Holographic superfluids and the dynamics of symmetry breaking

Jerome Gauntlett

- Part I (1/4)

  Equilibrium AdS/CFT - overview of different holographic phases

  Aristomenis Donos, Christiana Pantelidou

- Part II (3/4)

  Non-equilibrium AdS/CFT - quench of a holographic superfluid

  Joe Bhaseen, Ben Simons, Julian Sonner, Toby Wiseman

Part I: Equilibrium AdS/CMT

- One focus: use holography to analyse the equilibrium behaviour of CFTs when held at finite temperature and charge density and/or in a uniform magnetic field and try to make contact with different phases that are seen in condensed matter
- Approach:
  - Construct all AdS black hole solutions with the relevant asymptotic behaviour
  - Calculate the free energies and deduce the phase diagram
- Hard!

Part I: Equilibrium AdS/CMT

- One focus: use holography to analyse the equilibrium behaviour of CFTs when held at finite temperature and charge density and/or in a uniform magnetic field and try to make contact with different phases that are seen in condensed matter
- Approach:
  - Construct all AdS black hole solutions with the relevant asymptotic behaviour
  - Calculate the free energies and deduce the phase diagram
- Hard!

Questions:

- What type of phases are possible?
- What kind of zero temperature ground states are possible?
- Do we find interesting new emergent scaling behaviour in the far IR?
- Transport?

Top-down solutions of D=10/11 supergravity preferable

AdS/CMT has certainly led to new insights into string/M-theory - including rich new classes of black hole solutions

Superfluid Phases

- Key ingredient: charged bulk fields that spontaneously break a global U(1) symmetry
- s-wave superfluids have $l = 0$ order parameter
  Use charged bulk scalar fields. [Gubser][Hartnoll,Herzog,Horowitz] [Gauntlett,Sonner,Wiseman][Gubser,Herzog,Pufu,Tesileanu]
- p-wave superfluids have $l = 1$ order parameter
  Seen in eg He_3, heavy fermions, organics, Sr_2RuO_4
  - In D=4,5 use SU(2) gauge fields. Take the background to be charged with respect to $U(1) \subset SU(2)$ and then spontaneously break the U(1) [Gubser][Hartnoll,Roberts]
  - In D=5 can use a charged self-dual two-form [Aprile,Franco,Rodriguez,Russo]
- Back-reacted black hole solutions have been found by solving ODEs

- Above examples are electrically charged black holes corresponding to CFTs at finite chemical potential with respect to the global U(1) symmetry
- Holographic superconductivity also occurs for CFTs in a magnetic field
    - Examples using bulk charged scalars [Almuhairi,Polchinksi][Donos,Gauntlett,Pantelidou]
    - Examples using bulk charged vectors [Ammon,Erdmenger,Kerner,Strydom] [Almuhairi,Polchinksi][Donos,Gauntlett,Pantelidou] (similar to condensation of rho mesons in QCD? [Chernodub])
    - Both occur for N=4 d=4 SYM and N=8 d=3 SYM and there is an interesting interconnection with supersymmetry
    - Inferred using a linearised analysis - to construct back reacted black holes need to solve PDEs

Spatially Modulated Phases

- In condensed matter there is a variety of phases that are spatially modulated, spontaneously breaking translation invariance.
- For example: charge density waves and spin density waves.
- The modulation is fixed by an order parameter associated with non-zero momentum.
- Spatially modulated superconducting phases are also possible. FFLO phase is a variation of BCS. Perhaps seen in some heavy fermions (eg $CeCoIn_5$) and some organic superconductors

- Spatially modulated black holes are possible
- CFTs at finite chemical potential with respect to U(1) and we find examples of spatial modulated phases:
  D=5 [Nakamura,Ooguri,Park] then D=4 [Donos,Gauntlett]
- Also find spatial modulation in D=4,5 in the presence of magnetic fields [Donos,Gauntlett,Pantelidou]
- These examples have spatially modulated currents. Can also have [Donos,Gauntlett,Pantelidou]
  - Charge density waves for D=4
  - Spatially modulated superfluids for D=5. Specifically p-wave with a helical order
  - Top-down examples in D=10,11 supergravity

Spatially modulated phases are not exotic... typical?!

- The spatially modulated phases above inferred using a linearised perturbative analysis. Generically to go beyond this one needs to solve PDEs.....many interesting questions to look at

- An interesting exception[Donos,JPG]:

  - Back reacted black holes that describe p-wave superfluids with a helical structure.

  - The D=5 black holes are spatially homogeneous with a helical Bianchi VII_0 symmetry and we constructed black holes by solving ODEs!

  - At zero temperature the black holes become domain wall solutions and interpolate between AdS5 in the UV and a new scaling solution in the IR with helical symmetry (see also [Iizuka,Kachru,....])

Superconductor Week

Commercialization · Markets · Products
Business Developments · R&D · Cryogenics

June 30, 2012 Vol. 26, No. 10

June 30, 2012 Vol. 26, No. 10

Superconductor Week

Commercialization · Markets · Products
Business Developments · R&D · Cryogenics

- LH2 CARRIES 30 MW
- VNIIKP BSCCO CABLE PROJECT ONGOING
- NIMS SYNTHESIZES SC FULLERINE NANOWHISKERS
- NANOWHISKERS PREPARED WITH LLIP METHOD
- SC VOLUME FRACTIONS AS HIGH AS 80% OBSERVED
- U CALGARY MICROWAVE PULSE STORAGE METHOD
- CORNELL/BNL LINK MAGNETISM, IRON-BASED SC
- STUDY CONFIRMS MAGNETIC PAIRING THEORY
- IMPERIAL COLLEGE DESCRIBES P-WAVE SC PHASE
- ADS/CFT APPLIED TO HTS
- P-WAVE SC SIMILAR TO LIQUID CRYSTAL PHASES
- *U.S. SUPERCONDUCTIVITY PATENTS*

Part 2: Non equilibrium Dynamics

- Studying the far from equilibrium dynamics of any system is a very challenging problem
- Receiving much interest because of new experiments e.g. in the context of cold atoms
- Can we use AdS/CFT to obtain new insights?
- Basic idea is very simple: analyse time dependent black hole solutions.
- Technically challenging because it requires solving non-linear PDEs.
[.........][Chesler,Yaffe][Murata,Kinoshita,Tanahashi][Bizon,Rostworowski]
[Garfinkle,Pando Zayas][Bantilan,Pretorius,Gubser] [Heller,Janik,Witaszczyk]
[Buchel,Lehner,Myers][......]

- Quantum quench
- Start with a state of initial Hamiltonian that is abruptly changed e.g.

$$H = H_0 + g(t)H \qquad\qquad g(t) = g\theta(t)$$

How does the sate evolve?

  - a) Nature of thermalisation? Relaxation times?
  - b) Universal behaviour near critical points?

- Here we will study a quantum quench of superfluids. Analyse in the context of AdS/CFT.
- In addition we discover some universal features that apply more broadly for late time dynamics

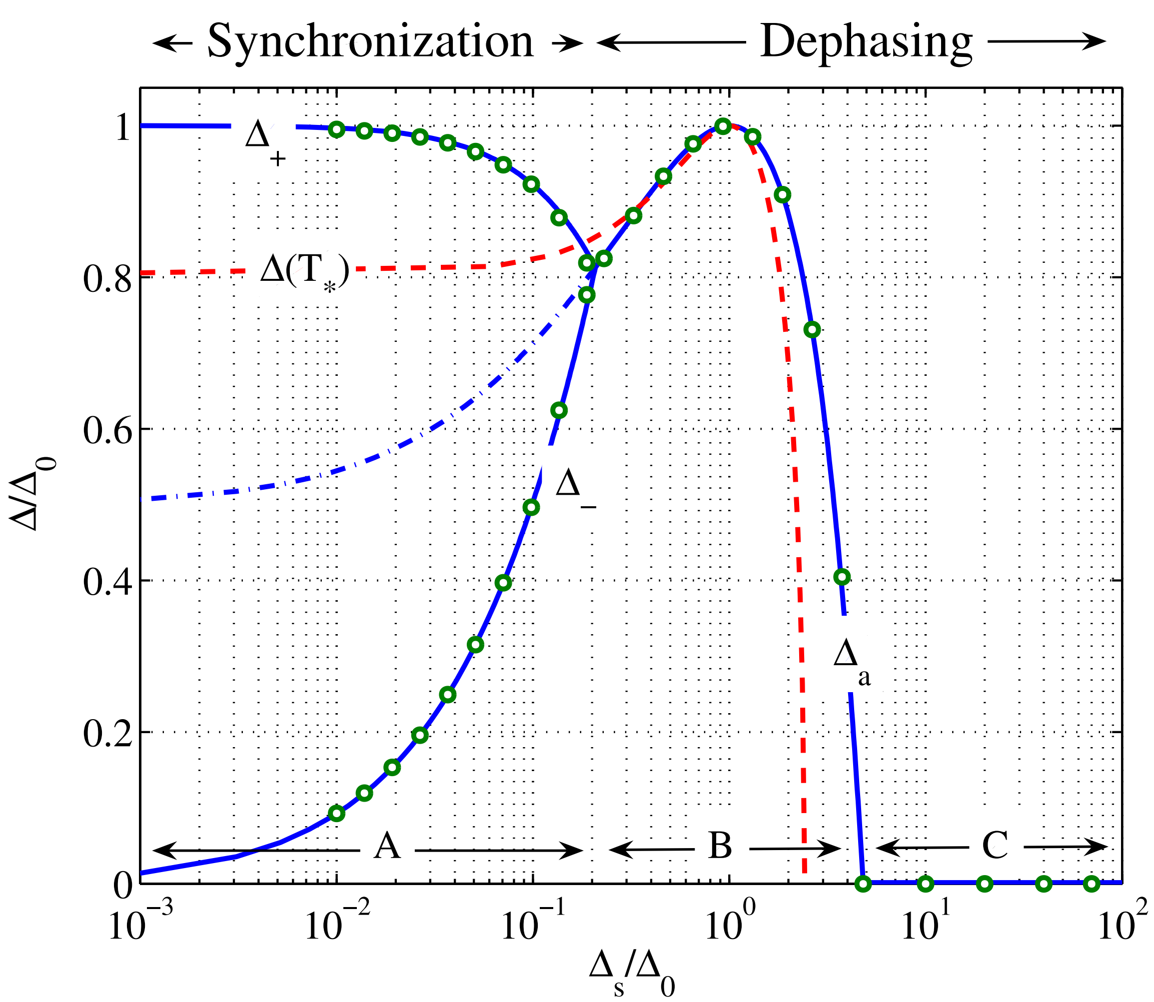
Quantum quench of a BCS superconductor

- Abruptly switch on pairing interactions in a weakly coupled BCS setting [Barankov,Levitov]
- Approximations:
    - collisionless
    - no thermal dissipation
    - no vortex production

B-L showed this is an integrable system

- B-L Dynamical phase diagram

Final pairing gap



Strength of quench

- Region A: persistent oscillations in a final superfluid state
- Region B: power-law decay to final superfluid state
- Region C: power-law decay to unbroken phase final state

- What happens if we relax the assumptions and consider the role of collisions, thermal damping, non-BCS, strong coupling ...
- What survives?
- Explore using AdS/CFT

THE MODEL [Hartnoll,Herzog,Horowitz]

$$S = \int d^4x \left(R + 6 - F^2 - |D\psi|^2 - m^2|\psi|^2\right)$$

$$D\psi = d\psi - iqA\psi$$

AdS4 vacuum is dual to a d=2+1 CFT with a global U(1) symmetry

Order parameter for superfluid $\psi \leftrightarrow \mathcal{O}$

For simplicity we choose $q = 2$ $m^2 = -2$ $\Delta(\mathcal{O}) = 2$

Unbroken phase black holes

- The electrically charged AdS-RN black hole

$$ds^2 = -g dt^2 + g^{-1} dr^2 + r^2 (dx^2 + dy^2)$$

$$A = \phi dt \qquad\qquad \psi = 0$$

$$g = r^2 - (r^2 + \mu^2)\frac{r_+}{r} + \frac{r_+^2 \mu^2}{r^2}$$

$$\phi = \mu\left(1 - \frac{r_+}{r}\right)$$

- Describes CFT at chemical potential μ and temperature T

Superfluid phase black holes in equilibrium

- Spatially homogeneous and isotropic

$$ds^2 = -ge^{-\beta}dt^2 + g^{-1}dr^2 + r^2(dx^2 + dy^2)$$

$$A = \phi dt \qquad \psi = \psi(r) \in \mathbb{R}$$

- UV asymptotics

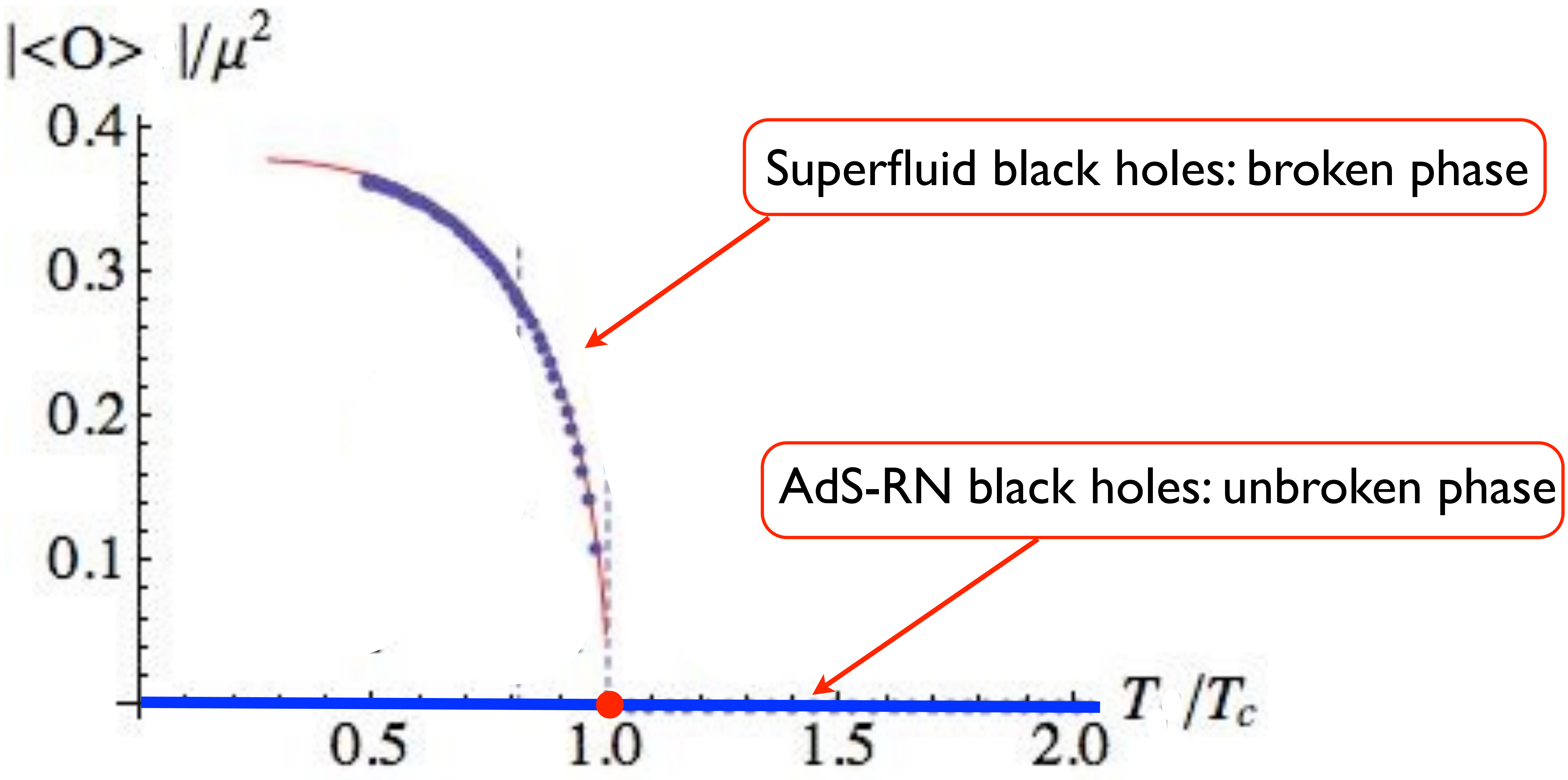
$$g = r^2 + \ldots \qquad \beta = 0 + \ldots \qquad \phi = \mu - \frac{q}{r} + \ldots$$

$$\psi = \frac{\psi_1}{r} + \frac{\psi_2}{r^2} + \ldots$$

$\psi_1 \leftrightarrow$ adding a source to CFT $\qquad \psi_2 \leftrightarrow \langle \mathcal{O} \rangle$

Superfluid black holes have $\psi_1 = 0$

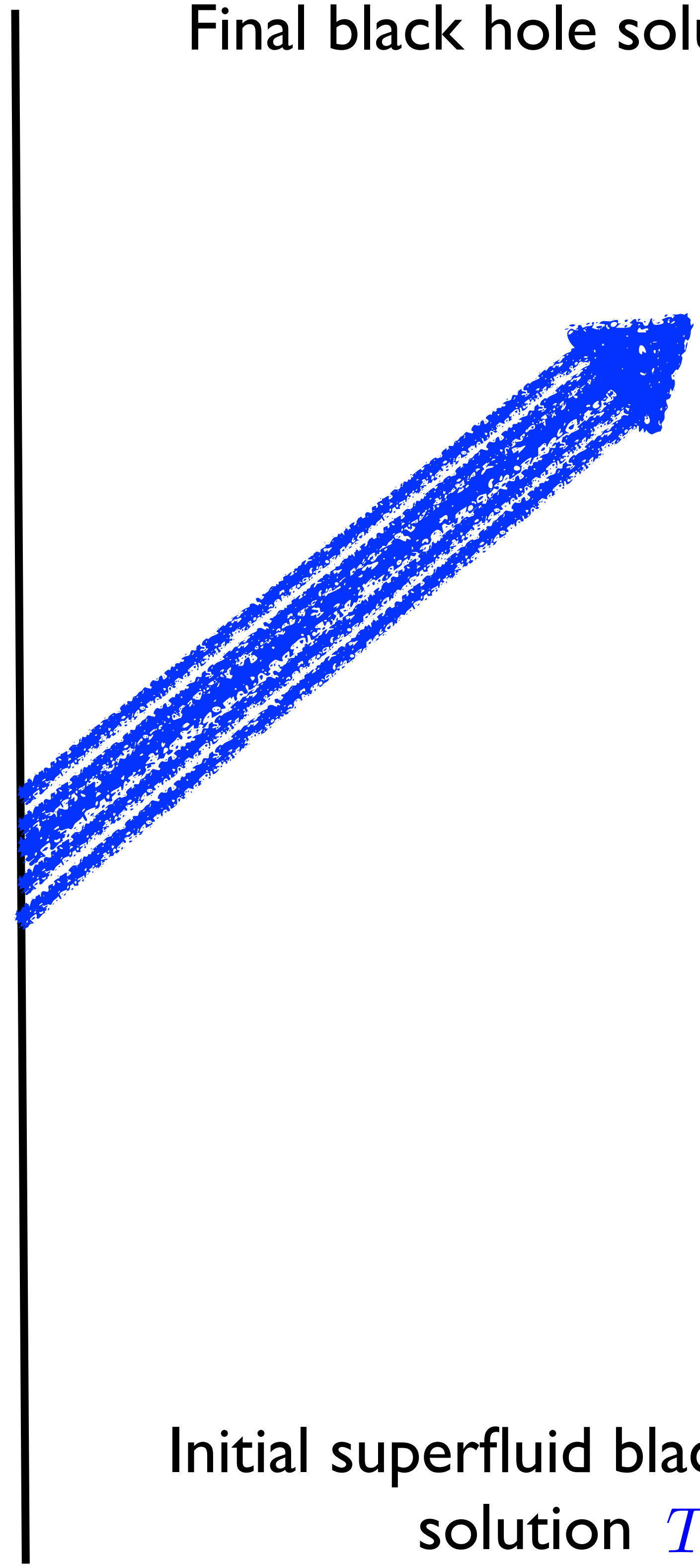
Equilibrium black holes: the superfluid phase transition



The dynamical quench from the superfluid phase

No source
$t = \infty$

Final black hole solution

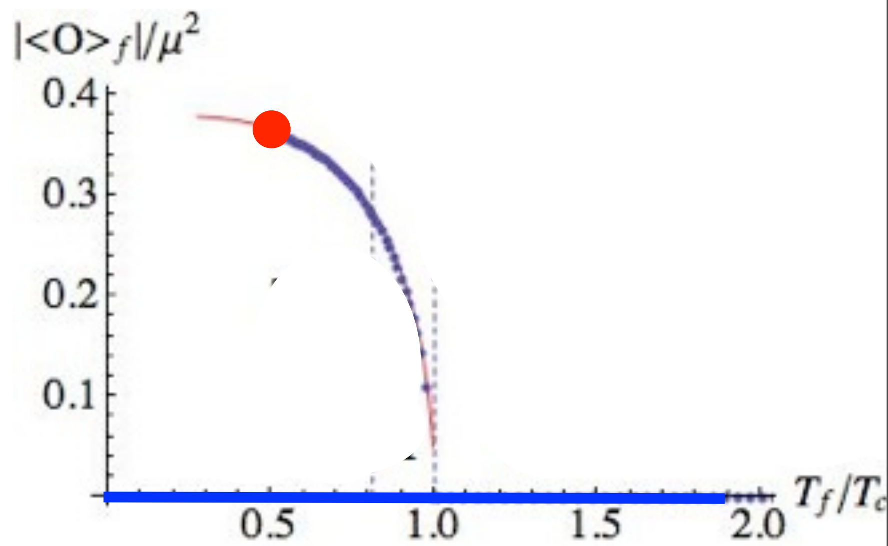


Gaussian pulse of
field dual to $\mathcal{O}$

No source
$t = -\infty$

Initial superfluid black hole
solution $T = 0.5T_c$

AdS4 boundary



Consider homogeneous and isotropic quenches

Use ingoing Eddington Finklestein coordinates

$$ds^2 = z^{-2}\left[-F\,dv^2 - 2\,dvdz + S^2(dx_1^2 + dx_2^2)\right]$$

$$F = F(v,z) \qquad S = S(v,z)$$

$$A = A_v(v,z)dv \qquad \psi = \psi(v,z)$$

[Kinoshita,Murata,Tanahashi]

- Asymptotic AdS4 expansion $z = 0$

$$\psi(v,z) = z\psi_1(v) + z^2\psi_2(v) + \ldots$$

$$\psi_1(v) = \delta e^{-10v^2}$$

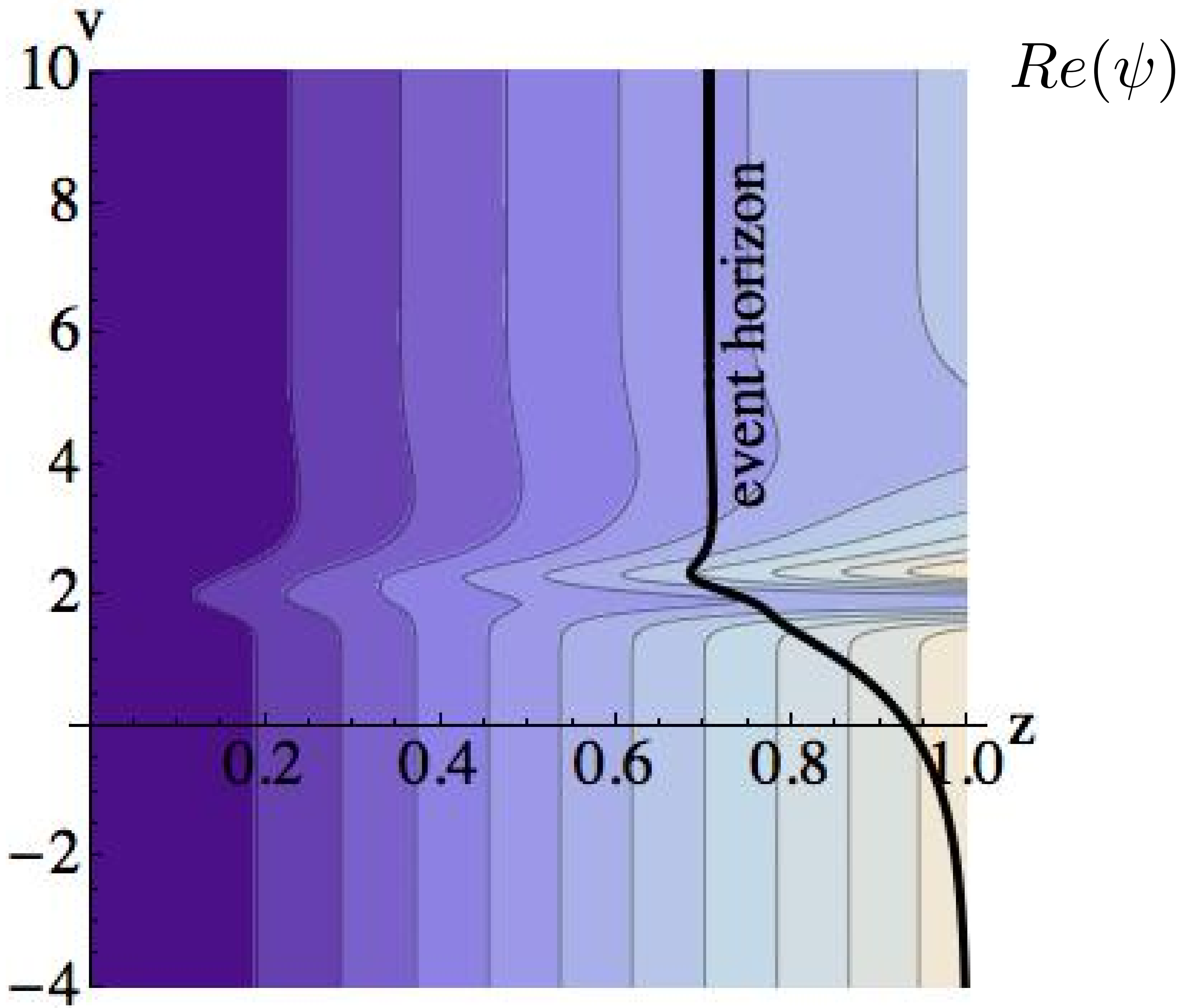
$$A_v(v,z) = \mu(v) - z\rho(v) + \ldots$$

$$\langle \mathcal{O} \rangle \sim \psi_2 - \mu\psi_1$$

$$F(v,z) = 1 + \ldots$$

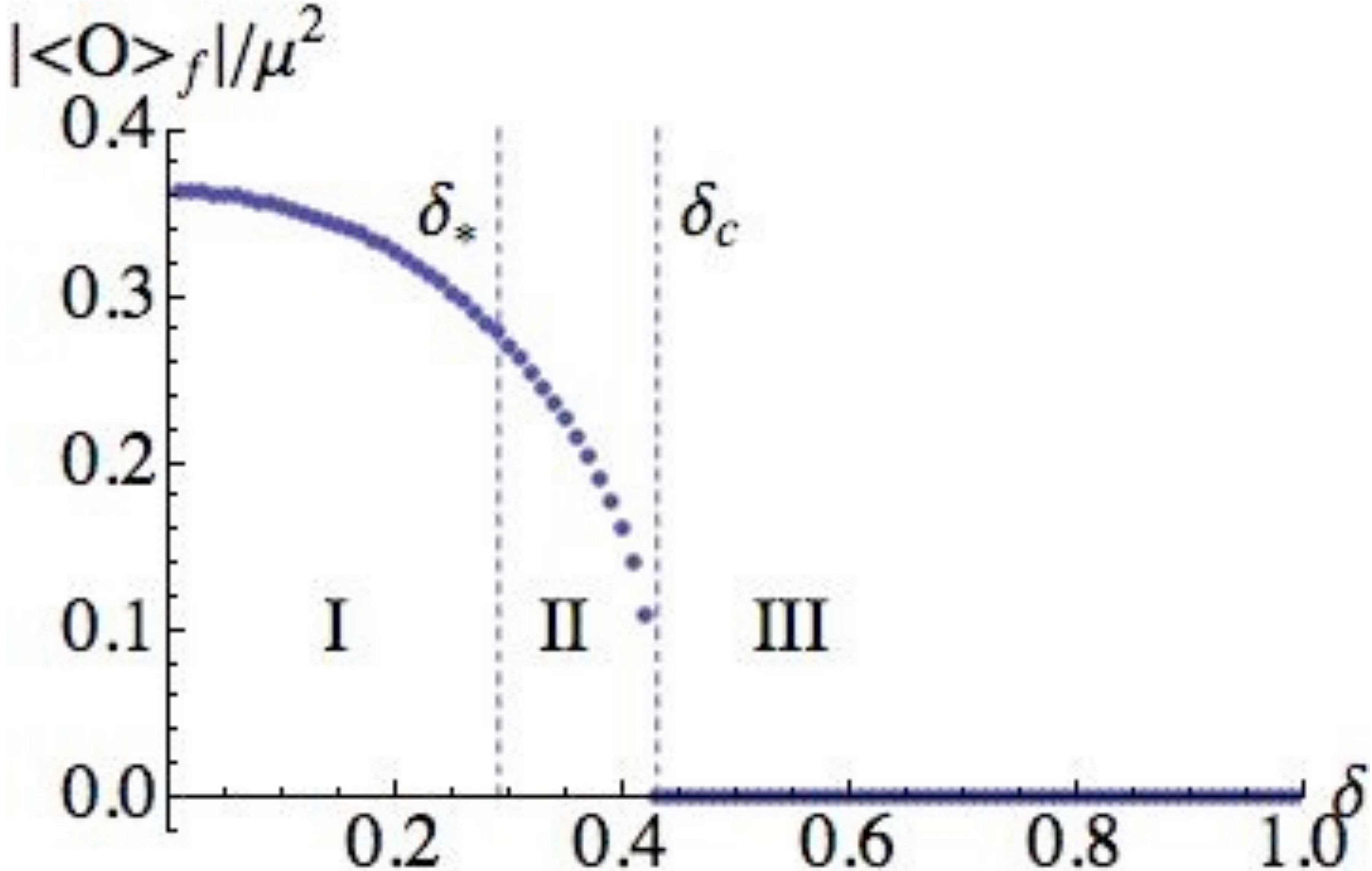
$$S(v,z) = 1 + \ldots$$

Results



At late times it settles down to an equilibrium black hole solution

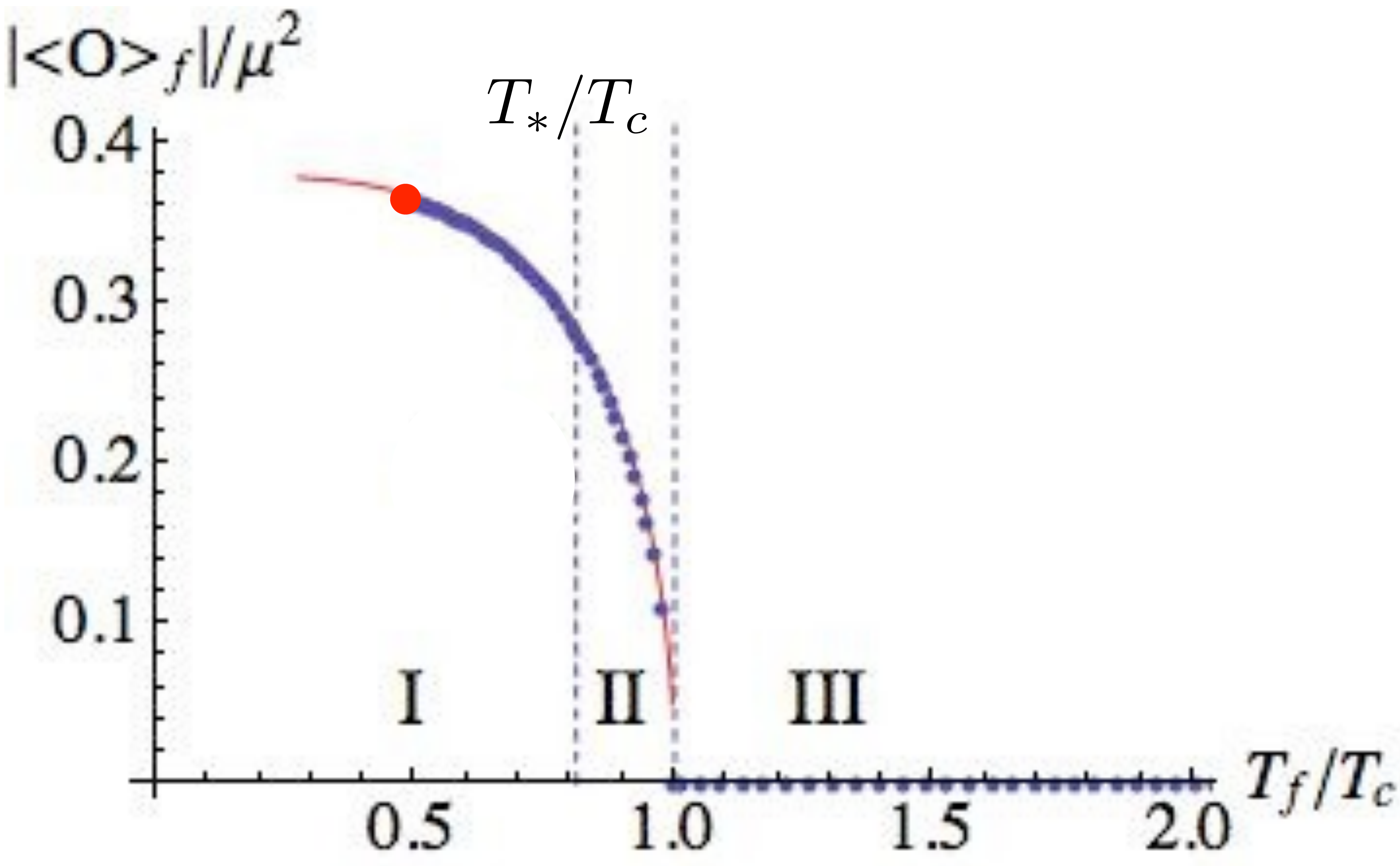
Dynamical phase diagram



Strength of quench

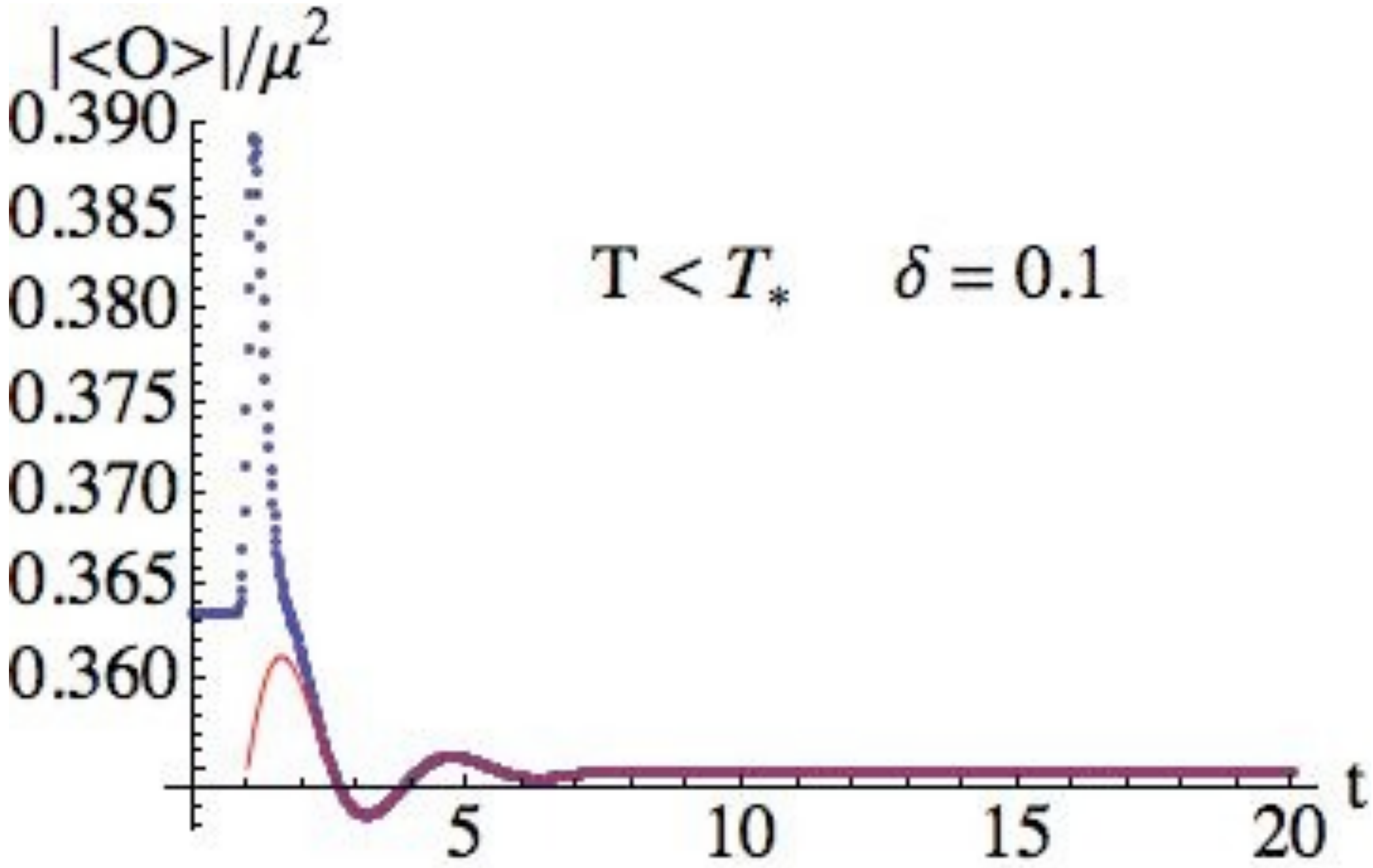
The quench adds energy and heats the superfluid decreasing $\langle \mathcal{O} \rangle$

Phase diagram as a function of temperature of final black hole

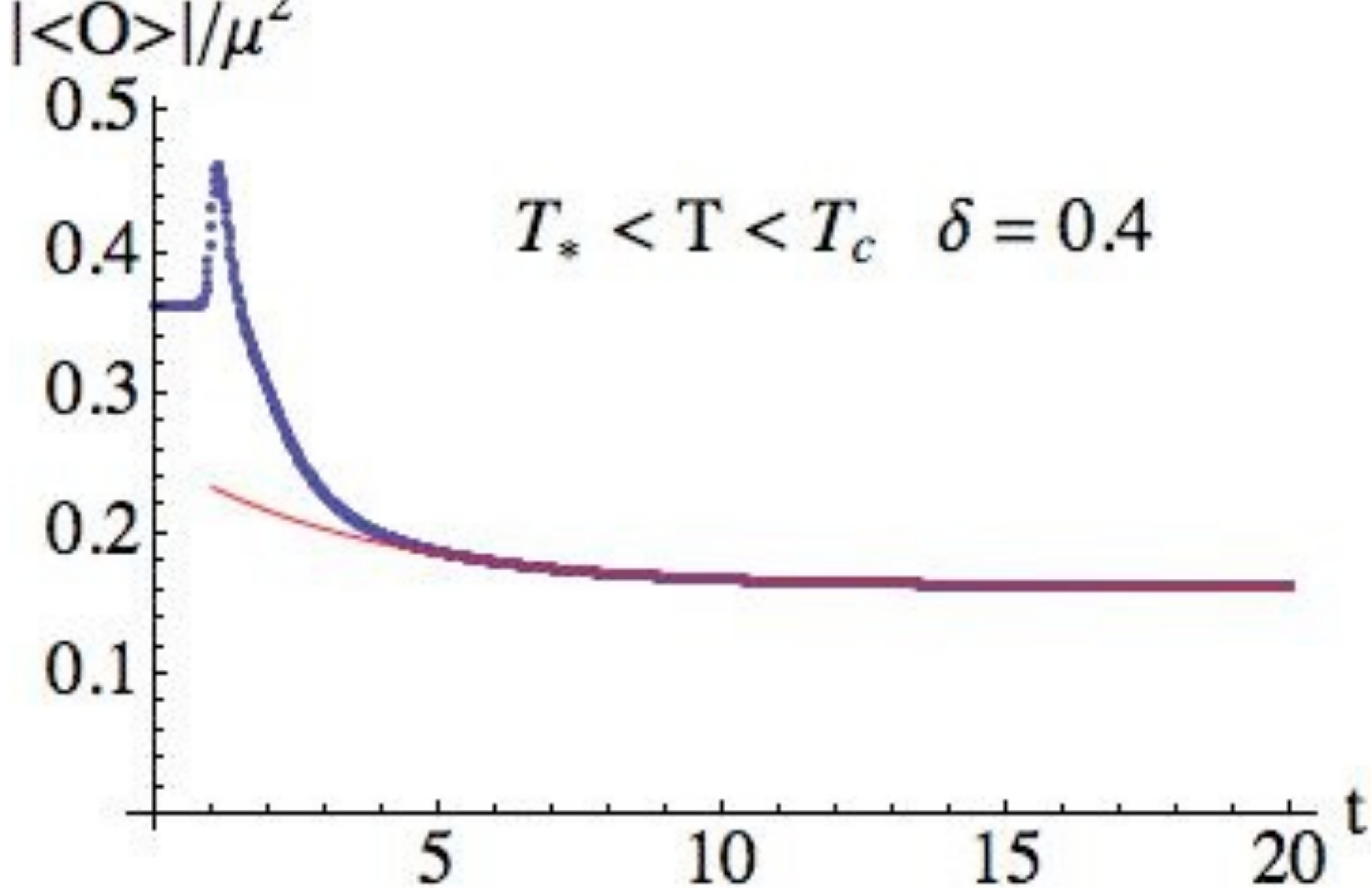


There is an emergent temperature T_* corresponding to the quench δ_*

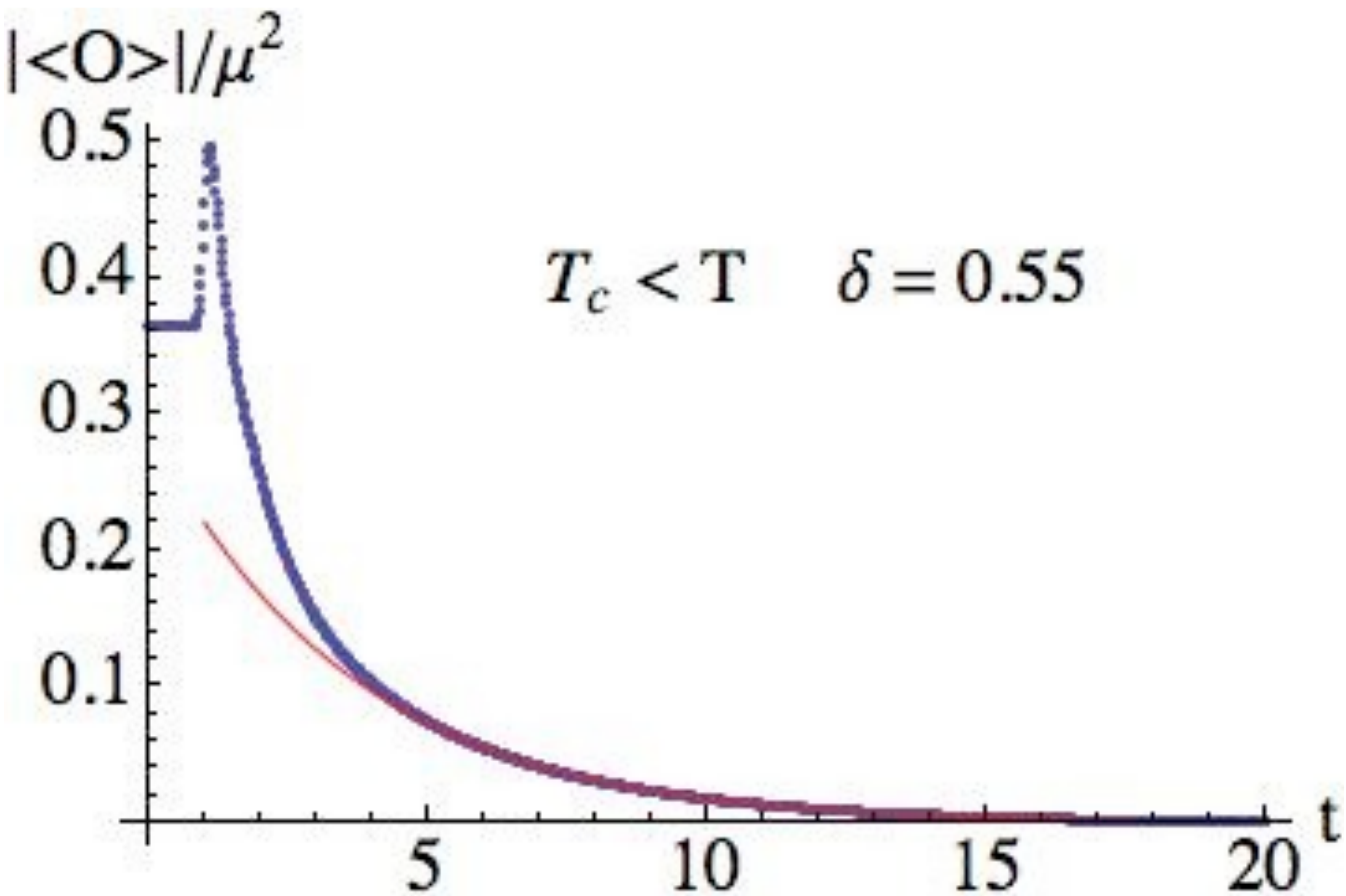
Region I: Small quenches give rise to damped oscillations to a final superfluid black hole

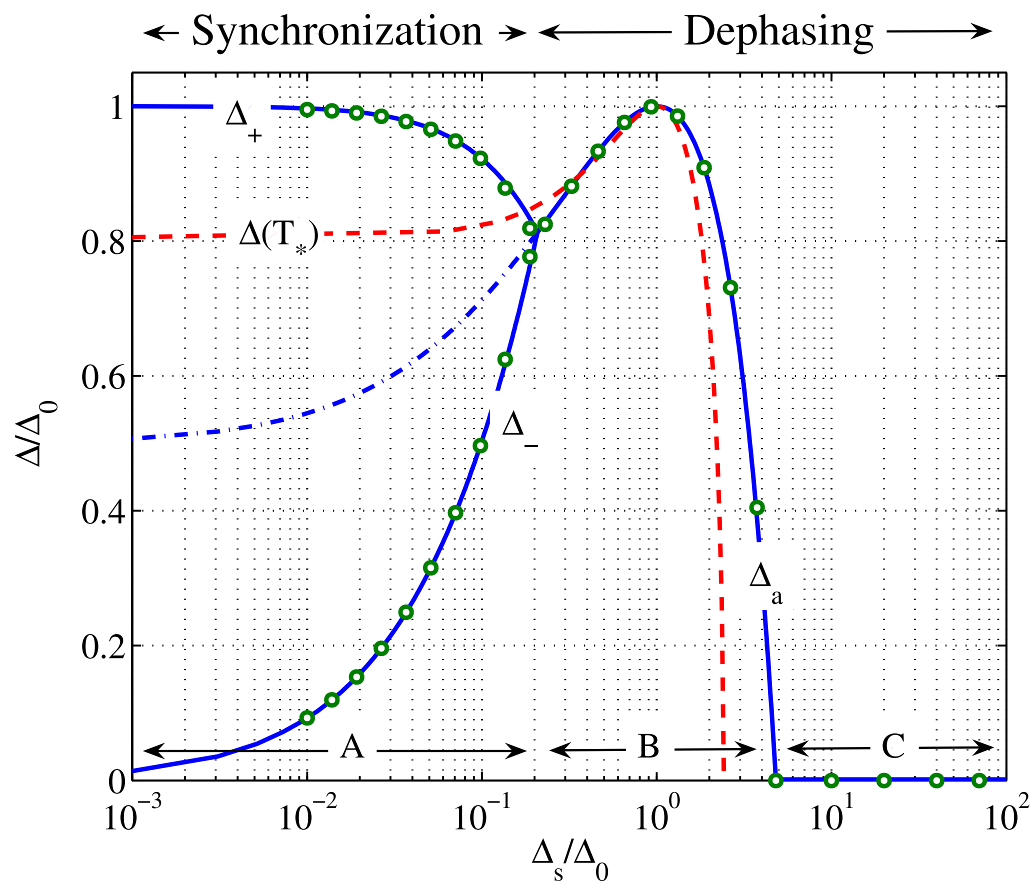


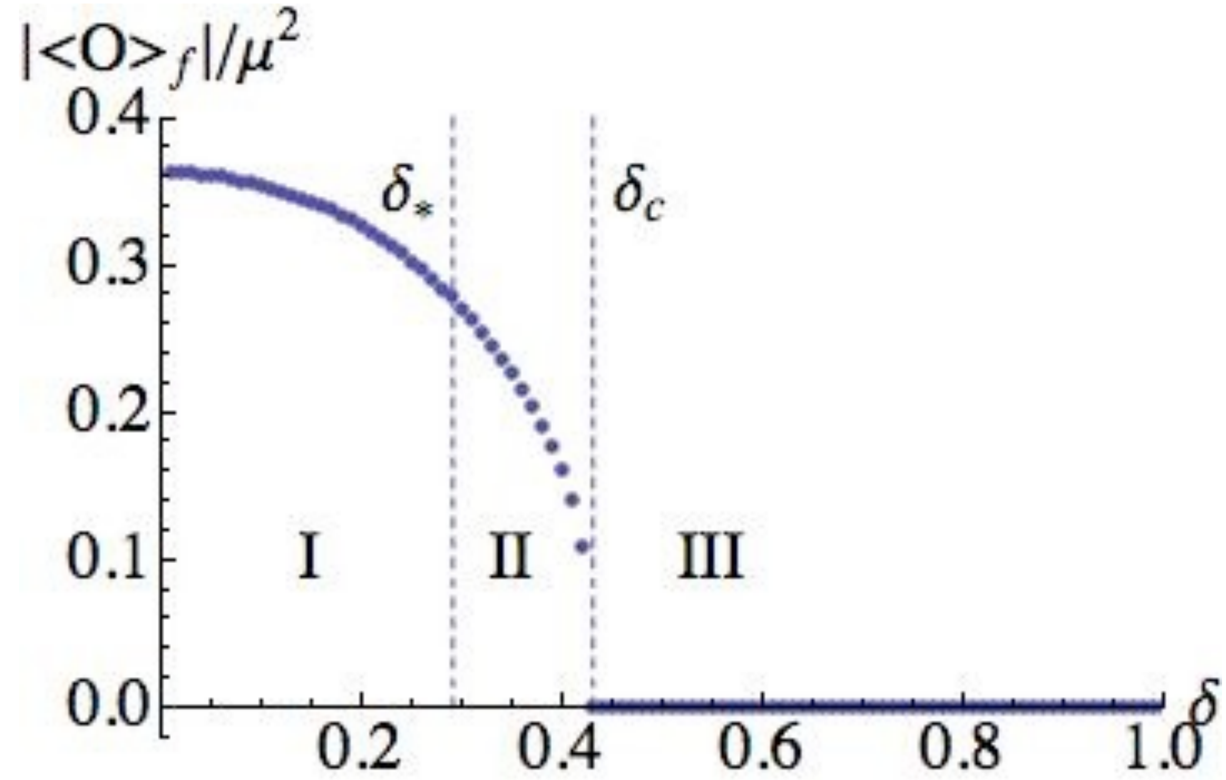
Region II: Larger quenches give rise to a decayed approach to a final superfluid black hole



Region III: Larger quenches still give rise to a decayed approach to the unbroken phase







- The three regions of B-L survive the inclusion of collisions, thermal affects and also in a strongly coupled set up
- Region I: Persistent oscillations in B-L replaced with exponentially damped oscillations
- Region II: power-law damped oscillations in B-L replaced with decay
- δ_* is an analogue of the B-L dephasing transition

Quasi normal modes

- The late time behaviour should be governed by linear response theory, which is determined by the quasi-normal modes of the black hole
- Provides excellent check of numerics and also leads to key insight into where the emergent temperature T_* comes from
- Recall that the QNMs are linearised perturbations with ingoing boundary conditions at the black hole event horizon and are normalisable at the AdS boundary
- They are functions of complex ω . For stable black holes they lie in the lower half plane
- The late time dynamics should be governed by the dominant quasi-normal modes i.e. the QNMs that are closest to the real axis

The QNM pole-dance

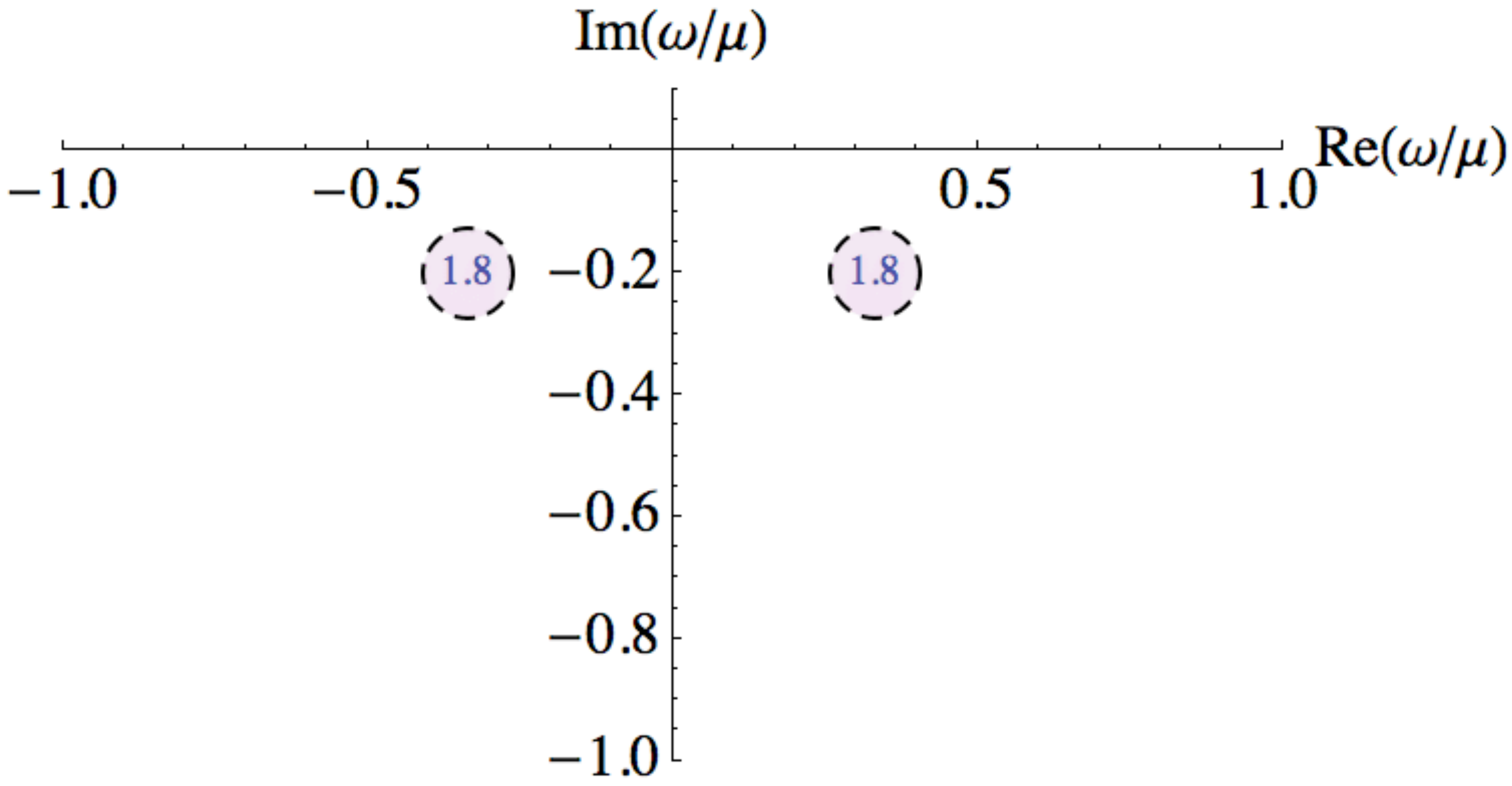
For the homogeneous and isotropic quenches we only consider the QNMs at zero momentum and only consider sector involving $\delta\psi$

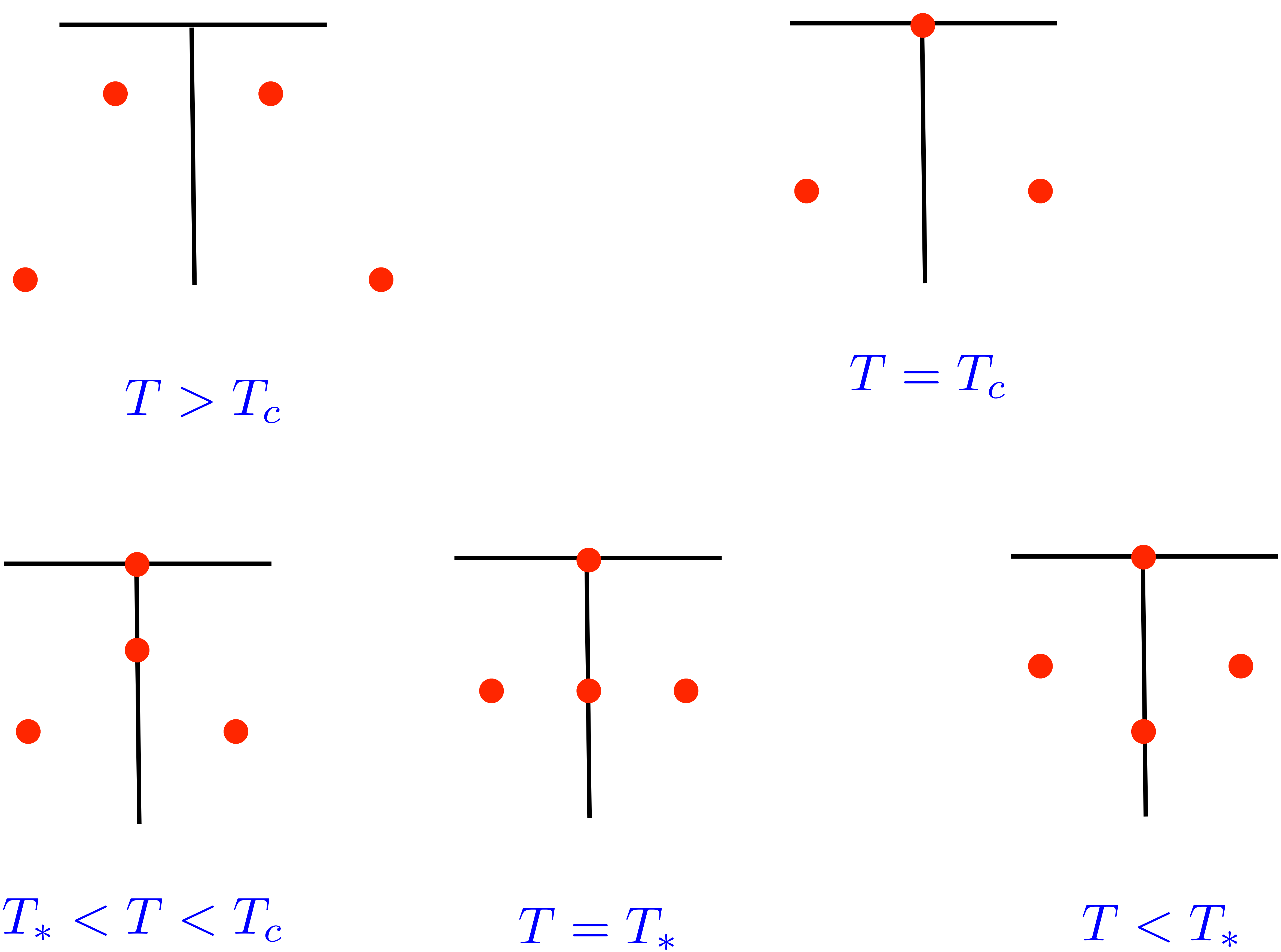
Start with QNMs at $T > T_c$ and follow motion as we decrease T

The QNM pole-dance

For the homogeneous and isotropic quenches we only consider the QNMs at zero momentum and only consider sector involving $\delta\psi$

Start with QNMs at $T > T_c$ and follow motion as we decrease T




$T > T_c$
$T = T_c$
$T_* < T < T_c$
$T = T_*$
$T < T_*$

- Notice symmetry under $\omega \to -\omega^*$ .This comes from time-reversal invariance of the system

- The QNM at $\omega = 0$ is the Goldstone mode. The one sailing down the imaginary axis is the amplitude mode

- The late time dynamics is determined by the dominant QNMs - the ones that lie closest to the real axis

- Roughly, expect the real part of the QNMs are associated with oscillations and the imaginary part with decay

- More precisely we have

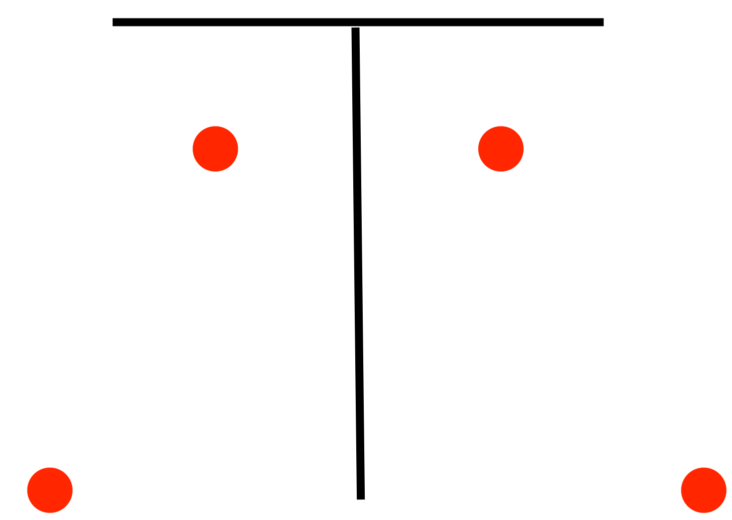
$$|\langle \mathcal{O}(t) \rangle| = |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|$$

where ω corresponds to the dominant QNMs

Region III: $T > T_c$ decayed approach to a final unbroken phase black hole

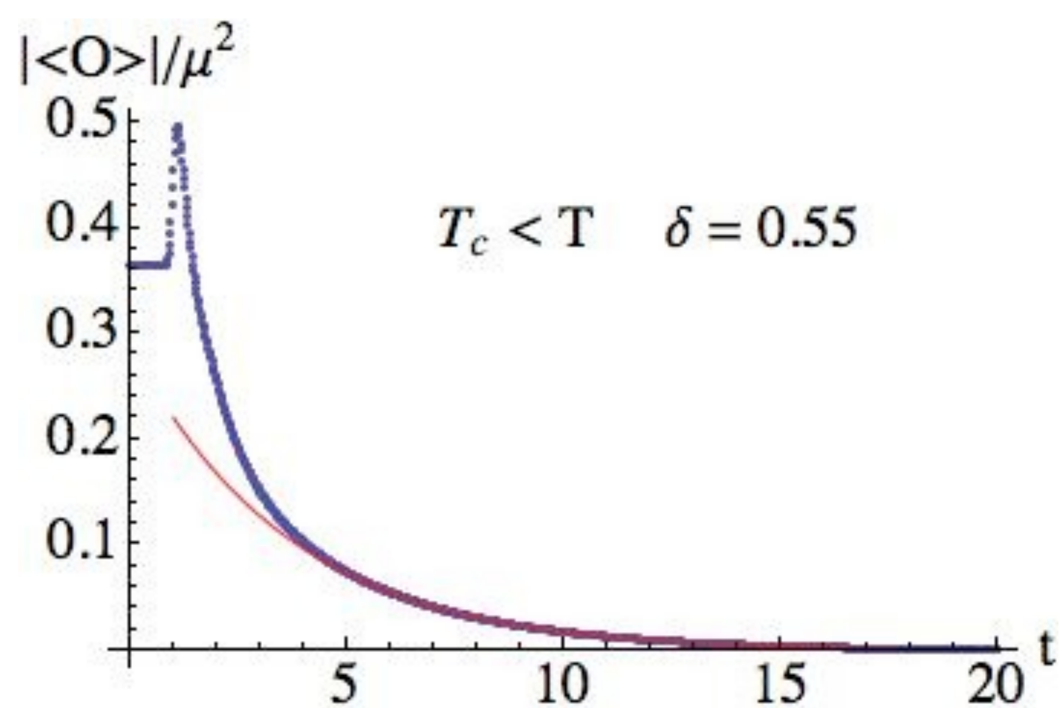
- The dominant QNMs have

$$Re(\omega) \neq 0 \quad Im(\omega) \neq 0$$



- But $\langle \mathcal{O} \rangle_f = 0$ so

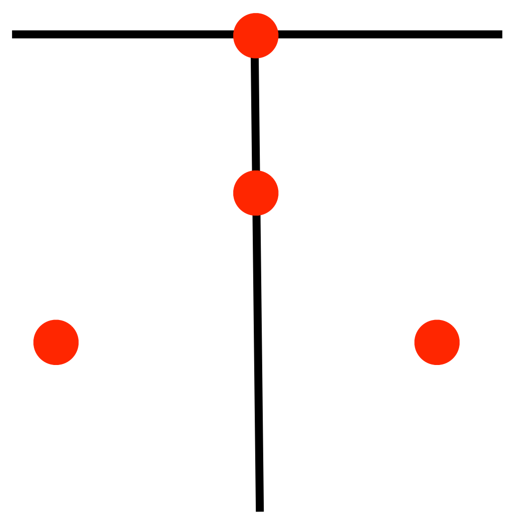
$$\begin{aligned} |\langle \mathcal{O}(t) \rangle| &= |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}| \\ &= |\mathcal{A}| e^{Im(\omega) t} \end{aligned}$$



Region II: $T_* < T < T_c$ decayed approach to a final superfluid phase black hole

- The dominant QNMs have

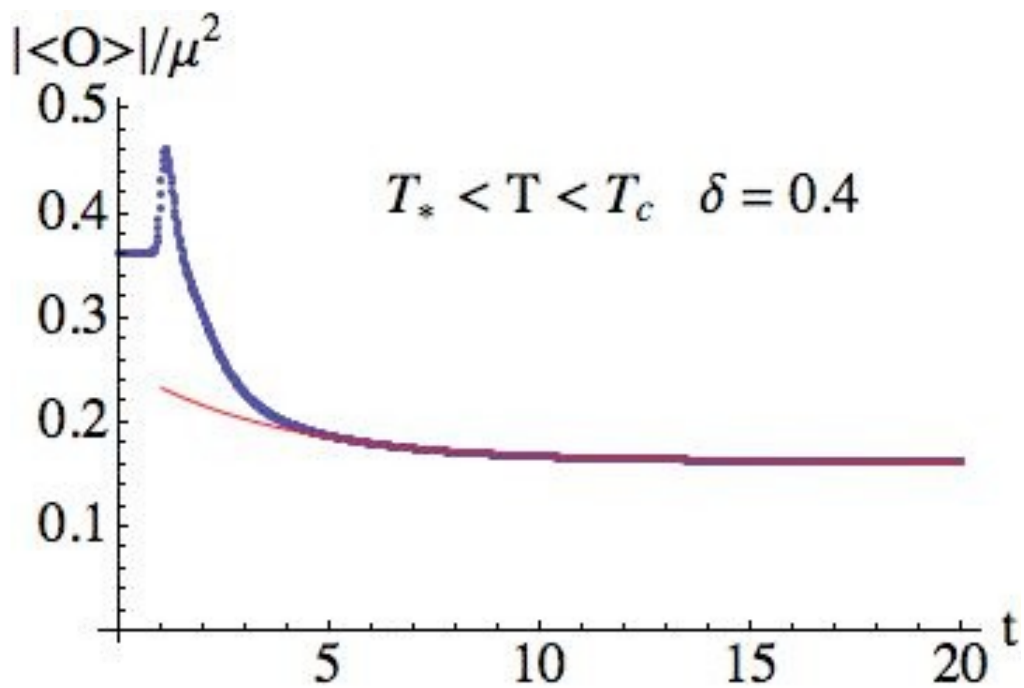
$$Re(\omega) = 0 \quad Im(\omega) \neq 0$$



- Now $\langle \mathcal{O} \rangle_f \neq 0$ so

$$|\langle \mathcal{O}(t) \rangle|^2 = |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|^2$$

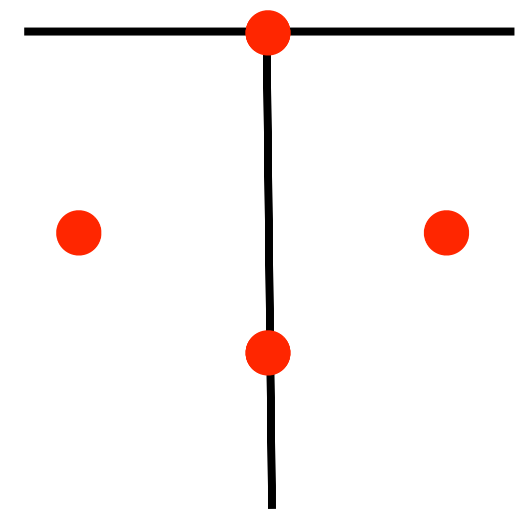
$$= |\langle \mathcal{O} \rangle_f|^2 + |\mathcal{A}|^2 e^{2Im(\omega)t} + 2e^{Im(\omega)t} Re[\langle \mathcal{O} \rangle_f A^*]$$



Region I: $T < T_*$ damped oscillations approaching a final superfluid phase black hole

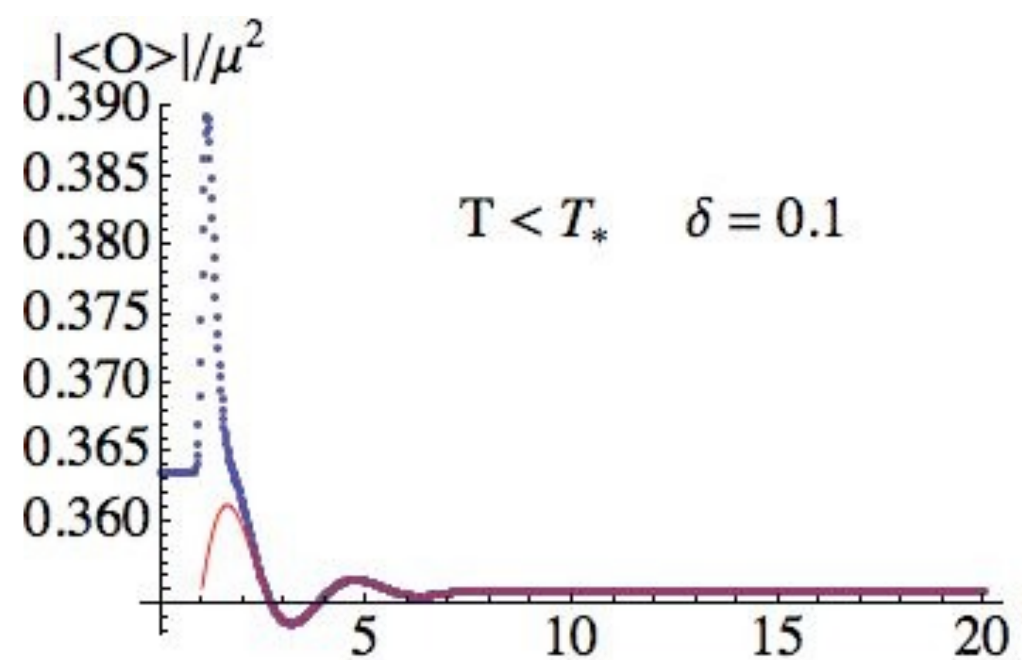
- The dominant QNMs have

$$Re(\omega) \neq 0 \qquad Im(\omega) \neq 0$$



- Now $\langle \mathcal{O} \rangle_f \neq 0$ so

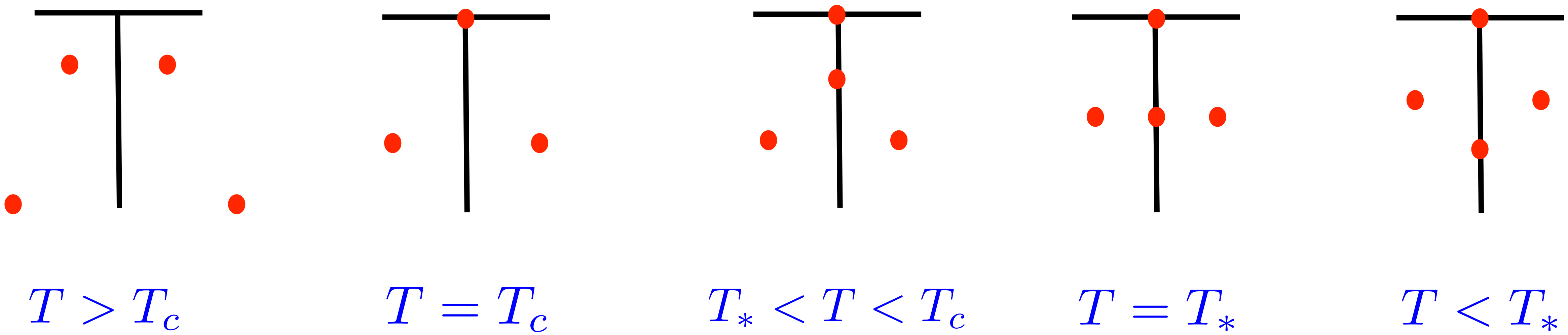
$$\begin{aligned} |\langle \mathcal{O}(t) \rangle|^2 &= |\langle \mathcal{O} \rangle_f + \mathcal{A} e^{-i\omega t}|^2 \\ &= |\langle \mathcal{O} \rangle_f|^2 + |\mathcal{A}|^2 e^{2Im(\omega)t} \\ &+ 2e^{Im(\omega)t}\Big(Re[\langle \mathcal{O} \rangle_f A^*] \cos[Re(\omega)t] \\ &\qquad -Im[\langle \mathcal{O} \rangle_f A^*] \sin[Re(\omega)t]\Big) \end{aligned}$$



Universality

- The main features, including the new dynamical temperature scale T_* in the superfluid phase are captured by the QNMs
- Expect similar results for other holographic superfluids both in d=2+1 and also in 3+1
- The results also have significance for non-holographic systems!
- Recall that the location of QNMs correspond to the location of the poles of the retarded Green's function for the operator $\mathcal{O}$ in the dual CFT
- Thus ANY system that has poles in the retarded Greens function as we have here will give rise to the same late time linear response under a quench

- **Key point**: the pole structure below is the generic structure we expect for a time-reversal invariant system that breaks a continuous symmetry



- The precise value of T_* will depend on the details of the system. There could also be additional temperature scales
- The phenomenon should also be seen if a local symmetry is broken
- Can this been seen in experiment? eg Cold atom experiments

Final Comments

- We have used AdS/CFT to obtain the far from equilibrium dynamics of a strongly coupled superfluid under a quantum quench
- Determined the dynamical phase diagram and explained how its late-time features are determined by the structure of QNMs
- Top down model of [JPG,Sonner,Wiseman]. This model captures infinite class of CFTs and can quench from arbitrary low temperature. Work in progress.
- A universal picture has emerged which covers holographic and non-holographic systems that have time-reversal invariance and assuming spatial homogeneity and isotropy.
- Can we calculate T_* in a weakly coupled theory?
- Can the phenomenology be verified experimentally?