Gauge artifacts in primordial bi-spectrum & holographic non-Gaussianity

Yuko Urakawa (U.Barcelona, Ochanomizu U.)

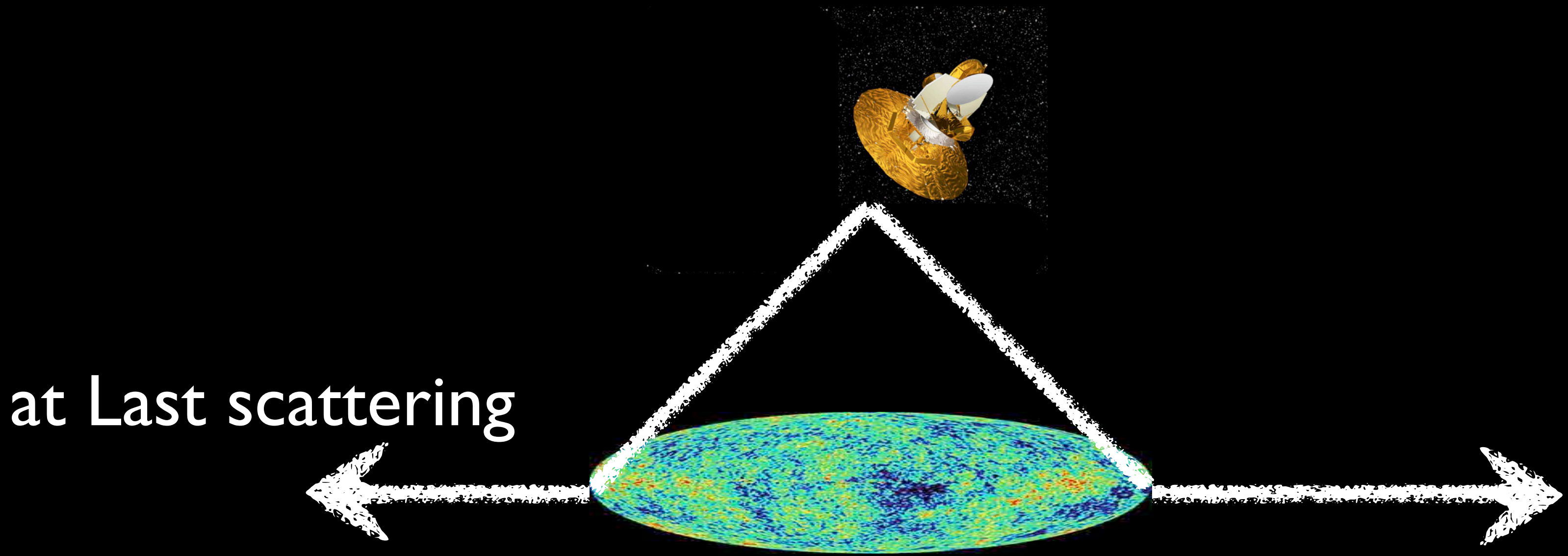
Y.U. & T. Tanaka 1007.0468[hep-th], 1009.2947[hep-th], 1103.1251[astro-ph]

Y.U. 1105.1078[hep-th]

J. Garriga & Y.U. in preparation

SI2011

Initial conditions of the universe



- ✔ Gaussian distribution
- ✔ Scale invariant
- ✔ Adiabatic

$$\Delta^2(k) = \Delta^2(k_0)\left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$\Delta^2(k_0) = (2.445 \pm 0.096) \times 10^{-9}$$

$$n_s = 0.960 \pm 0.013$$

I.C., the model of early universe should yield

⟶ INFLATION!!

Cosmological fluctuations we observe

$$\Updownarrow$$

Gauge-invariant perturbations

$$\begin{cases} \text{at universe wt infinite vol.} \\ \text{at universe wt finite vol.} \end{cases}$$

Gauge invariance

= Invariance under gauge-transformations

- Types of gauge transformations

$$x^\mu \;\longrightarrow\; x^\mu + \delta x^\mu$$

- Whole universe with infinite (3dim-)vol.

  $|\delta x^\mu| \ll 1$ at whole of universe

- A portion of universe with finite (3dim-)vol.

  $|\delta x^\mu| \ll 1$ at the portion (→ local universe)

No restrictions on outside the local universe

Gauge invariance 2

(ex) Scale transformation

$$x^i \to e^{f(t)} x^i = x^i + f(t) x^i + \dots$$

Allowed only in a local universe

- Gauge invariance in universe with infinite volume

  Invariance only for bounded trans. at infinity

- Gauge invariance in the local universe

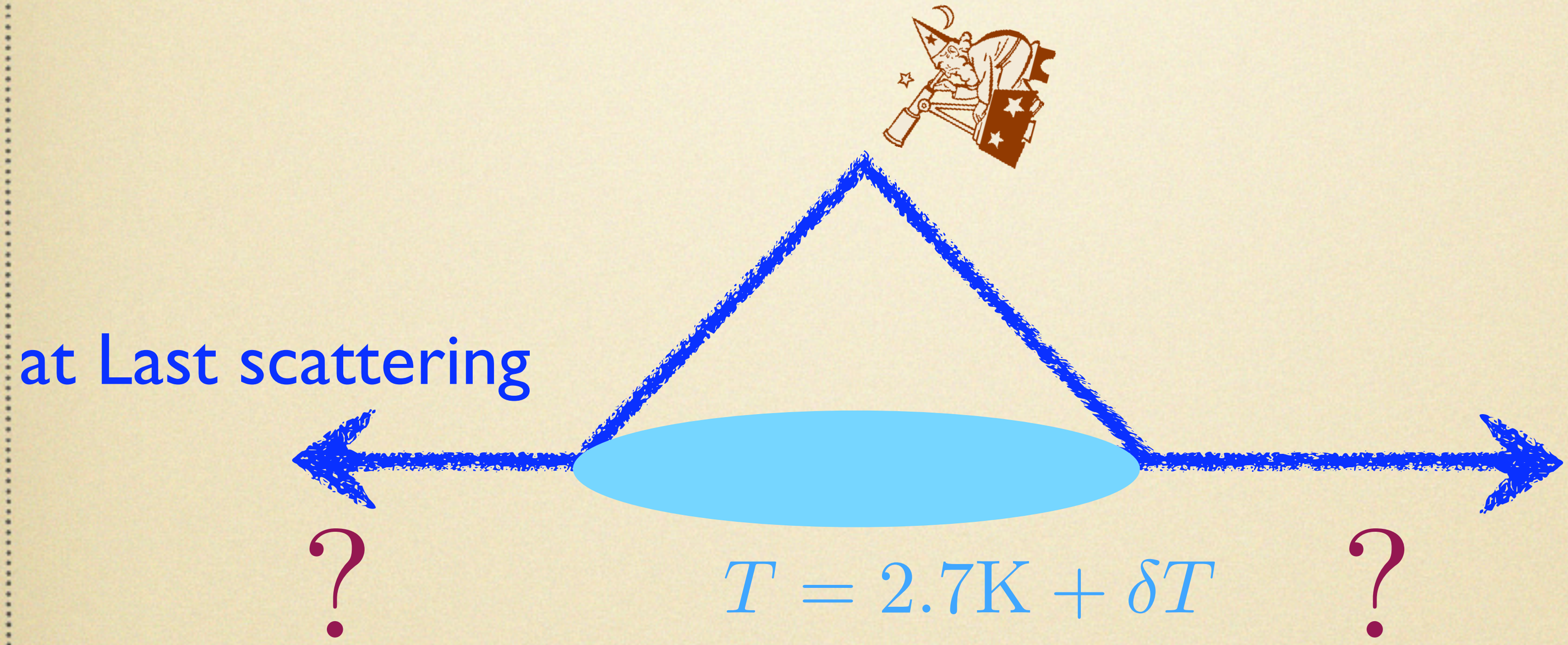
  Invariance for bounded&unbounded trans. at infinity

  "Genuine gauge-invariance"

Y.U.&T.Tanaka(10)

Why genuine gauge-invariance?

We can access only a portion of the universe.



Irrelevant to what happens out of our local universe!

As an example...

- Action

  Single field inflation

$$S_\phi = -\frac{1}{2}\int \sqrt{-g}\ [g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2V(\phi)]\mathrm{d}^4x$$

- ADM decomposition

$$\mathrm{d}s^2 = -N^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right)$$

Comoving gauge

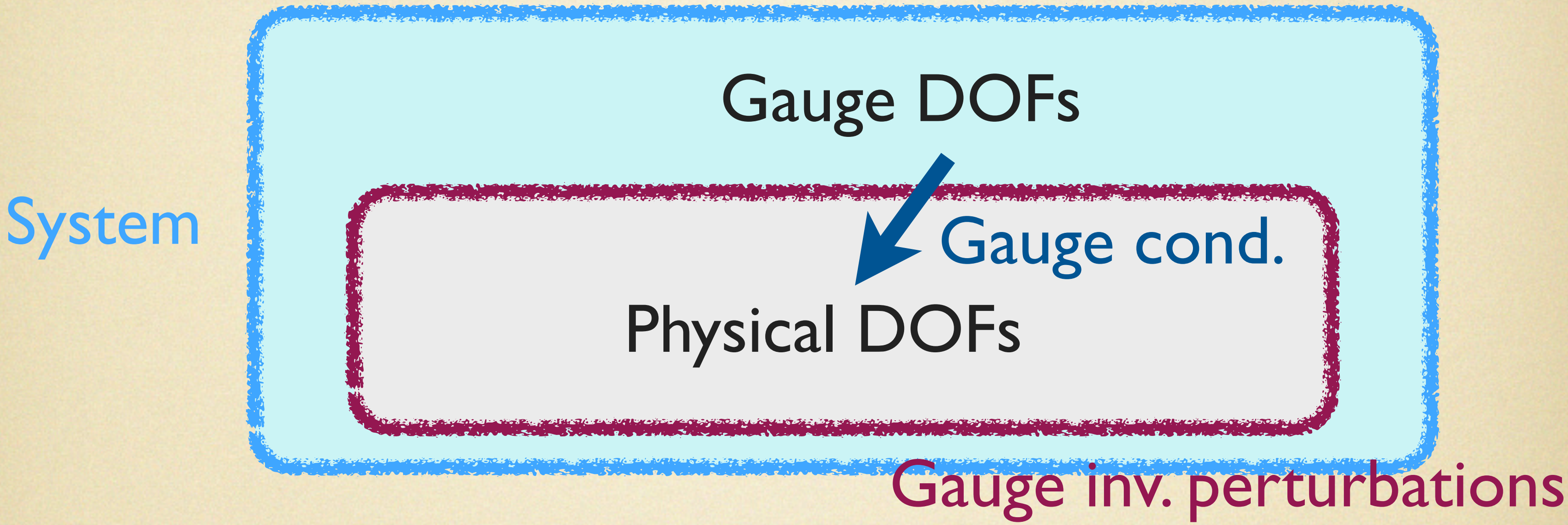
Maldacena (2002)

$$\delta\phi = 0 \quad h_{ij} = e^{2(\rho+\zeta)}\left[e^{\delta\gamma}\right]_{ij}$$

$$\delta\gamma_{ii} = 0 \quad \partial_j\delta\gamma_{ij} = 0$$

e^ρ: scale factor

Gauge-invariant perturbation



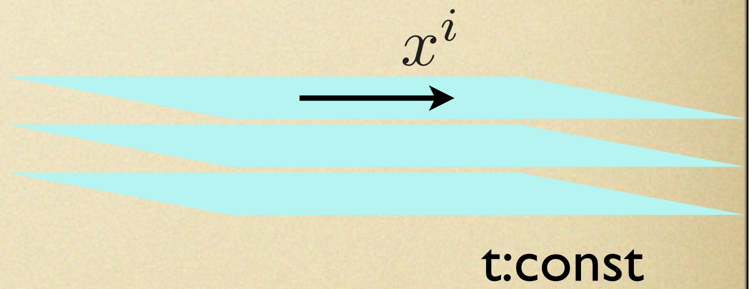
- Gauge invariance in infinite universe

$$\delta\phi(t,\, x^i) = 0$$

fixes time slicing

$$\delta\gamma_{ii}(t,\, x^i) = \partial_i \delta\gamma_{ij}(t,\, x^i) = 0$$

fixes spatial coordinates



→ Gauge invariance in universe with infinite volume

Local gauge-invariance

- Gauge invariance in local universe

  (i) Time slicing fixed

  (ii) Spatial coordinates not fixed

  Residual gauge DOFs $x^i \to x^i + \delta x^i$

$$\delta x^i \simeq C^i{}_{i_1 \ldots . i_n}(t) x^{i_1} \cdots x^{i_n}$$

Symmetric traceless

Un-bounded at spatial infinity

[Remark]

Residual gauge DOFs appear as the boundary cond. in solving Hamiltonian/Momentum constraints.

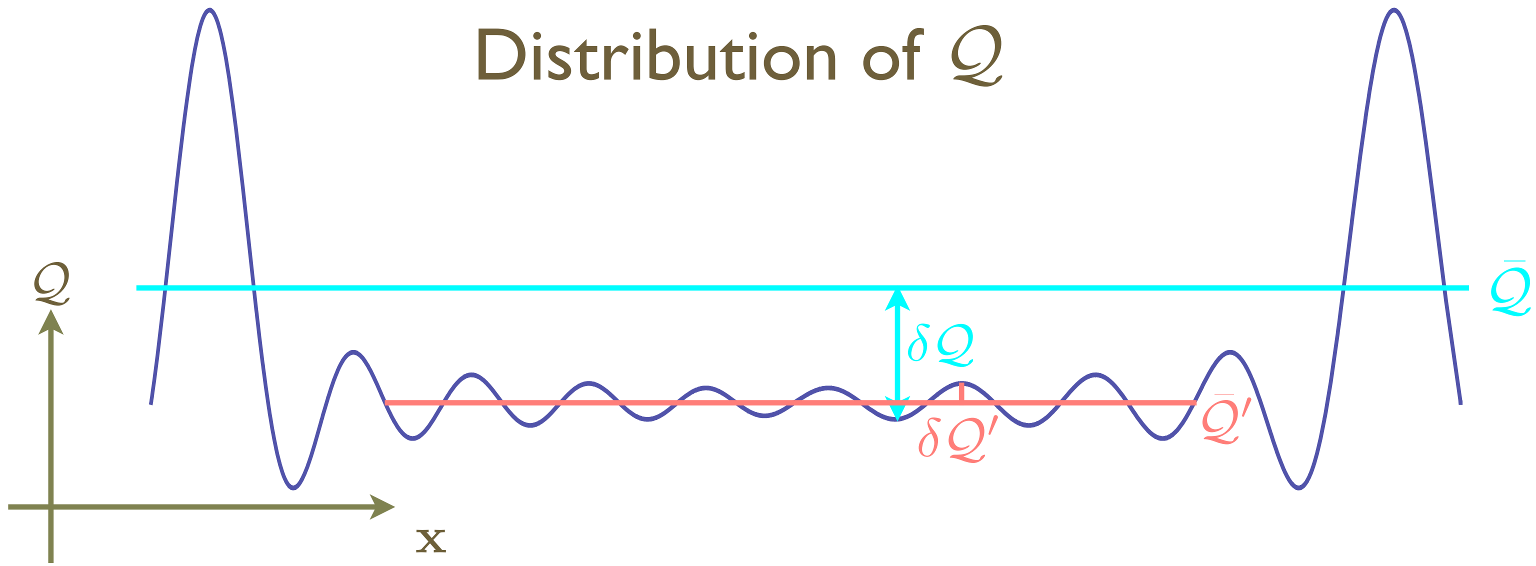
Elliptic type eqs.

Def. of fluctuations

$$\delta\mathcal{Q} := \mathcal{Q} - \bar{\mathcal{Q}} \qquad \mathcal{Q} = \zeta,\, \delta\gamma_{ij}$$

Averaged value $\quad \bar{\mathcal{Q}} := \int \mathrm{d}^3\mathbf{x}\, \mathcal{Q} \,/ \int \mathrm{d}^3\mathbf{x}$

Distribution of $\mathcal{Q}$



Gauge transformation : $\bar{\mathcal{Q}} \rightarrow \bar{\mathcal{Q}}'$

Aspects of genuine gauge invariance

1. Infrared divergence problem

Y.U.&T.Tanaka (09, 10^1, 10^2)

2. Primordial non-Gaussianity

T.Tanaka&Y.U. (11), Y.U. (11)

3. Genuine gauge-inv. in holographic universe

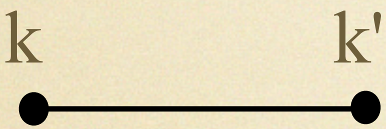
J. Garriga and Y.U. (in preparation)

Infrared(IR) divergence

- Two point function $\langle \zeta_k \, \zeta_{k'} \rangle$

$\mathcal{L}_{\rm int} \propto \zeta^4$ $\quad\quad$ ζ : mass-less field

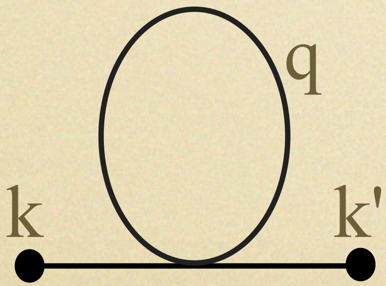
■ Leading order



$$\langle \zeta_k \, \zeta_{k'} \rangle = |\zeta_k|^2 \propto k^{-3}$$

Scale-invariant

■ Next to leading order



Momentum (Loop)integral

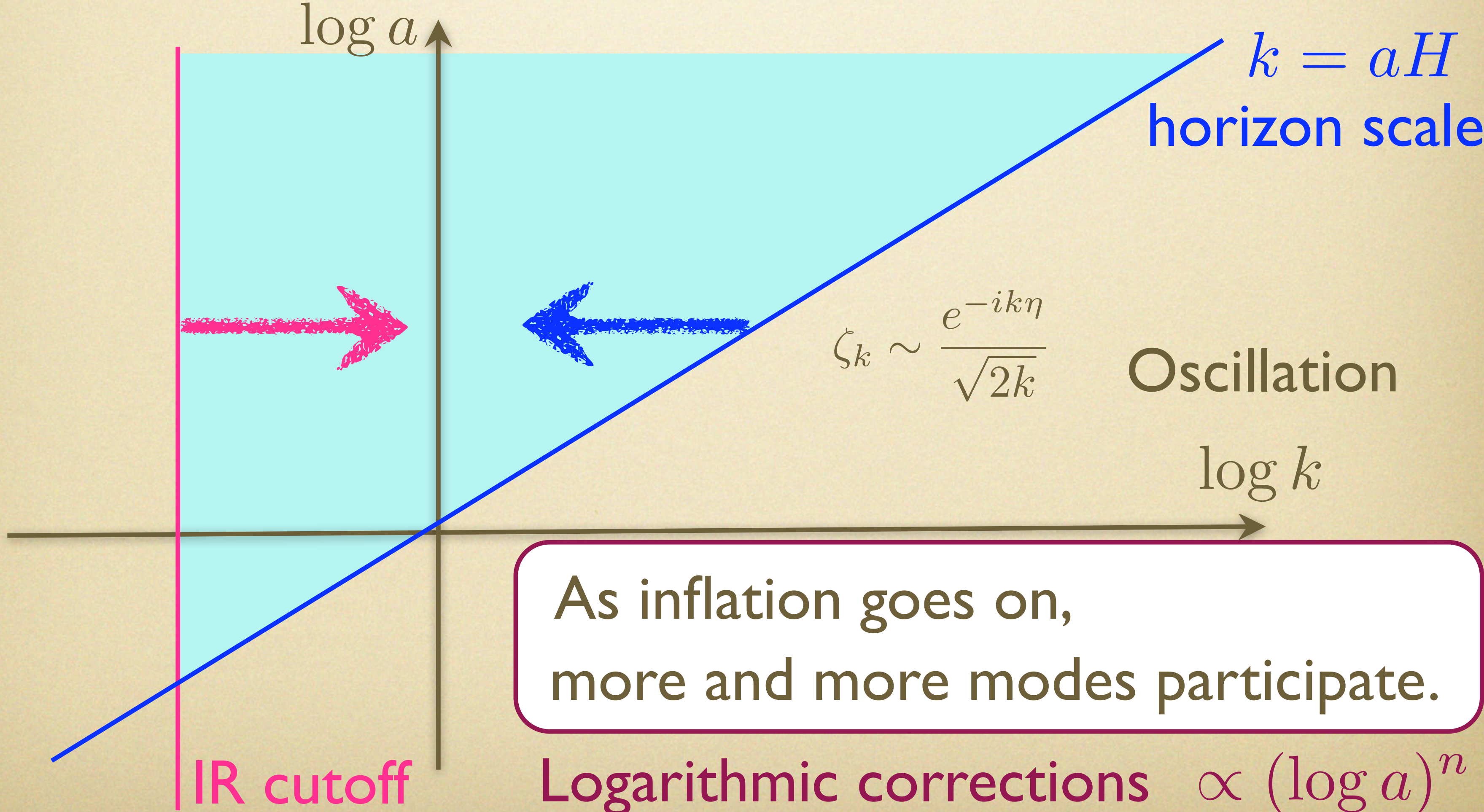
$$\int \mathrm{d}^3 q \, |\zeta_q|^2 = \int \mathrm{d}^3 q / q^3$$

Logarithmic divergence

Introduction of IR cutoff

$$\langle \zeta\zeta\ldots \rangle = \prod_i \int \mathrm{d}t_i \, \mathrm{d}^3\mathbf{k}_i \ldots$$

Which modes participate in loop corrections?



As inflation goes on,
more and more modes participate.

Logarithmic corrections $\propto (\log a)^n$

Gauge-inv. operator

Gauge invariance regarding $x^i \to \tilde{x}^i = x^i + \delta x^i$

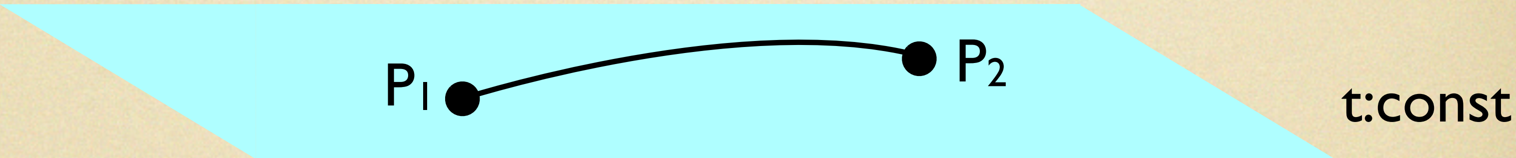
● Geodesic normal coordinate

Scalar quantity, labeled by the gauge-invariant argument

⟶ Gauge-invariant

■ 3D scalar curvature ${}^sR$ Y.U.&T.Tanaka(10)

Two-point function on t:const surface



$$\langle {}^sR(P_1){}^sR(P_2) \rangle \longrightarrow \langle {}^sR{}^sR \rangle (l)$$

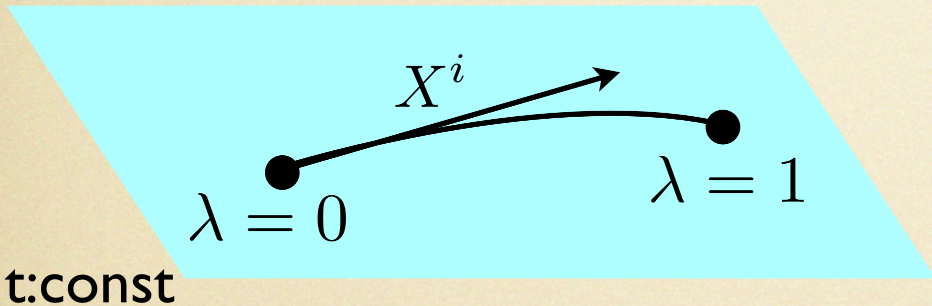
l: Geodesic distance between P_1 and P_2

Geodesic normal coordinate

$$\begin{cases} x^i & \text{: Global coordinates} \\ X^i & \text{: Geodesic normal coordinates} \end{cases}$$

$$\delta\gamma_{ii} = 0 \quad \partial_j \delta\gamma_{ij} = 0$$

- 3D geodesics



$$\frac{\mathrm{d}x^i}{\mathrm{d}\lambda^2} + {}^s\Gamma^i{}_{jk} \frac{\mathrm{d}x^j}{\mathrm{d}\lambda} \frac{\mathrm{d}x^k}{\mathrm{d}\lambda} = 0$$

$$X^i = \left. \frac{\mathrm{d}x^i}{\mathrm{d}\lambda} \right|_{\lambda=0}$$

spatial metric $\quad \mathrm{d}l^2 = e^{2\zeta} \left[\delta_{ij} + \delta\gamma_{ij}\right] \mathrm{d}x^i \mathrm{d}x^j$

large scale limit →

$$x^i(X) \simeq e^{-\zeta} \left[e^{-\delta\gamma/2}\right]^i{}_j X^j$$

$$\delta x^i := x^i - X^i \simeq \left(e^{-\zeta} \left[e^{-\delta\gamma/2}\right]^i{}_j - \delta^i_j\right) X^j$$

Gauge-invariant initial state

IR regularity cond., necessity cond. of gauge invariance

in interaction picture

(C1). Relates ζ and ζ_I

$$\zeta(t_i) = \zeta[\zeta_I(t_i)]$$

ζ_I: interaction picture field

(C2). Positive frequency fn. for ζ_I

ρ:e-folding

$$(1+\varepsilon)\,\partial_\rho \zeta_k - x^i \partial_i \zeta_k + \varepsilon \zeta_k + \ldots = -\left(\partial_{\log k} + 3/2\right)\zeta_k$$

roughly speaking...

almost scale-invariance and "usual" UV behavior

Remarks

- Slow-roll approximation
  - Leading order $\mathcal{O}(\varepsilon^0)$

    Bunch-Davies vacuum (C1), (C2) OK!
  - Higher orders

    Adiabatic vacuum & $\zeta_H(t_i) = \zeta_I(t_i)$

    → (C1), (C2) are not satisfied

- No dependence on the size of local region

  Potentially divergent terms disappear

  No IR cut-off

Aspects of genuine gauge invariance

1. Infrared divergence problem

Y.U.&T.Tanaka (09, 10^1, 10^2)

2. Primordial non-Gaussianity

→ 2. 1 Single-field models T.Tanaka&Y.U. (11), Y.U. (11)

2. 2 Multi-field models

3. Genuine gauge-inv. in holographic universe

J. Garriga and Y.U. (in preparation)

Non-Gaussianity

- Consistency relation

Maldacena (02), Creminell & Zaldarriaga (04)

$$^{g}\zeta_{\mathbf{k}}(\rho) = \int \frac{\mathrm{d}^3\mathbf{x}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}}\, {}^{g}\zeta(\rho,\, \mathbf{x})$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle \underset{k_1 \to 0}{\longrightarrow} -(2\pi)^3 \delta^{(3)}\left(\Sigma \mathbf{k}_i\right)(n_s - 1) P_{k_1} P_{k_2}$$

In general single field models of inflation,

bi-spectrum in $k \to 0$ is related with power-spectrum.

→ Number of light fields during inflation

Single or Plural?

Non-Gaussianity 2

T.Tanaka & Y.U.(11)

- Revisit of consistency relation

Require the local gauge invariance of bi-spectrum

- Gauge-invariant operator

Use of geodesic normal coordinate

- Gauge-invariant (vacuum) state

Leading terms in squeezed limit $(k_1/k_2, k_1/k_3 \to 0)$

$$\langle {}^g\zeta_{\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\zeta_{\mathbf{k}_3} \rangle \to 0$$

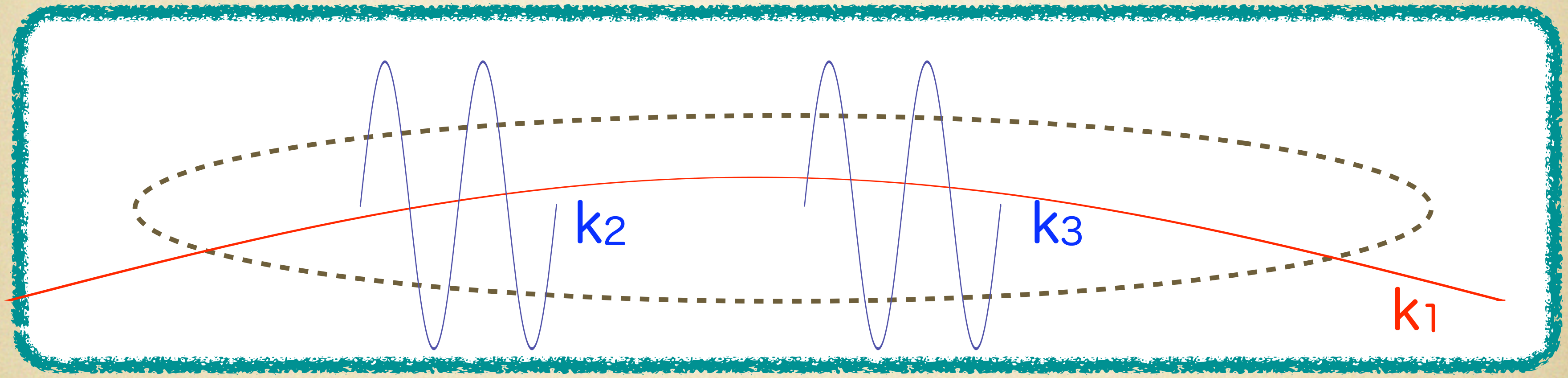
Leading contributions in consistency relation are dominated by gauge modes.

Dominance of gauge artifact

T.Tanaka & Y.U.(11)

- Local non-Gaussianity

$$\langle {}^g\zeta_{\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\zeta_{\mathbf{k}_3} \rangle \qquad k_1/k_2, k_1/k_3 \ll 1$$



k₁ modifies the background for k₂ & k₃

Gauged away by using unbounded gauge trans.

Aspects of genuine gauge invariance

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Y.U.&T.Tanaka (09, 10[1], 10[2])

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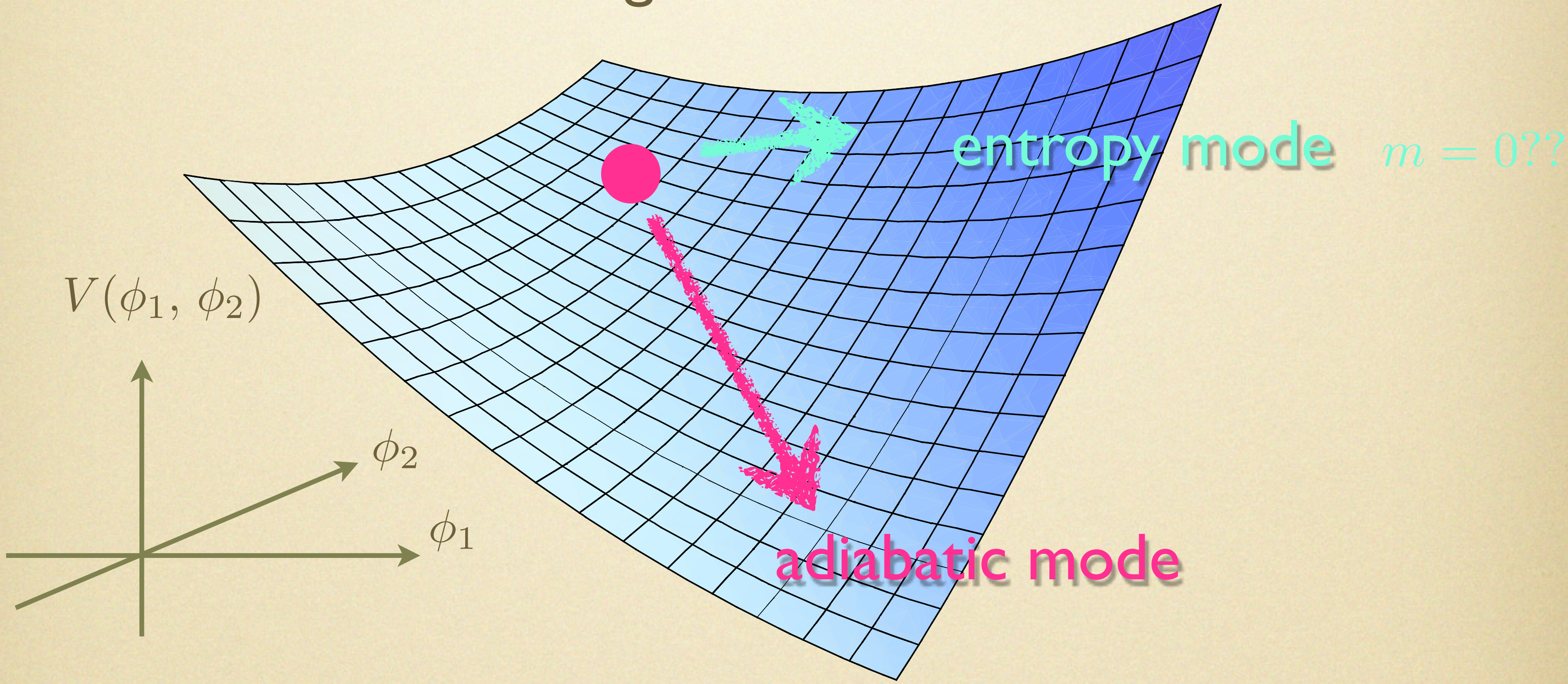
Multi-filed models

Inflation with multi light scalar fields



- Entropy mode

Intrinsically gauge independent

(If exist) IR div. for entropy mode $\neq$ Gauge artifact

Multi-filed models

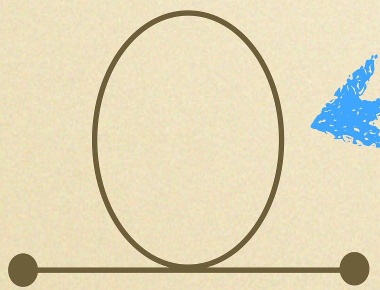
Y.U. (11)

- Gauge invariance condition in multi-field models

ζ : adiabatic mode $\mathcal{S}$: entropy mode

Two-point function of gauge-invariant operator ${}^gR$

- Potentially divergent terms in $\langle {}^gR\, {}^gR \rangle$

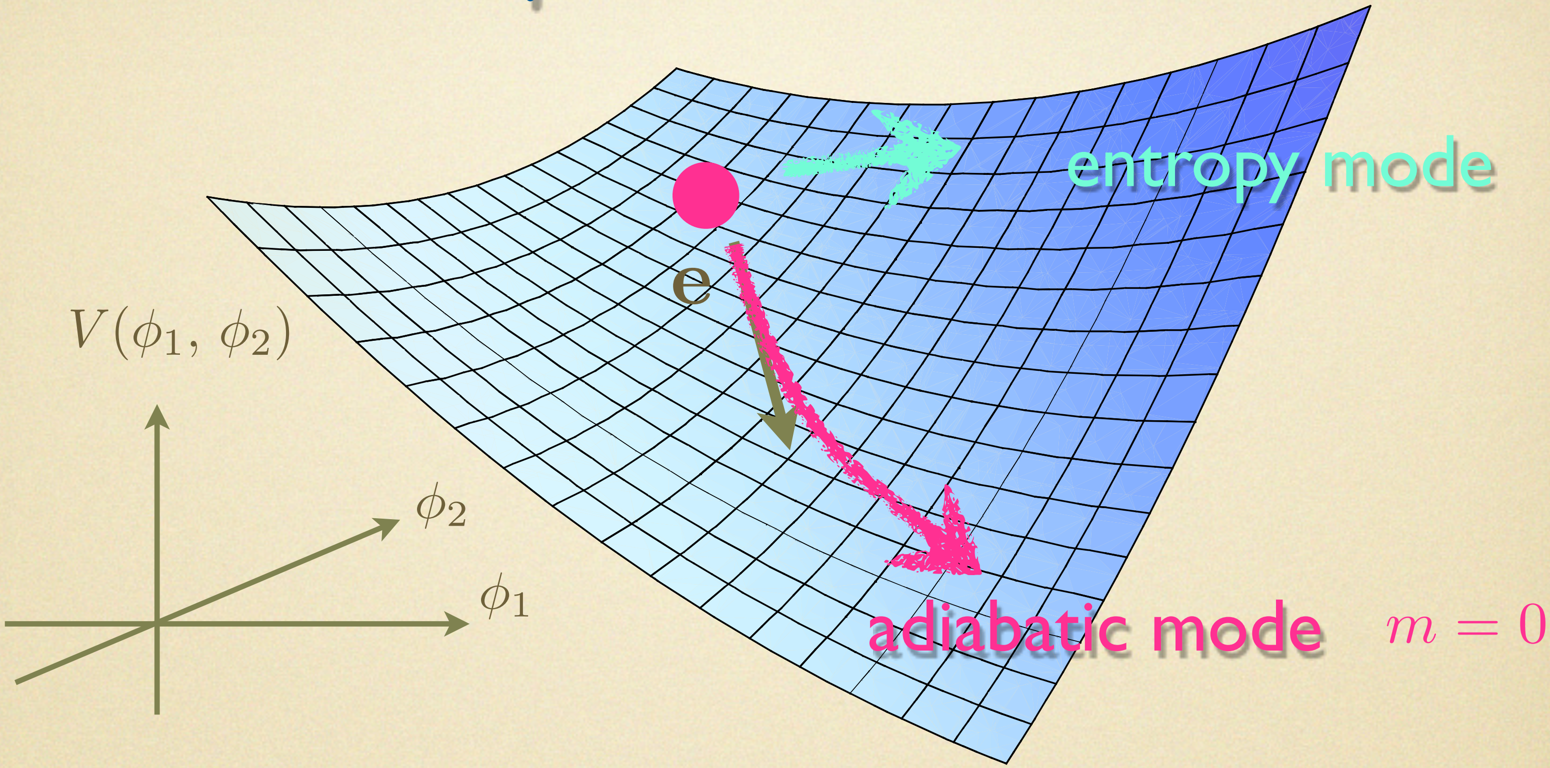


$\langle \zeta^2 \rangle (, \langle \zeta \mathcal{S} \rangle), \langle \mathcal{S}^2 \rangle$

Gauge artifact

Necessary condition for gauge-invariance
$\longrightarrow$ Loop corrections with $\langle \zeta^2 \rangle \, (, \langle \zeta \mathcal{S} \rangle)$ vanish.

Field decomposition



$$\zeta \sim \cos\theta\delta\phi_1 + \sin\theta\delta\phi_2 \propto \mathbf{e}\left(\mathbf{e}\cdot\delta\phi\right)$$

$$\mathcal{S} \sim -\sin\theta\delta\phi_1 + \cos\theta\delta\phi_2 \propto \delta\phi - \mathbf{e}\left(\mathbf{e}\cdot\delta\phi\right)$$

θ :Angle btwn (ϕ_1, ϕ_2) and $(\zeta, \mathcal{S})$

Gauge-invariance conditions

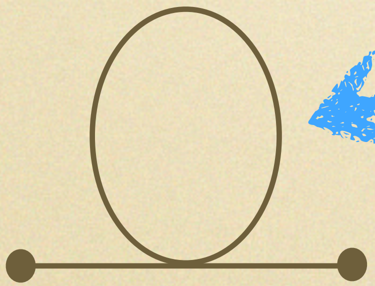
- Gauge invariance condition

  1. $\zeta(t_i) = \zeta[\zeta_I(t_i), \mathcal{S}_I(t_i)]$

  2. Positive frequency fn. for ζ_I

     * Constraints only on Adiabatic field

- Potentially divergent terms in $\langle\, {}^g\!R\, {}^g\!R \,\rangle$ Y.U. (11)



$\langle\zeta^2\rangle(,\ \langle\zeta\mathcal{S}\rangle),\ \langle\mathcal{S}^2\rangle$

Regularized

Non-Gaussianity

in the squeezed limit $k_1/k_2, k_1/k_3 \to 0$

$$\langle {}^g\zeta_{\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\zeta_{\mathbf{k}_3} \rangle \propto |\zeta_{k_2}|^2 \, \mathrm{Re}\left[\zeta_{k_1} \partial_\eta \zeta^*_{k_1}\right]$$

EOM for ζ

$$\zeta'' + 2\frac{z'}{z}\zeta' - \partial^2\zeta \propto \theta' \mathcal{S} + \cdots$$

¡Remark!

If trajectory is curved $i.e.\ \theta' \neq 0$

→ $\partial_\eta \zeta_{k_1} \not\to 0\,, \ \langle {}^g\zeta_{\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\zeta_{\mathbf{k}_3} \rangle \not\to 0$

Time variation in ζ → Non-vanishing contributions

Cannot be gauged away

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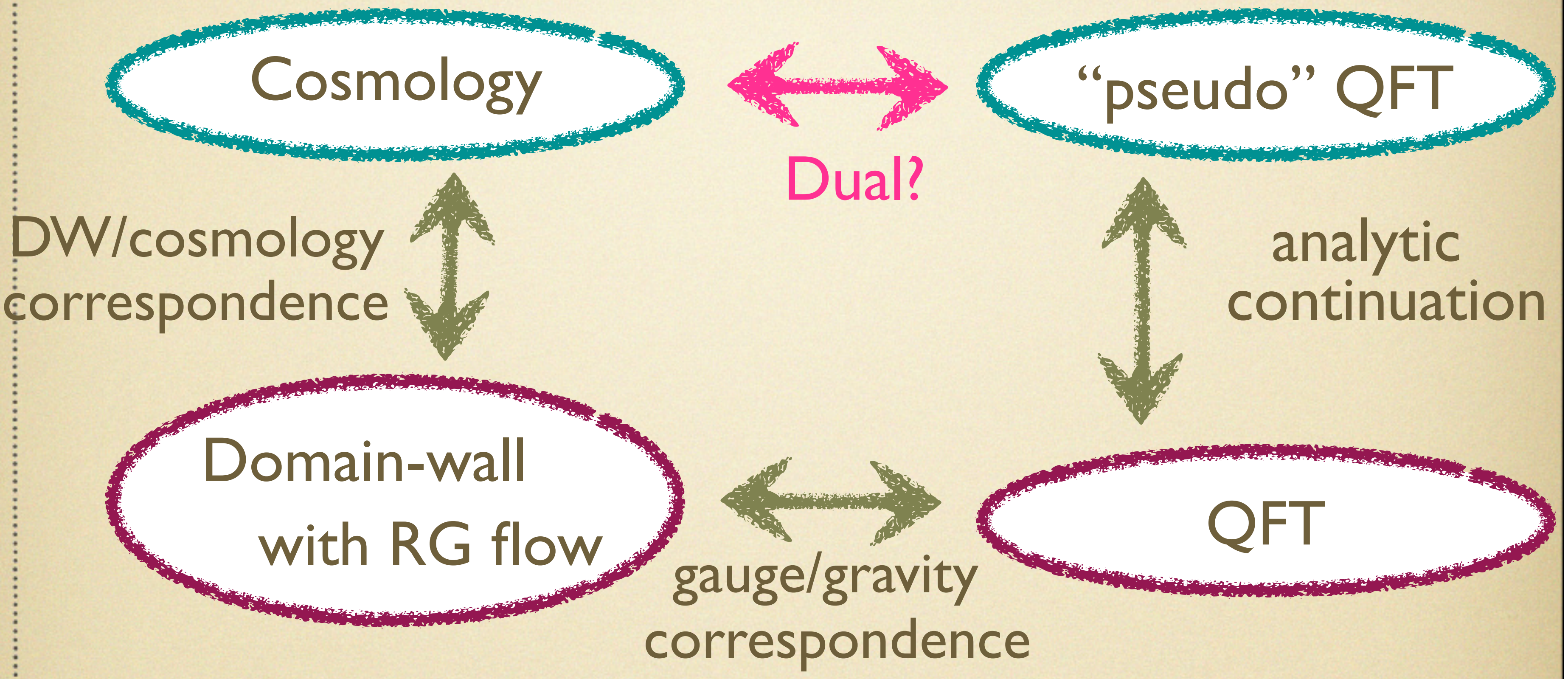
Holographic universe

McFadden&Skenderis(09)



- Strong coupling → Weak gravity
- Weak coupling → Strong gravity (stringy phase)

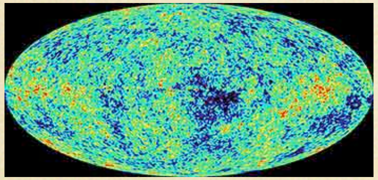
Phenomenological holography

String/M-theory?

Holographic dual
of early universe

holography as phenomenology

Precision cosmology



- Priomrdial pert. sourced by QFTs

K.Skenderis et al. (09, 10,11)

1. Power spectrum

   Nearly scale-invariant

2. Bi-spectrum

   Equilateral type? No local-type ???

Primordial pert. from holography

$$Z_{\rm bulk}[g_{ij},\varphi]=\int D[\Phi]\exp\left[-\int_{\partial M}(\mathcal{L}[\Phi,g_{ij}]+\varphi\mathcal{O}_\varphi[\Phi])\right]$$

ϕ :inflaton

$$(\varphi,\,\mathcal{O}_\varphi)\longrightarrow(g_{ij},\,T_{ij})\ \ (\phi,\,\mathcal{O}_\phi)$$

J. Garriga and Y.U.

$$\delta\phi=0\qquad\delta g_{ij}=e^{2(\rho+\zeta)}\left[e^{\delta\gamma}\right]_{ij}$$

1. Formal expression in holographic universe

$$\langle\zeta(x_1)\zeta(x_2)...\zeta(x_n)\rangle\longleftarrow\langle T^i{}_i(x_1)T^i{}_i(x_2)...T^i{}_i(x_n)\rangle$$

2. Gauge-invariant state

$\psi[\zeta]\sim e^{-S_{\rm bulk}[\zeta]}$

$\psi[\zeta]$ independent of $\zeta(\mathrm{k}=0)$

→ $\langle{}^g\zeta_{\mathbf{k}_1}{}^g\zeta_{\mathbf{k}_2}{}^g\zeta_{\mathbf{k}_3}\rangle\simeq0\qquad(k_2,k_3\gg k_1)$

Comments

If holographic description is possible...

Genuine gauge-invariance implies absence of local NG

- Weak coupling in bulk → T.Tanaka&Y.U. (11)
- Strong coupling in bulk

However.....

Several issues to be discussed

- Transition to Big Bang
- DW/Cosmology correspondence

Summary

- Observable fluctuations should be genuinely gauge invariant.

- Implications of genuine gauge invariance

  1. No IR divergence in single field models
  2. Consistency relation for bi-spectrum is dominated by gauge modes.
  3. Time variation of ζ can generate observable fluc.

- Genuine gauge-invariance requests the absence of of local-type NG also in holography.

  ＊Valid both in strong/weak coupling limit

Supplement

Residual gauge modes

Single field inflation

$$S_\phi = -\frac{1}{2}\int \sqrt{-g}\,[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2V(\phi)]\mathrm{d}^4x$$

- General solutions of δN, $\check{N}_i = e^{-\rho}N_i$

From Hamiltonian&Momentum constraints at 1st order

$$\delta N_1(x) = \frac{1}{\rho'}\left(\zeta_1'(x) - \frac{1}{4}\partial^i G_{i,1}(x)\right)$$

$$\partial^2 G_{i,1}(x) = 0$$

$$\check{N}_{i,1}(x) = \partial_i\left(\frac{\phi'^{\,2}}{2\rho'^{\,2}}\partial^{-2}\zeta_1'(x) - \frac{1}{\rho'}\zeta_1(x)\right) - \frac{1}{4}\left(1 + \frac{\phi'^{\,2}}{2\rho'^{\,2}}\right)\partial_i\partial^{-2}\partial^j G_{j,1}(x) + G_{i,1}(x)$$

DOFs in δN & N_i → Residual gauge DOFs

Residual gauge modes 2

$$(\delta N,\, \check{N}_i)\ \text{for}\, G_i = 0 \longrightarrow \left(\delta\tilde{N},\, \tilde{\check{N}}_i\right)\ \text{for}\, G_i \neq 0$$

- Gauge transformation: $(t,\, x^i) \to (t + \delta t,\, x^i + \delta x^i)$
  - Time coordinate Fixed by $\delta\phi = 0$
  - Spatial coordinates $\gamma^{ij}\delta\gamma_{ij} = 0$ $\partial^i \delta\gamma_{ij} = 0$

∃Residual gauge modes

$$\delta x_i = -\int \mathrm{d}\eta\, G_i + \frac{1}{4}\int \mathrm{d}\eta \partial_i \partial^{-2}\partial^j G_j + \frac{1}{4}\int \frac{\mathrm{d}\eta}{\rho'}\int \mathrm{d}\eta \partial_i \partial^j G_j + \cdots$$

(i) Scale transformation

$$x^i \to e^{f(\eta)} x^i \qquad \zeta(x) \to \tilde{\zeta}(x) = \zeta(x) - f(\eta) + \cdots$$

(ii) Shear transformation

$$x^i \to x^i + C^i{}_j(\eta) x^j \qquad C^i{}_i = 0,\, C_{ij} = C_{ji}$$

$$\delta\gamma_{ij}(x) \to \delta\tilde{\gamma}_{ij}(x) = \delta\gamma_{ij}(x) - 2C_{ij}(\eta) + \cdots$$

Residual gauge modes 2

Comoving gauge $\delta\phi = 0$ $h_{ij} = e^{2(\rho+\zeta)}\left[e^{\delta\gamma}\right]_{ij}$

$\delta\gamma_{ii} = 0$ $\partial_j \delta\gamma_{ij} = 0$

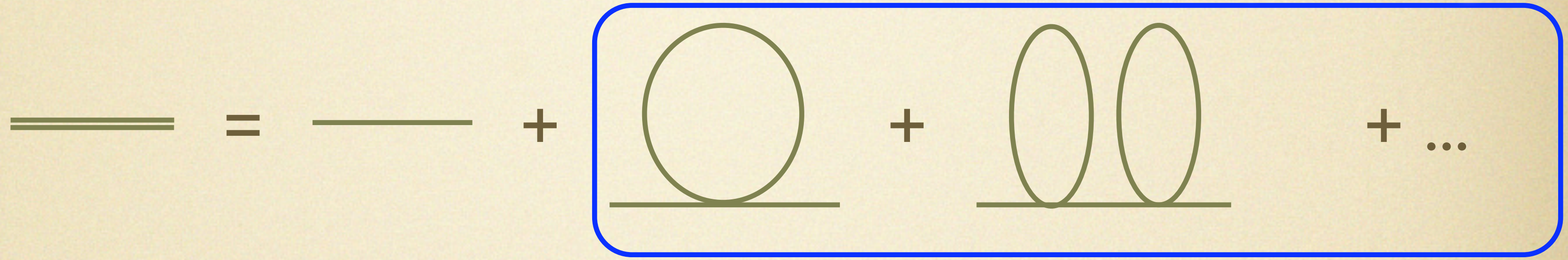
$$\delta x_i(x) = -\int \mathrm{d}\eta\, G_i(x) + \frac{1}{4}\int \mathrm{d}\eta \partial_i \partial^{-2}\partial^j G_j(x) + \frac{1}{4}\int \frac{\mathrm{d}\eta}{\rho'}\int \mathrm{d}\eta\, \partial_i \partial^j G_j(x) + H_i(\boldsymbol{x}) + \int \frac{\mathrm{d}\eta}{\rho'}\,\partial^2 H_i(\boldsymbol{x}) \,,$$

Vector fns. $\partial^2 G_i(x) = 0$

$$3\partial^2 H_i(\boldsymbol{x}) + \partial_i \partial^j H_j(\boldsymbol{x}) = 0$$

Other IR issues

- Re-summation can cure IR singularity? C. Burges et al. (09,10)



Generate effective mass

Singular behavior in mass-less limit

→ Break-down of perturbation theory?

- Effects of decoherence can cure IR singularity?

Y.U.&T.Tanaka (09)

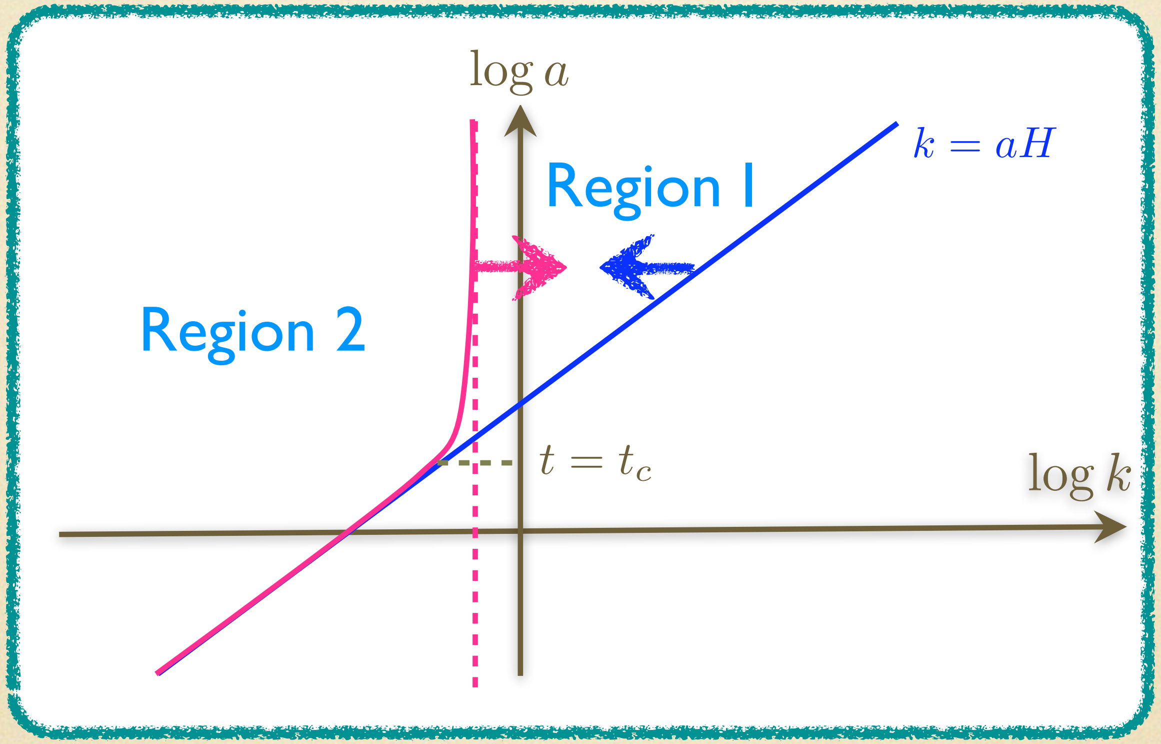
Momentum integral can be regularized. Time integral?

- Stability of de Sitter spacetime

Polyakov (07, 09), Marolf and Morrison (09,10)

Analytic continuation from Euclidean S^5

More about secular growth



ζ_n $\begin{cases} \text{Contributions from region I} & \left[\dfrac{H(t)}{M_{\rm pl}\varepsilon^{1/2}}\right]^n \\ \text{Contributions from region II} & \{a_i H_i L(t)\}\left[\frac{H_i}{M_{\rm pl}\varepsilon^{1/2}}\right]^n \end{cases}$

For $n > n_c$, suppression is not enough to eliminate the contributions from the distant past.

Gauge-invariant initial state

1. $\zeta(t_i) = \zeta\left[\zeta_I(t_i)\right]$

Y.U.&T.Tanaka(10)

Heisenberg eq. $\mathcal{L}\zeta = \mathcal{S}[\zeta]$

$$\zeta = \Sigma_i a_i F[\zeta_I] + \mathcal{L}^{-1}\mathcal{S}$$

$\mathcal{L}$: Derivative op.

homogeneous solution

$$\mathcal{L}F[\zeta_I] = 0$$

Conditions on a_i → (C1)

2. Positive frequency fn. for ζ_I

ρ:e-folding

$$(1+\varepsilon)\,\partial_\rho\zeta_k - x^i\partial_i\zeta_k + \varepsilon\zeta_k + \ldots = -\left(\partial_{\log k} + 3/2\right)\zeta_k$$

(C2)

Consistency relations

- Consistency relation in global coordinate

Maldacena (02), Creminell & Zaldarriaga (04)

$$^g\zeta_{\mathbf{k}}(\rho) = \int \frac{\mathrm{d}^3\mathbf{x}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{x}}\, {}^g\zeta(\rho,\, \mathbf{x})$$

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle \underset{k_1 \to 0}{\to} -(2\pi)^3\delta^{(3)}\left(\Sigma\mathbf{k}_i\right)(n_s - 1)P_{k_1}P_{k_2}$$

- Consistency relation for genuine gauge-inv. quantities

{ (1) Geodesic normal coordinate
  (2) Gauge-inv. initial vacuum

cf. $^gR_2 \simeq e^{-2\,{}^g\zeta_2}\partial^2\,{}^g\zeta_2$

$$^g\zeta(\eta,\, X^{(i)}) = \sum_{n=0}^{\infty} \frac{\delta x^{i_1}\cdots\delta x^{i_n}}{n!}\partial_{i_1}\cdots\partial_{i_n}\zeta(\eta,\, x^i)|_{x^i = X^{(i)}}$$

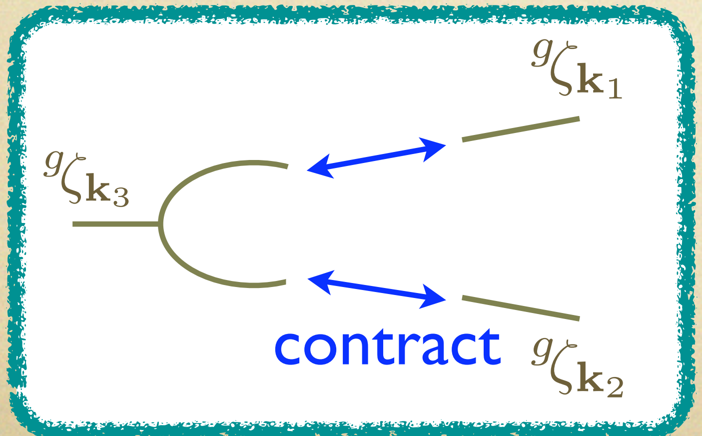
Non-Gaussianity

$$ {}^gF(\mathbf{k}_1,\, \mathbf{k}_2,\, \mathbf{k}_3) = \langle\, {}^g\zeta_{(\mathbf{k}_1}\, {}^g\zeta_{\mathbf{k}_2}\, {}^g\zeta_{\mathbf{k}_3)} \rangle $$

$$ {}^g\zeta_{\mathbf{k}}(\rho) = \int \frac{\mathrm{d}^3\mathbf{X}}{(2\pi)^{3/2}} e^{-i\mathbf{k}\cdot\mathbf{X}}\, {}^g\zeta(\rho,\, \mathbf{X}) $$

- Expansion by interaction picture field ψ

$$ {}^g\zeta(X) \simeq \psi + \frac{1}{2}\mu_1\psi^2 + (1+\mu_2)\,\psi\partial_\rho\psi - \mathcal{L}^{-1}\mu_3\psi^2 - (1+\lambda_2)\psi X^i\partial_{X^i}\psi $$



$$ \mu_1 := \varepsilon_1 + \frac{1}{2}\varepsilon_2 + 2\xi_2 $$

$$ \mu_2 := \varepsilon_1\,(1+\varepsilon_1+\varepsilon_2) + \lambda_2 $$

$$ \mu_3 := \frac{3}{4}\varepsilon_2(2\varepsilon_1+\varepsilon_2) $$

Revisit of consistency relation

Leading terms for $k_1 \ll k_2\,,k_3$

Y.U. & T. Tanaka (to appear soon)

$$
\begin{aligned}
&{}^gF(\mathbf{k}_1,\,\mathbf{k}_2,\,\mathbf{k}_3)\\
&\simeq -(1+\lambda_2)|v_{k_1}|^2(2\pi)^{-3/2}\left\{\frac{1}{2}\mathrm{Re}\left[v_{k_2}v^*_{|k_1+k_3|}+v_{k_3}v^*_{|k_1+k_2|}\right]+|v_{k_2}|^2+|v_{k_3}|^2\right\}\mathbf{k}_1\cdot\partial_{\mathbf{K}}\delta^{(3)}(\mathbf{K})\\
&\quad +\frac{1}{2}(1+\lambda_2)|v_{k_1}|^2(2\pi)^{-3/2}\mathrm{Re}\left[v_{k_2}v^*_{|k_3+k_1|}-v_{|k_2+k_1|}v_{k_3}\right](\mathbf{k}_2-\mathbf{k}_3)\cdot\partial_K\delta^{(3)}(\mathbf{K})\\
&\quad -2|v_{k_1}|^2(2\pi)^{-3/2}\delta^{(3)}(\mathbf{K})\sum_{a=2}^{3}\mathrm{Re}\left[v_{k_a}\left\{\mathcal{L}^{-1}_{|k_a+k_1|}\mu_3 v^*_{k_a}-\mathcal{L}^{-1}_{k_a}\mu_3 v^*_{k_a}\right\}\right]\\
&\quad +\mathcal{O}(\varepsilon^3)
\end{aligned}
$$

→ ${}^gF(\mathbf{k}_1,\,\mathbf{k}_2,\,\mathbf{k}_3)\to 0$ in the limit $k_1 \to 0$

Non-Gaussianity 2

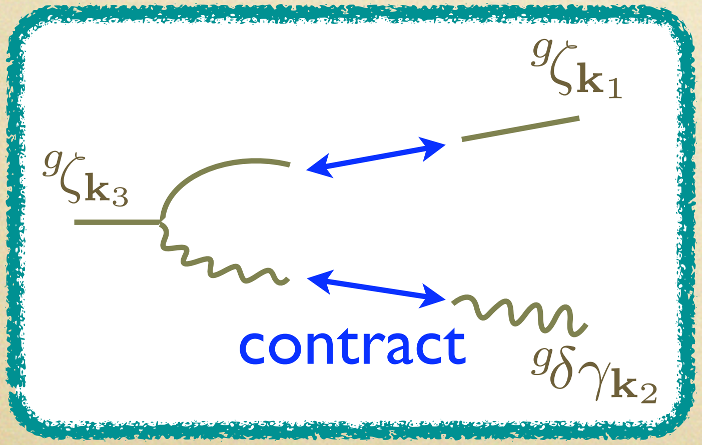
Contributions from GWs

$$ {}^gF'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) := \langle\, {}^g\zeta_{(\mathbf{k}_1} {}^g\zeta_{\mathbf{k}_2} {}^g\delta\gamma_{\mathbf{k}_3)} \rangle $$

Vertex $\zeta\zeta\delta\gamma$

$$ \zeta_2 \supset \tfrac{1}{2}\delta\gamma^{ij} X_i \partial_{X^j}\psi $$

$$ {}^g\zeta_2 = \zeta_2 + \delta x^i \partial_i \psi|_{x=X} = \zeta_2 - \tfrac{1}{2}\delta\gamma^{ij} X_i \partial_{X^j}\psi + \ldots $$



$$ {}^gF'(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) \to 0 $$

at large scales

DW/Cosmology correspondence

- FRW background

Domain-wall solution with potential $-V$

$$ds^2 = dz^2 + a^2\left[-\frac{d^2\tau}{1+K\tau^2} + \tau^2\left(d\psi^2 + \sinh^2\psi d\Omega^2_{D-2}\right)\right]$$

$$(t,\, r,\, \theta) = -i(z,\, \tau,\, \psi)$$

Cosmological solution with potential V

$$ds^2 = -dt^2 + a^2\left[\frac{d^2r}{1-Kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\tilde{\Omega}^2_{D-2}\right)\right]$$

- Perturbations

Correspondence is confirmed

McFadden&Skenderis(09)

Gauge/gravity correspondence

d-dim gauge theory ⟷ (d+1)-dim gravity theory

Dual?

J. Maldacena (97)

d-dim N=4 super YM-theory	TypeIIB super-string on $AdS_5 \times S^5$

- Examples in holographic universe

Two examples where holography is known

1. Asymptotically AdS

   Deformation of a CFT

2. Asymptotically power-law

   QFT with generalized conformal structure

Strong/Weak coupling limit

- Two approximations

J. Maldacena (97)

1) Neglect loop corrections of graviton

Large N $N \gg 1$

2) Neglect massive modes of string excitation

Large 't Hooft coupling $N g_{\rm YM}^2 \gg 1$

Strong coupling in CFT

in holographic cosmology

we adapt only condition 1)

we can consider both strong/weak couplings

Primordial pert. from holography

- Gauge/gravity correspondence

$$Z_{\rm bulk}[g_{ij},\varphi] = \int {\rm D}[\Phi] \exp\left[-\int_{\partial {\rm M}} (\mathcal{L}[\Phi, g_{ij}] + \varphi \mathcal{O}_\varphi[\Phi])\right]$$

$$(\varphi,\, \mathcal{O}_\varphi) \longrightarrow (g_{ij},\, T_{ij}) \;\; (\phi,\, \mathcal{O}_\phi)$$

- Wave function

J. Garriga and Y.U. (in preparation)

$$\delta\phi = 0, \qquad h_{ij} = e^{2(\rho+\zeta)}\delta_{ij}$$

$$S_{\rm bulk}[g_{ij},\, \varphi] \simeq -\log Z_{\rm bulk}[g_{ij},\, \varphi]$$

$$P[\zeta] = |\psi[\zeta]|^2 = e^{-2{\rm Re}[S_{\rm bulk}]}$$

n-point fns

$$\langle \zeta(x_1)\zeta(x_2)...\zeta(x_n)\rangle = \int {\rm D}[\zeta]\, \zeta(x_1)\zeta(x_2)...\zeta(x_n) P[\zeta]$$