Landau resonance and the core-cusp problem in cold dark matter halos

Go Ogiya
Masao Mori
(University of Tsukuba, Japan)
Structure Formation in the Universe

• Dark Matter (DM)
  – is the dominant element in mass
  – interacts only through gravity with others
  – assembles baryon (atoms) and DM
  – drives structure formation (DM halos, Galaxies etc.)

• Cold Dark Matter (CDM) cosmology
  – is the standard paradigm
  – matches observations in large scale
  – has serious problems in small scale

Core-Cusp problem

Ishiyama et al. (2011)
What is the Core-Cusp Problem?

Mass-density profile of DM halo

Observation vs Theory (CDM)

Constant : Core

Divergent : Cusp

van Eymeren et al. (2009)

Navarro et al. (1997)
Realistic Simulation

- Mashchenko et al. (2008)
  - Cosmological N-body+SPH simulation
    - Supernova feedback etc.
    - Blue: gas, Yellow: star

- Gas
  - Blown out (expansion)
  - Fall back towards center (contraction)
  - Repeat many times
  - Gas Oscillation
Idealized Model

1) Gas heating by supernovae
2) Gas expansion
3) Energy loss by radiative cooling
4) Contraction towards the center
5) Ignition of star formation again

Repetition of these processes

Gas Oscillation

Change of potential ⇒ DM halo is affected gravitationally
⇒ Cusp to Core transition?
Resonance Model

- Linear analysis of the resonance between the density wave and the particles: assumption and approximation
  - Perturb equilibrium system (0) by external force (ex) ⇒ induced values (ind)
  - Focus on the particle group with $\rho_0 = \text{const.}, v_0 = \text{const.}$
  - External force is oscillatory
    $$ - \frac{\partial \Phi_{\text{ex}}}{\partial r} = \sum_n A_n \cos (kr - n\Omega t) $$
    $(A: \text{strength}, \ k: \text{wavenumber}, \ \Omega: \text{frequency})$

- Linearized Euler eq.
  $$ \frac{\partial v_{\text{ind}}(t,r)}{\partial t} + v_0 \frac{\partial v_{\text{ind}}(t,r)}{\partial r} = -\frac{\partial \Phi_{\text{ex}}(t,r)}{\partial r} $$
  $$ v_{\text{ind}}(t,r) = -\sum_n \frac{A_n}{n\Omega - kv_0} \{\sin (kr - \Omega t) - \sin (kr - kv_0 t)\} $$

- Linearized eq. of continuity
  $$ \frac{\partial \rho_{\text{ind}}(t,r)}{\partial t} + v_0 \frac{\partial \rho_{\text{ind}}(t,r)}{\partial r} = -\rho_0 \frac{\partial v_{\text{ind}}(t,r)}{\partial r} $$
  $$ \rho_{\text{ind}}(t,r) = -\sum_n \frac{A_n \rho_0 k}{(n\Omega - kv_0)^2} \times \{\sin (kr - n\Omega t) - \sin (kr - kv_0 t) + (n\Omega - kv_0) t \cos (kr - kv_0 t)\} $$

- When $n\Omega \sim kv_0$, coefficients diverge **Resonance**
Resonance Condition

- Resonance between particles and density waves
  \[ n \Omega \sim k \nu_0 \]

- Some arithmetic calculations \((n = 1)\)

  Resonance occurs when the condition is satisfied
  \( \rightarrow \) efficient energy transfer
  \( \rightarrow \) system expands
  \( \rightarrow \) density change dramatically
  \( \Rightarrow \textbf{Cusp to Core transition} \)

\[ t_d(r) \sim T \]
\[ t_d(r) = \sqrt{\frac{3\pi}{32G\bar{\rho}(r)}} \]

\textbf{Our model} \( \Leftrightarrow \) \textbf{Observations}

\textbf{T} : Period of gas oscillation \( \Leftrightarrow \) Star formation histories of galaxies
\textbf{t}_d(r) : Local dynamical time \( \Leftrightarrow \) Density of DM halos
Core Scale

• Density profile of CDM halo
  \[ \rho(r) = \frac{\rho_0 R_{DM}^3}{r^\alpha (r + R_{DM})^{3-\alpha}} \]

• Mass profile
  \[ M(\alpha; x) = \frac{4\pi \rho_0 R_{DM}^3}{3-\alpha} x^{3-\alpha} \, _2F_1[3-\alpha, 3-\alpha, 4-\alpha; -x] \]
  \[ \text{↑ Gauss’s hyper-geometric function} \]
  \[ x \equiv r/R_{DM} \]

• Resonant condition

Dynamical time

\[ t_d(r) \sim T \]

\[ t_d(r) = \sqrt{\frac{3\pi}{32G \rho(r)}} \]

\[ r_{\text{core}} = R_{DM} \left( \frac{T}{T_c} \right)^2 / \alpha \]

\[ T_c^2 \equiv \frac{\pi^2}{8G} \frac{R_{DM}^3 c^{3-\alpha}}{M_{\text{vir}}} \, _2F_1[\alpha; -c] \]

• Virial mass \( M_{\text{vir}} \)
• Concentration \( c \)
• Oscillation period \( T \)

\[ \downarrow \]

We can predict the core scale created by resonance!!
Numerical Model

**DM halo (N-body system):**
NFW model (Navarro et al. 1997)

**Baryon (external potential):**
Hernquist potential (Hernquist 1990)

\[
\rho(r) = \frac{\rho_0 R_{DM}^3}{r(r + R_{DM}^2)}
\]

\[
\Phi_b(r, t) = -\frac{G M_b}{r + R_b(t)}
\]

\[
R_b(t) \propto \cos \left(\frac{2\pi t}{T}\right)
\]

**Property of DM halo**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of particles</td>
<td>( N ) 16M, 128M</td>
</tr>
<tr>
<td>Softening parameter ( \epsilon )</td>
<td>0.004kpc</td>
</tr>
<tr>
<td>Virial mass of a DM halo ( M_{\text{vir}} )</td>
<td>( 10^9 M_{\odot} )</td>
</tr>
<tr>
<td>Virial radius of a DM halo ( R_{\text{vir}} )</td>
<td>10kpc</td>
</tr>
<tr>
<td>Scale radius of a DM halo ( R_{DM} )</td>
<td>2kpc</td>
</tr>
<tr>
<td>Total baryon mass ( M_{b,\text{tot}} )</td>
<td>( 1.7 \times 10^8 M_{\odot} )</td>
</tr>
</tbody>
</table>

Oscillation period of the external force, \( T \)

\[
T = 1, 3, 10 \ t_d(0.2\text{kpc})
\]
The cusp-core transition and the resultant core scale depends on the oscillation period of the external potential, $T$.

$T = 10t_d = 10\text{Myr}$

$T = 3t_d \text{ (HR)}$

$T = 10t_d$

Initial condition

$\rho [M_\odot \text{ pc}^{-3}]$

$r [\text{kpc}]$

$\rho = R_{\text{DM}} \left( \frac{T}{T_c} \right)^{2/\alpha}$

Prediction of Core Scale
- Virial mass $M_{\text{vir}}$
- Concentration $c$
- Oscillation period $T$

$r_{\text{core}} = R_{\text{DM}} \left( \frac{T}{T_c} \right)^{2/\alpha}$
Density profiles of DM halos after several oscillation periods

Fourier spectrum of radial velocity

$$v_r(t, r) \rightarrow \hat{v}_r(\omega, r)$$

Peaks appear when $$\omega = 2\pi/T$$.

Each position of the peaks matches the core scale.
Result3 - Overtones -

\[ v_{\text{ind}}(t, r) = -\sum_{n} \frac{A_n}{n\Omega - k\nu_0} \{\sin(kr - \Omega t) - \sin(kr - k\nu_0 t)\} \]

\( \omega = n\Omega \rightarrow \text{Spectrum with peak (Resonance)} \)

- \( \omega = \frac{2\pi}{t_d} \)
- \( n=1 \) for \( T=1t_d \)
- \( \omega = \frac{2\pi}{2t_d} \)
- \( \omega = \frac{2\pi}{3t_d} \)
- \( \omega = \frac{2\pi}{4t_d} \)
- \( \omega = \frac{2\pi}{5t_d} \)
- \( \omega = \frac{2\pi}{6t_d} \)
- \( \omega = \frac{2\pi}{7t_d} \)
- \( \omega = \frac{2\pi}{8t_d} \)
- \( \omega = \frac{2\pi}{9t_d} \)
- \( \omega = \frac{2\pi}{10t_d} \)

\( \vec{r} \)

\( r \text{ [kpc]} \)
Summary

• Study the dynamical response of a DM halo
  – The Core-Cusp problem
  – Oscillatory change of gravitational potential

• Analytical Model
  – Resonance between particles and density waves
  – Resonant condition $t_d(r) \sim T$
    dynamical time oscillation period

• N-body simulations
  – Resonance plays a significant role to flatten central cusp
    • Resonance $\rightarrow$ Efficient energy transfer
    • $\rightarrow$ Cusp to Core transition
  – Core scale is well matched to our predictions