

山崎士人 3d N=2 SUSY gauge theory \leftrightarrow 3d $SL(2, \mathbb{R})$ Chern-Simons

3-mfd (eg hyperbolic mfd) geometry

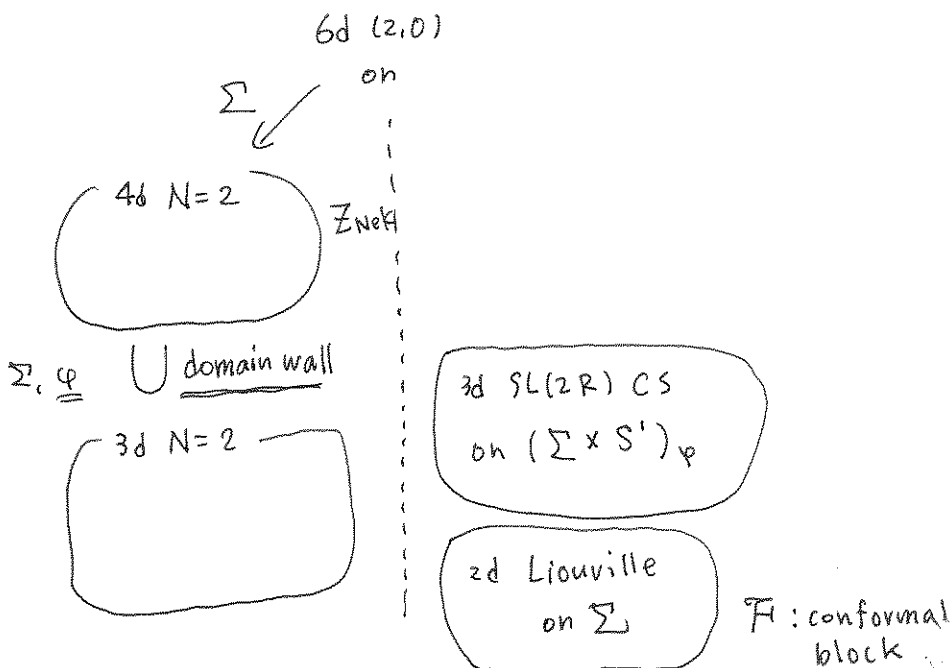
Refs	Drukker-Gaiotto-Gomis	1003
	Hosomichi-Lee-Park	1009
	Terashima-Y	1103 1106
	Dimofte-Gukov	1106

3d N=2 SUSY gauge theory \Leftrightarrow $SL(2, \mathbb{R})_{N=2}$ pure Chern-Simons on 3 mfd M
 Yang-Mills (+CS) + matter

- $\mathbb{Z}_{3d N=2} [S^3]$
 \parallel
 $\mathbb{Z}_{3d SL(2R) CS} [M]$
- exact computation of \mathbb{Z}
 4d on $S^4, S^3 \times S^1, \dots$
 3d on $S^3, S^2 \times S^1, \dots$
- relation between SUSY / non SUSY
 AGT, GRRY, ...

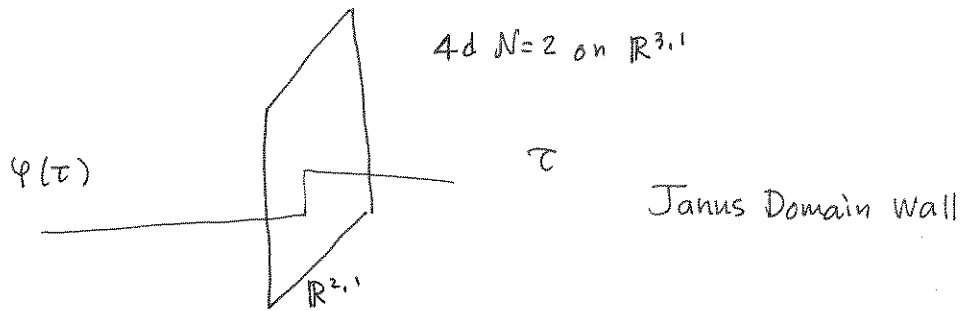
背後に 6d. (2,0) theory (M5-brane 上の理論)
 on $S^3 \times M$

M の例) $\cdot \Sigma_{g,h} \times I_{[0,1]}$ $\cdot (\Sigma_{g,h} \times S^1)_\varphi$
 $(\chi(\Sigma) < 0)$ $\varphi \in MCG(\Sigma) = Diff(\Sigma) / Diff_0(\Sigma)$



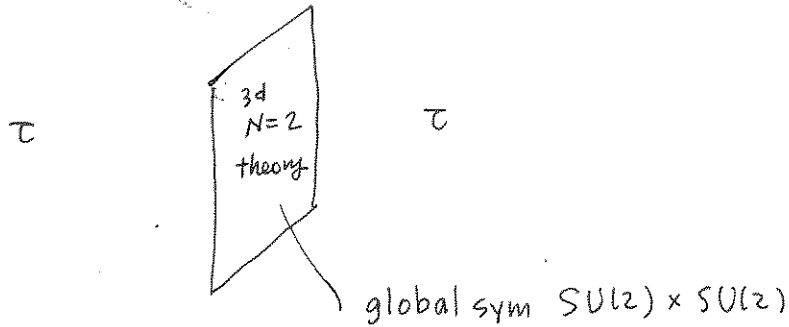
* duality domain wall

4d $N=2$ SCFT on complex coupling (τ)
 = Σ on complex structure (Gaiotto)



$\varphi \in \text{MCG}(\Sigma) \Leftrightarrow \varphi \in S\text{-duality of 4d SCFT}$

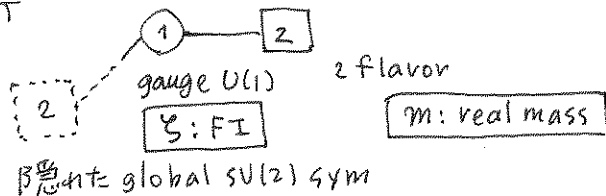
φ^{-1} on LHS



eg. $\Sigma = \Sigma_{1,1}$ (1 handle \rightarrow Σ)

\rightarrow 4d $N=2^*$, $\varphi = \text{PSL}(2, \mathbb{Z}) \ni S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

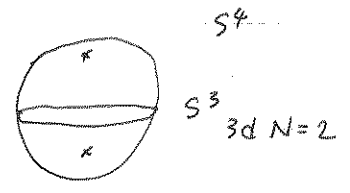
\rightarrow 3d $N=2$ SCFT



$m, p \Leftrightarrow$ mapping cylinder on CS theory on boundary condition

generalization of AGT

$$Z[S^4] = \int [da] \bar{Z}_{Nek}(a) Z_{Nek}(a)$$



$$\rightarrow \int [da][da'] \bar{Z}_{Nek}(a) Z_{3d}(a, a') Z_{Nek}(a')$$

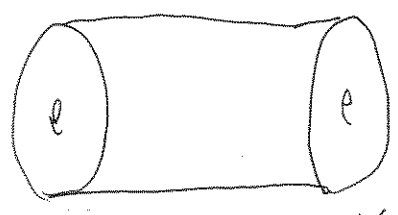
$\Rightarrow m, \xi$

Coulomb br
1^0 \rightarrow 1-9

$$= \int \frac{da}{da'} F_\alpha \left[\begin{array}{c} \uparrow \\ \text{conformal block} \\ \varphi_{\alpha, \alpha'} \end{array} \right] F_{\alpha'}$$

$\varphi \in M(G(\Sigma))$ の F_α の作用

$$F_\alpha(\varphi(\tau)) = \int da' \varphi_{\alpha\alpha'} F_{\alpha'}(\tau)$$



Liouville $\langle \alpha |$ $\hat{\varphi}$ $| \alpha' \rangle$

$$|\alpha\rangle, |\alpha'\rangle \in \mathcal{H}_{CS}(\Sigma) \rightarrow M(G(\Sigma))$$

CS理論の古典極限

CS level $k = b^{-2}$, $b \neq 0$ S^3 の squashing $1^0 \rightarrow 1-9$
 $F \in U(1)$

$$Z_{3d N=2} [S_b^3] = b^2 (x_1^2 + x_2^2) + b^{-2} (x_3^2 + x_4^2) = 1$$

3-manifold geometry \mathbb{H}^3 / Γ (hyperbolic 3-mfd)
 M_3

$$\text{Vol}(M_3) + i \text{CS}(M_3)$$

$$k \rightarrow \infty \text{ z'' } Z_{CS} \sim \exp [k (\text{Vol} + i \text{CS})]$$

S_b^3 z'' $b \rightarrow 0 \dots \rightarrow$ 2d (2,2) theory, Wess \leftrightarrow QIM, XY function