Momentum transfer in non-equilibrium steady states

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Collaboration with

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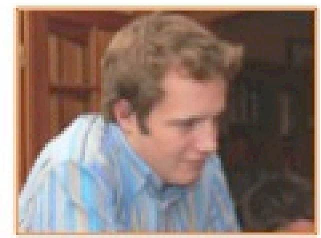
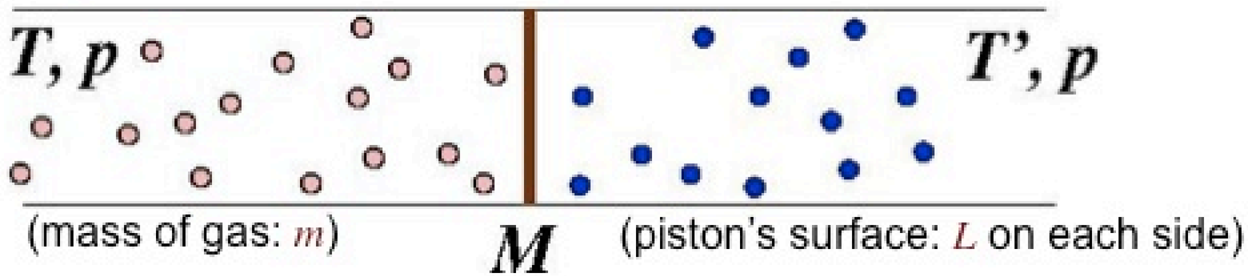


* PRL108,160601(2012)
* arXiv:1204.6536 (submitted)

Stochastic processes that cannot be described by standard *Langevin* equations

- Adiabatic piston



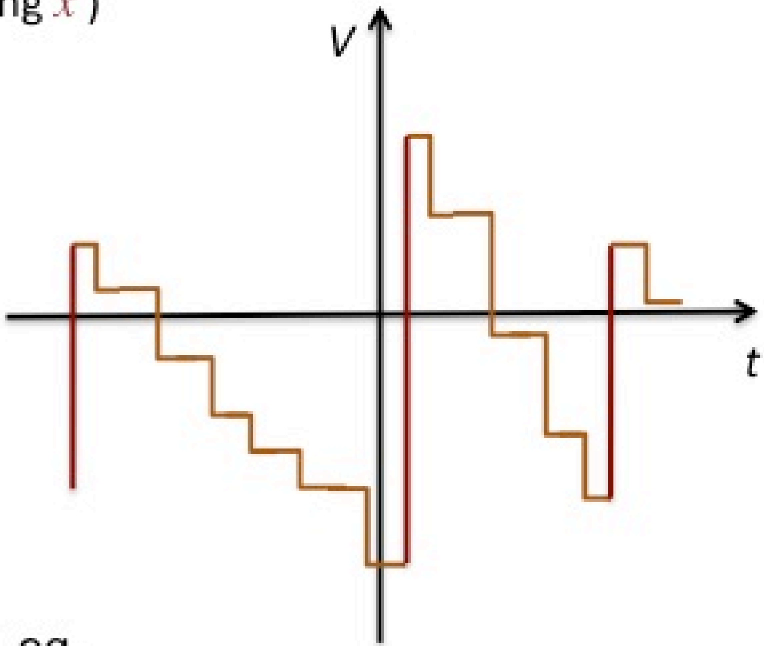
$T \neq T'$

how & why does the piston move?

Feynman (1963) : the fluctuations of piston transport "heat"
Callen (1985) : thermodynamics does not answer piston's motion

Simple example

Ideal 2D gas particles and a long triangle (moving along x)



(i) Detailed balance:

$$\{V(t)\} \simeq \{-V(-t)\}$$

=> No bias $\overline{V} = 0$

* The asymmetry is not captured by Langevin eq.

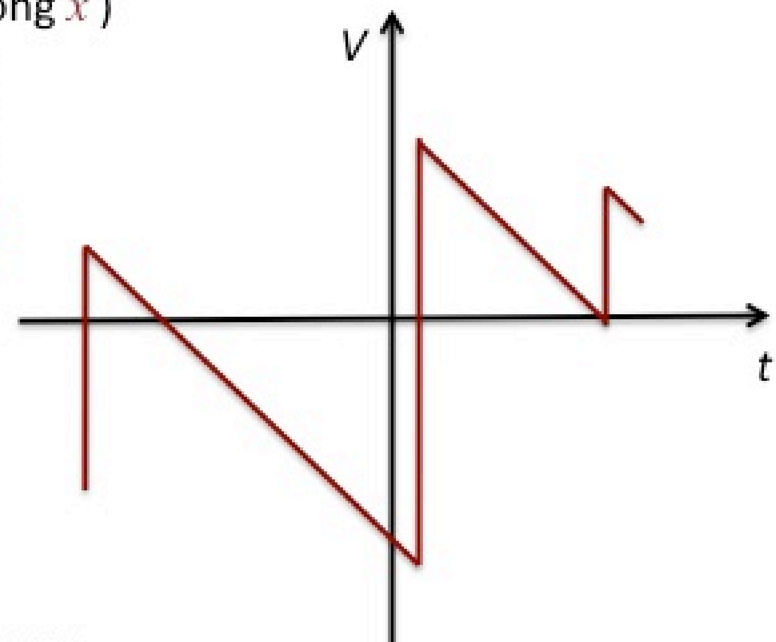
$$M\frac{dV}{dt} = -\gamma V + \sqrt{2\gamma k_{\mathrm{B}} T}\zeta(t)$$

(iii) $\theta \longrightarrow 0$: "Law of Large Number" (frequent but inefficient)

Simple example

Ideal 2D gas particles and a long triangle (moving along x)

constant force on *long sides*



(i) No bias:

$$\overline{V} = 0$$

(ii) Detailed balance:

$$\{V(t)\} \simeq \{-V(-t)\}$$

* The asymmetry is not captured by Langevin eq.

$$M\frac{dV}{dt} = -\gamma V + \sqrt{2\gamma k_{\mathrm{B}} T}\zeta(t)$$

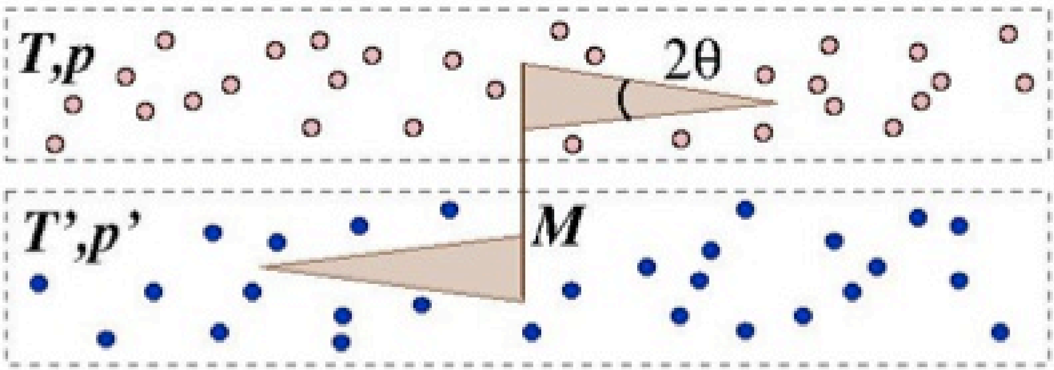
(iii) $\theta \longrightarrow 0$: "Law of Large Number" (frequent but inefficient)

=> constant force on *the base* (no fluctuations, no friction vs V)

variations

- Brownian ratchet

  [Van den Broeck, *et al*. PRL 2004]



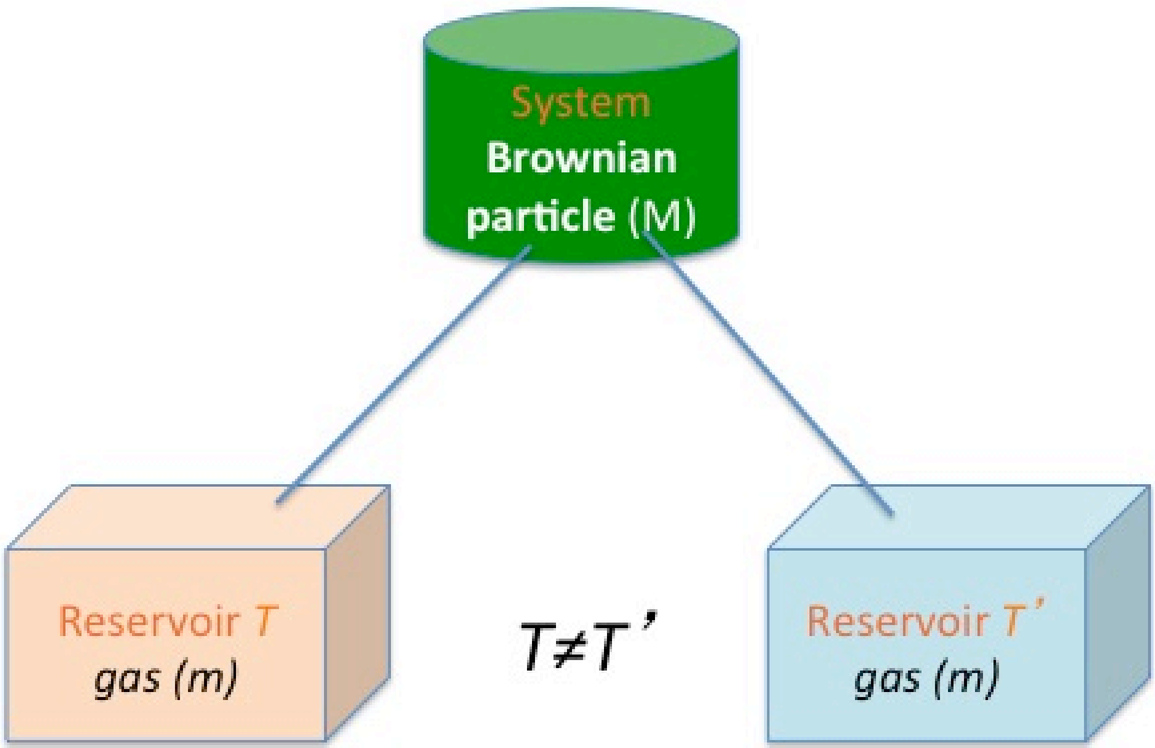
$T \neq T'$

(T,p) : Initial condition = Maxwell-Boltzmann distribution with density $\rho = p/k_B T$

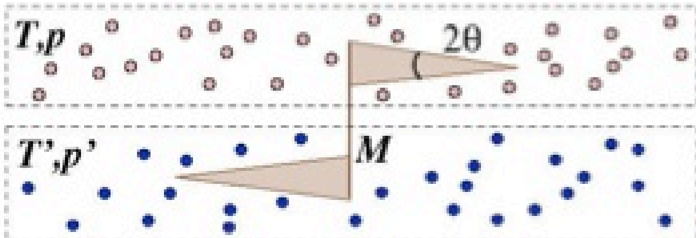
A class of phenomena that cannot be described by *Langevin* equation

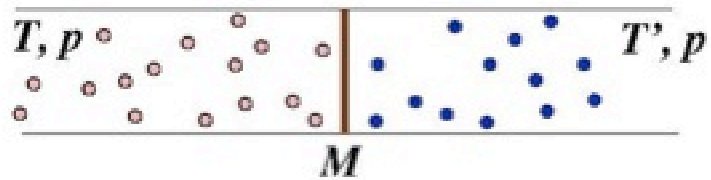
+

Non-equilibrium steady state



variations





etc.

"Curie's principle" => $\overline{V} \neq 0$

* no "how it moves",

* no mechanism

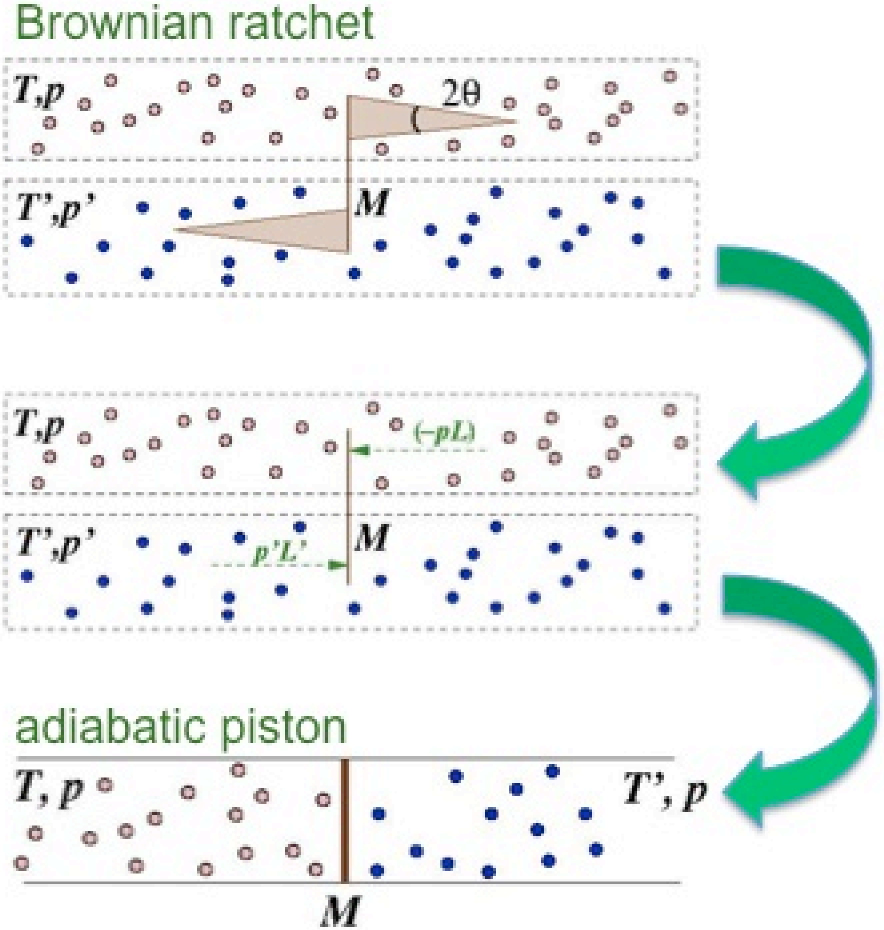
Conventional approach :

* case-by-case calculations to know "how"

* no physical explanations

Present talk : mechanism & generality

variations



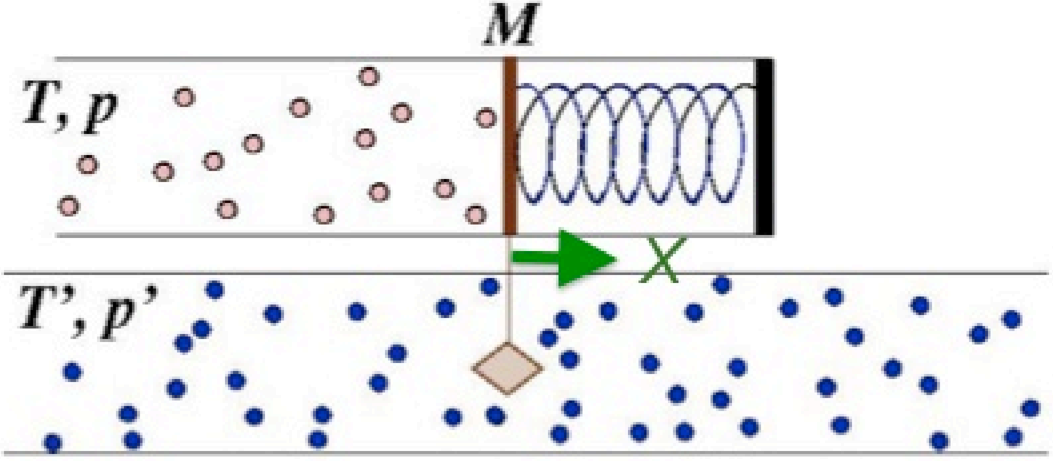
$$\theta \rightarrow 0$$

$$p'L' = pL$$

cancellation of forces on *long sides*

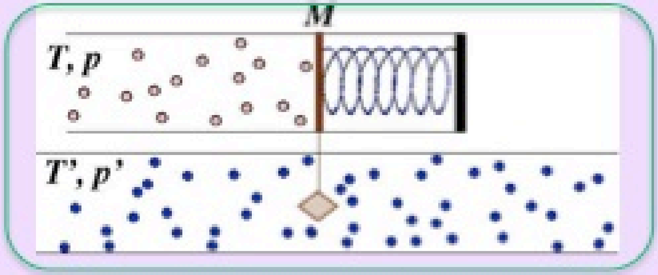
Core model

- Cooled [warmed] piston



$$T \neq T' \quad \Rightarrow \quad \overline{X} \neq \overline{X}_{\text{eq}}$$

Core model:



Conventonal approach

Master/Boltzmann equation for $P(X,V,t)$

$$\begin{aligned}\partial_t P(X,V,t) &= -V\partial_X P(X,V,t) - [-\gamma' V - \partial_X U(X)]\,\partial_V P(X,V,t) \\ &\quad - \int_{V'} W(V'|V)P(X,V,t) + \int_{V'} W(V|V')P(X,V',t) + \frac{k_\mathrm{B}T'}{\gamma'}\partial_X^2 P(X,V,t)\end{aligned}$$

with transition rate

$$W(V'|V)dV'dt = H(v_x - V)\times[dt(v_x - V)\rho L]\sqrt{\frac{m}{2\pi k_\mathrm{B}T}}e^{-\frac{m}{2k_\mathrm{B}T}v_x^2}\left(\frac{m+M}{2m}\right)dV'$$

binary elastic collision, $V' = V + \frac{2m}{m+M}(v_x - V)$

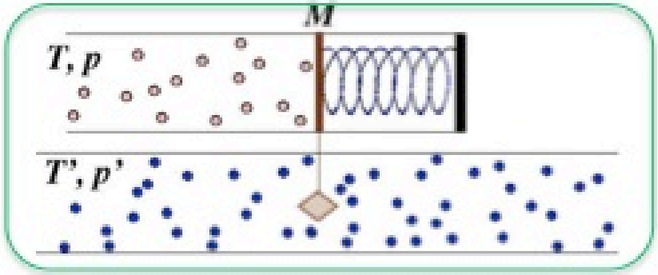
v_x : velocity of colliding particle

Moment hierarchy ($m<<M$) :

$$M\frac{d\langle V\rangle}{dt} = -\langle U'(X)\rangle - \gamma\langle V\rangle - \gamma'\langle V\rangle + \rho k_\mathrm{B}TL\frac{m\frac{M\langle V^2\rangle}{k_\mathrm{B}T} + M}{m+M}$$

$$\begin{aligned}M\frac{d\langle V^2\rangle}{dt} &= -\langle VU'(X)\rangle \\ &\quad -\gamma\left[\frac{2M-m}{2M+2m}\langle V^2\rangle - \frac{k_\mathrm{B}T}{M+m}\right] - \gamma'\left[\langle V^2\rangle - \frac{k_\mathrm{B}T'}{M}\right] + c\langle V\rangle\end{aligned}$$

Core model:



Strategy : *de*composition of problem

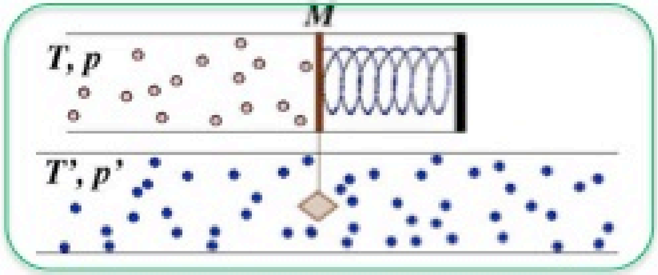
(1) Energy dissipation

(2) Momentum transfer

(1) Energy dissipation — of purely mechanical system.
We should not confront the "origin of irreversibility" issue.

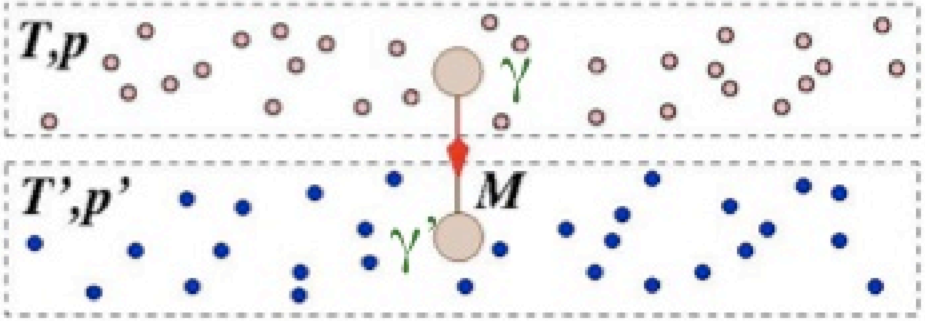
Core model:

(1) Energy dissipation



Observation-I :

Dissipation depends on micro-parameters through friction constants, γ and γ' (weak coupling & lowest order in $\frac{m}{M} \ll 1$)



➔ Langevin equation + Stochastic Energetics suffice

Heat:

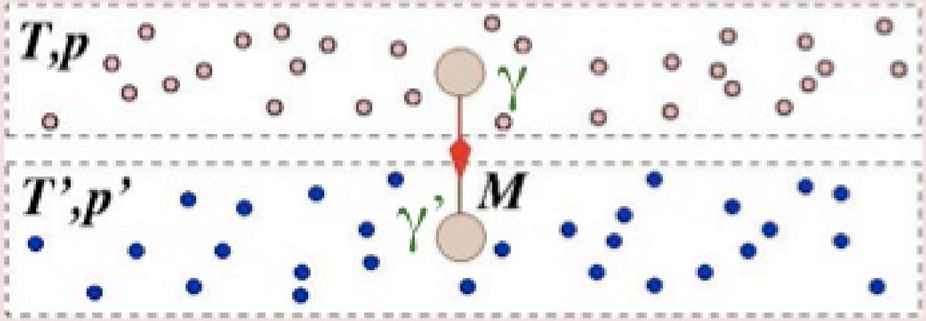
$$d'Q/dt = [-\gamma V + \sqrt{2\gamma k_{\mathrm{B}} T}\zeta(t)] \circ V$$

Dissipation rate

$$J^{(e)}_{\mathrm{diss}} = \frac{k_{\mathrm{B}}T - k_{\mathrm{B}}T'}{M(\gamma^{-1} + \gamma'^{-1})}$$

Heuristic derivation of heat flow $J^{(e)}_{\rm diss}$ (Parrondo,1996)

(i) Kinetic temperature of Brownian object: $T_{\rm kin}$



(ii) Linearity at interfaces:

$$J^{(e)}_{\rm diss} = \frac{\gamma}{M}(k_{\rm B}T - k_{\rm B}T_{\rm kin}) \qquad J^{(e)\prime}_{\rm diss} = \frac{\gamma'}{M}(k_{\rm B}T' - k_{\rm B}T_{\rm kin})$$

(iii) Energy conservation: $J^{(e)}_{\rm diss} + J^{(e)\prime}_{\rm diss} = 0$

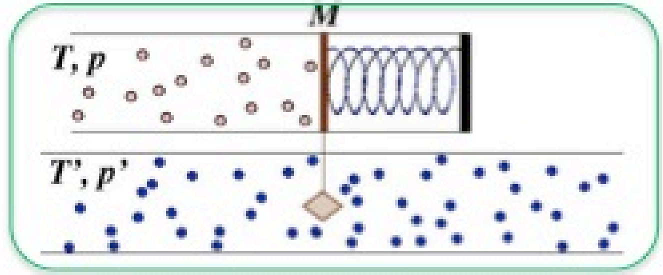
=> heat flow

$$J^{(e)}_{\rm diss} = -J^{(e)\prime}_{\rm diss} = \frac{k_{\rm B}T - k_{\rm B}T'}{M(\gamma^{-1} + \gamma'^{-1})}$$

(exact in the lowest order of m/M)

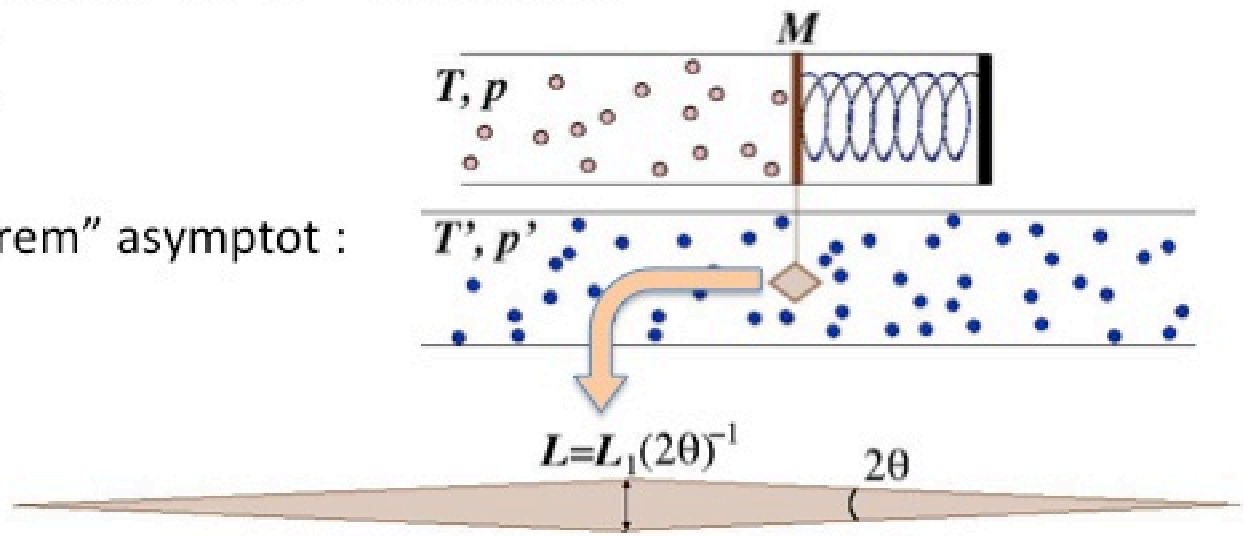
Core model:

(2) Momentum transfer



Observation-II :

Momentum transfer to 2nd reservoir is *not* essential



← "Central Limit Theorem" asymptot :

For $\theta \to 0$

Force on the rhombus $\longrightarrow$ $-\gamma' V + \sqrt{2\gamma' k_{\mathrm{B}} T'}\zeta'(t)$

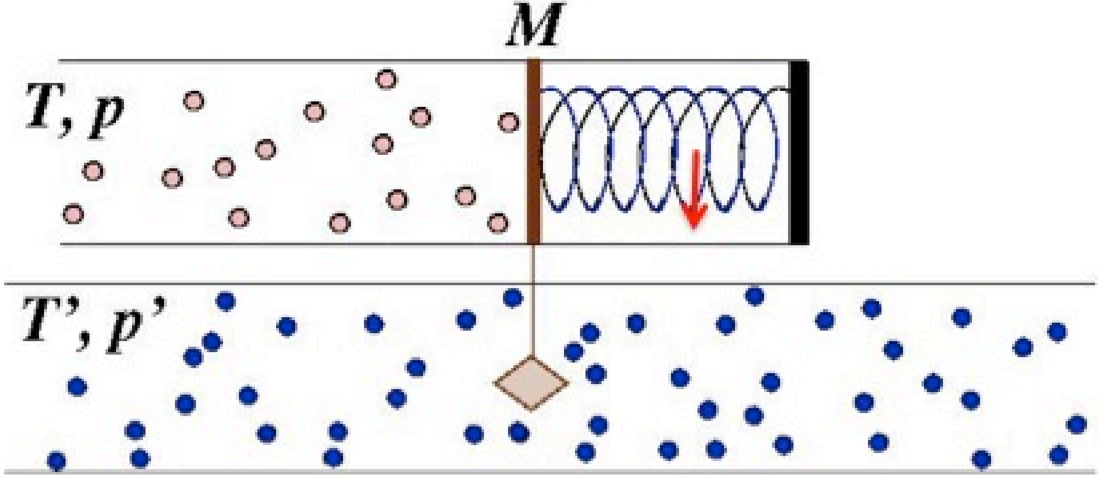
→ mean momentum transfer from the reservoir *T'* = $\overline{-\gamma' V + \sqrt{2\gamma' k_{\mathrm{B}} T'}\zeta'(t)} = 0$

Core model:

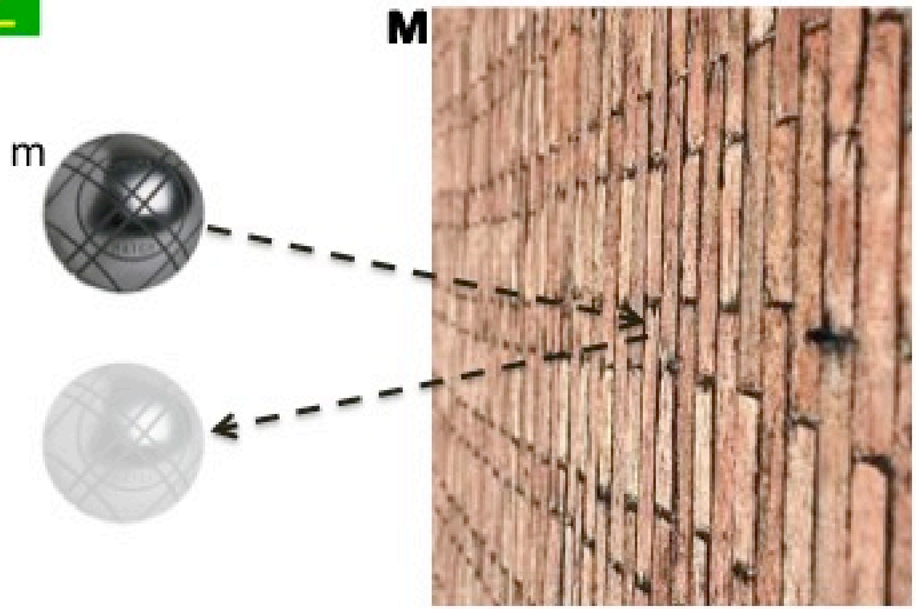
(2) Momentum transfer deficit

What force (= *momentum transfer*)

is on the energy-dissipating piston ?

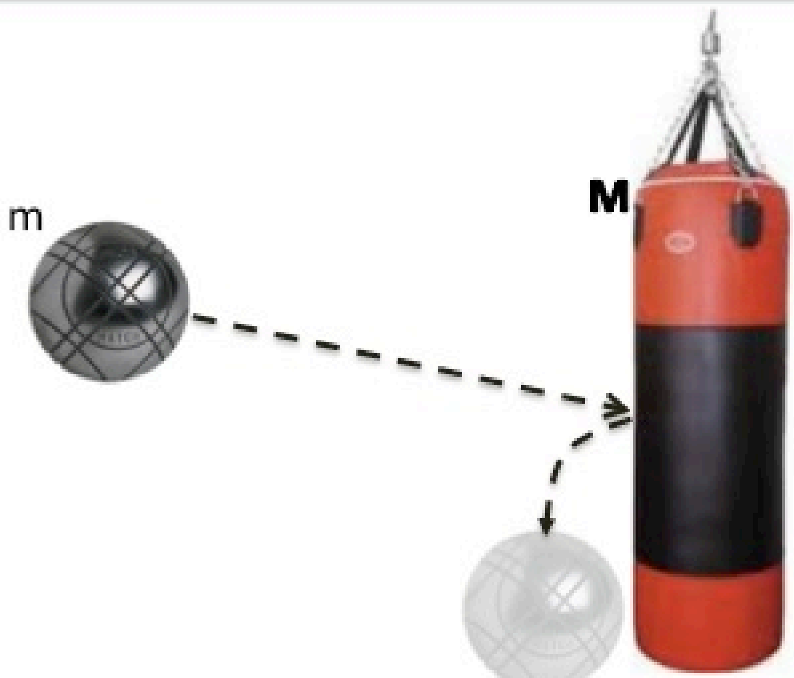


idea



Energy dissipation : small

Momentum transfer : large



Energy dissipation : large

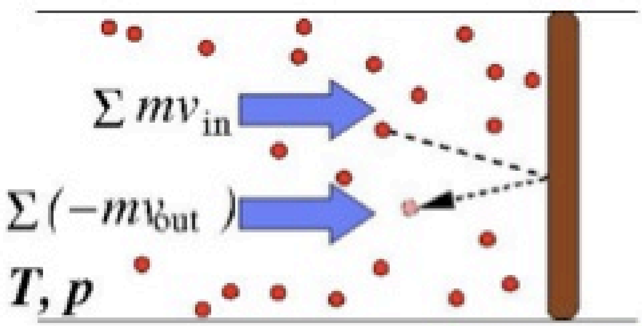
Momentum transfer : **small**

"Momentum transfer **deficit** due to dissipation"

Principle of MDD

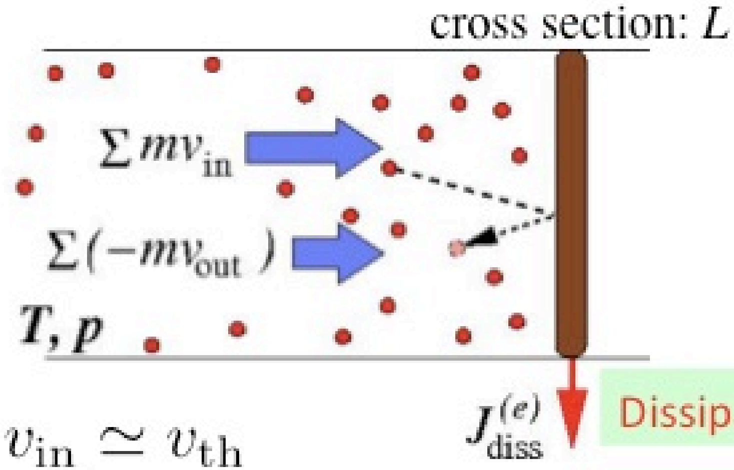
(2) Momentum transfer deficit

Estimation of momentum fluxes



Equilibre

$$v_{\text{in}} \simeq v_{\text{th}}$$
$$|v_{\text{out}}| \simeq |v_{\text{in}}| \simeq v_{\text{th}}$$



Dissipation

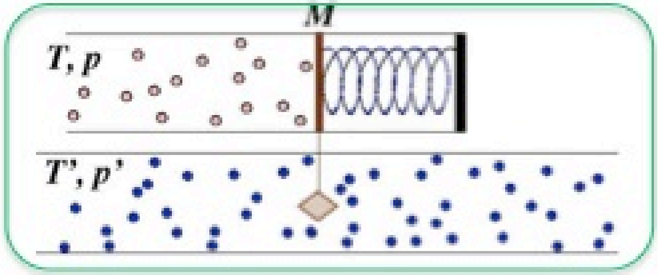
$$v_{\text{in}} \simeq v_{\text{th}}$$
$$|v_{\text{out}}| < |v_{\text{in}}| \simeq v_{\text{th}}$$

Energy loss/collision : $e_1 = \dfrac{J^{(e)}_{\text{diss}}}{w_{\text{coll}}}$

Collision tate : $w_{\text{coll}} = \dfrac{\rho}{2} L v_{\text{th}}$

Energy balance: $\dfrac{mv^2_{\text{out}}}{2} = \dfrac{m{v_{\text{th}}}^2}{2} - e_1$

A small calculation :



Energy loss/collision : $e_1 = \dfrac{J^{(e)}_{\rm diss}}{w_{\rm coll}}$

Collision tate : $w_{\rm coll} = \dfrac{\rho}{2} L v_{\rm th}$

Energy balance: $\dfrac{m v_{\rm out}^2}{2} = \dfrac{m {v_{\rm th}}^2}{2} - e_1$

$$\Leftrightarrow \quad (m v_{\rm th} - m|v_{\rm out}|) \frac{v_{\rm th} + |v_{\rm out}|}{2} = \frac{J^{(e)}_{\rm diss}}{w_{\rm coll}}$$

$\simeq v_{\rm th}$

Momentum transfer deficit /time :

$$(m v_{\rm th} - m|v_{\rm out}|) \times w_{\rm coll} \simeq \frac{J^{(e)}_{\rm diss}}{v_{\rm th}}$$

Total momentum transfer rate :

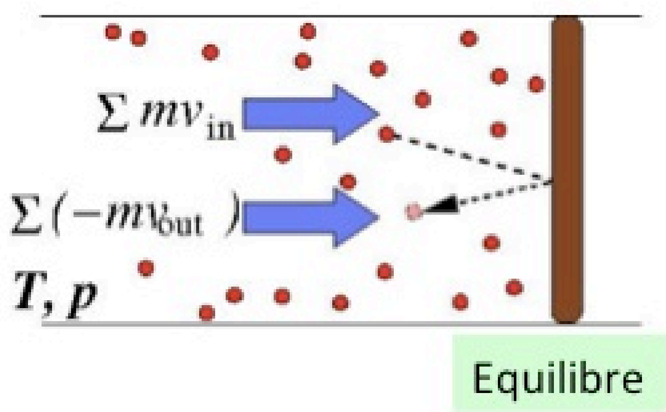
$$(m v_{\rm th} - m v_{\rm out}) \times w_{\rm coll} \simeq pL - \frac{J^{(e)}_{\rm diss}}{v_{\rm th}}$$

$$(\text{l.h.s.} = [2 m v_{\rm th} - (m v_{\rm th} - m|v_{\rm out}|)] \times w_{\rm coll})$$

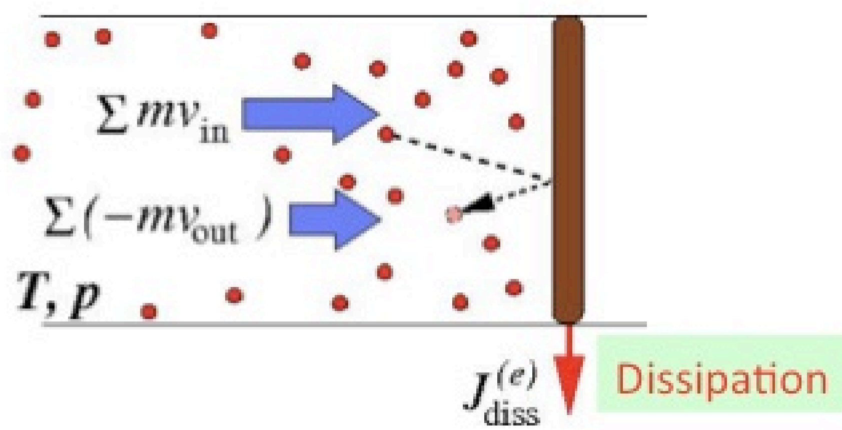
Principle of MDD

(2) Momentum transfer deficit

Estimation of momentum fluxes



$$\overline{\sum mv_{\rm in} + \sum(-mv_{\rm out})} = pL$$



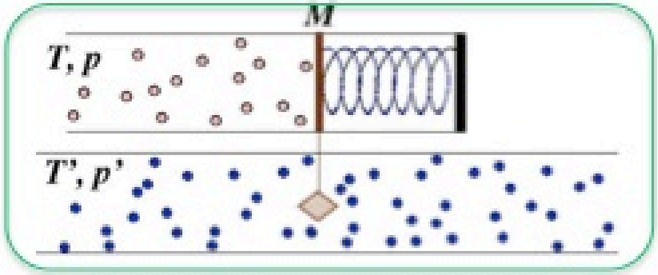
Result

$$\overline{\sum mv_{\rm in} + \sum(-mv_{\rm out})} = pL - c\frac{J^{(e)}_{\rm diss}}{v_{\rm th}}$$

Momentum transfer deficit due to dissipation (MDD)

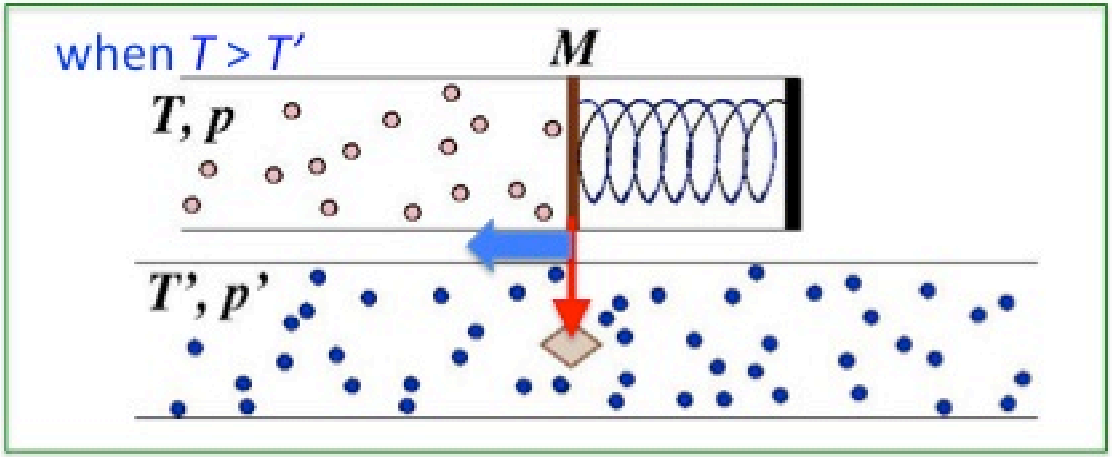
Core model:

(2) Momentum transfer deficit



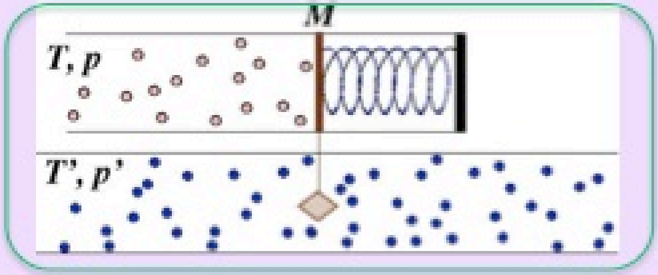
$$\overline{\sum m v_{\rm in} + \sum(-m v_{\rm out})} = pL - c\frac{J^{(e)}_{\rm diss}}{v_{\rm th}}$$

$c = 1$: simple argument, $c = \sqrt{\frac{\pi}{8}}$: gas kinetics calculation



"*Energy absorbing surfaces* receive *less pressure* than equilibrium"

Core model:



hindsight (あとから見れば書いてある)

Conventonal approach (Boltzmann/master eq.)

➔

1st moment: momenum equation

$$M\frac{d\langle V\rangle}{dt} = -\langle U'(X)\rangle - (\gamma+\gamma')\langle V\rangle + pL - c\frac{J^{(e)}_{\rm diss}}{v_{\rm th}}$$

$c=\sqrt{\pi/8}$

MDD

$$J^{(e)}_{\rm diss} = \frac{\gamma}{M}(k_{\rm B}T - k_{\rm B}T_{\rm kin})$$

$$\frac{M\overline{V^2}}{2} = \frac{k_B T_{\rm kin}}{2}$$

2nd moment: energy equation

$$M\frac{d\langle V^2\rangle}{dt} = -\langle VU'(X)\rangle - \frac{\gamma}{M}\left[M\langle V^2\rangle - k_{\rm B}T\right] - \frac{\gamma'}{M}\left[M\langle V^2\rangle - k_{\rm B}T'\right] + c'\langle V\rangle$$

small

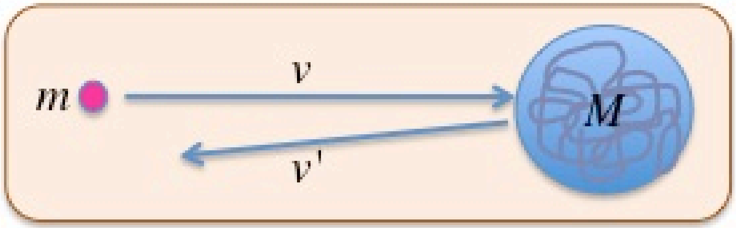
=0 : fixing of kinetic temperature

Reflection:

The inverse logic has been used in gas kinetics

(ex. *Qualitative Methods in Physical Kinetics and Hydrodynamics* (V.P. Krainov))

$$(v, V_{C.M.} = 0) \mapsto (v', V'_{C.M.})$$



knowledge of *v* and *v'* ➔ energy transfer *to* internal energy

$$\Rightarrow \quad \Delta\mathcal{E}_{\rm int} = \frac{m}{2}(v - v')[(1-\epsilon^2)v + (1+\epsilon^2)v']$$

$$\epsilon \equiv \sqrt{\frac{m}{M}}$$

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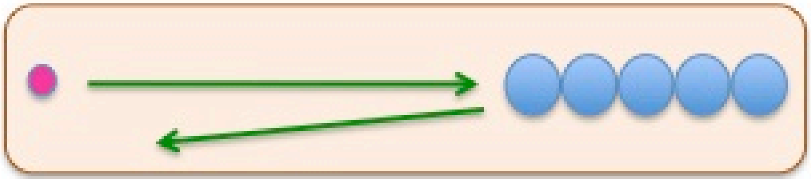
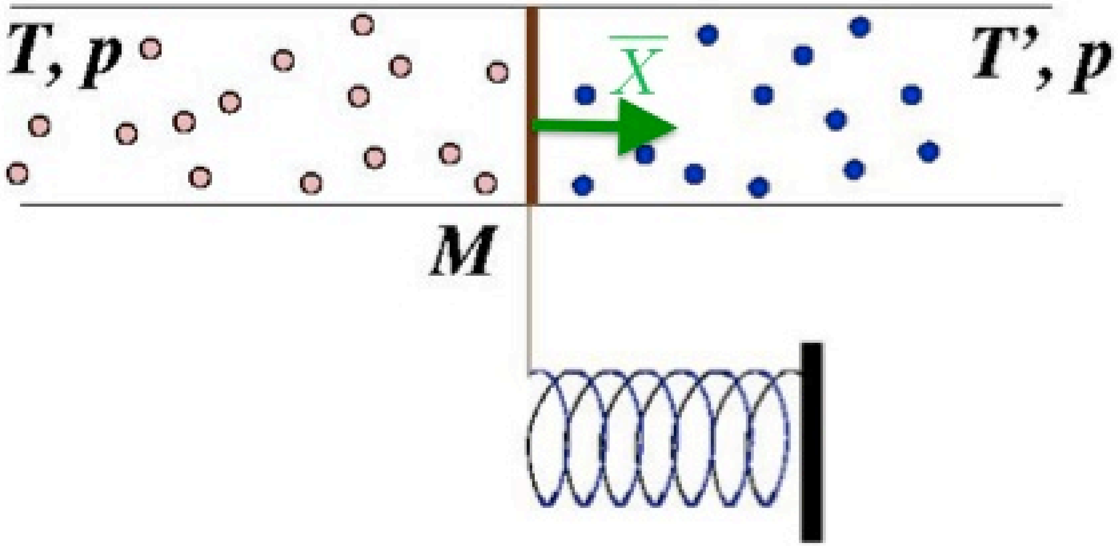
cf. an exotic motion of macroscopic objects *just in contact*



Applications

- *Trapped* adiabatic piston

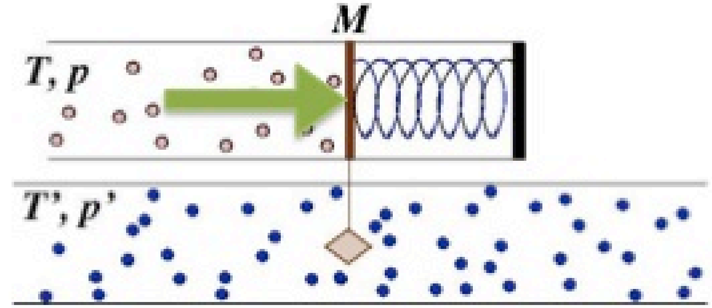
  [new]



$T \neq T'$

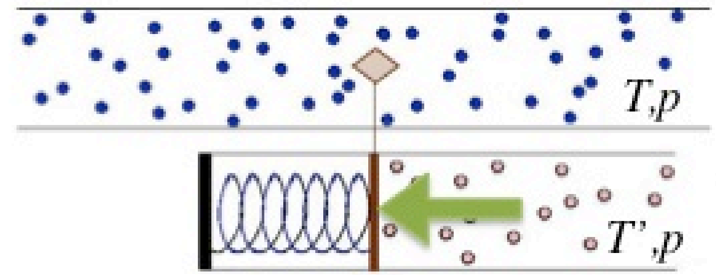
Note: fixed wall ➔ no dissipation ➔ no MDD ➔ no force

(i) Find force F_{left} on piston



$\overline{V} = 0$

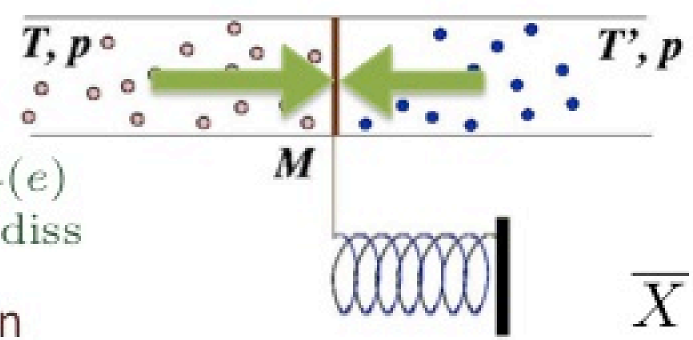
(i') Find force F_{right} on piston



$\overline{V} = 0$

$$F_{\mathrm{MDD}} = F_{\mathrm{left}} + F_{\mathrm{right}}$$
$$= -c\left(\frac{1}{v_{\mathrm{th}}} + \frac{1}{v'_{\mathrm{th}}}\right) J^{(\mathrm{e})}_{\mathrm{diss}}$$

force on trapped adiabatic piston

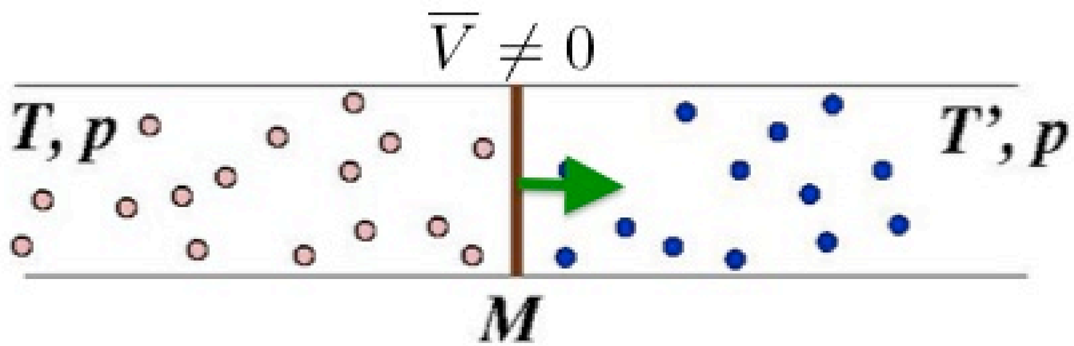


$\overline{V} = 0$

$\overline{X} \neq \overline{X}_{\mathrm{eq}}$

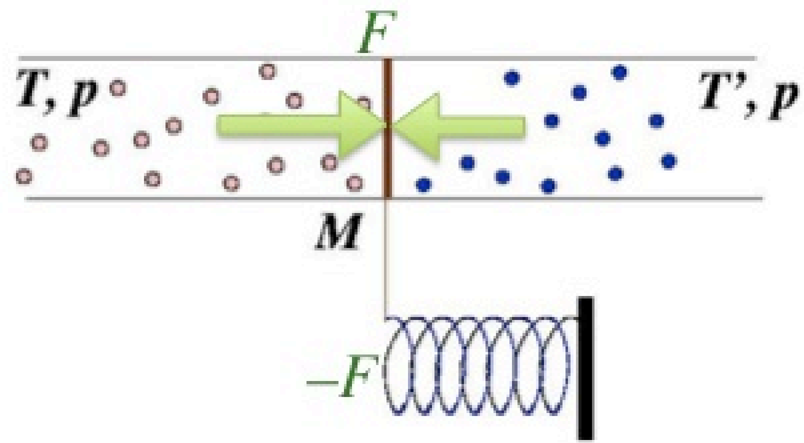
Applications

- Adiabatic piston



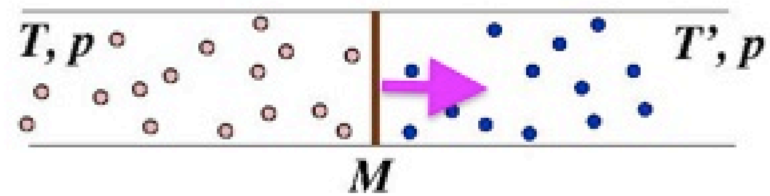
(i) Force on trapped adiabatic piston balanced by "spring"

$$F_{\mathrm{MDD}} + F_{\mathrm{spring}} = 0$$



(ii) Friction forces against motion at $\overline{V} \neq 0$

$$-\gamma\overline{V} - \gamma'\overline{V}$$



$\gamma,\ \gamma'$: friction constant of gas-piston coupling

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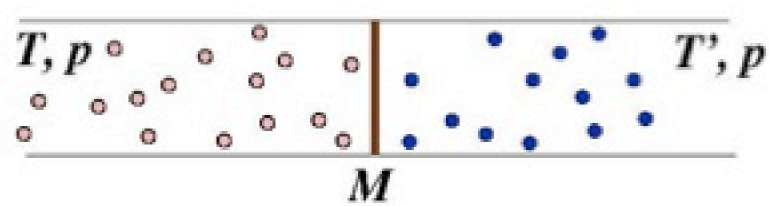
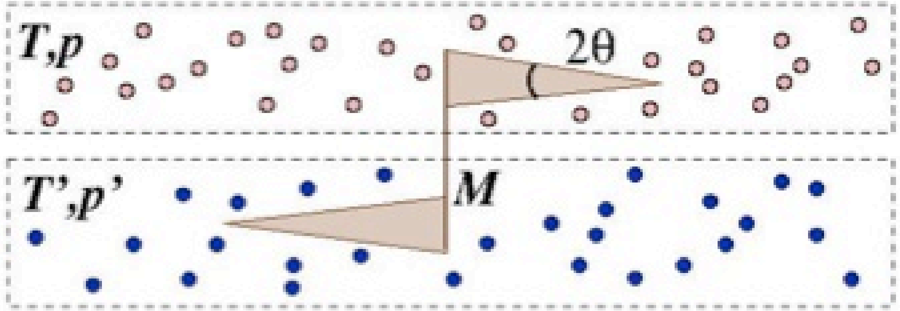
Velocity of adiabatic piston

$$F_{\mathrm{MDD}} - \gamma\overline{V} - \gamma'\overline{V} = 0$$

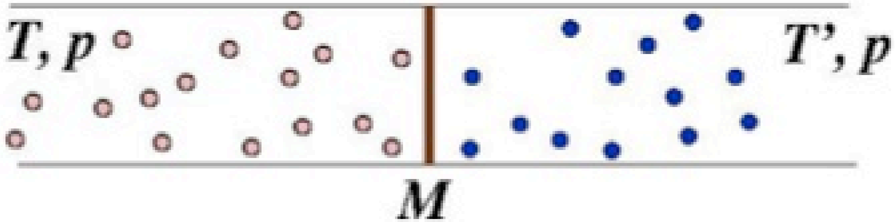


Applications

- Brownian ratchet



reduced to adiabatic piston
in the $\theta \to 0$ limit



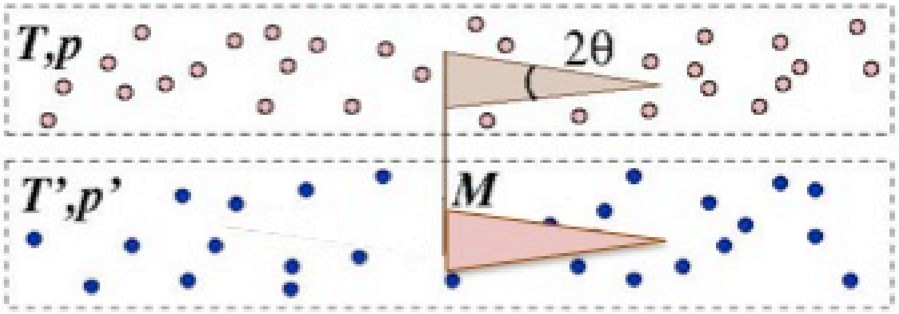
$$F = -c\left(\frac{1}{v_{\rm th}} + \frac{1}{v'_{\rm th}}\right) J^{(e)}_{\rm diss}$$

$$F - \gamma\overline{V} - \gamma'\overline{V} = 0$$

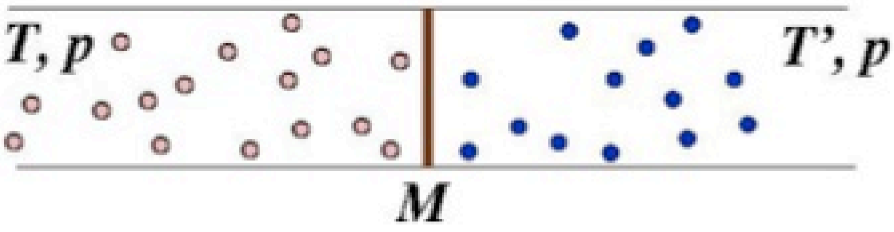
Applications

- What if

  ?



reduced to adiabatic piston
in the $\theta \to 0$ limit



$$F = -c\left(\frac{1}{v_{\rm th}} - \frac{1}{v'_{\rm th}}\right) J^{(e)}_{\rm diss}$$

$$F - \gamma \overline{V} - \gamma' \overline{V} = 0$$

Applications

Radiometer at extremely high vacuum. (cf. Sano's talk)

=> the radiometer turns in the opposite direction,
by the mechanism of MDD. (Experiment in 1911.)

Applications

- Inelatic triangle — non-trivial case

e : restitution coeff. T, p : uniform

Simplifications :
$\theta \ll 1$, $(1-e) \ll 1$

Two consequences of inelasticity

(i) absorption of **gas'** kinetic energy (house-keeping dissip.*)

➔ house-keeping mementum deficit $\pm\frac{1-e}{2}pL$ (hydrostatic)

(ii) modification of **triangle's CM** kinetic energy (excess dissip.)

$$k_{\rm B}T_{\rm eff} = \frac{1-e}{2}k_{\rm B}T \quad \Rightarrow \quad J^{(\rm e)}_{\rm diss,ex} = \frac{\gamma}{M}(k_{\rm B}T - k_{\rm B}T_{\rm eff})$$

➔ excess MDD => motion of triangle in MDD

Applications

- Inelatic triangle — non-trivial* case

house-keeping heat

Dissipation *from* the base : $J_{\text{diss}}^{(\text{e})} = \frac{m}{2}(1-e^2) \times w_{\text{coll}} + J_{\text{diss,ex}}^{(\text{e})}$

house-keeping MDD

Force on the sides : $F_{\text{right}} = -pL + \frac{1-e}{2}pL$

Force on the base : $F_{\text{left}} = pL - \frac{1-e}{2}pL - \frac{J_{\text{diss,ex}}^{(\text{e})}}{v_{\text{th}}}$

excess MDD

$$F_{\text{right}} + F_{\text{left}} - \gamma \overline{V} = 0$$

Summary

Mechanism of adiabatic piston is simply understood.

Key notions :

Energy dissipation is determined at Langevin level (indep. of geometry).

Momentum transfer deficit is then determined by energy dissipation.

"*Energy absorbing surfaces* receive *less pressure* than equilibrium"

Problems:

Hydrodnamic boundary condition of heat-absorbing wall" *(cf. Itami & Sasa)*

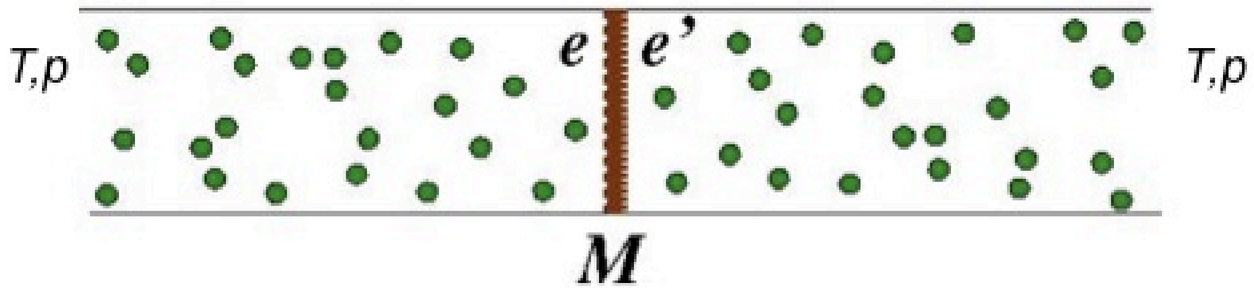
Effect of boundary thermostats on NESS

Contact value theorem out of equilibrium

Optical or quantum analogues,... soret, radiometer ?

Applications

- cf. Inelatic piston — trivial case



e, e' : restitution coefficients

Again

"*Energy absorbing surfaces* receive *less pressure* than equilibrium"

applies