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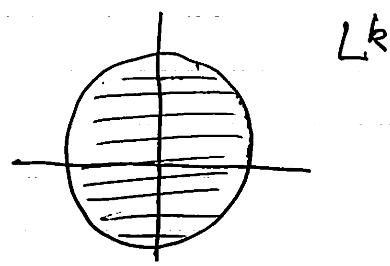
PAULI EXCLUSION @ STRONG COUPLING

Take a system with a global $U(1)$ symmetry.

Place at nonzero μ $\Rightarrow \langle J^t \rangle \neq 0$ [if not gapped, etc].

Free bosons $\rightarrow$ Bose condense $\cancel{U(1)}$

Free fermions $\rightarrow$ Fermi surface.

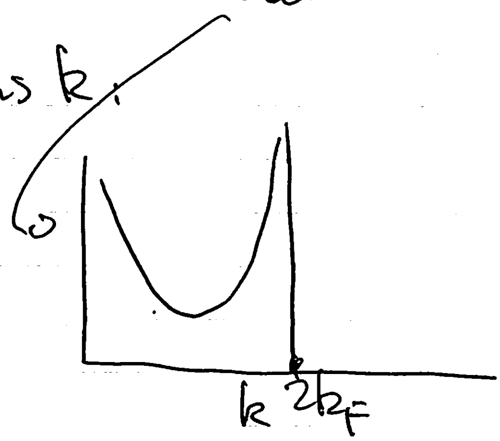


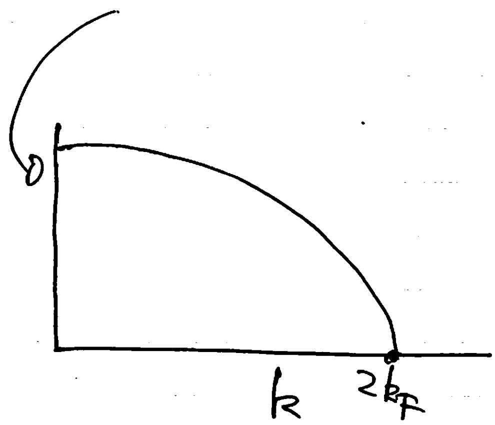
Pauli exclusion ① Prevents $\cancel{U(1)}$
② Pushes low energy excitations out to nonzero momentum.

How do we see the low energy excitations without assuming anything about the system:

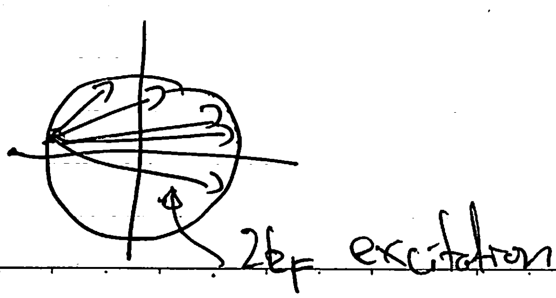
$$\lim_{\omega \to 0} \frac{\text{Im}\, G^R_{J^t J^t}(k,\omega)}{\omega} \quad \text{and} \quad \lim_{\omega \to 0} \frac{\text{Im}\, G^R_{J_\perp J_\perp}(k,\omega)}{\omega}$$

versus k:





why:



Kramers-Kronig.

Note $\lim_{\omega\to\infty} G^R(\omega,k)$ is real and $= \int_0^\infty d\omega \frac{\operatorname{Im} G^R(\omega,k)}{\omega}$

is not a universal low energy quantity.

Low energy spectral weight at finite k is phenomenologically crucial for Fermi liquids.

~~e.g. resistivity due to a lattice~~

theorem: If $\dot{P}=0$ (translation inv) and $\chi_{PJ}\neq 0$ (net charge $\langle J^t\rangle\neq 0$)

$\Rightarrow \sigma_{DC}=\infty$.

two ways to break $\dot{P}=0$: • heavy objects : $\bullet\longrightarrow$
• dilute : $\bullet\longrightarrow$

~~phe~~ most realistic for non-Fermi liquids: lattice.

scattering by a lattice violates momentum conservation: $\vec{k}\to\vec{k}+\vec{k}_L$

to add a lattice to a QFT, add coupling:

$$\Delta S = \int d^dx \, \underbrace{V(x)}_{\text{periodic coupling}} \, \mathcal{O} \;\; \to \text{operator}$$

if lattice coupling is irrelevant, can treat perturbatively in the IR.

result: $\rho_{DC} = \frac{1}{\sigma_{DC}} = \bar{V}^2 k_L^2 \lim_{\omega\to 0} \frac{\operatorname{Im} G^R_{\mathcal{OO}}(k_L,\omega)}{\omega}$

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For free bosons: dispersion $\omega\sim k$, no low energy degrees of freedom at $k=k_L$.

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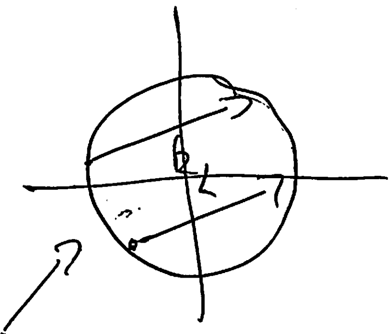
Boltzmann suppressed:

$$\text{Im}\, G^R_{00} \sim e^{-k_L/T} \Rightarrow \underline{\text{tiny resistivity}}$$

Fermi liquid:

umklapp.



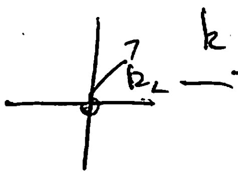
$\cancel{k_L < 2k_F}$

$$\Rightarrow \text{Im}\, G^R_{00} \sim T^2$$

$\mathcal{O} \sim \sum_{k} \psi^\dagger \psi^\dagger \psi \psi$ → first irrelevant operator that couples to lattice.

$r \sim T^2$, famous result, requires interplay of k_L and k_F.

[cf. boson



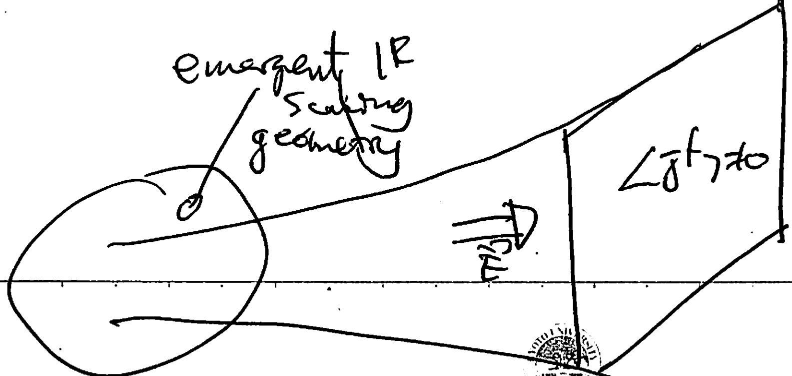
→ takes off shell]

To have any hope of getting e.g. $r \sim T$ from lattice scattering → need spectral weight at $k \neq 0$.

Pauli exclusion somewhat single-particle based. What of strong coupling?

Current AdS/CFT picture at $T \approx 0$:

Kiritsis et al, '10 ...



IR geometry:

hyperscaling violation exponent

$$ds^2 \sim r^{\frac{2\theta}{d}}\left(\frac{-dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{d\vec{x}^2}{r^2}\right)$$

dynamical critical exponent.

theory: eg: $\mathcal{L} \sim R + Z[\phi]F^2 + V[\phi] + (\nabla\phi)^2$

$$\left.\begin{array}{l} Z[\phi] \sim e^{\alpha\phi} \\ V[\phi] \underset{\text{as } \phi\to\infty}{\sim} e^{\beta\phi} \end{array}\right\}$$ has these spaces as solutions in the IR.

$\{z,\theta\}$ depend on $\{\alpha,\beta\}$.

As $\omega \to 0$, easy to show

$$\frac{\operatorname{Im} G^R_{JJ}(\omega,k)}{\omega} \simeq \frac{\operatorname{Im} G^R_{JJ}(\omega,k)}{\omega}\Big|_{IR}$$

can compute in IR.

[not true for real part]

<u>result</u> (A) if $z < \infty$, any θ: $\operatorname{Im} G^R_{JJ}$ is exponentially small as $\omega \to 0$, k finite.

Hartnoll-Shaghoulian 1203.4236

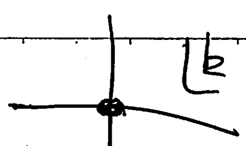
$$\sim e^{-\left(\frac{k^z}{\omega}\right)^{1/z-1}}$$

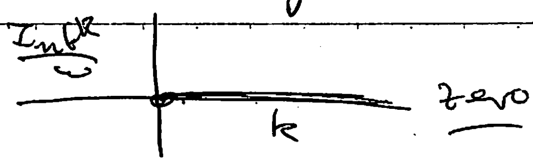
$\Rightarrow$ none of these show any structure in momentum space.

- do not look like Fermi liquids and do not behave like Fermi liquids.

[Ogawa, Takayanagi, Ugajin '11]

[these include the cases with a log entanglement entropy]





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Θ finite : $AdS_2 \times \mathbb{R}^2$

(B) if $z = \infty$

$\frac{\Theta}{z} = \eta$ finite : $r^{\#}(AdS_2 \times \mathbb{R}^2)$

then $$\frac{G^R_{JJ}(\omega)}{\omega} \sim \omega^{2f(k)-1}$$ [median finite T]

known

Semi-local criticality : k does not scale, but dimensions depend on k.

Iqbal, Liu, Mezei '11

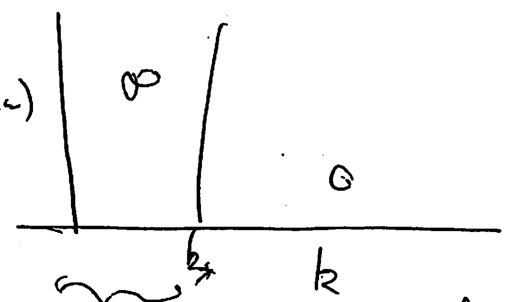
find : O_k relevant : $\Rightarrow 2f(k)-1<0$

irrelevant : $\Rightarrow 2f(k)-1>0$.

For $0<\eta<2$ we found, in transverse channel.

Anantua-Hartnoll-Martin-Ramirez 1210.1590

$\lim_{\omega\to 0} \frac{G^R_{J_\perp J_\perp}(k,\omega)}{\omega}$



↳ finite range of momenta for low energy spectral weight!

→ resistivity: $$r \sim T^{2f(k_L)}$$ power law

$$(z<\infty \Rightarrow r \sim e^{-1/T}).$$

Actually : $\eta=1$ interesting : • $AdS_4 \times S^7$ with 3 nonvanishing charges and 1 vanishing.

• lift to M-theory has AdS_3 factor.

→ extra symmetries.

conductivity due to disorder: $r \sim T^{2f(0)} \sim T$

linear in T! :)

In longitudinal channel, all modes irrelevant.

Punchline: • local quantum criticality with $\sigma < \eta$ 2 2 are only ~~the~~ models exhibiting Pauli-exclusion-like physics + corresponding phenomenology

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Another set of models: D3/D5 + generalizations.

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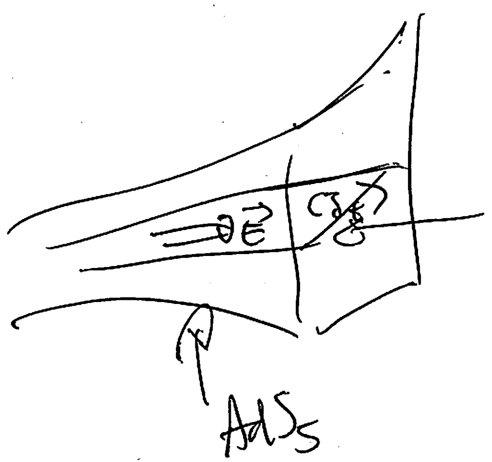


$AdS_4 \times S^2 \subset AdS_5 \times S^5$

• DBI action controls brane fluctuations.

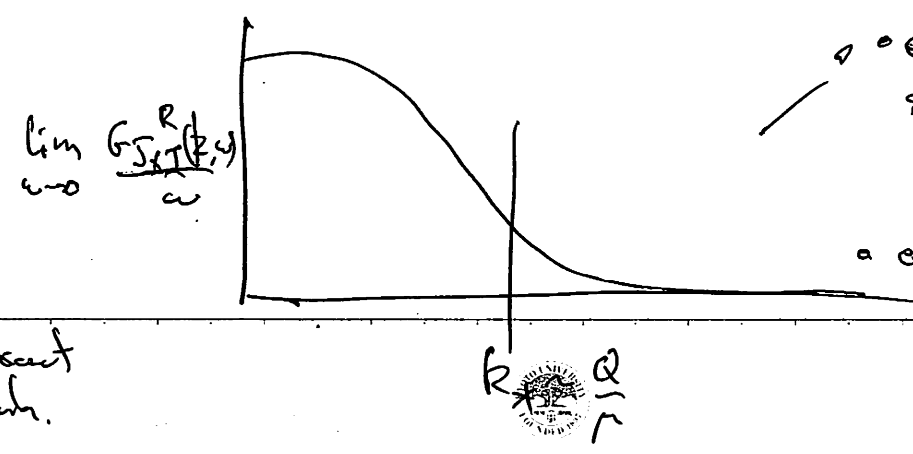
• "Baryonic" $U(1)$.

no local criticality (no backreaction) but nonlinearities in the action.

• found that in IR, k drops out of eqn.

• Find

found exact formula.



• exponentially small spectral weight for $k > k_f$

• order 1 for $k < k_f$.

Punchline: nonlinear DBI dynamics also leads to

spectral weight at $\omega \to 0$, k finite

$\to$ less clear what the RFT organizing principle is here.

(not local criticality)