \frac{5}{12} \eta_s - \frac{85}{54} \frac{s}{c_s^2} + \left( \frac{20}{81} \frac{1 + \lambda_3 X}{\epsilon_s} + \frac{65}{162 c_s^2 \epsilon_s} \right) \delta_{G3X}$$

$$+ \left( \frac{80}{81} \frac{3 + 2 \lambda_4 X}{\epsilon_s} + \frac{65}{27 c_s^2 \epsilon_s} \right) \delta_{G4XX} + \left( \frac{20}{81} \frac{5 + 2 \lambda_5 X}{\epsilon_s} + \frac{65}{162 c_s^2 \epsilon_s} \right) \delta_{G5XX}$$

For $c_s^2 \ll 1$ one can realize $|f_{\text{NL}}^{\text{equil}}| \gg 1$

Our analysis covers a wide variety of inflation models such as (i) $k$-inflation, (ii) nonminimal coupling models [scalar-tensor theories and $f(R)$ gravity], (iii) Galileon inflation, (iv) inflation with a Gauss-Bonnet coupling, etc.

It will be of interest to discriminate a host of inflationary models in future observations by using our formula of the nonlinear parameter as well as the spectral index and the tensor-to-scalar ratio.