

What Can We Learn From Coset CFTs and Their Duals

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3 > < 3 >

What We Don't Know (in AdS/CFT)?

In our best understood example of N = 4 SYM, we now know (in principle) the spectrum of all single trace operators (at $N = \infty$) for all λ \leftrightarrow perturbative string states.

But we still do not know:

- The spectrum at finite N (for generic λ). Relevant to understanding the physics of black holes and other geometries in AdS₅.
- Correlation functions of arbitrary operators (just for single trace operators) and even at the planar level (for generic λ). Presumably will see progress using integrability techniques.
- Even the spectrum of perturbative string states has no independent derivation yet from the string side apart from at large λ.

Clearly, a lot of work for us to do !

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- The spectrum at finite N (for generic λ). Relevant to understanding the physics of black holes and other geometries in AdS_5 .
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Even novel string geometry features not yet deciphered.

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Preliminary Remarks		Questions
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- Gravity ightarrow ($\lambda \gg 1$) Gauge Theory : Lots of Amazing Results.
- Gauge theory \rightarrow (Stringy/Quantum) Gravity: A Lot Less.



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- A bulk theory whose classical limit (N → ∞) is under reasonable control. A rich set of non-trivial classical solutions (and their gravitational interpretation).
- Compare the classical solutions with boundary description. Understand what stringy features translate into in the boundary language. (Entropy of thermal states, singularity resolution, locality)
- Use the boundary CFT at finite *N* to learn about quantum modifications (recovery of information).

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Boundary 2d CFT

- $\frac{G}{H}$ WZW theories are solvable. There is an intricate spectrum (for any N) and correlators computable. As a bonus, non-supersymmetric.
- Presence of a large unbroken higher spin symmetry (W_N or often much larger).

- Higher spin gauge fields (s = 2, 3...) in AdS₃ are non-propagating (though can have propagating scalars/fermions).
- Classical Vasiliev theories describing them are relatively tractable -"string-field-theory-lite". Many non-trivial classical solutions.
- Can already see novel string-like generalizations of geometry.
- Lots more to be uncovered and lessons to be learnt.

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Two kinds of Coset Dualities

Vector-Like Cosets: Central charge $c \propto N$. For example W_N minimal models

 $\frac{SU(N)_k \times SU(N)_1}{SU(N)_{k+1}}.$

Take $k, N \rightarrow \infty$ with ratio fixed. (Gaberdiel and R. G.)

Matrix-Like Cosets: Central charge $c \propto N^2$. For example, extended W_N models

 $\frac{SU(N)_k \times SU(N)_\ell}{SU(N)_{k+\ell}}.$

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Vector Coset Holography

The CFT: Minimal W_N series (generalising the Virasoro unitary series for N = 2)

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A line of fixed points when $k, N \to \infty$ (labelled by 'tHooft coupling $0 \le \lambda = \frac{N}{N+k} \le 1$) with $c_N(\lambda) = N(1 - \lambda^2)$.

The Bulk: Vasiliev higher spin theory in AdS_3 with gauge group $hs[\lambda]$ coupled to one complex scalar of mass

 $M^2 = -1 + \lambda^2.$

Central charge $c = \frac{3R_{AdS}}{2G_N}$ (Campoleoni et.al., Henneaux-Rey) i.e. $G_N \propto \frac{1}{N}$.

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What we know about the Duality

- Matching symmetries: Underlying W_∞[λ] symmetry. At finite N, (i.e. finite c) both sides have W_N symmetry due to a nontrivial equivalence of the quantum W_∞[λ] symmetry algebra (Gaberdiel-R.G.). Powerful non-linear symmetry.
- Matching Spectrum:

a) Perturbative excitations. (Gaberdiel-R.G.-Hartman-Raju)

- b) Non-perturbative excitations ("conical defects")
- (Castro-R.G.-Gutperle-Raeymakers; Perlmutter, Prochazka, Raeymakers).
- c) Black Hole States. (Kraus-Perlmutter, Gaberdiel-Hartman-Jin)
- Correlation functions: Matching of three point functions of scalars and currents. Large N factorisation of multi-trace operators (Chang-Yin, Kraus-Perlmutter, Papadodimas-Raju). Two point correlator on torus calculated for any N. Need to take large N limit (Chang-Yin).

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• $\mathcal{N} = 1, 2$ Kazama-Suzuki models (Creutzig-Hikida-Ronne).

Analogue of 3d/4d proposal relating vector models (in general with Chern-Simons interactions) to higher spin theories on AdS_4 . (Klebanov-Polyakov; Sezgin-Sundell; Giombi-Yin; GMPTWY; Aharony et.al.) Many parallels (as well as differences - cf. Maldacena-Zhiboedev) between the two cases.



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The Bulk Spectrum

Classification of Bulk Solutions

Let's look at known classical bulk solutions and their boundary interpretation. In increasing order of complexity they are:

- Perturbative quanta of the bulk higher spin fields and the scalar (Gaberdiel-R. G.-Saha; Gaberdiel-R.G.-Hartman-Raju). [In SUSY case (Creutzig-Hikida-Ronne, Candu-Gaberdiel)].
- Smooth conical defect like geometries with non contractible time circle. (Castro-R.G.- Gutperle-Raeymakers). With scalar - Perlmutter, Prochazka, Raeymakers. [In SUSY case (Tan; Datta-David)].
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The Perturbative Bulk Spectrum

$$Z_{\text{bulk}} = Z_{class} Z_{1-loop} = (q\bar{q})^{-c/24} Z_{\text{HS}} Z_{\text{scal}}(h)^2.$$

$$Z_{HS} = \prod_{s=2}^{\infty} \prod_{n=s}^{\infty} rac{1}{|1-q^n|^2} = \prod_{n=1}^{\infty} |1-q^n|^2 imes \prod_{n=1}^{\infty} rac{1}{|(1-q^n)^n|^2} \equiv | ilde{M}(q)|^2.$$

Bulk one loop determinants of higher spin fields (Gaberdiel-R. G.-Saha).

$$Z_{scal}(h) = \prod_{l=0,l'=0}^{\infty} \frac{1}{(1-q^{h+l}\bar{q}^{h+l'})}.$$
 (1)

Determinant of scalar - dual to operator of dim. $h = \frac{1}{2}(1 + \lambda)$. (Giombi-Maloney-Yin)

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The Bulk Spectrum		

- Z_{HS} matches with the CFT spectrum of the vacuum and its descendants in the large N 'tHooft limit.
- $Z_{scal}(h)^2$ matches with that of the CFT primaries (Λ_+ ; 0) and descendants where Λ_+ is an SU(N) rep. built from finite tensor powers of fundamental and antifundamental. (GGHR)
- Quite nontrivial that these CFT primaries behave like single and multi-particle states i.e. correlators obey large *N* factorisation. (Pappadodimas-Raju; Chang-Yin).
- However, these by themselves do not give a modular invariant spectrum nor account for all the operators of dimension of order one.
- Whole families of other states some forming a continuum down to zero ("light states") many of which also obey large N factorisation.
- QUESTION: What do these "non-perturbative" states correspond to in the bulk?

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- Quite nontrivial that these CFT primaries behave like single and multi-particle states i.e. correlators obey large N factorisation. (Pappadodimas-Raju; Chang-Yin).
- However, these by themselves do not give a modular invariant spectrum nor account for all the operators of dimension of order one.
- Whole families of other states some forming a continuum down to zero ("light states") many of which also obey large N factorisation.
- QUESTION: What do these "non-perturbative" states correspond to in the bulk?

	Vector Cosets	Questions
The Bulk Spectrum		

• Candidates for the dual of non-perturbative states.

- But interesting solutions of the bulk equations of motion in their own right. Found in $hs[\lambda = N] = SL(N)$ Chern-Simons theories with no scalar field turned on. Exist only for N > 2. (CGGR).
- Given by flat connections with holonomy in the spatial circle lying in non-trivial central elements of SL(N) (since gauge group is SL(N, C)/Z_N).

$$W_{\phi} = P \exp i \oint_{\phi} A \in \mathbb{Z}_N.$$

Thus smooth configuration since spatial (ϕ) circle is contractible.

• Discrete family of solutions where gauge field eigenvalues $\propto n_i - \frac{(\sum_i n_i)}{N}$ equivalent to label of SU(N) irrep. Λ_+ .

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Obstacle: we do not understand the centre of the group corresponding to the Lie algebra $hs[\lambda]$. Seems to contain a \mathbb{Z}_4 (Kraus-Perlmutter).

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$$Z_{\Lambda_-} = \sum_{\Lambda_+} |b_{(\Lambda_+,\Lambda_-)}(q)|^2.$$

Compelling reason therefore to identify the(Λ₊, Λ₋) states in the CFT as bound states of perturbative scalar quanta (Λ₊, 0) bound to (0, Λ₋) conical defect states in the SL(N) theory.

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$$a = (L_1 - \frac{2\pi}{k}\mathcal{L}L_{-1} - \frac{\pi}{2k}WW_{-2})dz - \mu(W_2 - \frac{4\pi\mathcal{L}}{k}W_0 + \frac{4\pi^2\mathcal{L}^2}{k^2}W_{-2} + \frac{4\pi\mathcal{W}}{k}L_{-1})d\bar{z}.$$

Here \mathcal{L}, \mathcal{W} are the spin two and spin three charge and μ the chemical potential for \mathcal{W} . Reduces to BTZ when $\mathcal{W} = 0$.

Demand the smoothness of geometry by requiring holonomy W_t in the time direction (contractible) to be trivial and equal to that for BTZ.
 Gives two relations between L, W, β and μ. Then obeys first law of thermodynamics. Also ∂L/∂μ = ∂W/∂β ⇒ Z(β, μ) exists.

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$$\ln Z_{BH}(\tau = \frac{i\beta}{2\pi}, \mu) = \frac{ic\pi}{12\tau} \left[1 - \frac{4\mu^2}{3\tau^4} + \frac{400}{27} \frac{\lambda^2 - 7}{\lambda^2 - 4} \frac{\mu^4}{\tau^8} - \frac{1600}{27} \frac{5\lambda^4 - 85\lambda^2 + 377}{(\lambda^2 - 4)^2} \frac{\mu^6}{\tau^{12}} + \ldots\right].$$

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If we keep ℓ finite as $k, N \to \infty$ then we have essentially ℓ flavors of the original theory.

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- Arises as IR description of two majorana fermions (in adjoint rep.) coupled to SU(N) gauge field. (In turn describes the high density limit of 2d Dirac fermion coupled to gauge field linearise about the Fermi surface). Interacting CFT 2d analogue of strange metal.
- Interesting feature is the emergent $\mathcal{N}=(2,2)$ SUSY not present in the microscopic theory.
- Partial analysis of modular invariant spectrum. Find operator dimension of density $Tr(\psi^{\dagger}\psi)$ to be $\frac{1}{3}$ for all *N*. Similarly pair operator also has $\Delta = \frac{1}{3}$. Quartic relevant operators with $\Delta = \frac{2(N-2)}{3N}$.
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Questions for the Bulk

- What is the temperature regime in which black hole states dominate the thermodynamics? Make a detailed identification of the CFT microstates ($h \propto N^{\alpha}$) which contribute to BH entropy. Note the unusual thermodynamics of CFT states (Banerjee et.al.) $F \propto NT \ln \frac{T}{\lambda^2}$ for small λ .
- Need to also study more solutions of the bulk hs[λ] theory. Black hole solutions with scalar field nonzero. Those corresponding to different thermodynamic branches (David et.al).
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Integrate these theories into the framework of gauge-string dualities and gain a general understanding of the domain of applicability of these ideas. Useful for connecting with non-SUSY real world systems.

In particular, one can envisage an increasing order of complexity of models:

 Integrable deformations of vector like CFTs. Flows between minimal models.

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