Is QCD ferromagnetic?

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Holographic Magnetars

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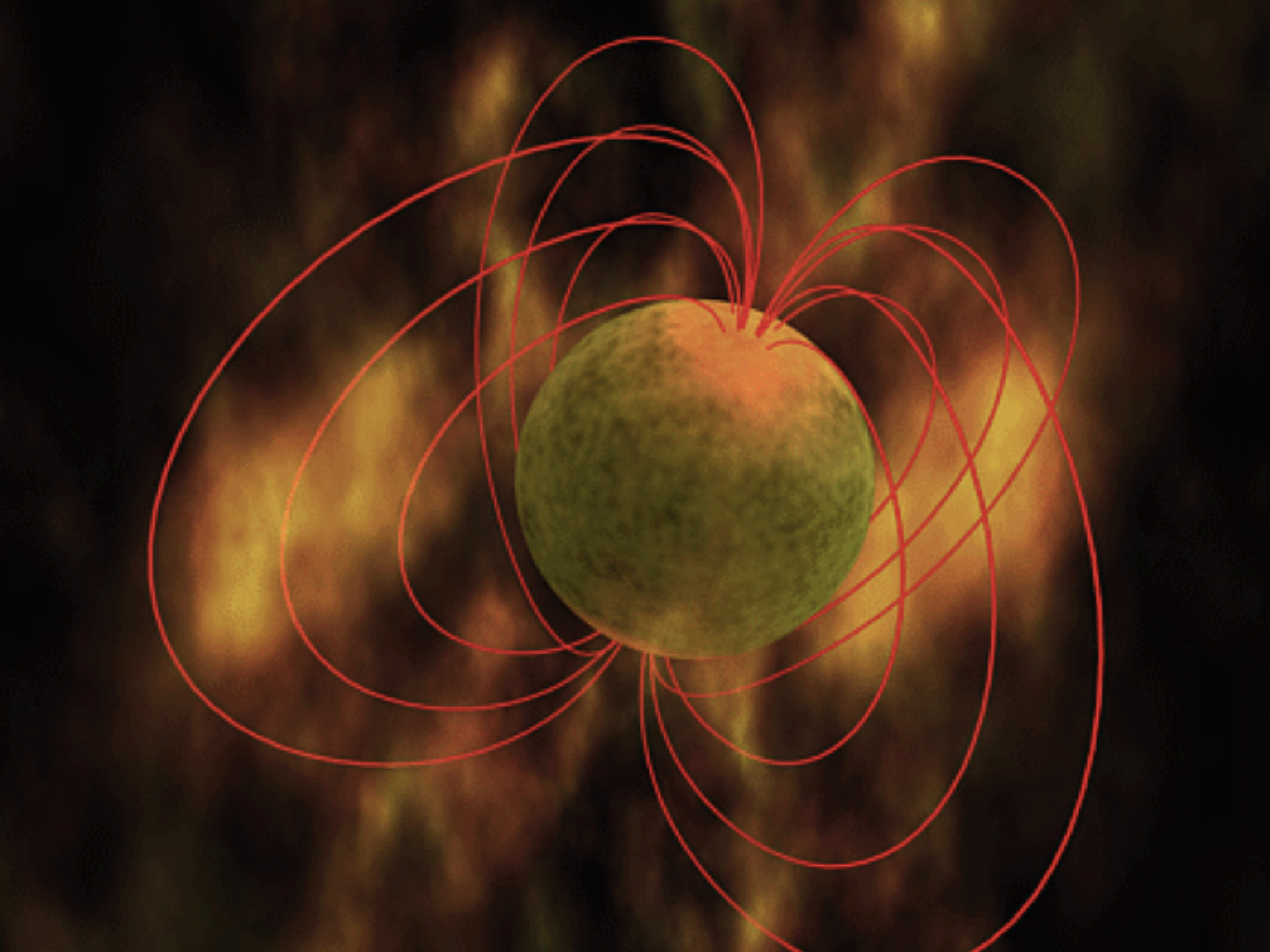
arXiv/1209.4814[hep-ph] + work in progress
w/ M. Eto (Yamagata), T.Hatsuda (RIKEN)



Problem

Origin of magnetars?

Cause

No magnetization at high density hadron phase?

Our solution

High density neutron matter

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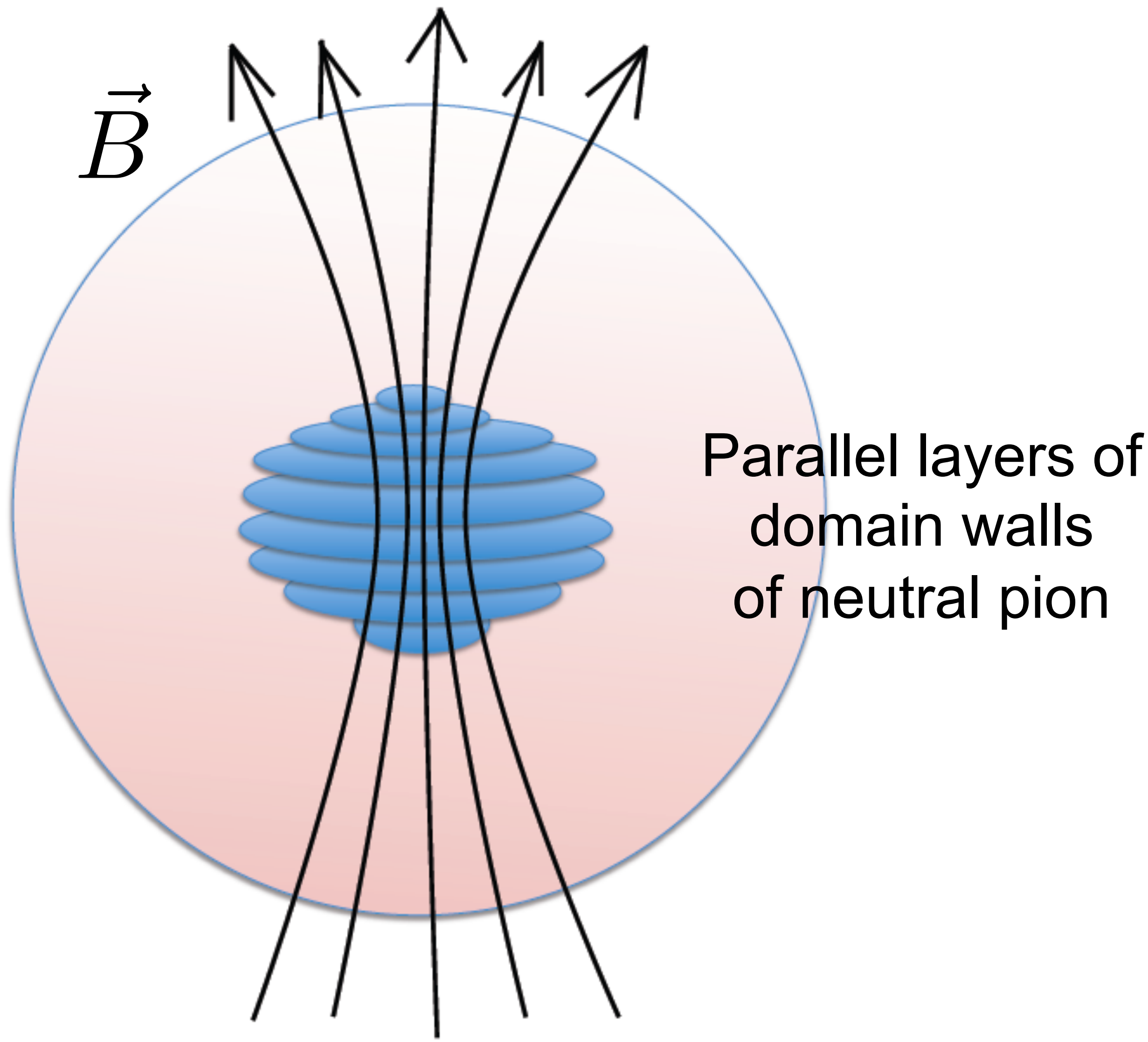
Axial anomaly + π^0 domain wall

↓

New phase : QCD Ferromagnetism


$\vec{B}$
Parallel layers of
domain walls
of neutral pion

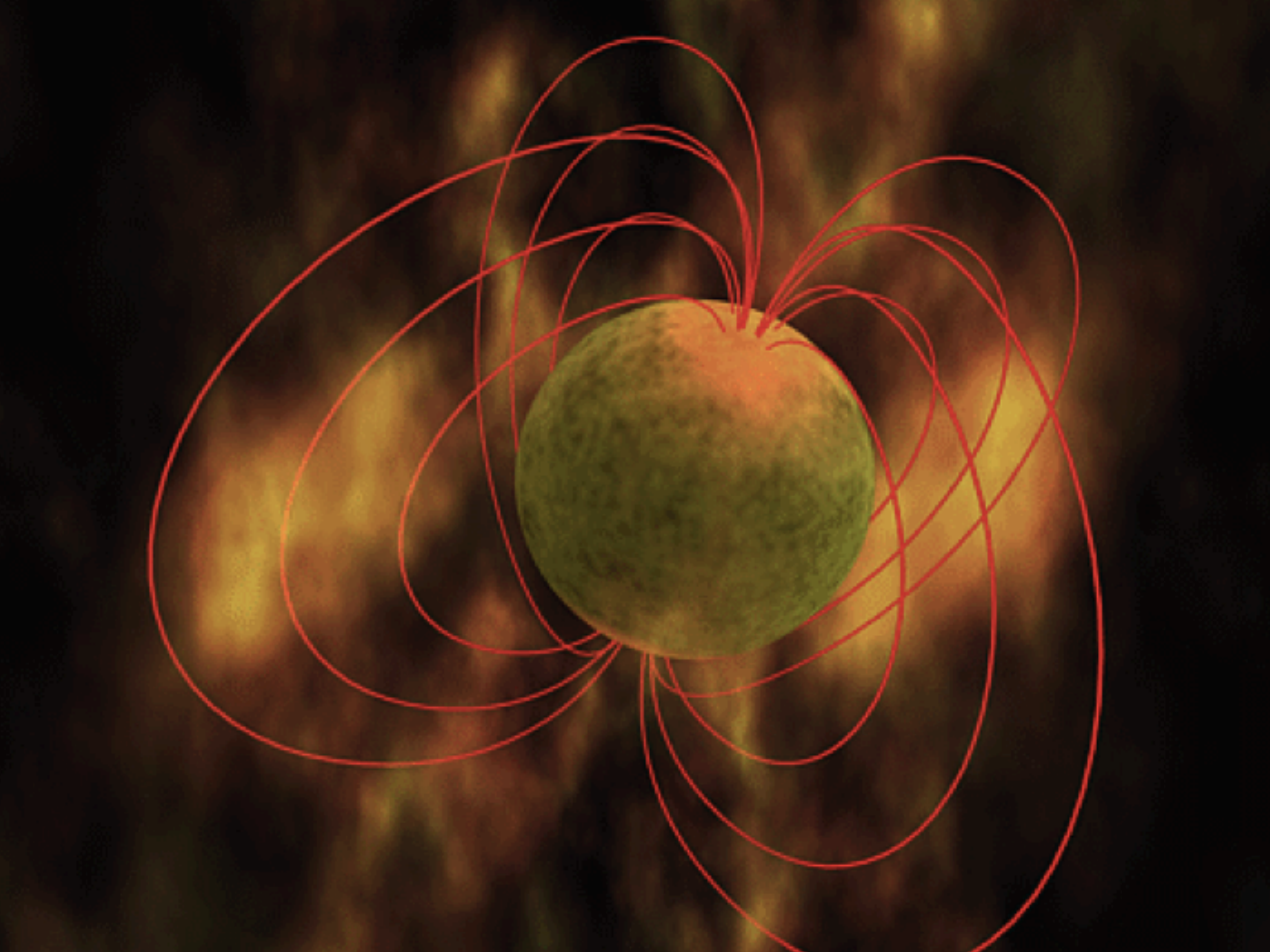
Plan

1. Magnetars. 3 pages

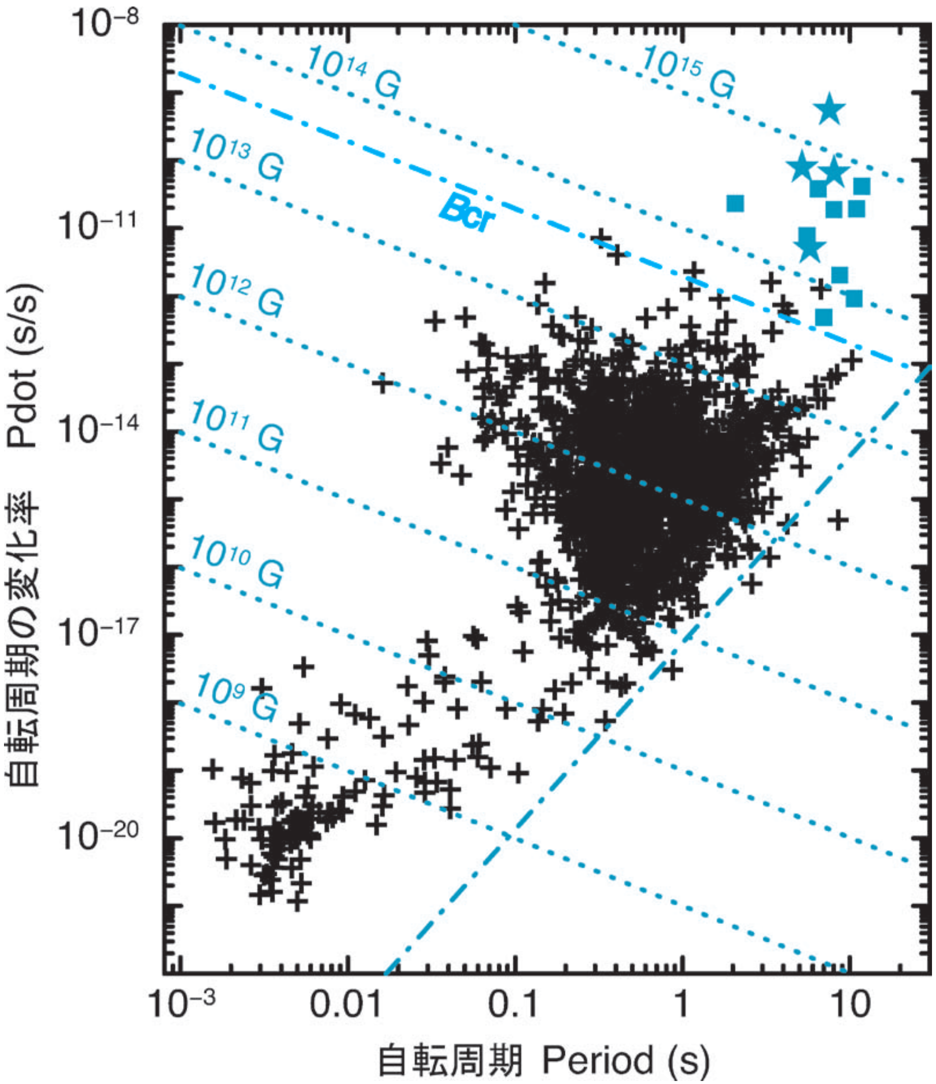
2. Pion domain wall. 3 pages

3. QCD Ferromagnetism. 3 pages

4. Holographization. 2 pages



Observation of Magnetars



[T.Enoto, 天文月報 2012.7]

Spindown luminocity

$$L = \partial_t (I\Omega^2) \propto \dot{P}/P^3$$

$$+$$

Magnetic dipole assumption

$$L = (2/3)\ddot{m}^2 \propto B^2/P^4$$

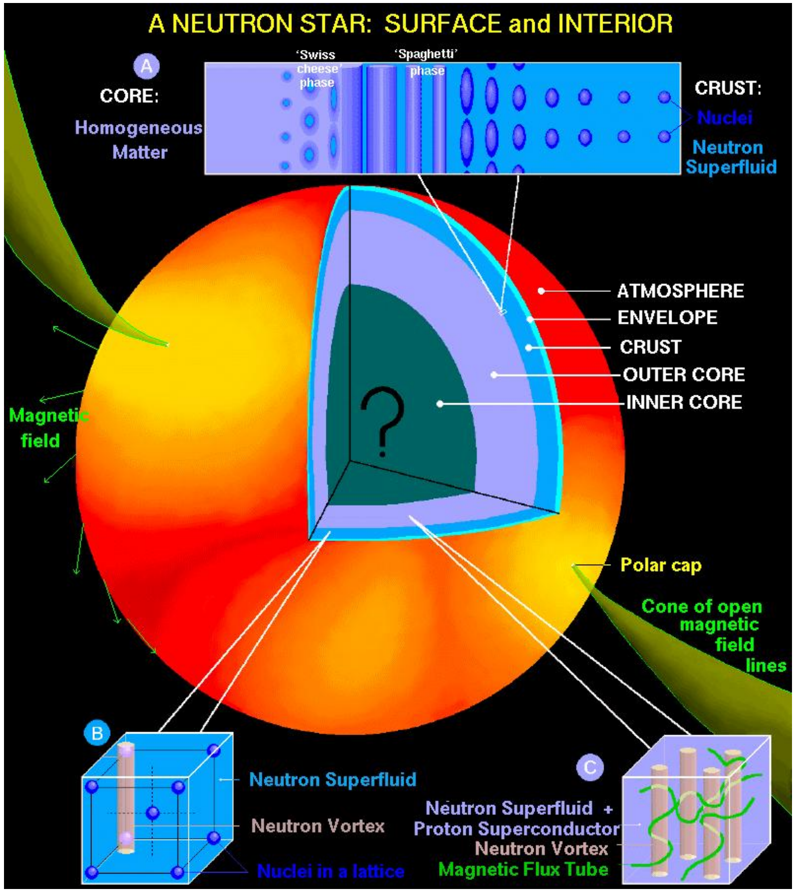
$$||$$

Surface magnetic field

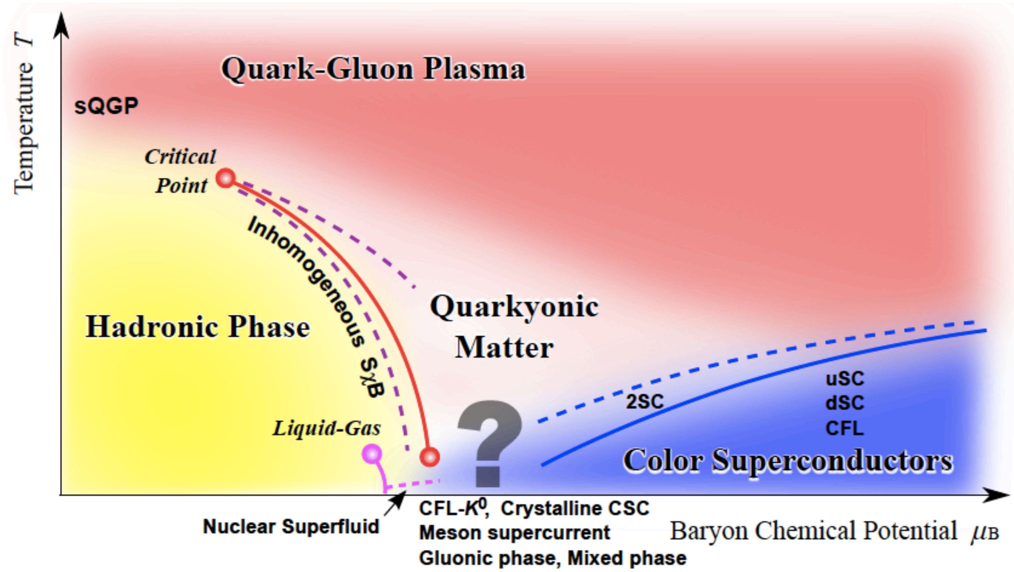
$$B = 3.2 \times 10^{19} \sqrt{P\dot{P}} \ [\mathrm{G}]$$

Magnetar hypothesis

Origin of the strong magnetic field?



[D.Page]



[Fukushima, Hatsuda 1005.4814]

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   --- What happens at the core?

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Describing high density

Chiral lagrangian for neutron and neutral pion

$$\mathcal{L} = \bar{n}(i\gamma^\mu\partial_\mu + \mu\gamma_0 - M_n)n - \frac{1}{2f_\pi}(\bar{n}\gamma_\mu\gamma_5 n)\partial^\mu\theta$$
$$+\frac{1}{2}(\partial_\mu\theta)^2 + f_\pi^2 m_\pi^2(\cos\theta - 1)$$

Chemical potential giving high density neutron matter

Weinberg's action : nucleons and all pions

$$\mathcal{L} = \bar{\psi}\,(iD_\mu\gamma^\mu - M_{\mathrm{N}})\,\psi - g\bar{\psi}\gamma_5\gamma^\mu\boldsymbol{\tau}\psi\cdot D_\mu\boldsymbol{\phi}$$
$$+\frac{1}{2}D_\mu\boldsymbol{\phi}\cdot D^\mu\boldsymbol{\phi} - \frac{1}{2}m_\pi^2(1+\boldsymbol{\phi}^2/4f_\pi^2)^{-1}\boldsymbol{\phi}^2$$

$$D_\mu\boldsymbol{\phi} \equiv (1+\boldsymbol{\phi}^2/4f_\pi^2)^{-1}\partial_\mu\boldsymbol{\phi}$$

$$D_\mu\psi \equiv \left[\partial_\mu + \frac{i}{4f_\pi^2}(1+\boldsymbol{\phi}^2/4f_\pi^2)^{-1}\boldsymbol{\tau}\cdot(\boldsymbol{\phi}\times\partial_\mu\boldsymbol{\phi})\right]\psi$$

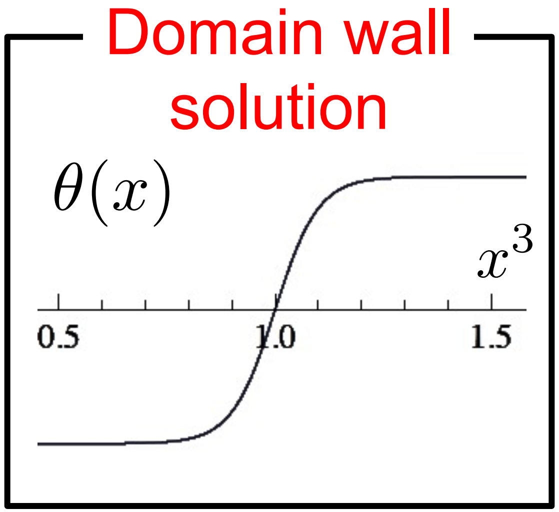
Domain wall of neutral pion

Chiral lagrangian = Sine-Gordon model

$$\mathcal{L} = \frac{1}{2} f_\pi^2 (\partial_\mu \theta)^2 - m_\pi^2 f_\pi^2 \cos\theta$$

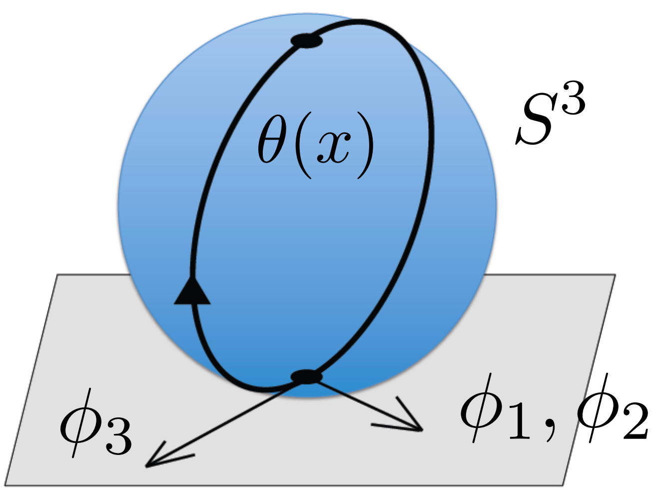
Classical solution connecting

$\theta = 0$ and $\theta = 2\pi$



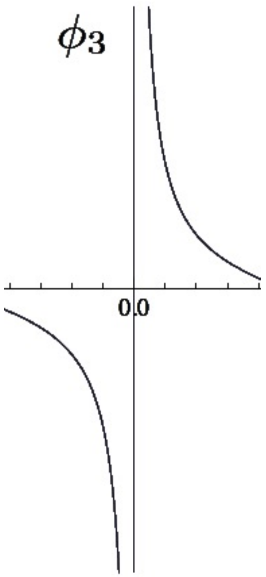
[Hatsuda 1986]

No topological obstruction against creation



$$\phi_3 = \frac{2 f_\pi}{\sinh[m_\pi x^3]}$$

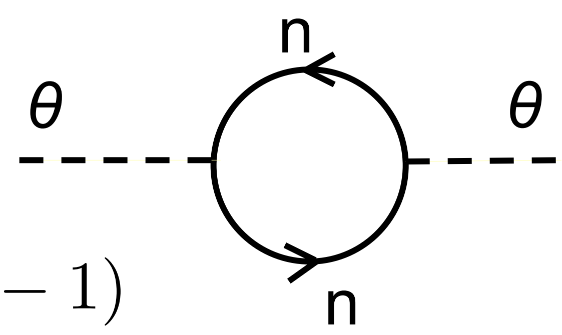
$$\phi_1 = \phi_2 = 0$$



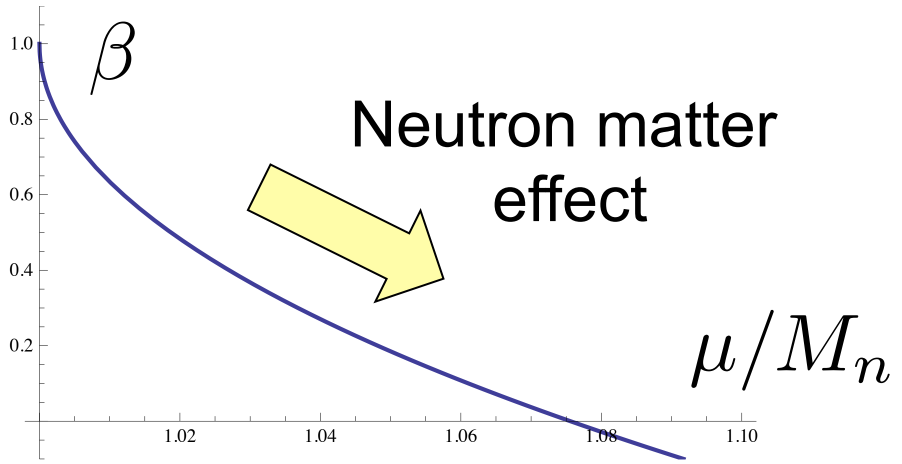
Light domain wall in neutron matter

[Hatsuda 1986]

Integration of neutron at 1-loop
→ pion effective action modified



$$\mathcal{L} = \frac{\alpha}{2}(\partial_0\theta)^2 - \frac{\beta}{2}(\partial_i\theta)^2 + f_\pi^2 m_\pi^2(\cos\theta - 1)$$



- β decreases at high density
- Light domain wall

$$\mathcal{E}/S = 8\sqrt{\beta} f_\pi^2 m_\pi$$

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   --- Wall are light at high density

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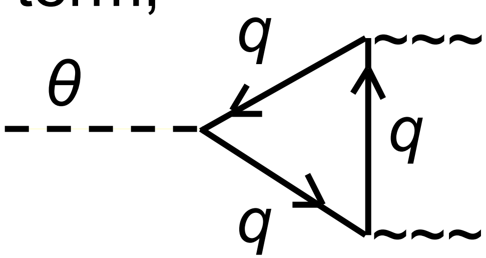
Baryonically charged domain wall

QCD axial anomaly induces "$\pi^0 \to 2\gamma$" term,

$$\mathcal{L}_{\rm anom} = \frac{e}{4\pi^2}\mu_B B_3 \partial_3 \theta$$



Domain wall inducing a baryon charge, under a magnetic field

$$N_B/S = eB_3/2\pi$$

[Son, Stephanov, 0710.1084]

Energy per baryon charge:

$$\mathcal{E}/N_B = \frac{16\pi f_\pi^2 m_\pi}{eB_i\hat{n}_i}\sqrt{\beta}$$

If smaller than neutron mass, domain wall dominates!

$$B \sim \sqrt{\beta} \times 10^{19}\ [\mathrm{G}]$$

$$L_{\rm WZW} = \frac{-ie}{16\pi^2} A_0^{(B)} B_3 \mathrm{tr}\left[\tau_3 (U\partial_3 U^\dagger + \partial_3 U^\dagger U)\right]$$

$$U \equiv (\sigma + i\boldsymbol{\tau}\cdot\boldsymbol{\pi})/f_\pi$$

Ferromagnetic QCD

Magnetization via wall $M = \frac{e\mu_{\rm B}}{4\pi^2}\partial_3\theta$

[Son, Zhitnitsky, hep-ph/0405216]

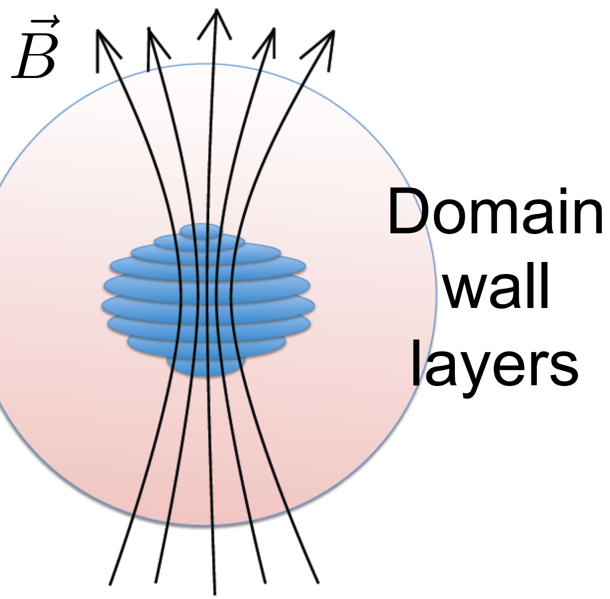
Wall layer with interval d : $M = \frac{e\mu_{\rm B}}{2\pi d}$

Induced baryon density : $\rho = \frac{eB_3}{2\pi}\frac{1}{d}$

$$M = \frac{\mu_{\rm B}\rho}{B_3}$$

Equilibrium

$$B_3 = M$$



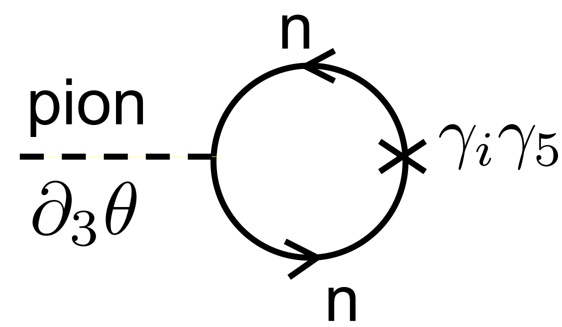
$$B_3 \sim 10^{19}[\mathrm{G}]$$

Spontaneous magnetization.
Very strong magnetic field,
QCD ferromagnetism

Spins align on the wall

Neutron spins accumulate on the wall:

$$\mathcal{S}_i(x) = \langle \bar{n}\gamma_i\gamma_5 n\rangle = (\beta - 1) f_\pi^2 \partial_i \theta(x)$$



→ Domain wall carries neutron spins:

$$s_3/S = 2\pi(\beta - 1) f_\pi^2$$

[Hatsuda 1986]

Neutrons have magnetic moment :

$$\mu = \frac{g e s_3}{2M_n} \quad \text{with} \quad g \sim -3.8$$

Domain walls are magnetized:

$$M = \frac{\pi |g| e f_\pi^2}{m_N} \frac{1}{d}$$

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   --- Anomaly magnetizes the core

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Full meson theory (酒井杉本) giving the same

Pions are gauge field A_z

$$S_{\rm D8} = -\frac{\lambda N_c}{216\pi^3}\int d^4x dz\,(1+z^2)F_{\mu z}^2$$

Quark mass effect : worldsheet instanton

$$S_{\rm mass} = c\int d^4x\,{\rm Ptr}\;\exp\left[-i\int_{-\infty}^{\infty} A_z dz\right] + {\rm c.c.}$$

[Hirayama, Lin, Yee, KH 0803.4192]
[Aharony, Kutasov, 0803.3547]

EOM for $A_\mu(x)$ is solved as

$$A_\mu = 0, \quad A_z = \frac{1}{1+z^2}\tau_3\,\theta(x^\mu)$$

Then the EOM for $A_z(x)$ reduces to sine-Gordon eq. of $\theta(x)$

Anomaly analysis and Magnetization

Anomaly = CS term of the D8-brane action

$$S_{\rm CS} = \frac{N_c}{24\pi^2}\int d^4x dz\, A_0^{(\rm B)} B_3^{(\rm em)} \partial_3 A_z .$$

Consider a chiral limit for light domain walls,

$$A_z = \frac{1}{1+z^2} a x^3$$

Total energy with all D8 action + EM :

$$\begin{aligned} E &\sim \lambda N_c a^2 + N_c \mu_q B a + B^2 \\ &= \lambda N_c \left(a + \frac{\mu_q B}{\lambda} \right)^2 - \frac{N_c \mu_q^2}{\lambda} B^2 + B^2 \end{aligned}$$

$\mu_q > \sqrt{\lambda / N_c}$ → Spontaneous magnetization

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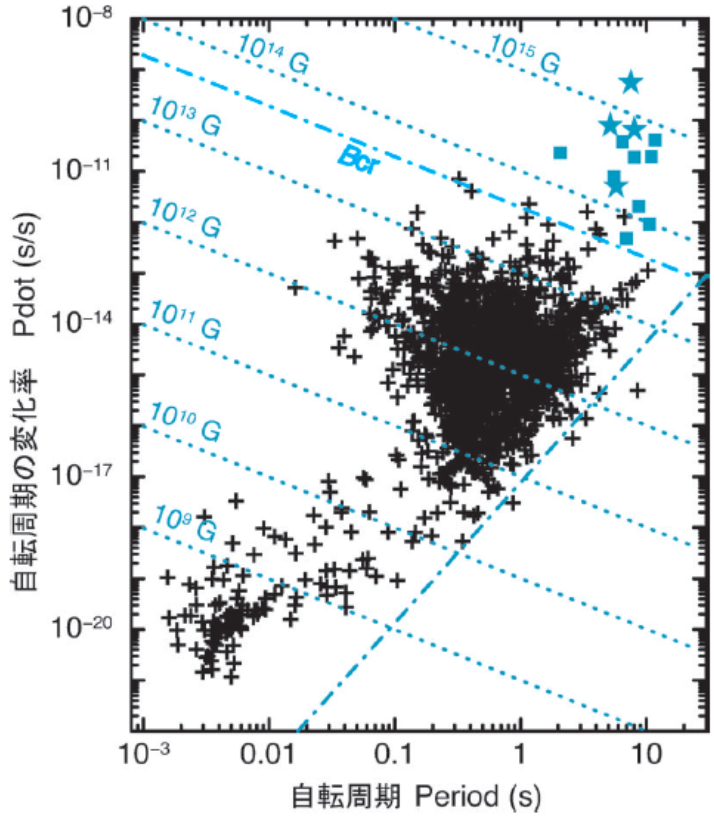
New phase : QCD Ferromagnetism

More to come…

- Stability of the walls?
  Charged pion condensates
- More realistic?
  Nuclear forces, hyperons
- EOS via walls?
- Neutron superfluidity?
  p-wave at high density,
  pairing on the walls
- Gravitational waves from
  isolated neutron stars?
- Classification of neutron stars?



[T.Enoto, 天文月報 (2012)]