Inflation in modified gravitational theories

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Inflationary models

Up to now many inflationary models have been proposed.
The conventional inflation is driven by a field potential with the Lagrangian

$$P = X - V(\phi) \qquad \text{where} \quad X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi/2$$

Meanwhile there are other types of single-field models such as

- K-inflation: the Lagrangian includes non-linear terms in X.

  $P = P(\phi, X)$ Examples: ghost condensate, DBI

- f(R) gravity: a simple example is $f(R) = R + R^2/(6M^2)$ [Starobinsky, 1980]

  f(R) gravity is equivalent to (generalized) Brans-Dicke theory with $\omega_{\rm BD} = 0$

- Non-minimal coupling models: a scalar field couples to the Ricci scalar.

  $F(\phi)R$ Example: Higgs inflation

- Galileon (G) inflation: the Lagrangian is constructed to satisfy the symmetry.

  $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$ (in the limit of flat space-time)

The most general single-field scalar-tensor theories having second-order equations of motion:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm pl}^2}{2} R + P(\phi, X) - G_3(\phi, X) \Box\phi + \mathcal{L}_4 + \mathcal{L}_5 \right].$$

Horndeski (1974)
Deffayet et al (2011)

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4,X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) \right]$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu} (\nabla^\mu \nabla^\nu \phi) - \frac{1}{6} G_{5,X} \left[(\Box\phi)^3 - 3(\Box\phi)(\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) + 2(\nabla^\mu \nabla_\alpha \phi)(\nabla^\alpha \nabla_\beta \phi)(\nabla^\beta \nabla_\mu \phi) \right]$$

This action covers most of the single-field scalar field models of inflation proposed in literature.

- K-inflation
- Non-minimal coupling models

$G_4 = F(\phi)$ ➡ Scalar-tensor theories (including f(R) gravity), Higgs inflation

$G_5 = F(\phi)$ ➡ Field-derivative coupling models ('New Higgs inflation')

- Galileon inflation

$$P = X - c\,\phi, \qquad G_3 \propto X, \qquad G_4 \propto X^2, \qquad G_5 \propto X^2$$

Even the Gauss-Bonnet coupling $\xi(\phi)\mathcal{G}$ can be recovered for a particular choice of P, G_3, G_4, G_5. (Kobayashi, Yamaguchi, Yokoyama, arXiv: 1105.5723)

Discrimination between single-field inflationary models

One can classify a host of inflationary models observationally from

1. The spectral index $n_{\mathcal{R}}$ of scalar curvature perturbations
2. The tensor-to-scalar ratio r

For the most general scalar-tensor theories, these observables are evaluated by Kobayashi, Yamaguchi, Yokoyama (arXiv: 1105.5723).

See also Naruko and Sasaki (CQG, 2011), De Felice and S.T. (JCAP, 2011), Gao (2011)

In those theories the scalar propagation speed c_s is in general different from 1.

In k-inflation the scalar non-Gaussianities are large for $c_s^2 \ll 1$

→ How about the scalar non-Gaussianities in the most general single-field scalar-tensor theories?

Gao and Steer (arXiv: 1107.2642), De Felice and S.T. (arXiv: 1107.3917)

The spectrum of curvature perturbations

We consider scalar metric perturbations $\alpha, \psi, \mathcal{R}$ with the ADM metric

$$ds^2 = -[(1+\alpha)^2 - a(t)^{-2}\, e^{-2\mathcal{R}}\, (\partial\psi)^2]\, dt^2 + 2\partial_i\psi\, dt\, dx^i + a(t)^2 e^{2\mathcal{R}} d\mathbf{x}^2$$

We choose the uniform field gauge: $\delta\phi = 0$

Using the momentum and Hamiltonian constraints, the second-order action for perturbations reduces to

$$S_2 = \int dt d^3x\, a^3 Q \left[\dot{\mathcal{R}}^2 - \frac{c_s^2}{a^2}(\partial\mathcal{R})^2\right]$$

$Q > 0$ and $c_s^2 > 0$ are required to avoid ghosts and Laplacian instabilities.

where $$Q = \frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}, \qquad c_s^2 = \frac{3(2w_1^2 w_2 H - w_2^2 w_4 + 4w_1\dot{w}_1 w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)}$$

$$w_1 = M_{\rm pl}^2 F - 4XG_{4,X} - 2HX\dot{\phi}G_{5,X} + 2XG_{5,\phi}$$

$$w_2 = 2M_{\rm pl}^2 HF - 2X\dot{\phi}G_{3,X} - 16H(XG_{4,X} + X^2 G_{4,XX}) + 2\dot{\phi}(G_{4,\phi} + 2XG_{4,\phi X})$$
$$-2H^2\dot{\phi}(5XG_{5,X} + 2X^2 G_{5,XX}) + 4HX(3G_{5,\phi} + 2XG_{5,\phi X})$$

$$w_3 = -9M_{\rm pl}^2 H^2 F + 3(XP_{,X} + 2X^2 P_{,XX}) + 18H\dot{\phi}(2XG_{3,X} + X^2 G_{3,XX}) - 6X(G_{3,\phi} + XG_{3,\phi X})$$
$$+18H^2(7XG_{4,X} + 16X^2 G_{4,XX} + 4X^3 G_{4,XXX}) - 18H\dot{\phi}(G_{4,\phi} + 5XG_{4,\phi X} + 2X^2 G_{4,\phi XX})$$
$$+6H^3\dot{\phi}(15XG_{5,X} + 13X^2 G_{5,XX} + 2X^3 G_{,5XXX}) - 18H^2 X(6G_{5,\phi} + 9XG_{5,\phi X} + 2X^2 G_{5,\phi XX})$$

$$w_4 = M_{\rm pl}^2 F - 2XG_{5,\phi} - 2XG_{5,X}\ddot{\phi}$$

The scalar power spectrum is $$\mathcal{P}_{\mathcal{R}} = \frac{H^2}{8\pi^2 Q c_s^3}$$

Kobayashi, Yamaguchi, Yokoyama (arXiv: 1105.5723)
De Felice and S. T. (arXiv: 1107.3917)

The spectrum of tensor perturbations

The second-order action for tensor perturbations is

$$S = \sum_{\lambda} \int dt d^3x \, a^3 Q_T \left[\dot{h}_{\lambda}^2 - \frac{c_T^2}{a^2} (\partial h_{\lambda})^2 \right]$$

λ corresponds to polarization modes

where

$$Q_T = \frac{w_1}{4} = \frac{1}{4} M_{\rm pl}^2 F \left[1 + \mathcal{O}(\epsilon)\right]$$

$$c_T^2 = w_4/w_1 = 1 + \mathcal{O}(\epsilon)$$

where $F = 1 + 2G_4/M_{\rm pl}^2$ and $\epsilon = -\dot{H}/H^2 \ll 1$.

This term comes from the nonminimal coupling in $\mathcal{L}_4$

$$\mathcal{L}_4 = G_4(\phi, X)\, R + G_{4,X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)(\nabla^\mu \nabla^\nu \phi) \right]$$

The no-ghost condition is satisfied for $F > 0$.

The tensor power spectrum is

$$\mathcal{P}_T = \frac{H^2}{2\pi^2 Q_T c_T^3} \simeq \frac{2H^2}{\pi^2 M_{\rm pl}^2 F}$$

The tensor-to-scalar ratio is

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_{\mathcal{R}}} \simeq 16 c_s \epsilon_s$$

where

$$\epsilon_s = \frac{Q c_s^2}{M_{\rm pl}^2 F} = \mathcal{O}(\epsilon)$$

Scalar non-Gaussianities

For the ADM metric with scalar metric perturbations the third-order perturbed action is given by (obtained after many integrations by parts)

$$S_3 = \int dt\, \mathcal{L}_3$$

where

$$\mathcal{L}_3 = \int d^3x \Big\{ a^3\mathcal{C}_1 M_{\rm pl}^2 \mathcal{R}\dot{\mathcal{R}}^2 + a\,\mathcal{C}_2 M_{\rm pl}^2 \mathcal{R}(\partial\mathcal{R})^2 + a^3\mathcal{C}_3 M_{\rm pl}\dot{\mathcal{R}}^3 + a^3\mathcal{C}_4\dot{\mathcal{R}}(\partial_i\mathcal{R})(\partial_i\mathcal{X}) + a^3(\mathcal{C}_5/M_{\rm pl}^2)\partial^2\mathcal{R}(\partial\mathcal{X})^2$$
$$+a\mathcal{C}_6\dot{\mathcal{R}}^2\partial^2\mathcal{R} + \mathcal{C}_7\left[\partial^2\mathcal{R}(\partial\mathcal{R})^2 - \mathcal{R}\partial_i\partial_j(\partial_i\mathcal{R})(\partial_j\mathcal{R})\right]/a + a(\mathcal{C}_8/M_{\rm pl})\left[\partial^2\mathcal{R}\partial_i\mathcal{R}\partial_i\mathcal{X} - \mathcal{R}\partial_i\partial_j(\partial_i\mathcal{R})(\partial_j\mathcal{X})\right]$$
$$+\mathcal{F}_1\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\Big|_1\Big\}$$

$\mathcal{C}_i$ $(i = 1, 2, \cdots, 8)$ are slowly varying relative to a, and $\partial^2\mathcal{X} = Q\dot{\mathcal{R}}$.

$\mathcal{F}_1$ are second order in perturbations and

$$\frac{\delta\mathcal{L}_2}{\delta\mathcal{R}}\Big|_1 \equiv -2\left[\frac{d}{dt}(a^3Q\dot{\mathcal{R}}) - aQc_s^2\partial^2\mathcal{R}\right]$$

➡ This vanishes at first order in $\mathcal{R}$.

The vacuum expectation value of $\mathcal{R}$ for the three-point operator at the conformal time $\tau = \tau_f$ is

$$\langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\rangle = -i\int_{\tau_i}^{\tau_f} d\tau\, a\,\langle 0|\left[\mathcal{R}(\tau_f,\mathbf{k}_1)\mathcal{R}(\tau_f,\mathbf{k}_2)\mathcal{R}(\tau_f,\mathbf{k}_3), \mathcal{H}_{\rm int}(\tau)\right]|0\rangle$$

where $\mathcal{H}_{\rm int} = -\mathcal{L}_3$.

We find that the three-point correlation function is given by

$$\langle \mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\rangle = (2\pi)^7 \delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)(\mathcal{P}_{\mathcal{R}})^2 \frac{\mathcal{A}_{\mathcal{R}}}{\prod_{i=1}^{3} k_i^3}$$

Gao and Steer (arXiv: 1107.2642),
De Felice and S.T. (arXiv: 1107.3917)

where

$$\begin{aligned}
\mathcal{A}_{\mathcal{R}} = \frac{M_{\rm pl}^2}{Q}\Bigg\{ & \frac{1}{4}\left(\frac{2}{K}\sum_{i>j} k_i^2 k_j^2 - \frac{1}{K^2}\sum_{i\neq j} k_i^2 k_j^3\right)\mathcal{C}_1 + \frac{1}{4c_s^2}\left(\frac{1}{2}\sum_i k_i^3 + \frac{2}{K}\sum_{i>j} k_i^2 k_j^2 - \frac{1}{K^2}\sum_{i\neq j} k_i^2 k_j^3\right)\mathcal{C}_2 \\
& + \frac{3}{2}\frac{H}{M_{\rm pl}}\frac{(k_1k_2k_3)^2}{K^3}\mathcal{C}_3 + \frac{1}{8}\frac{Q}{M_{\rm pl}^2}\left(\sum_i k_i^3 - \frac{1}{2}\sum_{i\neq j} k_i k_j^2 - \frac{2}{K^2}\sum_{i\neq j} k_i^2 k_j^3\right)\mathcal{C}_4 \\
& + \frac{1}{4}\left(\frac{Q}{M_{\rm pl}^2}\right)^2 \frac{1}{K^2}\left[\sum_i k_i^5 + \frac{1}{2}\sum_{i\neq j} k_i k_j^4 - \frac{3}{2}\sum_{i\neq j} k_i^2 k_j^3 - k_1k_2k_3\sum_{i>j} k_i k_j\right]\mathcal{C}_5 + \frac{3}{c_s^2}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{(k_1k_2k_3)^2}{K^3}\mathcal{C}_6 \\
& + \frac{1}{2c_s^4}\left(\frac{H}{M_{\rm pl}}\right)^2 \frac{1}{K}\left(1 + \frac{1}{K^2}\sum_{i>j} k_i k_j + \frac{3k_1k_2k_3}{K^3}\right)\left[\frac{3}{4}\sum_i k_i^4 - \frac{3}{2}\sum_{i>j} k_i^2 k_j^2\right]\mathcal{C}_7 \\
& + \frac{1}{8c_s^2}\frac{H}{M_{\rm pl}}\frac{Q}{M_{\rm pl}^2}\frac{1}{K^2}\left[\frac{3}{2}k_1k_2k_3\sum_i k_i^2 - \frac{5}{2}k_1k_2k_3K^2 - 6\sum_{i\neq j} k_i^2 k_j^3 - \sum_i k_i^5 + \frac{7}{2}K\sum_i k_i^4\right]\mathcal{C}_8 \Bigg\},
\end{aligned}$$

The terms $\mathcal{C}_i$ $(i = 1, \cdots, 5)$ appear in k-inflation (Seery and Lidsey, Chen et al), which can be well approximated as an equilateral estimator.

The shape of the non-Gaussianities can be also approximated as an equilateral one even in the presence of $\mathcal{C}_6$ (Mizuno and Koyama) and $\mathcal{C}_7$, $\mathcal{C}_8$ (De Felice and S.T.).

The nonlinear parameter

We define the nonlinear parameter $f_{\rm NL}$, as $f_{\rm NL} = \frac{10}{3}\frac{\mathcal{A}_{\mathcal{R}}}{\sum_{i=1}^{3} k_i^3}$

For the equilateral configuration ($k_1 = k_2 = k_3$) one has

$$f_{\rm NL}^{\rm equil} = \frac{40}{9}\frac{M_{\rm pl}^2}{Q}\left[\frac{1}{12}\mathcal{C}_1 + \frac{17}{96c_s^2}\mathcal{C}_2 + \frac{1}{72}\frac{H}{M_{\rm pl}}\mathcal{C}_3 - \frac{1}{24}\frac{Q}{M_{\rm pl}^2}\mathcal{C}_4 - \frac{1}{24}\left(\frac{Q}{M_{\rm pl}^2}\right)^2\mathcal{C}_5 + \frac{1}{36c_s^2}\left(\frac{H}{M_{\rm pl}}\right)^2\mathcal{C}_6 - \frac{13}{96c_s^4}\left(\frac{H}{M_{\rm pl}}\right)^2\mathcal{C}_7 - \frac{17}{192c_s^2}\frac{H}{M_{\rm pl}}\frac{Q}{M_{\rm pl}^2}\mathcal{C}_8\right]$$

Under the expansion of the slow-variation parameters it follows that

$$f_{\rm NL}^{\rm equil} = \frac{85}{324}\left(1 - \frac{1}{c_s^2}\right) - \frac{10}{81}\frac{\lambda}{\Sigma} + \frac{55}{36}\frac{\epsilon_s}{c_s^2} + \frac{5}{12}\frac{\eta_s}{c_s^2} - \frac{85}{54}\frac{s}{c_s^2} + \left(\frac{20}{81}\frac{1+\lambda_{3X}}{\epsilon_s} + \frac{65}{162c_s^2\epsilon_s}\right)\delta_{G3X}$$
$$+\left(\frac{80}{81}\frac{3+2\lambda_{4X}}{\epsilon_s} + \frac{65}{27c_s^2\epsilon_s}\right)\delta_{G4XX} + \left(\frac{20}{81\epsilon_s} + \frac{65}{162c_s^2\epsilon_s}\right)\delta_{G5X} + \left(\frac{20}{81}\frac{5+2\lambda_{5X}}{\epsilon_s} + \frac{65}{162c_s^2\epsilon_s}\right)\delta_{G5XX}$$

where

(De Felice and S.T., arXiv: 1107.3917)

$$\eta_s = \frac{\dot{\epsilon}_s}{H\epsilon_s}, \quad s = \frac{\dot{c}_s}{Hc_s}, \quad \delta_{G3X} = \frac{G_{3,X}\dot{\phi}X}{M_{\rm pl}^2 HF}, \quad \delta_{G4XX} = \frac{G_{4,XX}X^2}{M_{\rm pl}^2 F}, \quad \delta_{G5X} = \frac{G_{5,X}H\dot{\phi}X}{M_{\rm pl}^2 F}, \quad \delta_{G5XX} = \frac{G_{5,XX}H\dot{\phi}X^2}{M_{\rm pl}^2 F}$$

$$\lambda_{3X} \equiv \frac{XG_{3,XX}}{G_{3,X}}, \quad \lambda_{4X} \equiv \frac{XG_{4,XXX}}{G_{4,XX}}, \quad \lambda_{5X} \equiv \frac{XG_{5,XXX}}{G_{5,XX}}, \quad \Sigma = \frac{w_1(4w_1w_3 + 9w_2^2)}{12M_{\rm pl}^4}.$$

$$\lambda = (F^2/3)[3X^2P_{,XX} + 2X^3P_{,XXX} + 3H\dot{\phi}(XG_{3,X} + 5X^2G_{3,XX} + 2X^3G_{3,XXX}) - 2(2X^2G_{3,\phi X} + X^3G_{3,\phi XX})$$
$$+6H^2(9X^2G_{4,XX} + 16X^3G_{4,XXX} + 4X^4G_{4,XXXX}) - 3H\dot{\phi}(3XG_{4\phi,X} + 12X^2G_{4,\phi XX} + 4X^3G_{4,\phi XXX})$$
$$+H^3\dot{\phi}(3XG_{5,X} + 27X^2G_{5,XX} + 24X^3G_{5,XXX} + 4X^4G_{5,XXXX})$$
$$-6H^2(6X^2G_{5,\phi X} + 9X^3G_{5,\phi XX} + 2X^4G_{5,\phi XXX})]$$

Standard inflation: $P = X - V(\phi), c_s^2 = 1, \lambda = 0$ ➡ $f_{\rm NL}^{\rm equil} = \frac{55}{36}\epsilon_s + \frac{5}{12}\eta_s$

The scalar propagation speed

If $c_s^2 \ll 1$, the large non-linear parameter $|f_{\rm NL}^{\rm equil}| \gg 1$ can be realized.

Expansion in terms of slow-variation parameters gives

$$c_s^2 \simeq \frac{\delta_{PX}+4\delta_{G3X}-2\delta_{G3\phi}+6\delta_{G4X}+20\delta_{G4XX}+4\delta_{G5X}+4\delta_{G5XX}-6\delta_{G5\phi}}{\delta_{PX}(1+2\lambda_{PX})+6\delta_{G3X}(1+\lambda_{3X})-2\delta_{G3\phi}+6\delta_{G4X}+24\delta_{G4XX}(2+\lambda_{4X})+6\delta_{G5X}+2\delta_{G5XX}(7+2\lambda_{5X})-6\delta_{G5\phi}}$$

where

$$\delta_{PX} = \frac{P_{,X}X}{M_{\rm pl}^2 H^2 F}, \quad \lambda_{PX} = \frac{XP_{,XX}}{P_{,X}}, \quad \delta_{G3\phi} = \frac{G_{3,\phi}X}{M_{\rm pl}^2 H^2 F}, \quad \delta_{G4X} = \frac{G_{4,X}X}{M_{\rm pl}^2 F}, \quad \delta_{G5\phi} = \frac{G_{5,\phi}X}{M_{\rm pl}^2 F}.$$

If either of the following conditions is satisfied, it is possible to realize the large non-Gaussianities:

$$\lambda_{PX} \gg 1\,, \qquad \lambda_{3X} \gg 1\,, \qquad \lambda_{4X} \gg 1\,, \qquad \lambda_{5X} \gg 1\,.$$

- In k-inflation one has

$$c_s^2 = \frac{1}{1+2\lambda_{PX}} = \frac{P_{,X}}{P_{,X}+2XP_{,XX}} \qquad\qquad \epsilon = \delta_{PX} = \frac{P_{,X}X}{3M_{\rm pl}^2 H^2}$$

Since k-inflation occurs around $P_{,X} \approx 0$, it follows that $c_s^2 \ll 1$. ➡ $|f_{\rm NL}^{\rm equil}| \gg 1$

If P is function of X only, a de Sitter solution with $P_{,X} = 0$ is problematic because the power spectrum $\mathcal{P}_{\mathcal{R}}$ diverges.

➡ We require the ϕ-dependence in P [like DBI inflation] or additional terms G_i $(i = 3, 4, 5)$ [like G-inflation].

K-inflation + Galileon terms

- The ghost condensate model plus the Galileon G_3 term

$$P = -X + X^2/(2M^4)\,, \quad G_3 = \mu X/M^4$$

(Kobayashi, Yamaguchi, Yokoyama, PRL, 2010)

There is a de Sitter solution with $1 - x \simeq \sqrt{3}\mu/M_{\rm pl}$ for $x = X/M^4$ close to 1.

→ $f_{\rm NL}^{\rm equil} \simeq 5/[6(1-x)] \simeq 4.62 r^{-2/3}$ (Mizuno and Koyama, PRD, 2010)

r is the tensor-to-scalar ratio.

- Let us consider the Galileon G_4 term

$$P = -X + X^2/(2M^4)\,, \quad G_4 = \mu X^2/M^7$$

For $x = X/M^4$ close to 1 there is a de Sitter solution characterized by

$$H^2 = \frac{M^3}{36\mu}\frac{1-x}{x}\,, \qquad \frac{\mu M}{M_{\rm pl}^2} = \frac{1-x}{6x^2(3-2x)} \qquad \text{and} \qquad c_s^2 = 2(1-x)/9$$

The inflationary observables are

$$\mathcal{P}_{\mathcal{R}} \simeq \frac{3\sqrt{2}}{256\pi^2}\left(\frac{M}{M_{\rm pl}}\right)^4 \frac{1}{(1-x)^{3/2}}\,, \quad r \simeq \frac{128\sqrt{2}}{9}(1-x)^{3/2}\,, \quad f_{\rm NL}^{\rm equil} \simeq 1.28 r^{-2/3}$$

→ For $r = 0.01$, $f_{\rm NL}^{\rm equil} = 9.4$

(De Felice and S.T., arXiv: 1107.3917)

- Let us consider the Galileon G_5 term

$$P = -X + X^2/(2M^4)\,, \quad G_5 = \mu X^2/M^{10}$$

→ $f_{\rm NL}^{\rm equil} \simeq 0.17 r^{-2/3}$

Our formula of $f_{\rm NL}^{\rm equil}$ can be applied to most of single-field inflation models proposed so far.

Let us consider nonminimal coupling models.

(i) $G_4 = F(\phi)$ → $\mathcal{L}_4 = F(\phi)R$ including scalar-tensor gravity and f(R) gravity

$\delta_{G4X} = \delta_{G4XX} = 0$ → $c_s^2 = 1/(1+2\lambda_{PX})$

If P does not have non-linear terms in X we have $c_s^2 = 1$ and

$$f_{\rm NL}^{\rm equil} = \mathcal{O}(\epsilon_s, \eta_s)$$

This is the case for Higgs inflation ($P = X - V(\phi)$) and Brans-Dicke theories ($P = \omega_{\rm BD} X/\phi - V(\phi)$).

(ii) $G_5 = F(\phi)$ → $\mathcal{L}_5 = F(\phi) G_{\mu\nu}(\nabla^\mu \nabla^\nu \phi)$

New Higgs inflation (with the coupling $G_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$) corresponds to the choice $F(\phi) \propto \phi$.

$\delta_{G5X} = \delta_{G5XX} = 0$ → $c_s^2 = \dfrac{\delta_{PX} - 6\delta_{G5\phi}}{\delta_{PX}(1+2\lambda_{PX}) - 6\delta_{G5\phi}}$

If P does not have non-linear terms in X we have $c_s^2 = 1$ and

$$f_{\rm NL}^{\rm equil} = \mathcal{O}(\epsilon_s, \eta_s)$$

The above two types of nonminimal couplings themselves do not give rise to large equilateral non-Gaussianities.

Potential-driven G-inflation

Let us consider the following model

$$P = X - V(\phi)\,, \qquad G_3 = -\frac{1}{M^{4n-1}} e^{\mu\phi/M_{\rm pl}} X^n$$

The Galileon coupling corresponds to $\mu = 0$ and $n = 1$.

In this case the scalar propagation speed squared is

$$c_s^2 \simeq \frac{\delta_{PX} + 4\delta_{G3X}}{\delta_{PX} + 6n\delta_{G3X}} \quad \Rightarrow \quad c_s^2 \simeq 2/(3n) \text{ for } \delta_{G3X} \gg \delta_{PX}\,.$$

For the specific theories with $n \gg 1$ one has $c_s^2 \ll 1$ and

$$f_{\rm NL}^{\rm equil} \simeq -\frac{865}{3888} n \quad \Rightarrow \quad |f_{\rm NL}^{\rm equil}| \gg 1$$

- In the following let us consider the theories with $n = 1$ (in which case the non-Gaussianities cannot be large).

For the potential $V(\phi) = V_0 (\phi/M_{\rm pl})^p$ the spectral index and the tensor-to-scalar ratio in the regime $\delta_{G3X} \gg \delta_{PX}$ are

$$n_{\rm s} \simeq 1 - \frac{3(p+1)}{(p+3)N+p} \left[1 - \frac{2(p-1)}{3(p+1)(p+5)} \mu x\right]$$

$$r \simeq \frac{64\sqrt{6}}{9} \frac{p}{(p+3)N+p} \left(1 - \frac{\mu x}{p+5}\right)$$

where $x \equiv \phi/M_{\rm pl} \ll 1$ for $\delta_{G3X} \gg \delta_{PX}$ and $\mu = \mathcal{O}(1)$.

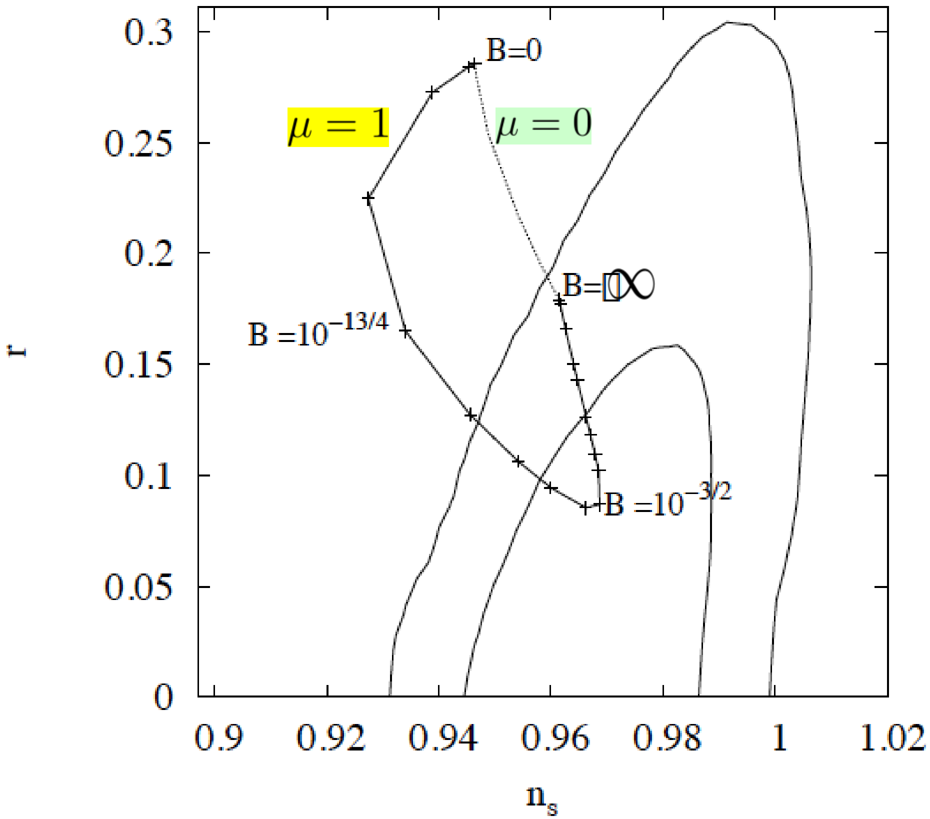
N is the number of e-foldings from the end of inflation.

For $N = 55$, in the limit where $\mu \to 0$, $n_{\rm s} = 0.9614$ and $r = 0.1791$ for $p = 4$.

⇒ The quartic potential can be saved (Kamada et al, PRD, 2010).

Observational constraints on G-inflation with the quartic potential

$$V(\phi) = V_0(\phi/M_{\rm pl})^4 \quad \text{and} \quad G_3 = -\frac{1}{M^3} e^{\mu\phi/M_{\rm pl}} X$$



The left figure corresponds to

$\mu = 0$ and $\mu = 1$
where $B = [V_0/(M^3 M_{\rm pl})]^{1/4}$

The quartic potential is saved by the Galileon-like corrections.

Kamada, Kobayashi, Yamaguchi, Yokoyama, PRD, 2010 $(\mu = 0)$

De Felice, S.T., Elliston, Tavakol, JCAP to appear $(\mu \neq 0)$

For $\mu > 0$ the intermediate values of B are within 1σ observational bound.

$B \to 0$ corresponds to $\delta_{PX}/\delta_{G3X} \gg 1$. (standard inflation)

$B \to \infty$ corresponds to $\delta_{PX}/\delta_{G3X} \ll 1$. (G-inflation)

Gauss-Bonnet couplings

Our general action covers the Gauss-Bonnet coupling $-\xi(\phi)\mathcal{G}$ by choosing

$$P = -8\xi^{(4)}(\phi)X^2(3 - \ln X)\,, \quad G_3 = -4\xi^{(3)}(\phi)X(7 - 3\ln X)\,,$$
$$G_4 = -4\xi^{(2)}(\phi)X(2 - \ln X)\,, \quad G_5 = 4\xi^{(1)}(\phi)\ln X$$

(Kobayashi, Yamaguchi, Yokoyama, arXiv: 1105.5723)

where $\xi^{(n)}(\phi) = \partial^n \xi(\phi)/\partial\phi^n$

For the potential-driven inflation (i.e. in the presence of the term $P = X - V(\phi)$) one has

$$c_s^2 = 1 - 64\delta_\xi^2(6\delta_\xi + \delta_X)/\delta_X$$

→ very close to $c_s^2 = 1$

where $\delta_X = X/(M_{\rm pl}^2 H^2)$ and $\delta_\xi = H\dot{\xi}/M_{\rm pl}^2$.

The nonlinear parameter is estimated as

$$f_{\rm NL}^{\rm equil} = \frac{55}{36}\delta_X + \frac{5}{12}\eta_X + \frac{275}{81}\frac{\delta_\xi}{\delta_X}(4\epsilon + 2\eta_\xi - \eta_X)$$

→ small

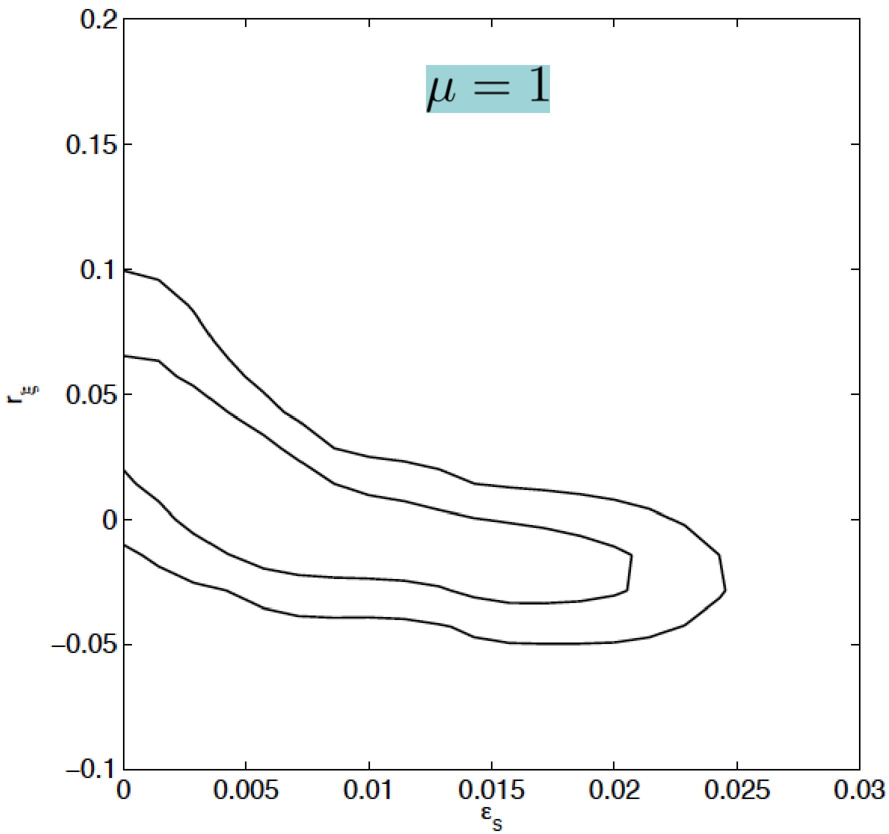
where $\eta_\xi = \dot{\delta}_\xi/(H\delta_\xi)$ and $\eta_X = \dot{\delta}_X/(H\delta_X)$.

This result matches with that derived explicitly in the presence of the Gauss-Bonnet coupling (De Felice and S.T., JCAP, 2011).

Observational constraints on the Gauss-Bonnet coupling

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + X - V(\phi) - \xi(\phi)\mathcal{G}\right]$$

where $V(\phi) = m^2\phi^2/2, \qquad \xi(\phi) = \xi_0 e^{\mu\phi/M_{\rm pl}}$



CMB likelihood analysis by CAMB

De Felice, S.T., Elliston,Tavakol, JCAP to appear (2011)

From the observational data of the scalar spectral index and the tensor-to-scalar ratio the GB Coupling is constrained by varying two parameters:

$$\epsilon_s (= \delta_X), \qquad r_\xi = \delta_\xi/\delta_X$$

The CMB likelihood analysis shows that the Gauss-Bonnet contribution needs to be suppressed:

$$r_\xi \equiv \delta_\xi/\delta_X < 0.1 \qquad (95~\%~\mathrm{CL})$$

Conclusions

We have evaluated the primordial non-Gaussianities for the very general single-field models characterized by

$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + P(\phi, X) - G_3(\phi, X)\Box\phi + \mathcal{L}_4 + \mathcal{L}_5\right].$$

We found that the equilateral nonlinear parameter is given by

$$f_{\rm NL}^{\rm equil} = \frac{85}{324}\left(1-\frac{1}{c_s^2}\right) - \frac{10}{81}\frac{\lambda}{\Sigma} + \frac{55}{36}\frac{\epsilon_s}{c_s^2} + \frac{5}{12}\frac{\eta_s}{c_s^2} - \frac{85}{54}\frac{s}{c_s^2} + \left(\frac{20}{81}\frac{1+\lambda_{3X}}{\epsilon_s} + \frac{65}{162c_s^2\epsilon_s}\right)\delta_{G3X}$$
$$+\left(\frac{80}{81}\frac{3+2\lambda_{4X}}{\epsilon_s} + \frac{65}{27c_s^2\epsilon_s}\right)\delta_{G4XX} + \left(\frac{20}{81\epsilon_s} + \frac{65}{162c_s^2\epsilon_s}\right)\delta_{G5X} + \left(\frac{20}{81}\frac{5+2\lambda_{5X}}{\epsilon_s} + \frac{65}{162c_s^2\epsilon_s}\right)\delta_{G5XX}$$

→ For $c_s^2 \ll 1$ one can realize $|f_{\rm NL}^{\rm equil}| \gg 1$

Our analysis covers a wide variety of inflation models such as (i)k-inflation, (ii) nonminimal coupling models [scalar-tensor theories and f(R) gravity], (iii) Galileon inflation, (iv) inflation with a Gausss-Bonnet coupling, etc.

It will be of interest to discriminate a host of inflationary models in future observations by using our formula of the nonlinear parameter as well as the spectral index and the tensor-to-scalar ratio.