Variation of fundamental constants

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08/08/11 @ Summer Institute 2011

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References

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- J.P. Uzan, Rev. Mod. Phys. 75, 403 (2003).
- C. M. Will, Liv. Rev. Rel. 9, 3 (2006).
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- …

Introduction

What are constants in physics ?

A constant = a parameter that is input by hand and cannot be explained by the theory

What is a constant depends on the theory we are considering.

e.g. The Fermi constant G_F was a constant in the Fermi's four interaction theory. But, the Fermi's theory is now replaced by the gauge theory and $G_F = 1 / (\sqrt{2}\, v^2)$ for $k \ll v$, where $v = (246.7 \pm 0.2)$ GeV is a vacuum expectation value of the Higgs field. For $k \gg v$, $G_F \sim k^{-2}$.

What are "fundamental" constants ?

A "fundamental" constant = a parameter that is input by hand and cannot be explained by the "fundamental" theory

→ **What is a fundamental constant depends on the fundamental theory we are considering.**

→ **If the variation of a quantity that is regarded as a constant in the fundamental theory is observed, it implies that the theory is not fundamental and the constant is replaced by a (dynamical) field whose dynamics cannot be neglected.**

List of the fundamental constants in the "standard" model of particle physics

Constant	Symbol	Value
Speed of light	c	$299\,792\,458$ m s^{-1}
Planck constant (reduced)	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$ J s
Newton constant	G	$6.674\,28(67) \times 10^{-11}$ m^2 kg^{-1} s^{-2}
Weak coupling constant (at m_Z)	$g_2(m_Z)$	0.6520 ± 0.0001
Strong coupling constant (at m_Z)	$g_3(m_Z)$	1.221 ± 0.022
Weinberg angle	$\sin^2\theta_W(91.2\ \mathrm{GeV})_{\overline{\mathrm{MS}}}$	0.23120 ± 0.00015
Electron Yukawa coupling	h_e	2.94×10^{-6}
Muon Yukawa coupling	h_μ	0.000607
Tauon Yukawa coupling	h_τ	0.0102156
Up Yukawa coupling	h_u	0.000016 ± 0.000007
Down Yukawa coupling	h_d	0.00003 ± 0.00002
Charm Yukawa coupling	h_c	0.0072 ± 0.0006
Strange Yukawa coupling	h_s	0.0006 ± 0.0002
Top Yukawa coupling	h_t	1.002 ± 0.029
Bottom Yukawa coupling	h_b	0.026 ± 0.003
Quark CKM matrix angle	$\sin\theta_{12}$	0.2243 ± 0.0016
	$\sin\theta_{23}$	0.0413 ± 0.0015
	$\sin\theta_{13}$	0.0037 ± 0.0005
Quark CKM matrix phase	δ_{CKM}	1.05 ± 0.24
Higgs potential quadratic coefficient	$\hat{\mu}^2$	?
Higgs potential quartic coefficient	λ	?
QCD vacuum phase	θ_{QCD}	$< 10^{-9}$

(Uzan 2011)

We have 22 fundamental (unknown) constants !!

Fundamental constants well constrained from experiments and observations

$$\begin{cases} \alpha_{EM} = \alpha = \dfrac{e^2}{4\pi\epsilon_0 \hbar c} = 1/137.03599976(50), \\ G = 6.673(10) \times 10^{-11} \mathrm{m}^3\ \mathrm{kg}^{-1}\ \mathrm{s}^{-2}, \\ \mu \equiv \dfrac{m_e}{m_p} = 5.44617 \times 10^{-4}. \end{cases}$$

$$e = 1.602176462(63) \times 10^{-19} \mathrm{C},$$

$$m_e = 9.10938188(72) \times 10^{-31} \mathrm{kg},$$

$$m_p = 1.67262158(13) \times 10^{-27} \mathrm{kg}.$$

Effects of renormalization

Quantum field theory predicts the running
of coupling constants and masses depending on the energy scale.

e.g.

$$\alpha_i^{-1}(E) = \alpha_{\mathrm{GUT}}^{-1} - \frac{b_i}{2\pi} \ln\left(\frac{E}{\Lambda_{\mathrm{GUT}}}\right).$$

In this sense, the observed coupling constants and masses are not constant.
Then, what we would like to probe is
the variation of bare coupling constants and masses.

But, in cosmological and terrestrial experiments, such running is almost negligible because the mass of the lightest charged particle, that is the electron, is near 0.5 MeV, which determines the infrared point of the renormalization equation of α.

Extensions of the standard model

We have a lot of reasons to extend the standard model.

- **The presence of the masses of neutrinos**

  additional 4 constants (Maki-Nakagawa-Sakata Matrix)
- **The hierarchy problem**

  additional 111 constants (MSSM soft breaking terms)
- **The suggestion of the grand unified theory**

  the number of gauge coupling constants are reduced, but …
- **The incompatibility of general relativity and quantum field theory**

  suggestion of string theory.

  Can we expect the small number of fundamental constants ?

How many fundamental constants finally?

It is usually said that there are three basic quantities in nature :
Length, Time, and Mass

Constant	Symbol	Value
Speed of light	c	$299\,792\,458$ m s^{-1}
Planck constant (reduced)	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$ J s
Newton constant	G	$6.674\,28(67) \times 10^{-11}$ m^2 kg^{-1} s^{-2}

These three constants are sufficient to represent all physical quantities.

- $[c] = LT^{-1}$: relativisitic or non-relativistic, causality
- $[hbar] = L^2MT^{-1}$: quantum or classical
- $[G] = L^3M^{-1}T^{-2}$: determines the energy scale (Planck scale).

String tension is the only input and determines the energy scale in string theory.

Two fundamental constants in string theory ???

Veneziano, Europhys. Lett., 2, 199 (1986).
Duff, Okun, & Venezaino, JHEP, 03, 023 (2002).

- **Veneziano argued that string theory only needs c & λ s : one fundamental unit of speed and one length.**

causality (relativity) **quantization**

Nambu-Goto action:

$$S = \frac{T}{c} \int d(\text{Area}), \quad \frac{S}{\hbar} = \lambda_s^{-2} \int d(\text{Area}).$$

T (string tention) appears in the action, but only a combination S/hbar, that is, string length $\lambda_s = \sqrt{\hbar c / T}$ may be relevant.

- **Zeldovich and Novikov noticed that pure quantum gravity would contain only two fundamental units, c & $l_P = \sqrt{G\hbar / c^3}$. G & hbar do not appear separately.**

$$\left(\frac{S}{\hbar} = \int d^4x \sqrt{-g} \frac{R}{16\pi} \frac{c^3}{G\hbar} = \frac{1}{16\pi l_P^2} \int d^4x \sqrt{-g} R \right)$$

Novikov, I.D. and Zel'dovich, Y.B., Relativistic Astrophysics: The structure and evolution of the universe, 2, (University of Chicago Press, Chicago, 1983).

Implications of variation of fundamental constants

What if the variation of a "constant" is observed ?

- The theory we are considering must be extended to a new theory, where the constant is replaced by a field that can vary.

⇒ Constancy or variation of a constant is a crucial test to the theory.

- If there is no gravity, the situation is quite simple. But, in fact, such an observation is a test to general relativity though we have implicitly assumed the Minkowski (flat) background thus far.

⇒ What happened if the theory formulated in the flat spcetime is considered in the curved spacetime ?

(Weak) Equivalence principle (WEP)

The universality of free fall (Weak equivalence principle) :

The trajectory of a freely falling “test” body is independent of its internal structure and composition.

On the other hand, variation of constants often leads to the violation of WEP because dilaton (modulus) typically has a direct coupling to matter.

Constancy or variation of a constant is a crucial test to EP or metric theory of gravity.

Test of the universality of free fall (WEP)

The universality of free fall (WEP) can be tested by comparing the accelerations of two test bodies in an external gravitational field.

$$\eta_{12} := 2\left|\frac{a_1 - a_2}{a_1 + a_2}\right|.$$

Table 3: Summary of the constraints on the violation of the universality of free fall.

Constraint	Body 1	Body 2	Ref.
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[4]
$(0.1 \pm 2.7 \pm 1.7) \times 10^{-13}$	Earth-like rock	Moon-like rock	[23]
$(-1.0 \pm 1.4) \times 10^{-13}$	Earth	Moon	[543]
$(0.3 \pm 1.8) \times 10^{-13}$	Te	Bi	[451]
$(-0.2 \pm 2.8) \times 10^{-12}$	Be	Al	[482]
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[482]
$(5.1 \pm 6.7) \times 10^{-12}$	Si/Al	Cu	[482]

(Uzan 2011)

Universality of free fall is confirmed with very good accuracy.

Implication of variation of fundamental constants on Λ

Cosmological constant Λ :

$$G_{\mu\nu} + 8\pi G\Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \Longleftrightarrow \langle 0 | T_{\mu\nu} | 0 \rangle = -\Lambda g_{\mu\nu}.$$

$\Rightarrow$ $\Lambda_0 \simeq 2.7 \times 10^{-47}\ \mathrm{Gev}^4$ **: vacuum (zero-point) energy density**

Variation of fundamental constants induces the shift of Λ.

e.g. Variation of alpha leads to variation of Λ through its coupling to charged particles. Each charged particle receives a shift in its mass due to electromagnetism so that such a mass shift influences the zero-point energy.

$$\frac{\delta\Lambda}{\delta\alpha} = \sum_i \frac{\delta\Lambda}{\delta m_i}\frac{\delta m_i}{\delta\alpha}$$

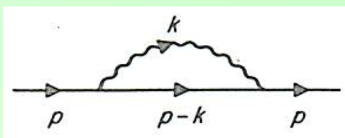
Variation of Λ due to variation of α

Banks, Dine, & Douglas, PRL 88, 131301, 2002
Donoghue, JHEP 03, 052, 2003

$$\frac{\delta\Lambda}{\delta\alpha} = \sum_i \frac{\delta\Lambda}{\delta m_i}\frac{\delta m_i}{\delta\alpha},$$

i : all (standard model) charged particles

- $\dfrac{\delta m_i}{\delta\alpha}$: **mass shift due to electromagnetic effect by the self energy**



$$ie_0^2\Sigma(p) = \frac{(ie_0)^2}{(2\pi)^4}\int d^4k\, iD_{F\alpha\beta}(k)\gamma^\alpha iS_F(p-k)\gamma^\beta$$

$$\Rightarrow \quad \left.\frac{\delta m_i}{\delta\alpha}\right|_{E<v} = \frac{m_i}{2\pi}\ln\left(\frac{v^2+m_i^2}{m_i^2}\right).$$

v : cut-off scale

- $\dfrac{\delta\Lambda}{\delta m_i}$: **zero-point energy shift due to the variation of mass**

$$\Lambda|_{E<v} = \sum_i\left(2\int^v \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{2}\omega_k^i\right), \quad \omega_k^i = \sqrt{m_i^2+\boldsymbol{k}^2}.$$

$$\Rightarrow \quad \left.\frac{\delta\Lambda}{\delta m_i}\right|_{E<v} = 2\int^v \frac{d^3\boldsymbol{k}}{(2\pi)^3}\frac{1}{2}\frac{m_i}{\omega_k^i} = \frac{m_i}{4\pi^2}\left[v\sqrt{v^2+m_i^2} - m_i^2\ln\left(\frac{v+\sqrt{v^2+m_i^2}}{m_i}\right)\right].$$

Variation of Λ due to variation of α II

$$\left.\frac{\delta\Lambda}{\delta\alpha}\right|_{E<v} = \sum_i \frac{\delta\Lambda}{\delta m_i}\frac{\delta m_i}{\delta\alpha} = \sum_i \frac{m_i^2}{8\pi^3}\left[v\sqrt{v^2+m_i^2} - m_i^2 \ln\left(\frac{v+\sqrt{v^2+m_i^2}}{m_i}\right)\right]\ln\left(\frac{v^2+m_i^2}{m_i^2}\right)$$

$$\sim \quad 10^{52}\Lambda_0 \quad \text{for } v \sim 100\,\text{GeV}.$$

$$\Rightarrow \quad \frac{\delta\alpha}{\alpha} \ll 10^{-50}.$$

Variation of alpha is strongly constrained.

Even if we take SUSY into account, the situation is not improved drastically because the SUSY must be broken in our Universe.

$$\delta\Lambda = (\delta\alpha)^2 M_{\text{SUSY}}^4, \sim \left(\frac{\delta\alpha}{\alpha}\right)^2\left(\frac{M_{\text{SUSY}}}{100\text{GeV}}\right)^4 10^{50}\Lambda_0,$$ **optimistic estimate**

$$\Rightarrow \quad \frac{\delta\alpha}{\alpha} \ll 10^{-25}.$$

Variation of Λ due to variation of fundamental constants

- **variation of electron mass**

$$m_e \frac{\delta\Lambda}{\delta m_e}\Big|_{E<v} = \frac{m_2^2 v^2}{8\pi^2} \sim 10^{42}\Lambda_0 \quad \text{for } v \sim 100\text{ GeV}.$$

- **variation of Higgs expectation value ~ v**

$$v \frac{\delta\Lambda}{\delta v}\Big|_{E<v} = m_t \frac{\delta\Lambda}{\delta m_t} \sim 10^{52}\Lambda_0 \quad \text{for } v \sim 100\text{ GeV}.$$

Thus, variations of fundamental constants induce huge shift of Λ.
Without miracle cancellation,
such variations are strongly constrained.

Note also that the estimate given here is not the overall Λ but it variation. Even if we can cancel the overall Λ to fit the observed value, we need other miracles to suppress its variations.

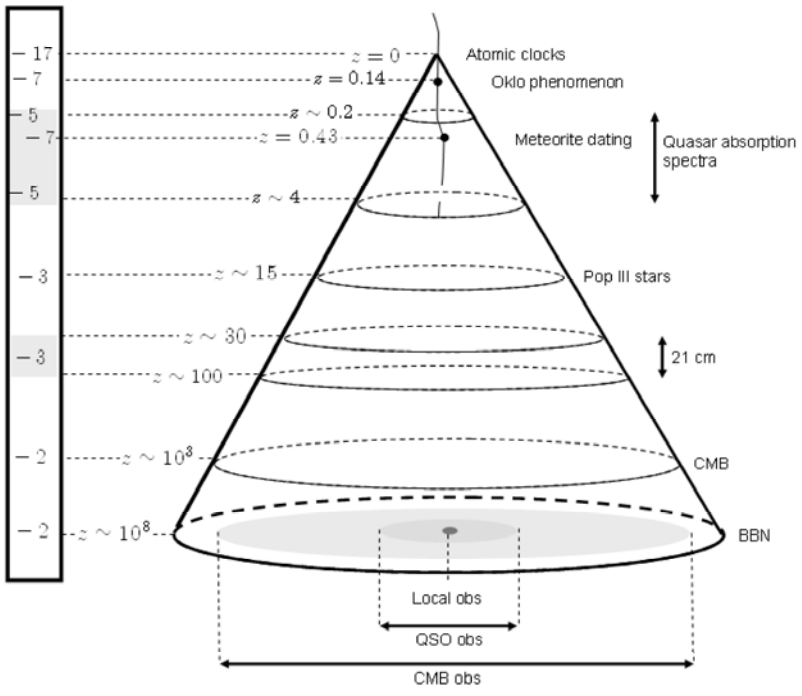
Summary of introduction

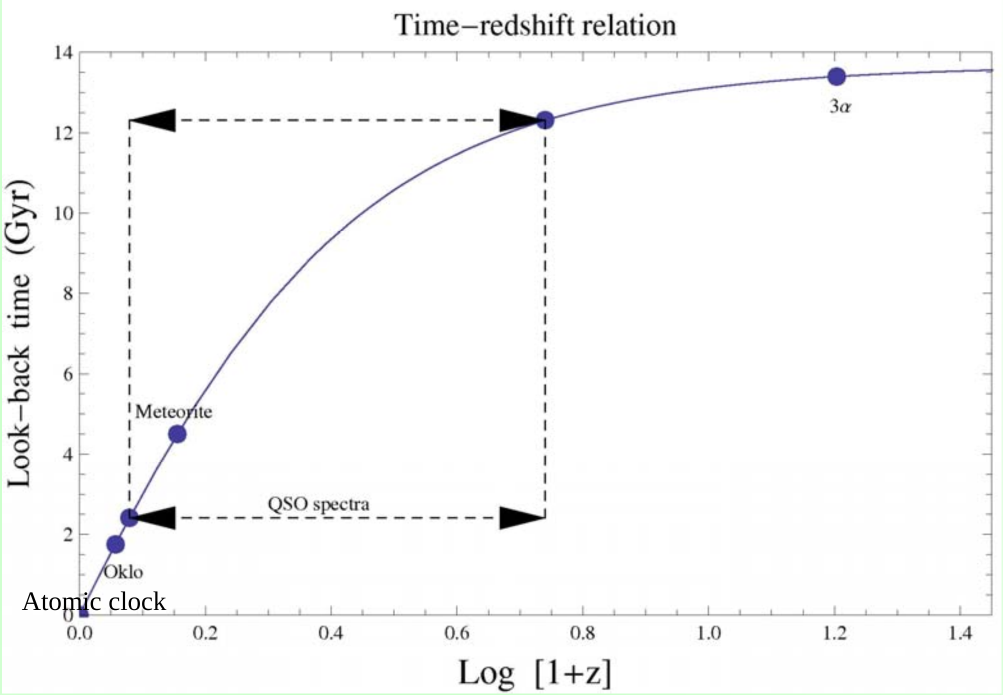
- Constancy or variation of a "constant" is a crucial test to the theory we are considering.

- Variations of fundamental constants have significant implications on equivalence principle and quantum field theory. Such variations are strongly constrained so that it is a quite non-trivial task to explain such variations theoretically even if any.

Experimental and observational constraints on variation of fine structure constant α

Experimental and observational constraints on fundamental constants

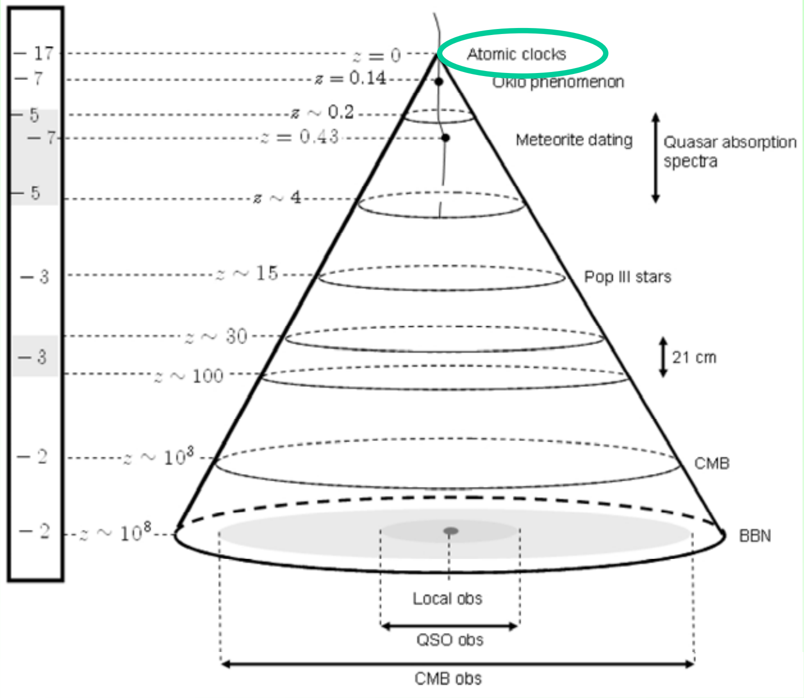


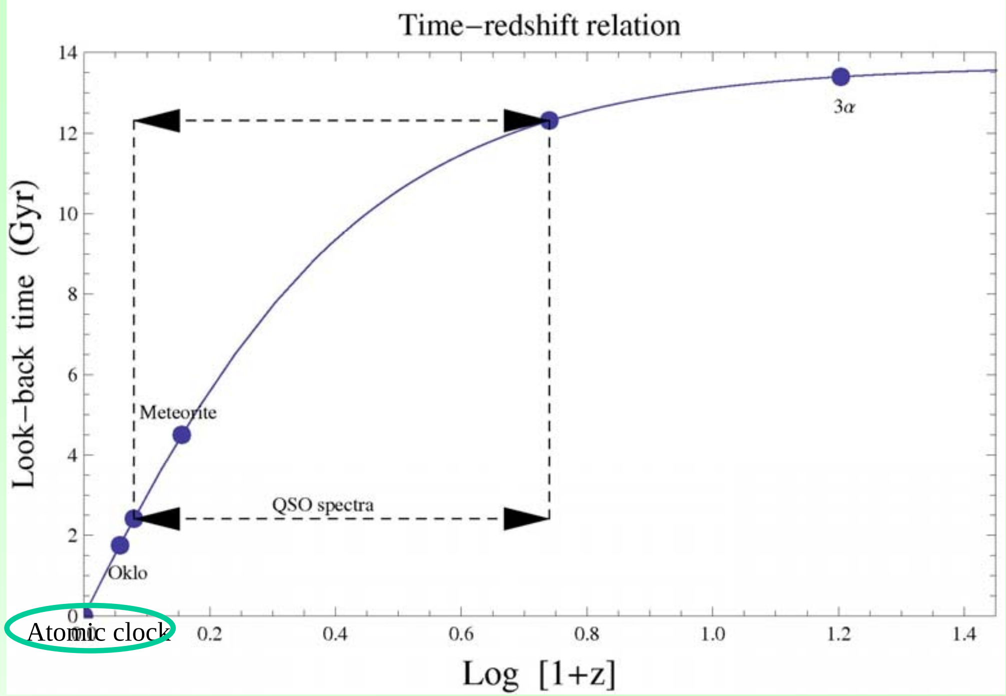


System	Observable	Primary constraints	Other hypothesis
Atomic clock	$\delta \ln \nu$	g_i, α_{EM}, μ	–
Oklo phenomenon	isotopic ratio	$E_r \leftrightarrow \alpha_{EM}$	geophysical model
Meteorite dating	isotopic ratio	$\lambda \leftrightarrow \alpha_{EM}$	–
Quasar spectra	atomic spectra	g_p, μ, α_{EM}	cloud physical properties
Stellar physics	element abundances	B_D	stellar model
21 cm	T_b/T_{CMB}	g_p, μ, α_{EM}	cosmological model
CMB	$\Delta T/T$	μ, α_{EM}	cosmological model
BBN	light element abundances	$Q_{np}, \tau_n, m_e, m_N, \alpha_{EM}, B_D$	cosmological model

(Uzan 2011)

Atomic clock





Atomic spectra (Hydrogen atom)

- **The non-relativistic approximation :**

$$\begin{cases} \psi_{nlm} = R_n(r) Y_{lm}(\theta, \phi), \\ E_n = -\frac{E_I}{n^2}\left(1 + \frac{m_e}{m_p}\right)^{-1}. \end{cases} \qquad \begin{cases} E_I \equiv \frac{1}{2} m_e c^2 \alpha^2 = 13.60580 \text{ eV}, \\ R_\infty \equiv \frac{E_I}{hc} = 1.097373 \times 10^7 \text{ m}^{-1}. \end{cases}$$

$$\Rightarrow \quad \nu = cR_\infty \left(\frac{1}{n'^2} - \frac{1}{n^2}\right).$$

(n, n' : principal quantum number)

- **The relativistic correction (fine-structure) ~ $\alpha^2 E_I$: (2p3/2 - 2p1/2)**

(spin-orbit interaction, $(v/c)^2$ correction, Darwin term)

$$E_{nlJ} = m_e c^2 - \frac{E_I}{n^2} - \frac{m_e c^2}{2n^4}\left(\frac{n}{J + 1/2} - \frac{3}{4}\right)\alpha^4 + \cdots \quad \Rightarrow \quad \nu \propto cR_\infty \alpha^2.$$

(J : total angular momentum)

- **Hyperfine structure ~ $\alpha^2 E_I \mu$: (21 cm)**

(interaction between the spins of the electron, S, and the proton, I)

$$M_S = \frac{e\hbar}{2m_e}\frac{g_e}{2}\frac{\boldsymbol{S}}{\hbar}, \quad M_I = -\frac{e\hbar}{2m_p}\frac{g_p}{2}\frac{\boldsymbol{I}}{\hbar} \quad \Rightarrow \quad \nu \propto cR_\infty \alpha^2 g_p \mu.$$

(g_e ~ 2.002 & g_P ~ 5.585 : gyromagnetic factors, $e = \sqrt{\hbar c \alpha}$.)

Atomic clock

- **Hyperfine frequency of an alkali-like atom :**

$$\nu_{\rm hfs} \simeq cR_\infty \alpha^2 g_i \mu A_{\rm hfs} F_{\rm hfs}(\alpha).$$

gi : gyromagnetic factor depending on an atom
Ahfs : numerical factor depending on an atom
Fhfs : relativistic corrections including many-body effects

- **frequency of an electronic transition :**

$$\nu_{\rm elec} \simeq cR_\infty \alpha^2 F_{\rm elec}(\alpha).$$

Aelec : numerical factor depending on an atom
Felec : relativistic corrections including many-body effects

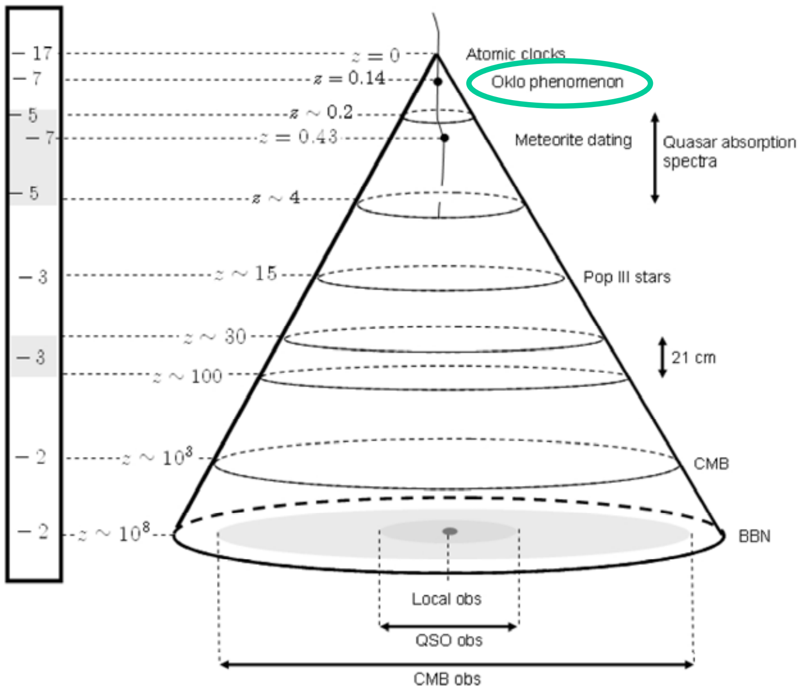
Comparison of different atomic clocks can constrain variation of α

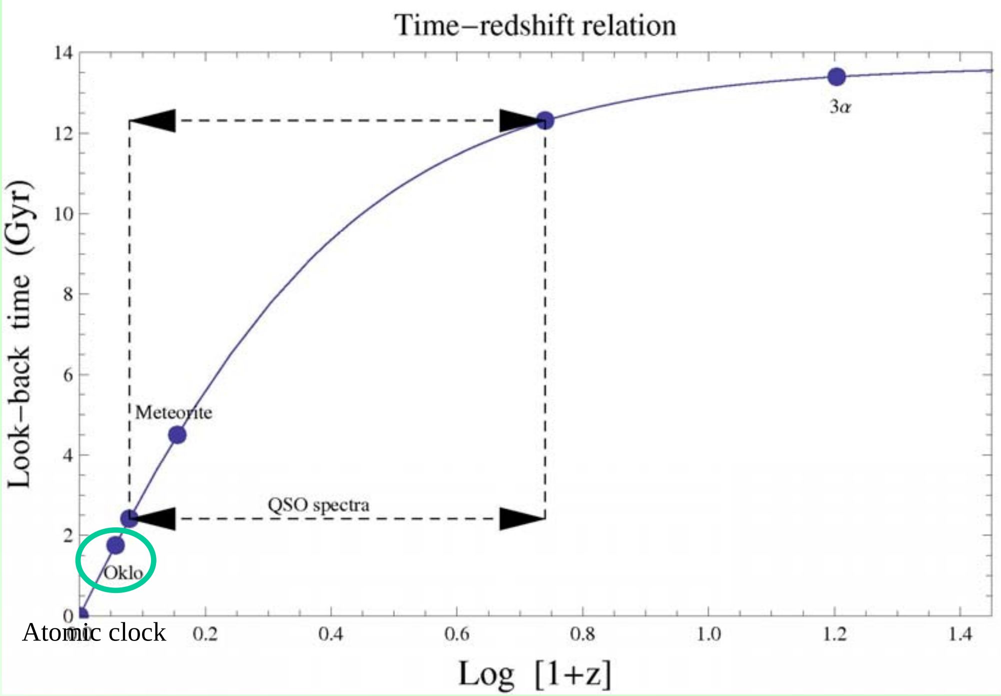
(Uzan 2011)

	Clock 1	Clock 2	Constraint (yr^{-1}) $\frac{d}{dt}\ln\left(\frac{\nu_{\rm clock\,1}}{\nu_{\rm clock\,2}}\right)$	Constants dependence	
(hyperfine)	^{87}Rb (hyperfine)	^{133}Cs	$(0.2 \pm 7.0) \times 10^{-16}$	$\frac{g_{\rm Cs}}{g_{\rm Rb}}\alpha_{\rm EM}^{0.49}$	
	^{87}Rb	^{133}Cs	$(-0.5 \pm 5.3) \times 10^{-16}$		
(1s - 2s)	^{1}H	^{133}Cs	$(-32 \pm 63) \times 10^{-16}$	$g_{\rm Cs}\bar{\mu}\alpha_{\rm EM}^{2.83}$	
(^{2}S1/2 – ^{2}D5/2)	^{199}Hg$^+$	^{133}Cs	$(0.2 \pm 7) \times 10^{-15}$	$g_{\rm Cs}\bar{\mu}\alpha_{\rm EM}^{6.05}$	
	^{199}Hg$^+$	^{133}Cs	$(3.7 \pm 3.9) \times 10^{-16}$		
(^{2}S1/2 – ^{2}D3/2)	^{171}Yb$^+$ (ytterbium)	^{133}Cs	$(-1.2 \pm 4.4) \times 10^{-15}$	$g_{\rm Cs}\bar{\mu}\alpha_{\rm EM}^{1.93}$	
	^{171}Yb$^+$	^{133}Cs	$(-0.78 \pm 1.40) \times 10^{-15}$		
(^{1}S0 – ^{3}P0)	^{87}Sr (strontium)	^{133}Cs	$(-1.0 \pm 1.8) \times 10^{-15}$	$g_{\rm Cs}\bar{\mu}\alpha_{\rm EM}^{2.77}$	
(opposite parities)	^{162}Dy (dysprosium)	^{163}Dy	$(-2.7 \pm 2.6) \times 10^{-15}$	$\alpha_{\rm EM}$	PRL, 98, 040801 (2008)
(^{1}S0 – ^{3}P0)	^{27}Al$^+$	^{199}Hg$^+$	$(-5.3 \pm 7.9) \times 10^{-17}$	$\alpha_{\rm EM}^{-3.208}$	Science, 319, 1808 (2008)

$$\frac{\dot\alpha_{EM}}{\alpha_{EM}} = (-2.7 \pm 2.6) \times 10^{-15}\ {\rm yr}^{-1}, \qquad \frac{\dot\alpha_{EM}}{\alpha_{EM}} = (-1.65 \pm 2.46) \times 10^{-17}\ {\rm yr}^{-1}.$$

Oklo phenomenon





Oklo natural nuclear reactor

- Oklo is the name of a town in the Gabon republic, where an open-pit uranium mine is situated.
- The French CEA (Commission for Atomic Energy) discovered that, about 1.8 x 10^9 years ago (z ~ 0.14), a natural nuclear reactor went critical, consumed a portion of its fuel and then shut a few million years later.
- The isotope ratio of $^{149}Sm/^{147}Sm$ is 0.02 in Oklo rather than 0.9 as in natural samarium due the following reaction: $n + ^{149}_{62}\mathrm{Sm} \rightarrow ^{150}_{62}\mathrm{Sm} + \gamma.$
- Shlyakhter pointed out that the capture cross section of the above reaction is dominated by a resonance of a neutron with the energy (Er^0 ~ 97.3 meV today) and is well described by the Breit-Wigner formula : $\sigma_{n,\gamma}(E) = \frac{g_0\pi}{2}\frac{\hbar^2}{m_n E}\frac{\Gamma_n\Gamma_\gamma}{(E-E_r)^2+\Gamma^2/4}.$
- From an analysis of nuclear and geochemical data, thermally averaged cross section on the neutron flux, σ_{149}, is inferred, which was translated to Er^{Oklo}.
- nuclear mass : m(A,Z) = Zm_p + (A-Z)m_n + E_S + E_{EM}.
  Bethe-Weizacker formula : $E_{\mathrm{EM}} = 98.25\frac{Z(Z-1)}{A^{1/3}}\alpha$ MeV.
- More refined analysis gives $\Delta Er = Er^{Oklo} - Er^0 = -1.1\mathrm{MeV}\ \Delta\alpha/\alpha$.

Constraint on α from Oklo

Table 8: Summary of the analysis of the Oklo data. The principal assumptions to infer the value of the resonance energy E_r are the form of the neutron spectrum and its temperature.

Ore	neutron spectrum	Temperature (°C)	$\hat{\sigma}_{149}$ (kb)	ΔE_r (meV)	Ref.
?	Maxwell	20	55 ± 8	0 ± 20	[466]
RZ2 (15)	Maxwell	$180-700$	75 ± 18	-1.5 ± 10.5	[123]
RZ10	Maxwell	$200-400$	91 ± 6	4 ± 16	[220]
RZ10				-97 ± 8	[220]
–	Maxwell + epithermal	327	91 ± 6	-45^{+7}_{-15}	[306]
RZ2	Maxwell + epithermal		73.2 ± 9.4	-5.5 ± 67.5	[417]
RZ2	Maxwell + epithermal	$200-300$	71.5 ± 10.0	–	[234]
RZ10	Maxwell + epithermal	$200-300$	85.0 ± 6.8	–	[234]
RZ2+RZ10				7.2 ± 18.8	[234]
RZ2+RZ10				90.75 ± 11.15	[234]

Gould et al.,
PRC, 74, 024607 (2006)

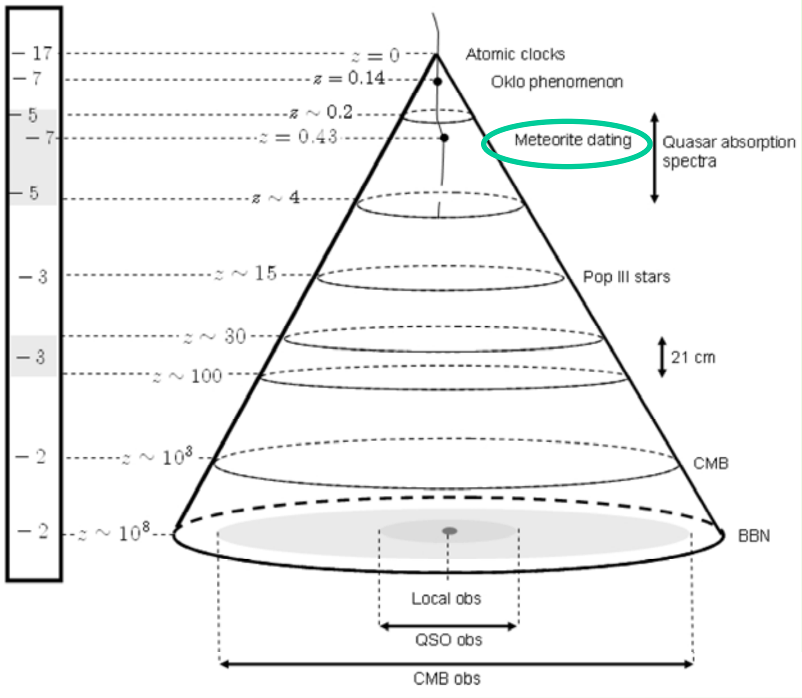
(Uzan 2011)

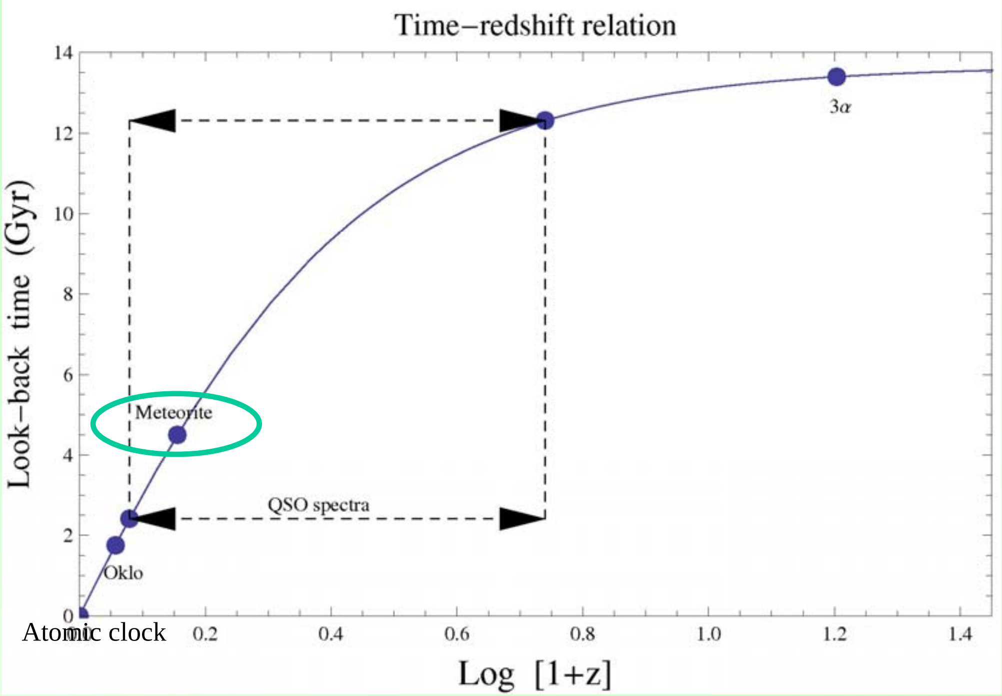
The most recent analysis given by Gould et al. gives

$$\frac{\Delta\alpha}{\alpha} = (-0.65 \pm 1.75) \times 10^{-8}$$ **at 95 C.L.**

⇒ $$\frac{\dot{\alpha}}{\alpha} = (-0.4 \pm 1.0) \times 10^{-17}\ \mathrm{yr}^{-1}$$ **assuming dot{α} is constant.**

Meteorite dating





Meteorite dating

- In meteorites, which were typically formed at the birth of the solar system (4~5) G yrs ago (z~ 0.43), long-lived isotopes can make α-decay or β-decay.
- Assume some meteorites containing an isotope X that decay into Y are formed at a time t_*. Then, the present abundances are given by

  $N_X(t_0) = N_{X*} \exp(-\lambda(t_0-t_*))$, $N_Y(t_0) = N_{X*}[1-\exp(-\lambda(t_0-t_*))]+N_{Y*}$.

  ➔ $N_Y(t_0) = [\exp(\lambda(t_0-t_*)) - 1]\, N_X(t_0) + N_{Y*}$.

  The data should lie on a line called an isochron, the slope of which determines λ(t0-t*) (λ : averaged decay rate).
- If we can have a good estimate t_0-t_* and the dependence of α_{EM} on the decay rate, the constraint on α_{EM} is obtained by comparing the above estimated decay rate with a laboratory measurement value of the decay rate.
- The α decay rate is governed by the penetration of the Coulomb barrier and is described by the Gamow theory : $\lambda \simeq \Lambda(\alpha_{\rm EM}, v) \exp\left(-4\pi Z \alpha_{\rm EM} \frac{c}{v}\right)$.

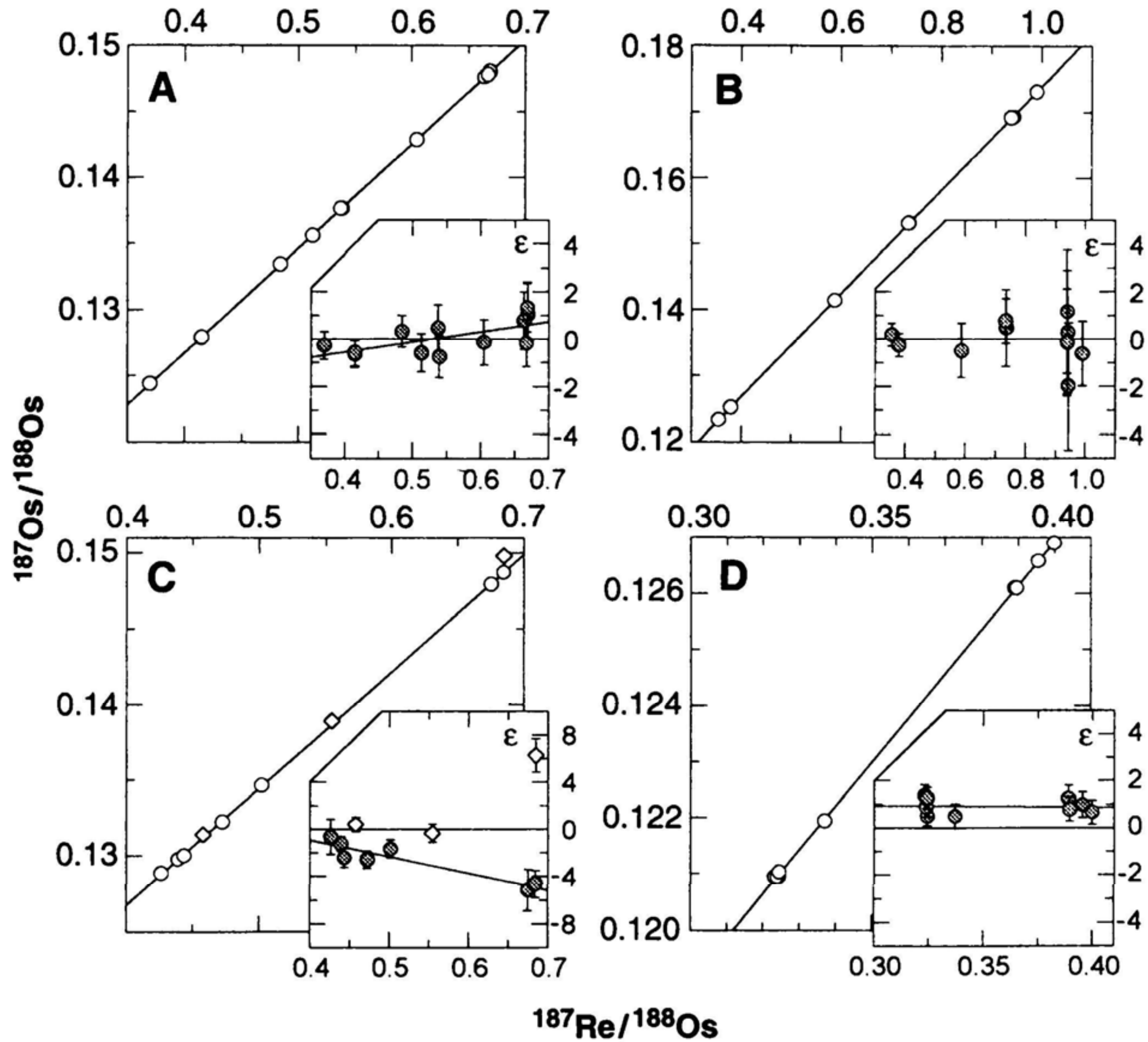
  E : the decay energy, $v/c = \sqrt{E/(2m_p c^2)}$: the escape velocity Λ : the slowly varying function.

  The dependence of α_{EM} on E is roughly given by the Bethe-Weizacker formula.
- The β decay rate with small decay energy E is given by a non-relativistic approximation : $\lambda = \Lambda_\pm E^{p\pm}$, $p_+ = l + 3$, $p_- = 2l + 2$.

Isochron

Smoliar et al., Science 271, 1099 (1996)

Fig. 1. Re-Os isochrons for (**A**) IIIA, (**B**) IIA, (**C**) IVA, and (**D**) IVB iron meteorites. The insets show the deviation in parts per 10,000 of data points from the best fit line in ε units: $\varepsilon = [(^{187}Os/^{188}Os) - S(^{187}Re/^{188}Os - I_O)] \times 10^4$, where S and I_O denote the isochron parameters of slope and initial $^{187}Os/^{188}Os$ ratio, respectively. For all meteorites, ε was calculated relative to the IIA isochron (represented by horizontal lines on the insets). Error bars on the insets account for uncertainties in both $^{187}Os/^{188}Os$ and $^{187}Re/^{188}Os$ ratios. The three IVA irons that were omitted in the isochron regression are shown as open diamonds.



Rhenium and osmium

Constraint on α_{EM} from meteorites

- α -decay : (Wilkinson, 1958)

$$^{238}_{92}\mathrm{U} \rightarrow ^{234}_{90}\mathrm{Th} + ^{4}_{2}\mathrm{He}, \quad (\Delta E \simeq 4.27 \text{ MeV})$$

Dyson (1972) claimed that the decay rate has not Changed by more than 20% during the past 2 Gyr. ⇒ $\left|\frac{\Delta\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}}\right| < 4 \times 10^{-4}.$

$$^{147}_{62}\mathrm{Sm} \rightarrow ^{143}_{60}\mathrm{Nd} + ^{4}_{2}\mathrm{He}, \quad (\Delta E \simeq 2.31 \text{ MeV})$$

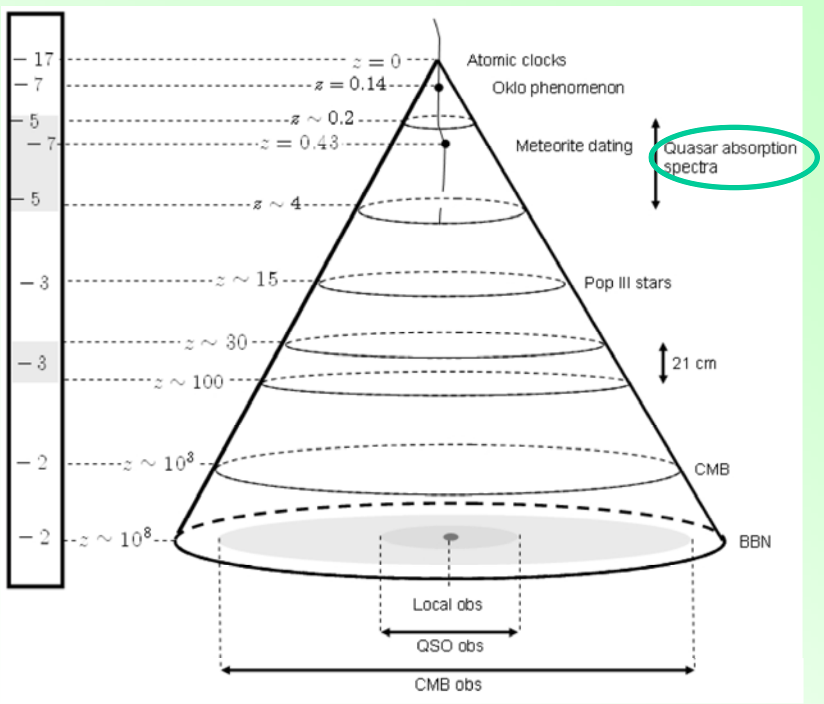
Olive et al. (2002) claimed that the fractional change of the decay rate is less than 7.5 x 10^{-3}. ⇒ $\left|\frac{\Delta\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}}\right| < (0.8 - 5) \times 10^{-7}.$

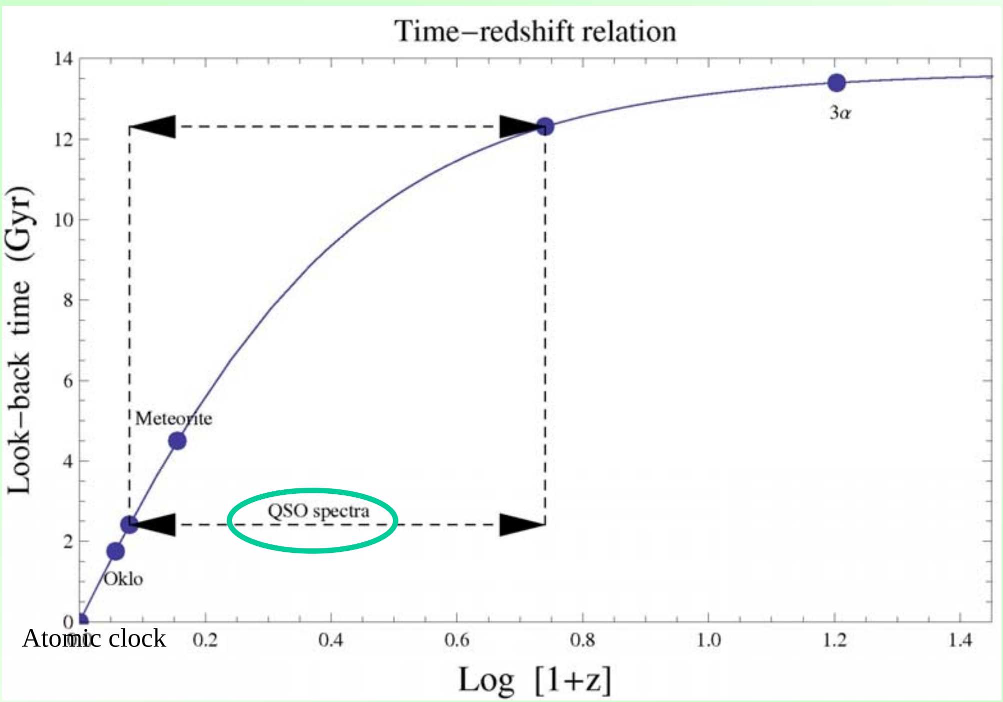
- β -decay : (Dicke, 1959)

$$^{187}_{75}\mathrm{Re} \rightarrow ^{187}_{76}\mathrm{Os} + \overline{\nu}_e + e^-, \quad (\Delta E \simeq 2.5 \text{ keV})$$

Olive et al. (2002) claimed that the fractional change of the decay rate is less than 0.5% in the past 4.6 Gyr. ⇒ $\left|\frac{\Delta\alpha_{\mathrm{EM}}}{\alpha_{\mathrm{EM}}}\right| < 3 \times 10^{-7}.$

Quasar absorption spectra



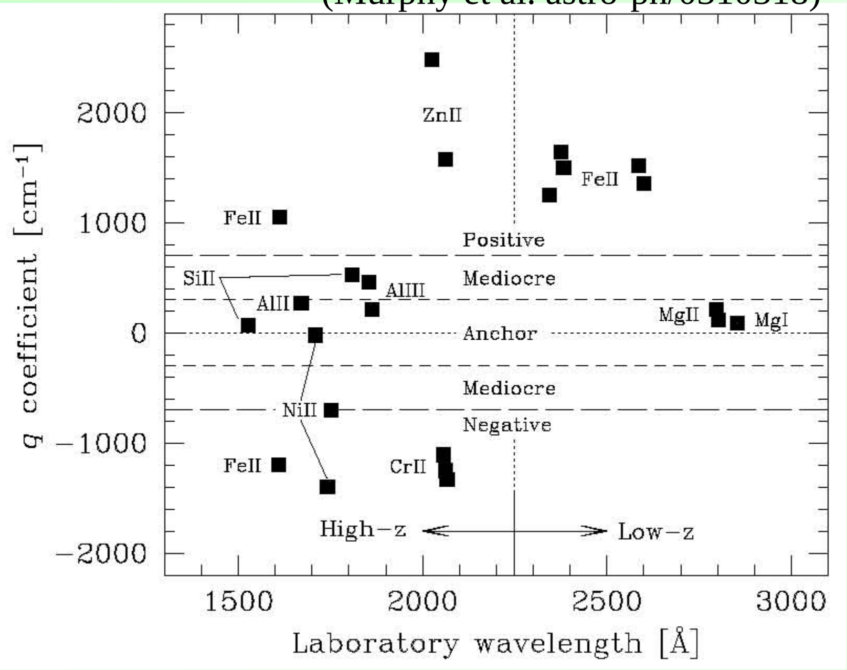


Quasar (QSO) absorption spectra

- Quasar is a very energetic and distant AGN, and is extremely luminous.
- Absorption lines in intervening clouds along the line of sight reflect the spectra of the atoms existing in the clouds.
- If we use a single transition, then we cannot discriminate the redshift effect of the expansion of the Universe from the effect of the variation of α.

  ➔ we need to consider various transitions and understand the dependencies of various transitions on α.

(Murphy et al. astro-ph/0310318)



$$\omega = \omega_0 + q\left[\left(\frac{\alpha}{\alpha^0}\right)^2 - 1\right] + q_2\left[\left(\frac{\alpha}{\alpha^0}\right)^4 - 1\right].$$

ω : the energy in the rest-frame of the cloud
$\omega 0$: the energy measured today in the laboratory
q, q2 : coefficients that determine the dependence

Only q is relevant in many cases.

Three methods

- ## Alkali doublet method (AD) :

This method focuses on the fine structure doublet of alkali-type ion with one outer electron. e.g. $2S_{1/2} \rightarrow 2P_{3/2}$ & $2S_{1/2} \rightarrow 2P_{1/2}$: $\Delta\nu = \frac{\alpha^2 Z^4 R_\infty}{2n^3}$ cm^{-1}.
This method compares transitions with respect to the same ground state.

- ## Many multiplet method (MM) : (Webb et al. 1999)

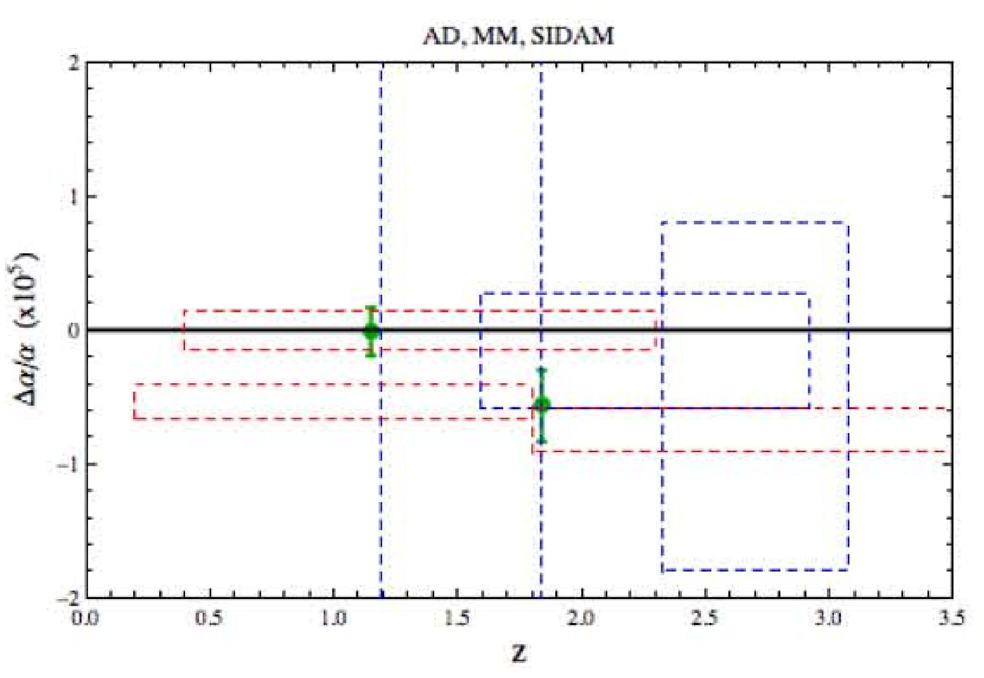
This method relies on the combination of transitions from different species (Mg : anchor) in the same cloud (but maybe in different regions of the cloud), which increases the sensitivity of α.
This method compares transitions relative to different ground states.

- ## Single ion differential measurement (SIDAM) :

(Levshakov et al. 2006)

This method is a variant of MM method to use a single ion (mainly used FeII, which provides transition with positive and negative q) in individual exposure.
This method can avoid spectral shift due to ionization inhomogeneities within absorbers as well as non-zero offsets between different exposures.

Constraint on α from QSO

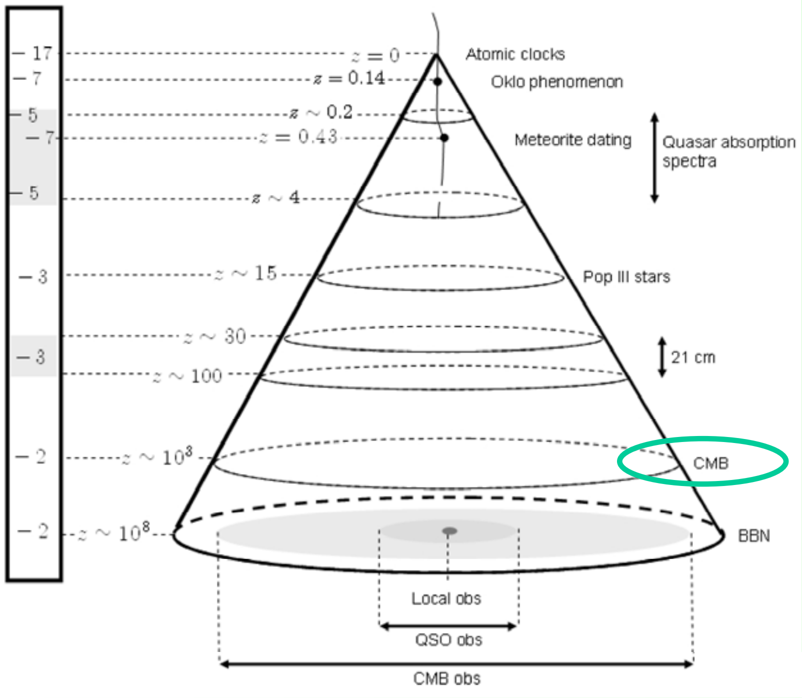


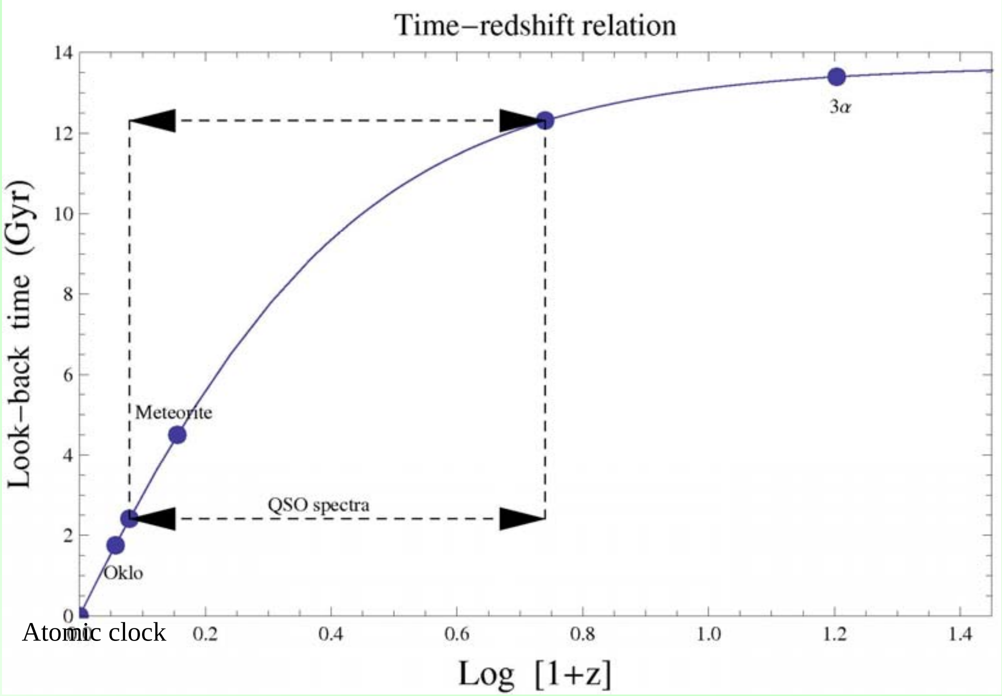
AD : blue
MM : red
SIDAM : green

(Uzan 2011)

Constant	Method	System	Constraint ($\times 10^{-5}$)	Redshift
α_{EM}	AD	21	(-0.5 ± 1.3)	2.33 – 3.08
	AD	15	(-0.15 ± 0.43)	1.59 – 2.92
	AD	9	(-3.09 ± 8.46)	1.19 – 1.84
	MM	143	(-0.57 ± 0.11)	0.2 – 4.2
	MM	21	(0.01 ± 0.15)	0.4 – 2.3
	SIDAM	1	(-0.012 ± 0.179)	1.15
	SIDAM	1	(0.566 ± 0.267)	1.84

Cosmic microwave background (CMB)





Cosmic microwave background (CMB)

- **CMB radiation is composed of photons last scattered around recombination.**
- **The last scattering surface (LSS) is defined by the peak of visibility function, g(z) = exp(- τ (z)) d τ / dz, which measures the differential probability that a photon last scattered at z. Here, τ is the optical depth and** $\dot{\tau} = x_e n_e c \sigma_T$.

  xe : the ionization fraction, $x_e^{\mathrm{EQ}} \propto \left(\frac{m_e}{T}\right)^{\frac{3}{2}} \exp\left(-\frac{\alpha^2 m_e}{2T}\right)$.

  ne : the total number density of electron

  $\sigma_T = \frac{8\pi}{3}\hbar^2 m_e^2 c^2 \alpha^2$: Thomson scattering cross section.

- **Variation of α changes the visibility function through xe & σ_T. In particular, increasing α raises the redshift of the LSS and decrease its thickness because of the exponential dependence on xe.**
- **These effects change the spectrum of CMB anisotropies :**
  (i) The peak position of anisotropies shifts to smaller scale (higher l) due to the increase of the redshift of the LSS.
  (ii) The amplitudes of anisotropies (Cl) are increased due to a smaller Silk damping.

Constraint on α from CMB

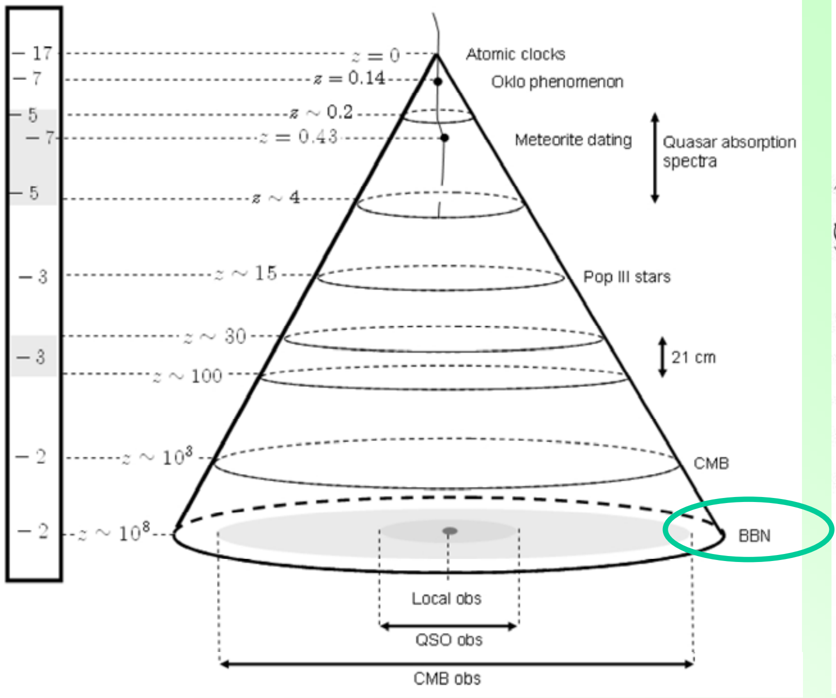
Constraint ($\alpha_{EM} \times 10^2$)	Data	Comment
[–9, 2]	BOOMERanG-DASI-COBE + BBN	BBN with α_{EM} only
		$(\Omega_{mat}, \Omega_b, h, n_s)$
[–1.4, 2]	COBE-BOOMERanG-MAXIMA	$(\Omega_{mat}, \Omega_b, h, n_s)$
[–5, 2]	WMAP-1	$(\Omega_{mat}h^2, \Omega_b h^2, \Omega_\Lambda h^2, \tau, n_s, \alpha_s)$
[–6, 1]	WMAP-1	same + $\alpha_s = 0$
[–9.7, 3.4]	WMAP-1	$(\Omega_{mat}, \Omega_b, h, n_s, \tau, m_e)$
[–4.2, 2.6]	WMAP-1 + HST	same
[–3.9, 1.0]	WMAP-3 (TT,TE,EE) + HST	$(\Omega_{mat}, \Omega_b, h, n_s, z_{re}, A_s)$
[–1.2, 1.8]	WMAP-5 + ACBAR + CBI + 2df	$(\Omega_{mat}h^2, \Omega_b h^2, \Theta, \tau, n_s, A_s, m_e)$
[–1.9, 1.7]	WMAP-5 + ACBAR + CBI + 2df	$(\Omega_{mat}h^2, \Omega_b h^2, \Theta, \tau, n_s, A_s, m_e)$
[–5.0, 4.2]	WMAP-5 + HST	$(\Omega_{mat}h^2, \Omega_b h^2, h, \tau, n_s, A_s)$
[–4.3, 3.8]	WMAP-5 + ACBAR + QUAD + BICEP	$(\Omega_{mat}h^2, \Omega_b h^2, h, \tau, n_s)$
[–1.3, 1.5]	WMAP-5 + ACBAR + QUAD + BICEP+HST	$(\Omega_{mat}h^2, \Omega_b h^2, h, \tau, n_s)$
[–0.83, 0.18]	WMAP-5 (TT,TE,EE)	$(\Omega_{mat}h^2, \Omega_b h^2, h, \tau, n_s, A_s, m_e, \mu)$
[–2.5, –0.3]	WMAP-7 + H_0 + SDSS	$(\Omega_{mat}h^2, \Omega_b h^2, \Theta, \tau, n_s, A_s, m_e$

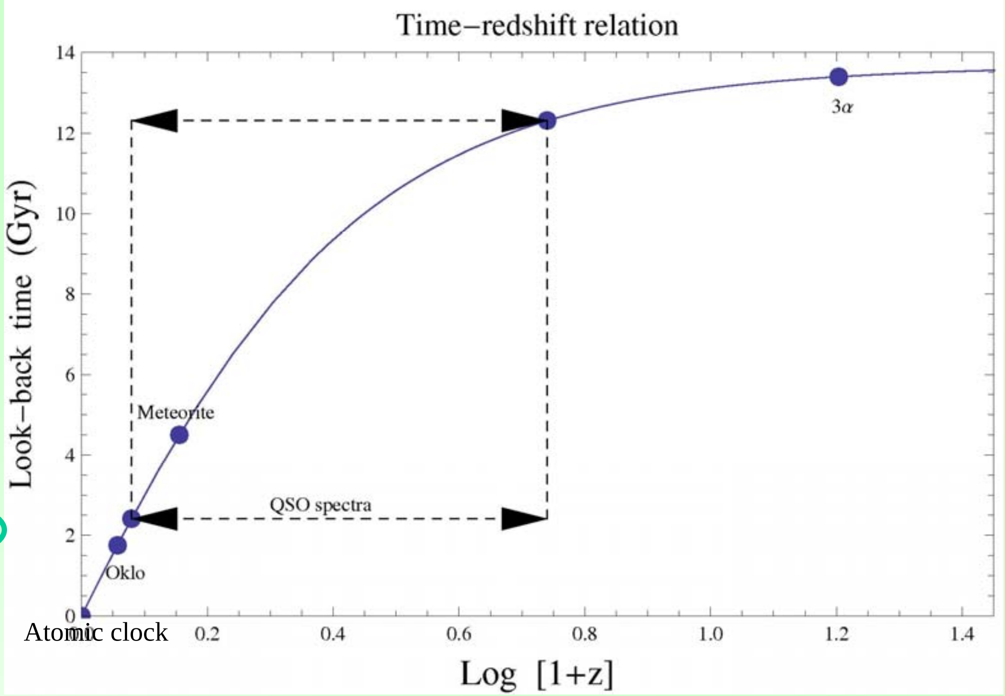
(Uzan 2011)

The latest WMAP 7 year data including polarization and SDSS data yield -0.025 < $\Delta\alpha/\alpha$ < -0.003 at 1 sigma level.

(Landau and Scoccola, Astron. Astrophys., 517, A62, (2010).)

Big Bang Nucleosynthesis (BBN)





Big Bang Nucleosynthesis (BBN)

- Light elements were synthesized in the early Universe.
  (Heavy elements are synthesized in the star and SN)
- Almost all neutrons are incorporated into ^{4}He.

How to estimate the abundance of ^{4}He ?

(i) β equilibrium between p & n :

$$n \leftrightarrow p + e^- + \overline{\nu}_e, \quad n + \nu_e \leftrightarrow p + e^-, \quad n + e^+ \leftrightarrow p + \overline{\nu}_e.$$

$$(n/p) = \exp\left(-Q_{np}/T\right), \quad Q_{np} \equiv (m_n - m_p)c^2 = 1.29 \text{ MeV}.$$

(ii) β equilibrium freezes out when the expansion rate dominates.

$$\Gamma_\beta \simeq G_F^2 T^5 = H \simeq T^2/M_G^2 \implies T_f \simeq 1/\left(G_F^2 M_G\right)^{1/3} \simeq 1 \text{ MeV} \implies (n/p) \simeq 1/6.$$

(iii) Nucleosynthesis starts from D : p+n ➔ D+ γ, B_D =2.22 MeV.

$$(D/n) \simeq 7.2\eta\,(T/m_n)^{3/2} \exp\left(B_D/T\right) \implies T_D \sim 0.07 \text{ Mev} \implies (n/p) \simeq 1/7.$$

$$\left(\ \eta = n_b/n_\gamma \simeq 6 \times 10^{-10}\right)$$

(Decay of n with τ n = 890 s)

$$Y \equiv \frac{\text{total mass of } {}^4\text{He}}{\text{total masses of p \& n}} = \frac{\frac{n_n}{2} \times 4m_p}{n_p m_p + n_n m_n} \simeq \frac{2\frac{n_n}{n_p}}{1 + \frac{n_n}{n_p}} \simeq 0.25.$$

Constraint on α from BBN

The abundance of 4He is the most sensitive to the change of Qnp.

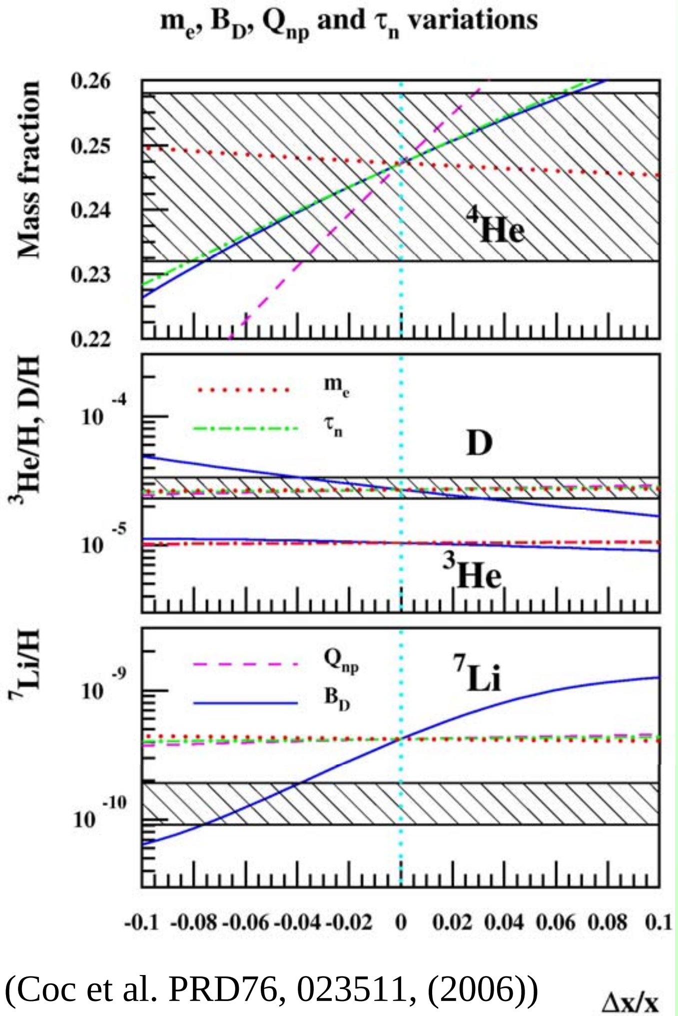
$$Q_{np} = a\alpha_{EM}\Lambda_{QCD} + (m_d - m_u), \quad (m_d - m_u) = 2.05 \text{ MeV}.$$

$$Q_{np} \simeq (1.29 - 0.76\Delta\alpha_{EM}/\alpha_{EM}) \text{ MeV}.$$

$$\frac{\Delta Y}{Y} \simeq -\frac{\Delta Q_{np}}{Q_{np}} \simeq 0.6\frac{\Delta\alpha_{EM}}{\alpha_{EM}}.$$

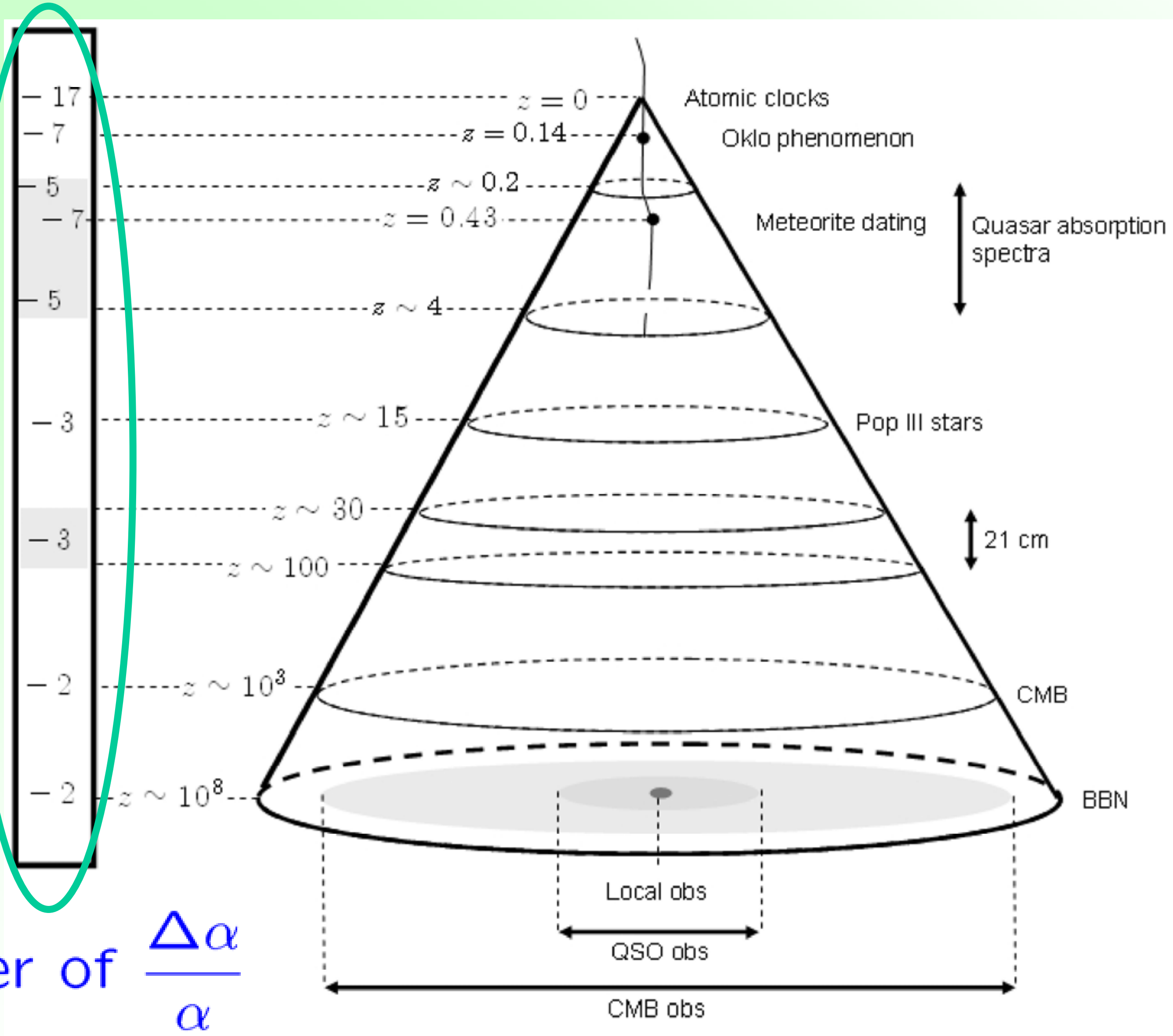
From ^{4}He data, $-4\times10^{-2} \lesssim \frac{\Delta Q_{np}}{Q_{np}} \lesssim 2.7\times10^{-2},$

$$-4.5\times10^{-2} \lesssim \frac{\Delta\alpha_{EM}}{\alpha_{EM}} \lesssim 6.7\times10^{-2}.$$



(Coc et al. PRD76, 023511, (2006))

Summary of constraint on α



Order of $\frac{\Delta\alpha}{\alpha}$

Spatial and temporal variation of fine structure constant α

(Chiba & MY, JCAP, 03, 044, 2011)

Observations of temporal variation of the fine structure constant

A nonvanishing temporal variation of the fine structure constant α was reported.

$$\frac{\Delta\alpha}{\alpha} = (-0.543 \pm 0.116) \times 10^{-5} \quad \text{for} \quad 0.2 < z < 3.7$$

⇨ **4.7 σ evidence for a varying α** (Murphy et al. 2003)

For Keck/HIRES (High Resolution Echelle Spectrometer) 143 absorption systems with 0.2 < z < 3.7, they compare the absorption wavelengths of magnesium and iron atoms in the same absorbing cloud.

Observations of spatial variation of the fine structure constant

(Webb et al. 2010)

For 2004 Keck sample,

$$\left.\frac{\Delta\alpha}{\alpha}\right|_{z<1.8} = (-0.54 \pm 0.12) \times 10^{-5}, \quad \left.\frac{\Delta\alpha}{\alpha}\right|_{z>1.8} = (-0.74 \pm 0.17) \times 10^{-5}.$$

α was smaller in the past !!

For 2010 VLT (the ESO Very Large Telescope) sample,

$$\left.\frac{\Delta\alpha}{\alpha}\right|_{z<1.8} = (-0.06 \pm 0.16) \times 10^{-5}, \quad \left.\frac{\Delta\alpha}{\alpha}\right|_{z>1.8} = (+0.64 \pm 0.20) \times 10^{-5}.$$

α was larger in the past !!

→ **They claimed the spatial variation of α.**

Observations of spatial variation of the fine structure constant II

Keck (Mauna Kea, Hawaii), VLT(Paranal, Chile) (Webb et al. 1008.3907)

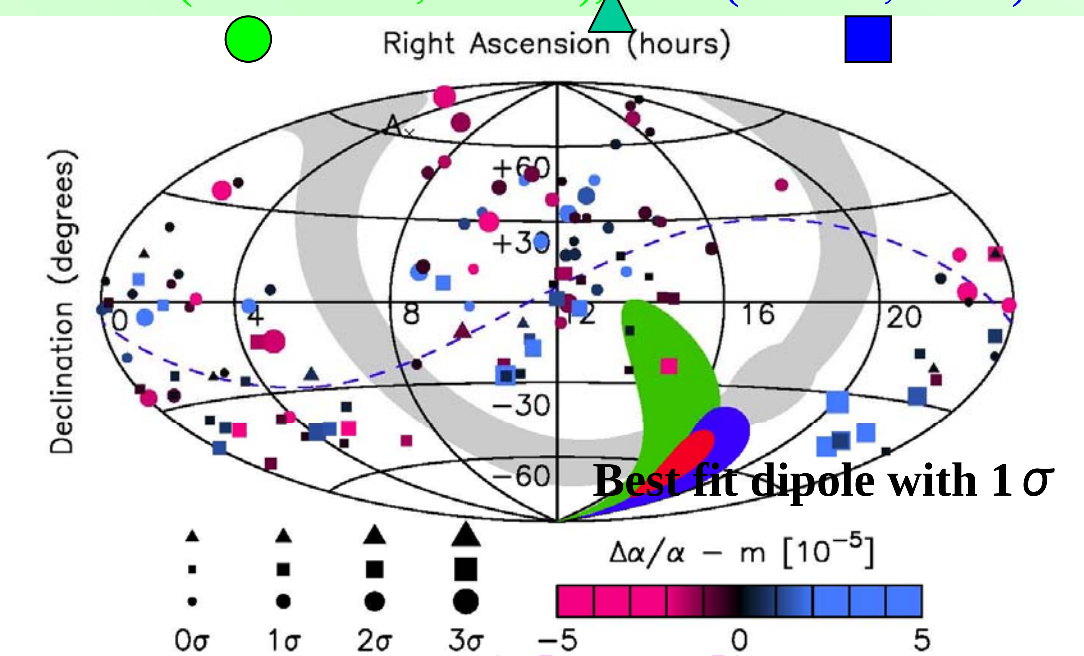


FIG. 5. Supplementary figure. All-sky illustration of the combined Keck and VLT $\Delta\alpha/\alpha$ measurements. Squares are VLT points. Circles are Keck points. Triangles are quasars observed at both Keck and VLT. Symbol size indicates deviation of $\Delta\alpha/\alpha$ from the monopole value m in $\Delta\alpha/\alpha = A\cos\Theta + m$ (Figures 2 and 3). The grey shaded area represents the Galactic plane with the Galactic centre indicated as a bulge. The blue dashed line shows the equatorial region of the α-dipole. More and larger blue squares are seen south of the equatorial region and more and larger red circles are seen north of it.

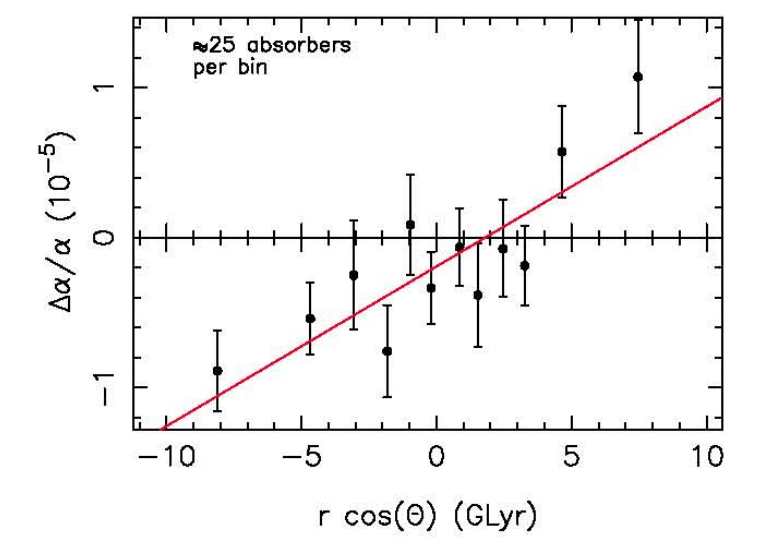


FIG. 3. $\Delta\alpha/\alpha$ vs $Br\cos\Theta$ for the model $\Delta\alpha/\alpha = Br\cos\Theta + m$ showing the gradient in α along the best-fit dipole. The best-fit direction is at right ascension 17.4±0.6 hours, declination −62±6 degrees, for which $B = (1.1\pm0.2)\times10^{-6}$ GLyr^{-1} and $m = (-1.9\pm0.8)\times10^{-6}$. This dipole+monopole model is statistically preferred over a monopole-only model also at the 4.1σ level. A cosmology with parameters $(H_0, \Omega_M, \Omega_\Lambda) = (70.5, 0.2736, 0.726)$ was used [14].

$$\frac{\Delta\alpha}{\alpha} = (1.1 \pm 0.2)\times 10^{-6}\left(\frac{r}{\mathrm{GLyr}}\right)\cos\theta + (-1.9\pm0.8)\times10^{-6}.$$

Variation of fundamental "constants" from the theoretical view point

- **Superstring theory** predicts the existence of a scalar partner, called **dilaton,** of the tensor Graviton.

[• But its **direct coupling to matter induces the violation of the (weak) equivalence principle.**]

⇨ **Temporal motion** of dilaton induces **time variation** of fundamental constants.

- **During inflation, all light scalars acquire quantum fluctuations.**

⇨ **Such fluctuations** of dilaton induce **spatial variation** of fundamental constants.

Difficulties in explaining both temporal and spatial variation of α

$\frac{\Delta\alpha}{\alpha}$: temporal variation across the Hubble time

~ **same order**

$\frac{\Delta\alpha}{\alpha}$: spatial variation across the horizon scale

A light scalar field ϕ is (slowly) evolving in cosmological time scale.

→ **Quantum fluctuations ~ H are obtained during inflation.**

different order

$\Delta\phi$ is not far from the Planck scale. $\neq$ **$\Delta\phi$ ~ H << the Planck scale (Tensor-to-scalar ratio r < 0.24)**

Homogeneous evolution and quantum fluctuation of a light scalar cannot explain both temporal and spatial variation.

→ **Spatial variation can be associated with domain walls.**

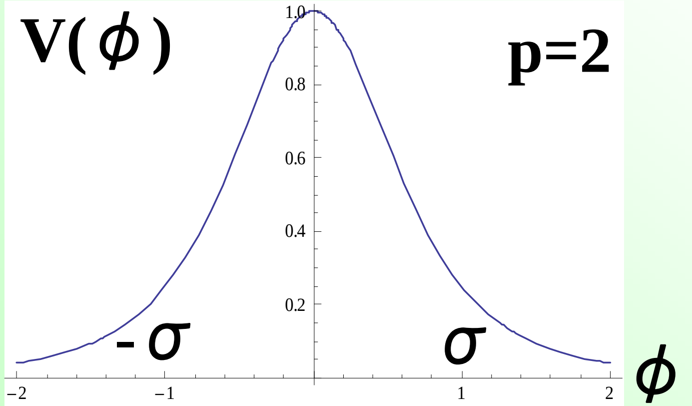
(Olive et al. 2010)

Runaway domain wall

$$S=\int d^4x\sqrt{-g}\left[\frac{{M_G}^2}{2}R-\frac{1}{2}(\nabla\phi)^2-V(\phi)-\frac{1}{16\pi\alpha_0}B(\phi)F_{\mu\nu}F^{\mu\nu}\right]+S_m.$$

α_0 : the bare fine structure constant

$$V(\phi)=\frac{M^{2p+4}}{\left(\phi^2+\sigma^2\right)^p}$$ **: runaway potential**



Runaway domain walls are formed.

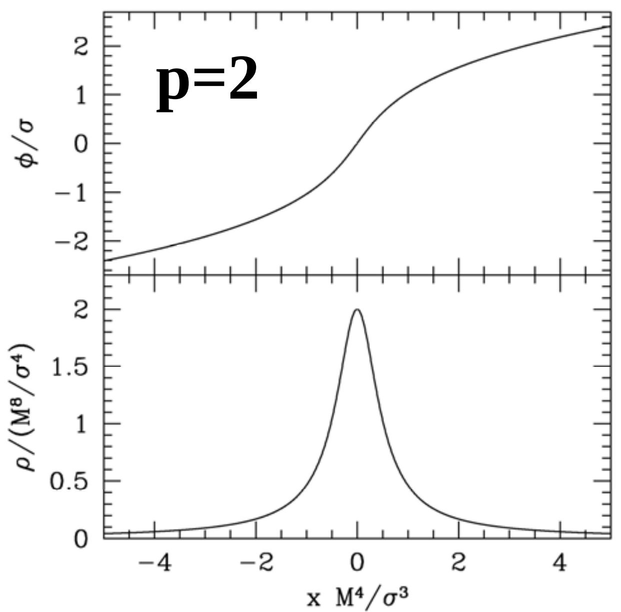
(Cho & Vilenkin 1999)

$$\delta\simeq\sigma^{p+1}/M^{p+2}$$ **: width of wall**

$$\mu\simeq\frac{M^{2P+4}}{\sigma^{2p}}\delta\simeq\frac{M^{p+2}}{\sigma^{p-1}}$$ **: tension of wall**

x ≫ δ → $$\phi\simeq(M^{p+2}x)^{1/(p+1)}.$$

x = H^{-1} → $$\phi\propto a^{3/2(p+1)}\ \text{for MD}$$



Variation of α

$$S=\int d^4x\sqrt{-g}\left[\frac{M_G^2}{2}R-\frac{1}{2}(\nabla\phi)^2-V(\phi)-\frac{1}{16\pi\alpha_0}B(\phi)F_{\mu\nu}F^{\mu\nu}\right]+S_m.$$

$$\begin{cases}\alpha(\phi)=\alpha_0/B(\phi), \\ B(\phi)=e^{-\xi\phi/M_G}\end{cases}$$

α_0 **: the bare fine structure constant**

: dilaton-like coupling function

small ξ ⇒

$$\alpha(\phi)\simeq\alpha_0\left(1+\xi\frac{\phi}{M_G}\right).$$

- **Time variation in either side of wall**
- **Spatial variation across wall**

⇒

$$\frac{\dot{\alpha}}{\alpha_0}=\xi\frac{\dot{\phi}}{M_G}=\pm\frac{3}{2(p+1)}\xi\frac{|\phi|}{M_G}H,$$

$$\frac{\Delta\alpha}{\alpha_0}=\xi\frac{\Delta\phi}{M_G}=2\xi\frac{\phi}{M_G}.$$

The opposite time variation of α between the Keck (dot α >0) and the VLT (dot α <0) as well as the spatial variation comparable to the time variation ($\Delta\alpha/\alpha$ ~|dot α|/αH^{-1}) is explained.

Variation of α II

$$\alpha(\phi) \simeq \alpha_0 \left(1 + \xi \frac{\phi}{M_G}\right).$$

Fix p=2 for definiteness

- **Time variation in either side of wall**

$$\frac{\dot{\alpha}}{\alpha_0} = \xi \frac{\dot{\phi}}{M_G} = \pm \frac{\xi}{2} \frac{|\phi|}{M_G} H,$$

- **Spatial variation across wall**

$$\frac{\Delta\alpha}{\alpha_0} = \xi \frac{\Delta\phi}{M_G} = 2\xi \frac{\phi}{M_G}.$$

Time variation can be fitted by $(\alpha(a) - \alpha_0)/\alpha_0 \simeq -6.2 \times 10^{-6}(1 - a^{1/2}).$

$$\Rightarrow \quad \frac{\dot{\alpha}}{\alpha_0} \simeq 3.1 \times 10^{-6} a^{1/2} H.$$

$$\Rightarrow \quad \xi \frac{\phi}{M_G} \simeq 3.5 \times 10^{-6}.$$

(z ~ 2)

$$\Rightarrow \quad \frac{\Delta\alpha}{\alpha_0} \simeq 7 \times 10^{-6}$$

: spatial variation

Our model explains naturally the largeness of the spatial variation.

Experimental constraints

After relaxation period, domain walls evolve according to the scaling solution, in which their typical scale is comparable to the Hubble scale, $\rho_{\rm wall} \sim \mu H \propto \frac{1}{t}.$

● Overdomination of walls :

$$\rho_{\rm wall} \simeq \mu H_0^{-2}/(4\pi H_0^{-3}/3) \ll \rho_0 = 3M_G^2 H_0^2. \quad \Rightarrow \quad M < 5.7 \times 10^2 {\rm GeV} \left(\frac{\sigma}{10^{15}{\rm GeV}}\right)^{1/4}.$$

● Sachs-Wolfe effect :

Domain walls induce the temperature anisotropy by SW effects.

$$\Phi \sim 2\pi G\mu H_0^{-1} \simeq (1/4) M_G^{-2} \mu H_0^{-1} \ll 10^{-5}. \quad \Rightarrow \quad M < 30 {\rm GeV} \left(\frac{\sigma}{10^{15}{\rm GeV}}\right)^{1/4}.$$

● Violation of the weak equivalence principle :

The light field ϕ mediates a (composite dependent) long-range force via the coupling to nucleons, leading to violation of the WEP.

$$\left\{ \begin{array}{l} \delta m_p = B_p \delta\alpha/\alpha_0 = 0.63 {\rm MeV} \delta\alpha/\alpha_0, \\ \delta m_n = B_n \delta\alpha/\alpha_0 = -0.13 {\rm MeV} \delta\alpha/\alpha_0. \end{array} \right. \quad \Rightarrow \quad \xi < 2.6 \times 10^{-3}.$$

(Eotvos-Dicke-Braginsky type experiments)

Violation of weak equivalence principle

Dilaton ϕ directly couples to n & p through electromagnetic corrections to nucleon masses.

$$\begin{cases} \delta m_p = B_p \dfrac{\Delta\alpha}{\alpha}, \quad B_p = 0.63 \text{ MeV} \quad : \text{ Born mass for proton,} \\ \delta m_n = B_n \dfrac{\Delta\alpha}{\alpha}, \quad B_n = -0.13 \text{ MeV} \quad : \text{ Born mass for neutron.} \end{cases}$$

(Gasser & Leutwyler, PRT, 87, 77 1982)

Since the mass of ϕ is very light, its exchange mediates a long-range force.

Such a force is isotope dependent due to the different dependence of ϕ on their masses and leads to the violation of (weak) equivalence of principle.

Φ exchange force

- **The nucleon-ϕ coupling induces the effective Yukawa coupling :**

$$\mathcal{L}_{\text{int}} = m_i(\phi)\overline{\psi}_i\psi_i.$$

$$\begin{cases} m_i(\phi) = \overline{m}_i + \delta m_i(\phi), \\ \delta m_i(\phi) = B_i \dfrac{\Delta\alpha}{\alpha} \equiv g_i \Delta\phi, \quad g_i = \xi \dfrac{B_i}{M_G}. \end{cases}$$

- **Yukawa potential :**

$$V(r) = -\sum_i \sum_j \frac{g_i g_j}{4\pi} \frac{e^{-m_\phi r}}{r} N_i^E N_j \simeq -\sum_i \sum_j \frac{g_i g_j}{4\pi r} N_i^E N_j \quad \text{for} \quad r \gg m_\phi^{-1}.$$

N_i^E(N_j) : the numbers of the nucleons in the earth (test body)
r : the distance between the Earth and the test body

- **The acceleration ($a=a_\phi+a_g$) of the test body with the mass m :**

$$\begin{cases} a_\phi = \dfrac{1}{m}\dfrac{dV(r)}{dr} = \dfrac{1}{4\pi m}\displaystyle\sum_i \sum_j \frac{g_i g_j}{r^2} N_i^E N_j = \frac{\xi^2}{4\pi M_G^2 m r^2}\left(N_p^E B_p + N_n^E B_n\right)\left(N_p B_p + N_n B_n\right), \\ a_g = G\dfrac{M_E}{r^2} \quad : \quad \text{Newton acceleration.} \end{cases}$$

Eotvos ratio

- **For test bodies 1 & 2 with almost equal mass $m_1 \sim m_2$,**

$$\eta_{12} := 2\left|\frac{a_1 - a_2}{a_1 + a_2}\right|.$$

- **For $a_\phi \ll a_g$,**

$(Np^1\text{-}Np^2)$ **$(Nn^1\text{-}Nn^2)$**

$$\begin{cases} a_1 + a_2 \simeq a_g^1 + a_g^2 = 2a_g, \\ a_1 - a_2 = a_\phi^1 - a_\phi^2 = \dfrac{\xi^2}{4\pi M_G^2 m r^2}\left(N_p^E B_p + N_n^E B_n\right)\left(\Delta N_p B_P + \Delta N_n B_n\right). \end{cases}$$

$$\begin{aligned} \eta_{12} &= \frac{2\xi^2}{M_E m}\left(N_p^E B_p + N_n^E B_n\right)\left(\Delta N_p B_P + \Delta N_n B_n\right) \\ &\quad \left(M_E = \left(N_p^E + N_n^E\right) m_{nuc}, \quad m = (N_p + N_n)\, m_{nuc}\right) \\ &= \frac{2\xi^2}{m_{nuc}^2}\left(R_p^E B_p + R_n^E B_n\right)\left(\Delta R_p B_P + \Delta R_n B_n\right) \\ &\quad \left(R_{p(n)}^E \equiv \frac{N_{p(n)}^E}{N_p^E + N_n^E} \simeq \frac{1}{2}, \quad \Delta R_{p(n)} \equiv \frac{\Delta N_{p(n)}}{N_p + N_n} \simeq (0.06 - 0.1)\right) \\ &\simeq (0.06 - 0.1)\,\xi^2 \left(\frac{B_p + B_n}{m_{nuc}}\right)^2 \simeq 3 \times 10^{-14}\left(\frac{\xi}{10^{-3}}\right)^2. \end{aligned}$$

$$\eta < 2 \times 10^{-13}$$

$$\xi < 2.6 \times 10^{-3}.$$

Test of the universality of free fall (WEP)

The universality of free fall (WEP) can be tested by comparing the accelerations of two test bodies in an external gravitational field.

$$\eta_{12} := 2\left|\frac{a_1 - a_2}{a_1 + a_2}\right|.$$

Table 3: Summary of the constraints on the violation of the universality of free fall.

Constraint	Body 1	Body 2	Ref.
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[4]
$(0.1 \pm 2.7 \pm 1.7) \times 10^{-13}$	Earth-like rock	Moon-like rock	[23]
$(-1.0 \pm 1.4) \times 10^{-13}$	Earth	Moon	[543]
$(0.3 \pm 1.8) \times 10^{-13}$	Te	Bi	[451]
$(-0.2 \pm 2.8) \times 10^{-12}$	Be	Al	[482]
$(-1.9 \pm 2.5) \times 10^{-12}$	Be	Cu	[482]
$(5.1 \pm 6.7) \times 10^{-12}$	Si/Al	Cu	[482]

(Uzan 2011)

Universality of free fall is confirmed with very good accuracy.

Constraints

- **The present value of ϕ :**

$$\phi \simeq (M^3 H_0^{-1})^{1/3} > \sigma. \quad \Rightarrow \quad 6\text{GeV}\left(\frac{\sigma}{10^{15}\text{GeV}}\right)^{3/4} < M.$$

Combined with Sachs-Wolfe constraint : $M < 30\text{GeV}\left(\frac{\sigma}{10^{15}\text{GeV}}\right)^{1/4}.$

$$\Rightarrow \quad \sigma < 2 \times 10^{16}\text{GeV} \quad \text{and} \quad M < 70\text{GeV}.$$

- **On the other hand, the present value of ϕ :**

$$\xi \frac{\phi}{M_G} \simeq 3.5 \times 10^{-6} \quad \Rightarrow \quad M \simeq 30\text{GeV}\left(\frac{\xi}{10^{-3}}\right)^{-3/4}.$$

$$\Rightarrow \quad 3 \times 10^{-4} < \xi < 2.6 \times 10^{-3}$$

(present constraint from violation of WEP)

$$\Leftrightarrow \quad 3 \times 10^{-15} < \eta_{12} < 2 \times 10^{-13}$$

Order of magnitude improvements may detect violation of WEP.

Allowed parameter region

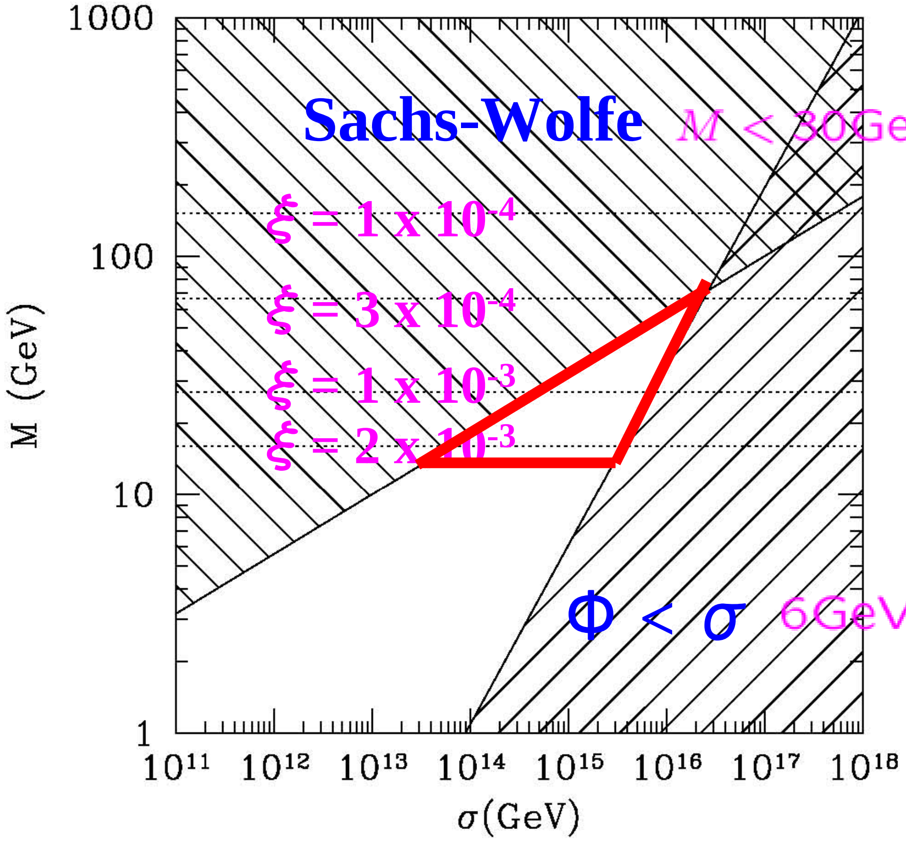


Figure 2. Allowed parameter space. Upper region is excluded due to the Sachs-Wolfe effect (or large density parameter) eq. (3.1); lower region is excluded because of $\phi < \sigma$ and the absence of the scaling solution, eq. (3.10). Dotted lines explain the QSO data (eq. (3.9)) with $\xi = 2\times10^{-3}, 10^{-3}, 3\times10^{-4}, 10^{-4}$ from bottom to top.

$$V(\phi) = \frac{M^{2p+4}}{\left(\phi^2+\sigma^2\right)^p}$$

$$B(\phi) = e^{-\xi\phi/M_G}$$

$$3\times10^{-4} < \xi < 2.6\times10^{-3}$$

Summary

- Constancy or variation of fundamental constant is a crucial test to the standard model of particle physics
- If any, such a variation has significant implications on cosmological constant and equivalence principle.
- Recent observations claim temporal and spatial variation of fine structure constant α though additional and independent observations are necessary.
- It is not so easy to explain such variations simultaneously.