Transport in Holographic Lattices

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Gauge/gravity duality can reproduce many properties of condensed matter systems, even in the limit where the bulk is described by classical general relativity:

1) Fermi surfaces
2) Non-Fermi liquids
3) Superconducting phase transitions
4) …

It is not clear why it is working so well.

Can one do more than reproduce qualitative features of condensed matter systems?

Can gauge/gravity duality provide a quantitative explanation of some mysterious property of real materials?

We will see evidence that the answer is yes!

Most previous applications have assumed translational symmetry.

Unfortunate consequence: Any state with nonzero charge density has infinite DC conductivity. (A boost produces a nonzero current with no applied electric field.)

This can be avoided in a probe approximation (Karch, O'Bannon, 2007;….).

Plan: Add a lattice to the simplest holographic model of a conductor and calculate transport properties.

A perfect lattice still has infinite conductivity due to Bloch waves. So we work at nonzero T and include dissipation. (Cf: Kachru et al; Maeda et al; Hartnoll and Hoffman; Liu et al.)

Main result: We will find a surprising similarity to the optical conductivity in the normal phase of the cuprates.

Simple model of a conductor

Suppose electrons in a metal satisfy

$$m\frac{dv}{dt} = eE - m\frac{v}{\tau}$$

If there are n electrons per unit volume, the current density is J = nev. Letting $E(t) = Ee^{-i\omega t}$, find J = σ E, with

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

where $K = ne^2/m$. This is the Drude model.

$$\mathrm{Re}(\sigma) = \frac{K\tau}{1+(\omega\tau)^2}, \quad \mathrm{Im}(\sigma) = \frac{K\omega\tau^2}{1+(\omega\tau)^2}$$

Note:

(1) For $\omega\tau \gg 1, \quad |\sigma| \approx K/\omega$

(2) In the limit $\tau \to \infty$:

$$\mathrm{Re}(\sigma) \propto \delta(\omega), \quad \mathrm{Im}(\sigma) = K/\omega$$

This can be derived more generally from Kramers-Kronig relation.

Our gravity model

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{2} F_{\mu\nu} F^{\mu\nu} - 2 \nabla_\mu \Phi \nabla^\mu \Phi + \frac{4\Phi^2}{L^2} \right]$$

We work in Poincare coordinates with boundary at z=0. Then

$$\Phi \to z\phi_1 + z^2 \phi_2 + \mathcal{O}(z^3)$$

We introduce the lattice by requiring:

$$\phi_1(x) = \mathcal{A}_0 \cos(k_0 x)$$

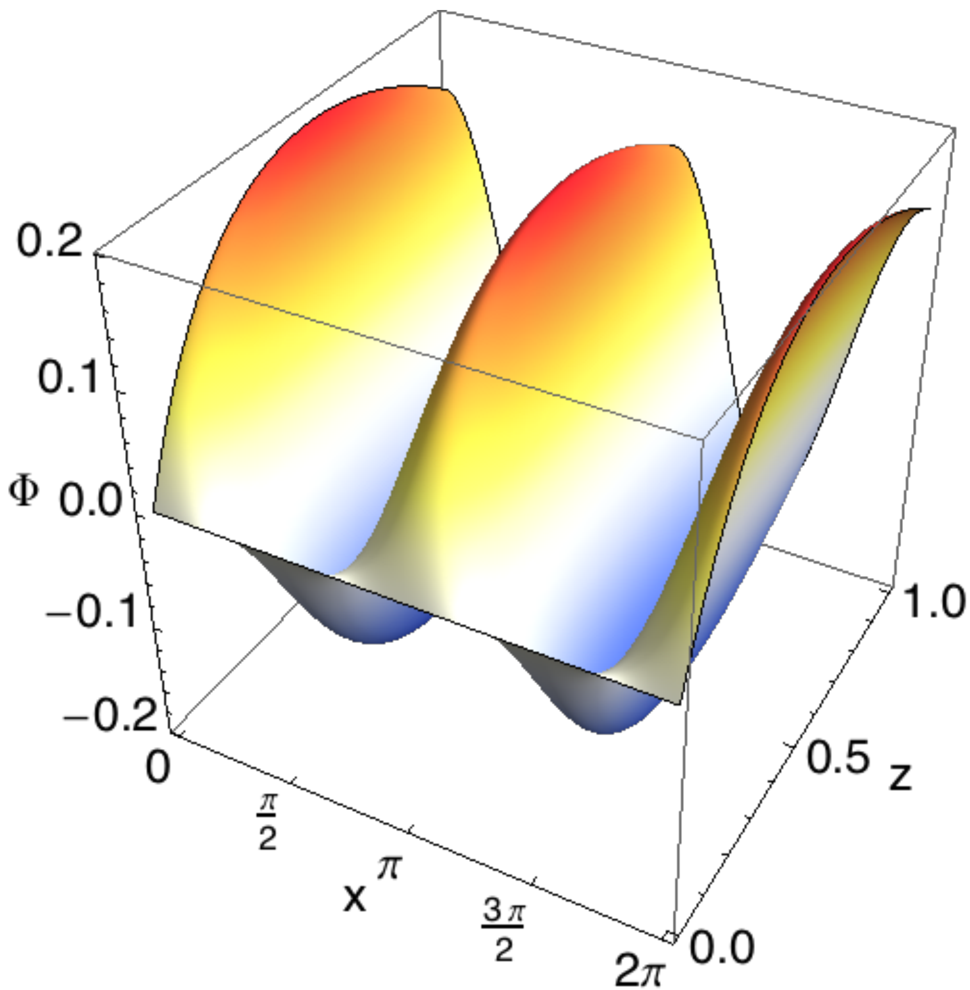
Want finite temperature: Add black hole

Want finite density: Add charge to the black hole. The chemical potential is then

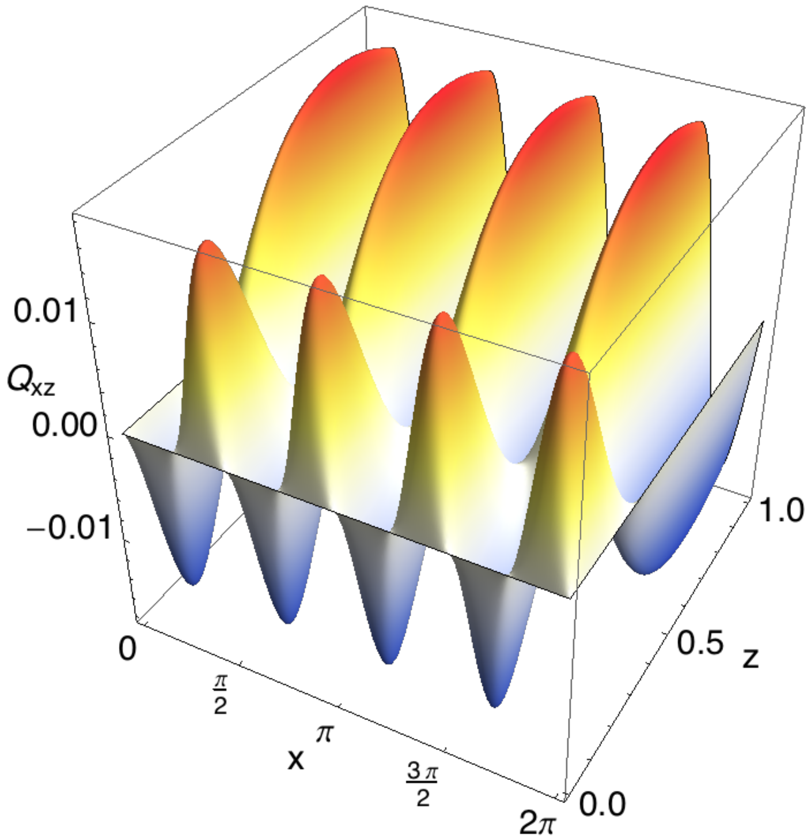
$$A_t\,(z=0) = \mu$$

We numerically find solutions with smooth horizons that are static and translationally invariant in one direction. (Have to solve 7 coupled nonlinear PDE's in 2D.)

Scalar field in holographic lattice

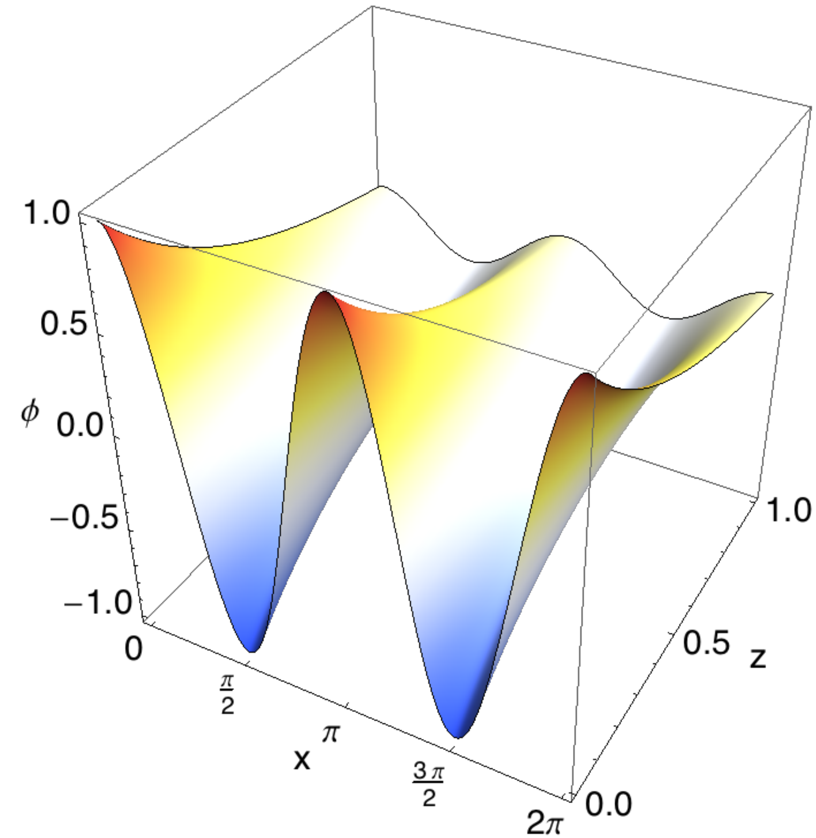


Solution with $T/\mu = .1$, $k_0 = 2$ and unit amplitude



metric component

Lattice induced on metric has k_0 = 4.



scalar field

Lattice with k_0 = 2

Conductivity

To compute the optical conductivity using linear response, we perturb the solution

$$g_{\mu\nu} = \hat{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad A_\mu = \hat{A}_\mu + \delta A_\mu, \quad \Phi = \hat{\Phi} + \delta\Phi$$

Boundary conditions:

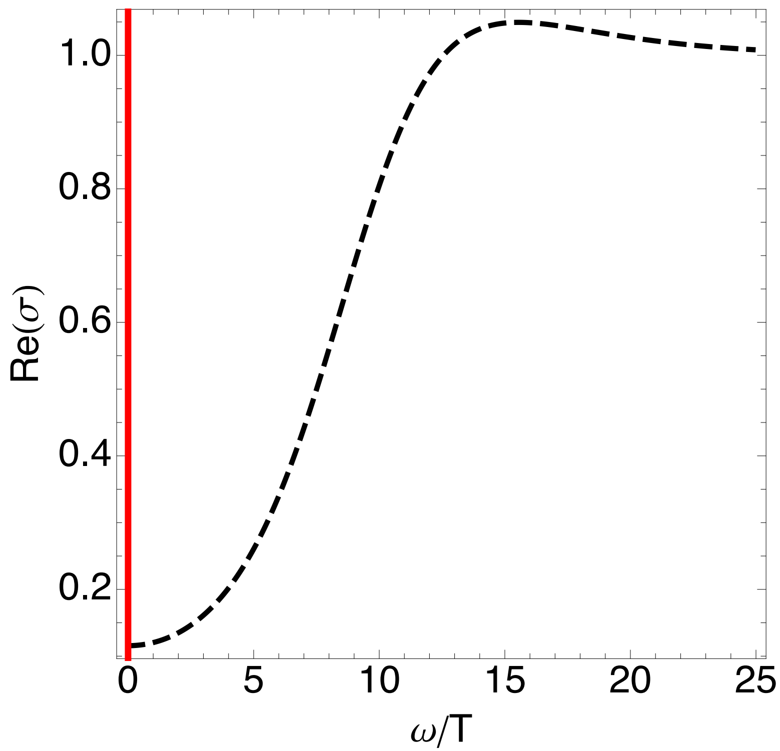
ingoing waves at the horizon

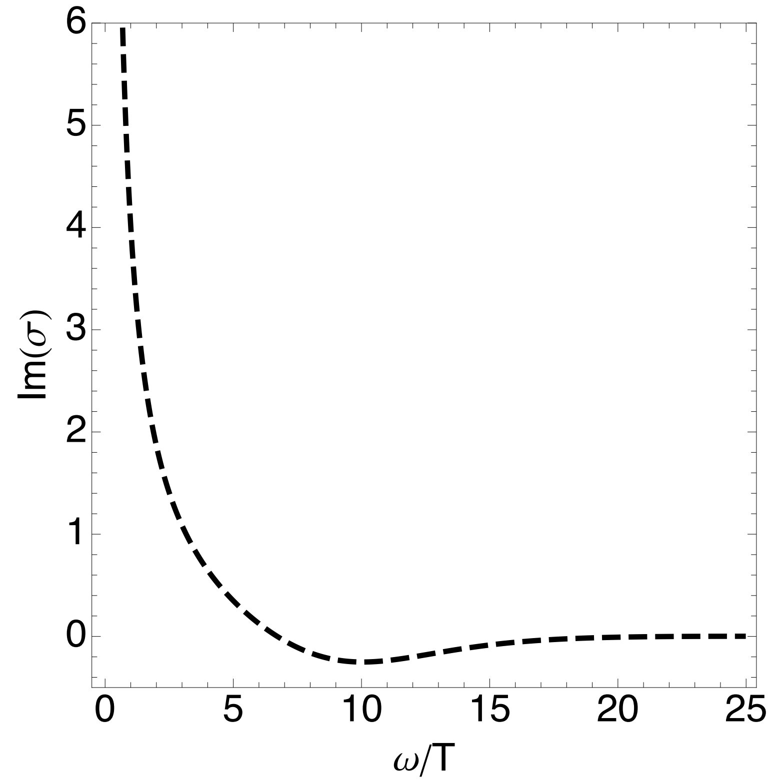
$\delta g_{\mu\nu}$ and $\delta\Phi$ normalizable at infinity

$\delta A_t \sim O(z)$, $\delta A_x = e^{-i\omega t} [E/i\omega + J z + \ldots]$

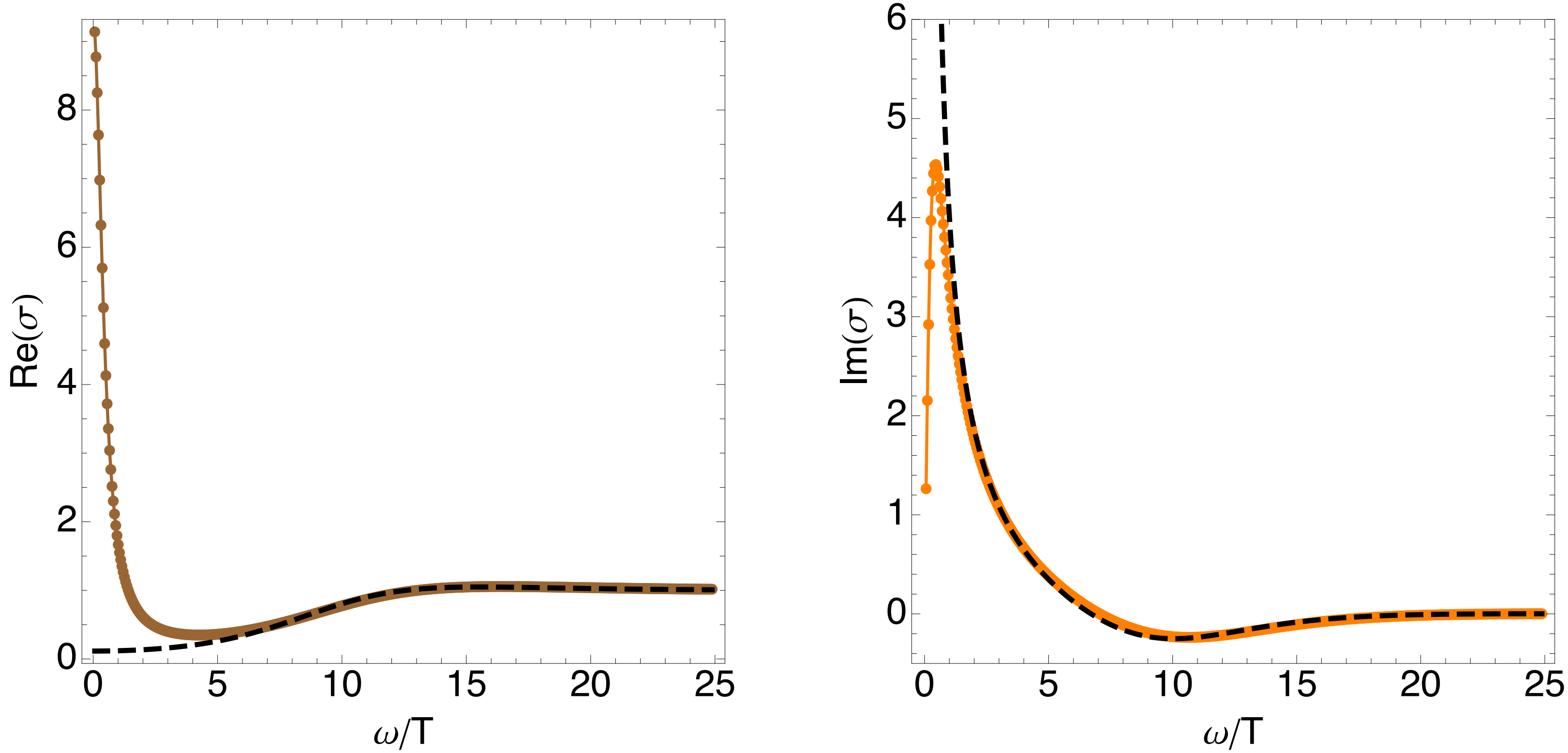
induced current

Review: optical conductivity with no lattice ($T/\mu = .115$)



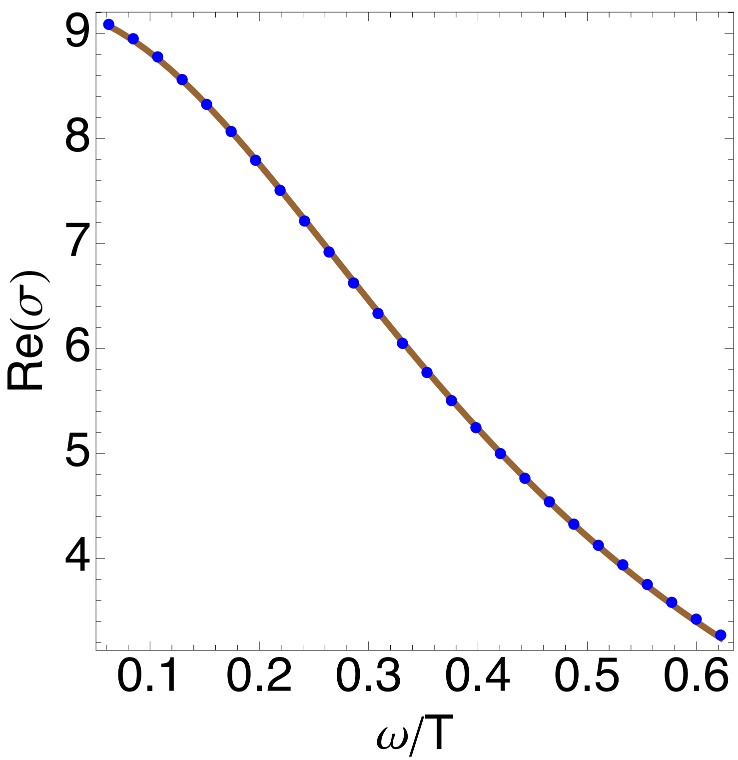


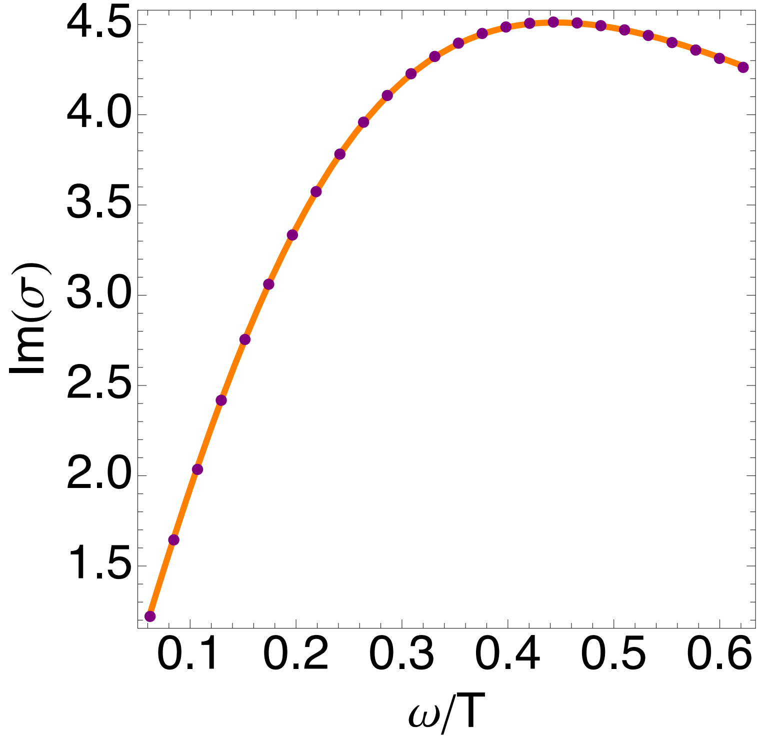
With the lattice, the delta function is smeared out



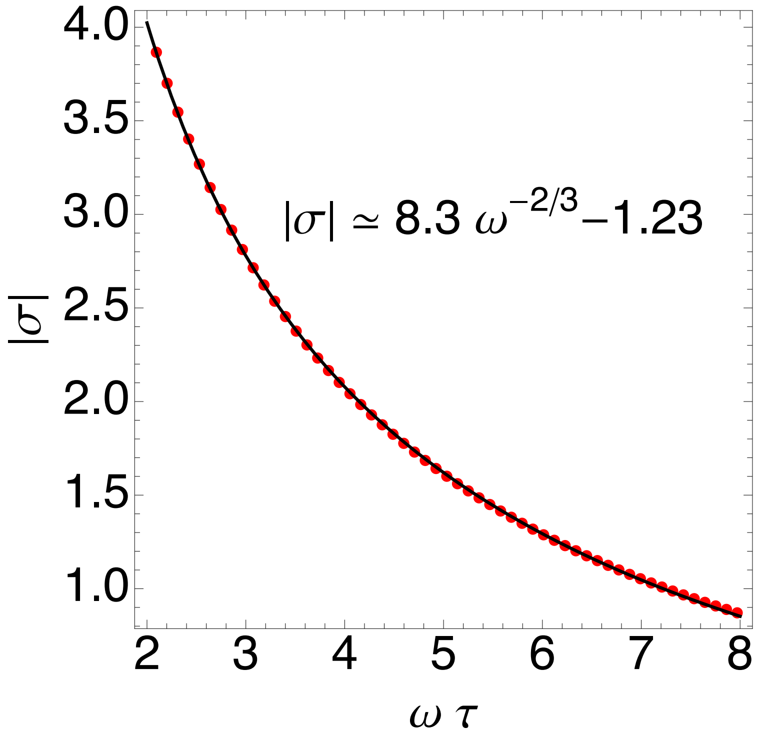
The low frequency conductivity takes the simple Drude form:

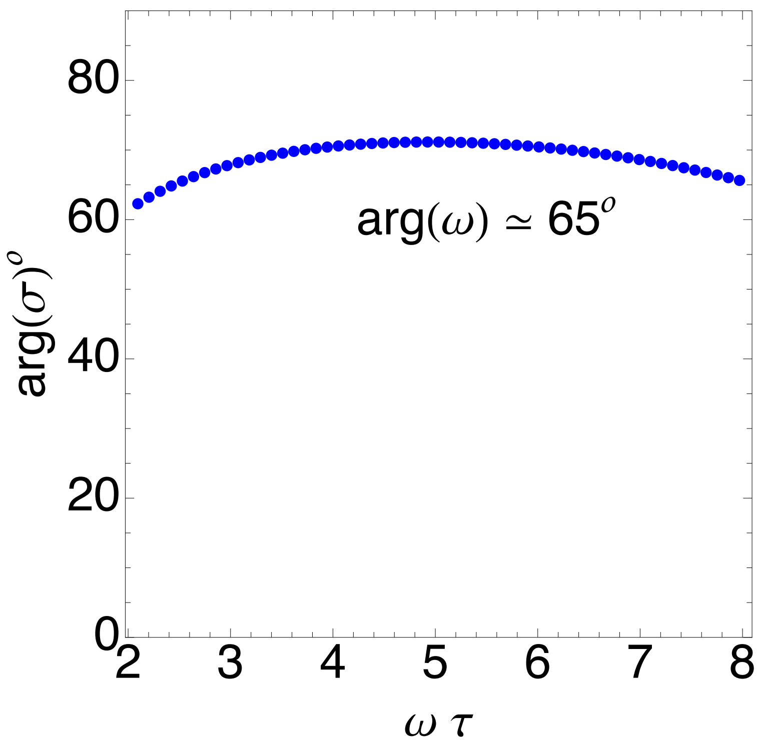
$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$





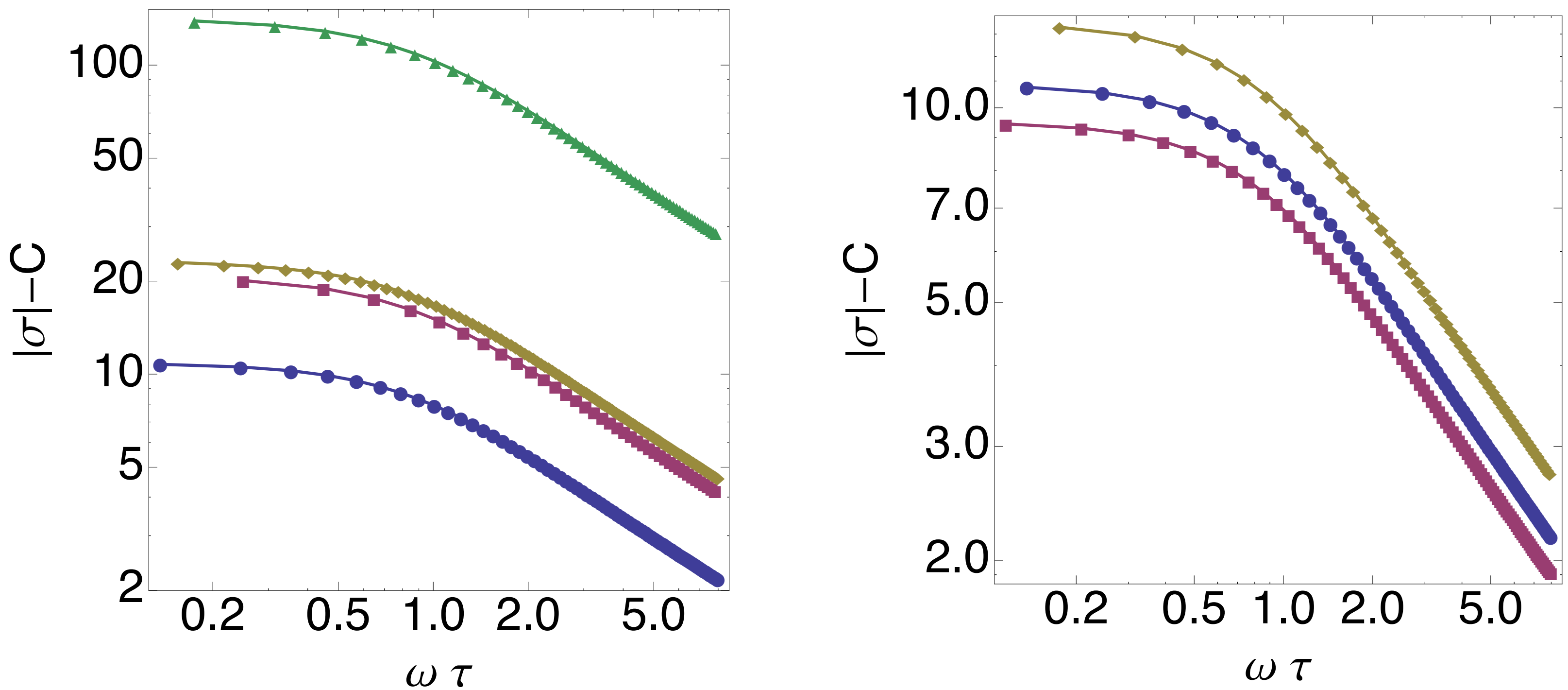
Intermediate frequency shows scaling regime





The data is very well fit by $$|\sigma| = \frac{B}{\omega^{2/3}} + C$$

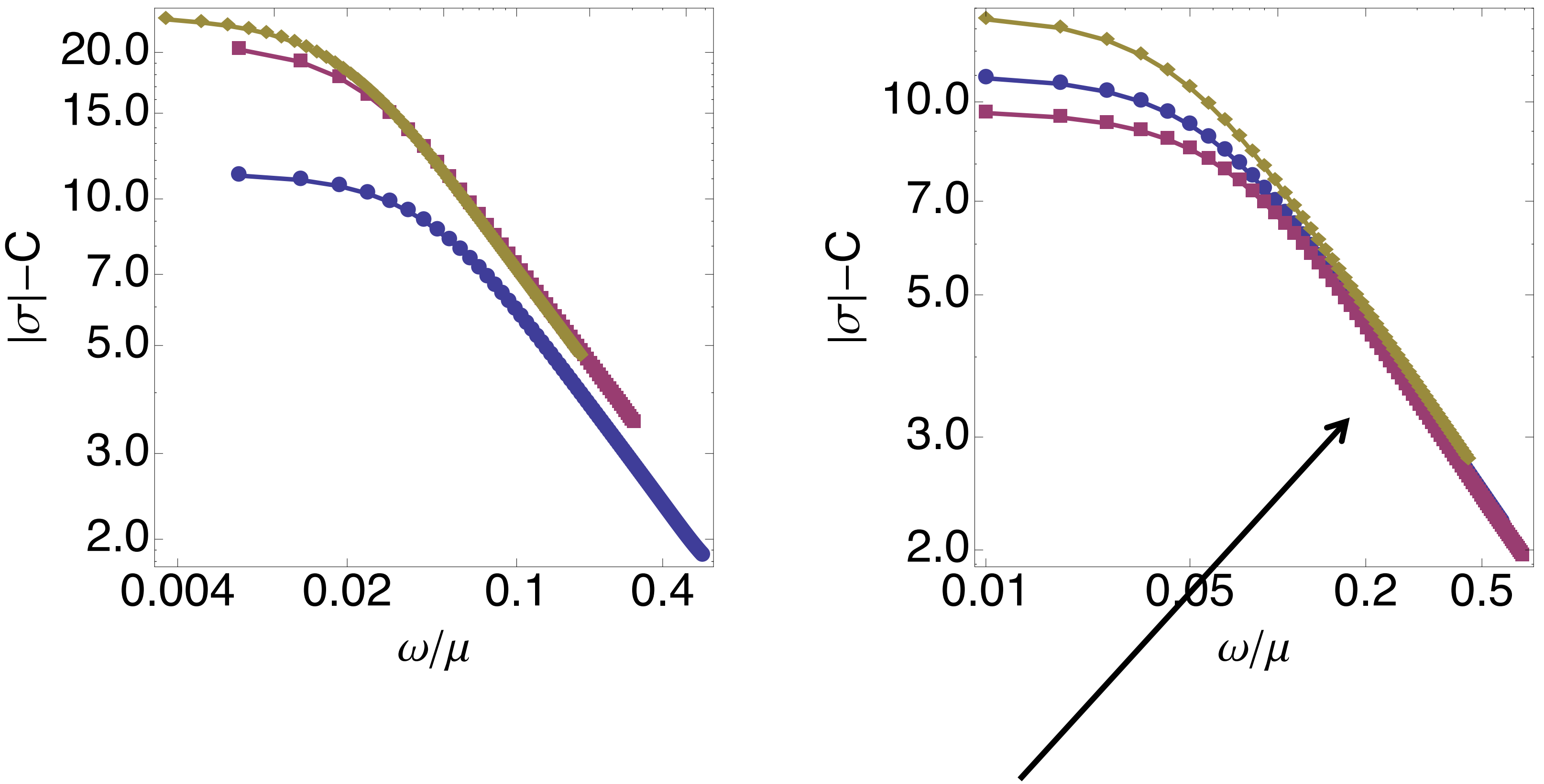
The exponent 2/3 is robust



different wavenumbers
k_0 = .5, 1, 2, 3

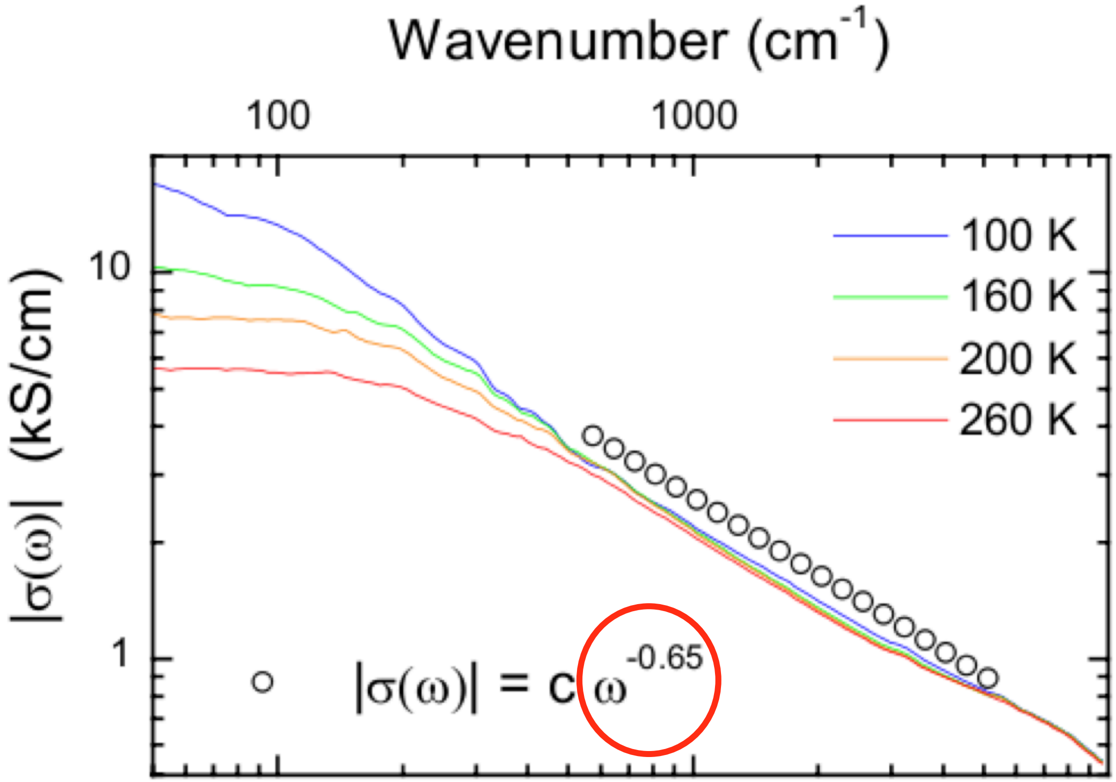
different temperatures
T/μ = .098, .115, .13

Same conductivity plotted as a function of ω/μ



Coefficient of power law is independent of T.

Comparison with the cuprates
(van der Marel, et al 2003)



$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

Ionic Lattice

We now take our bulk action to be just the 4D Einstein-Maxwell theory (no scalar field).

Introduce the lattice by making the chemical potential be a periodic function:

$$A_t \to \mu(x) \equiv \bar{\mu}\left[1 + \mathcal{A}_0 \cos(k_0 x)\right]$$

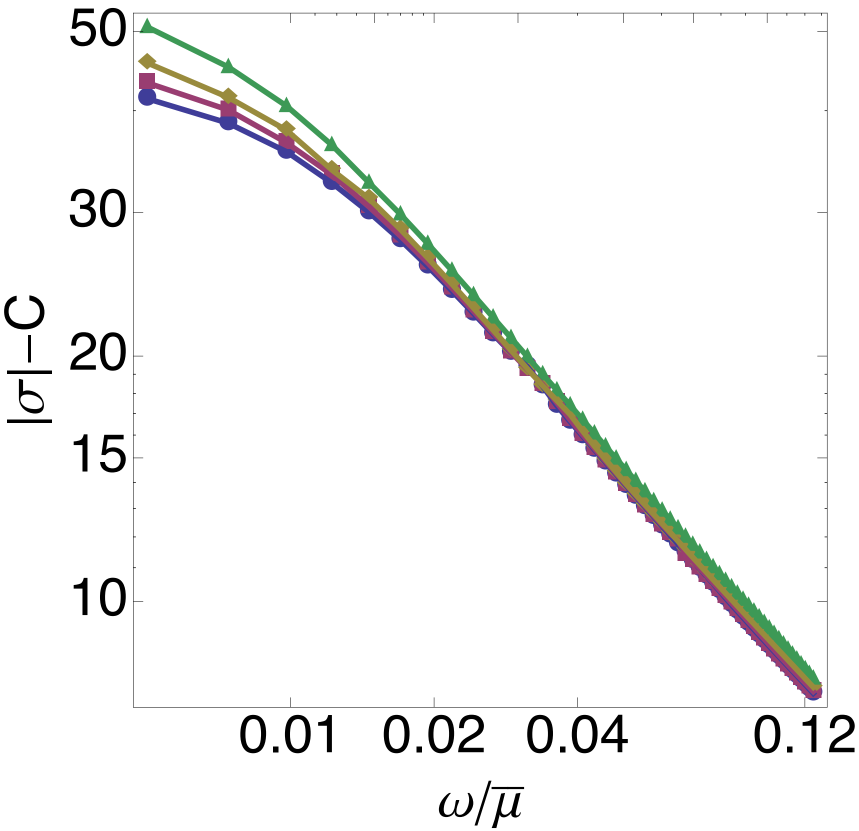
Two changes in bulk lattice:

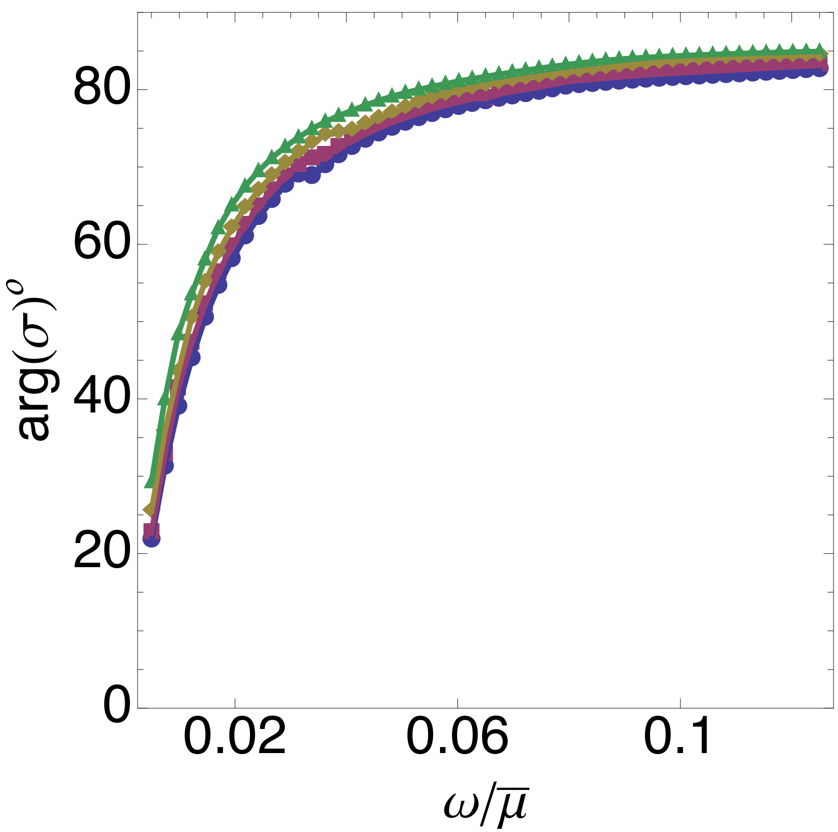
1) Amplitude is larger
2) Wavenumber is k_0 rather than $2k_0$

No change in the optical conductivity. Still get:

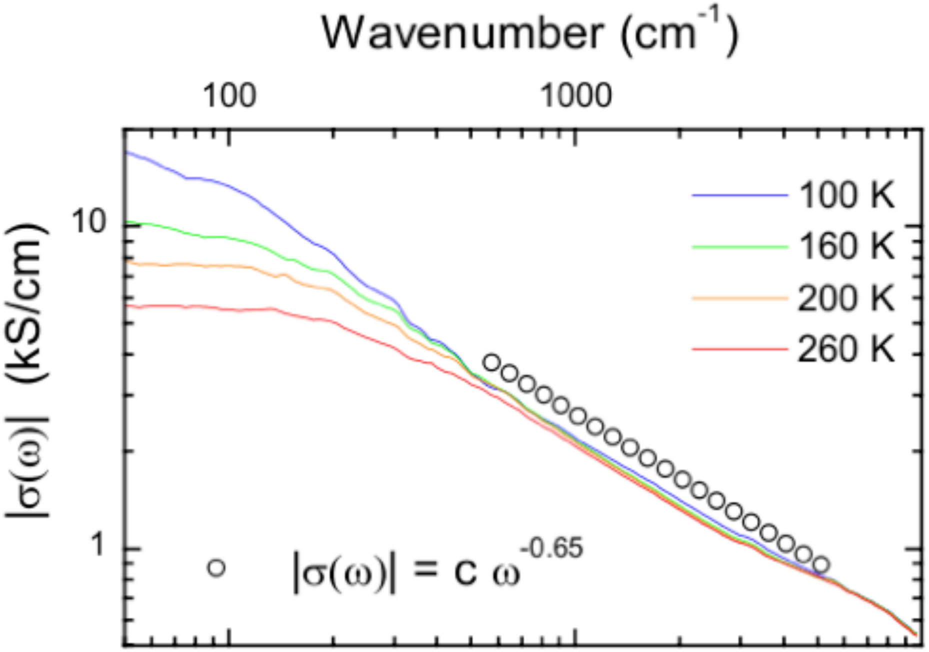
1) Drude behavior at low frequency
2) Power law fall-off with exponent 2/3 for $2 < \omega\tau < 8$

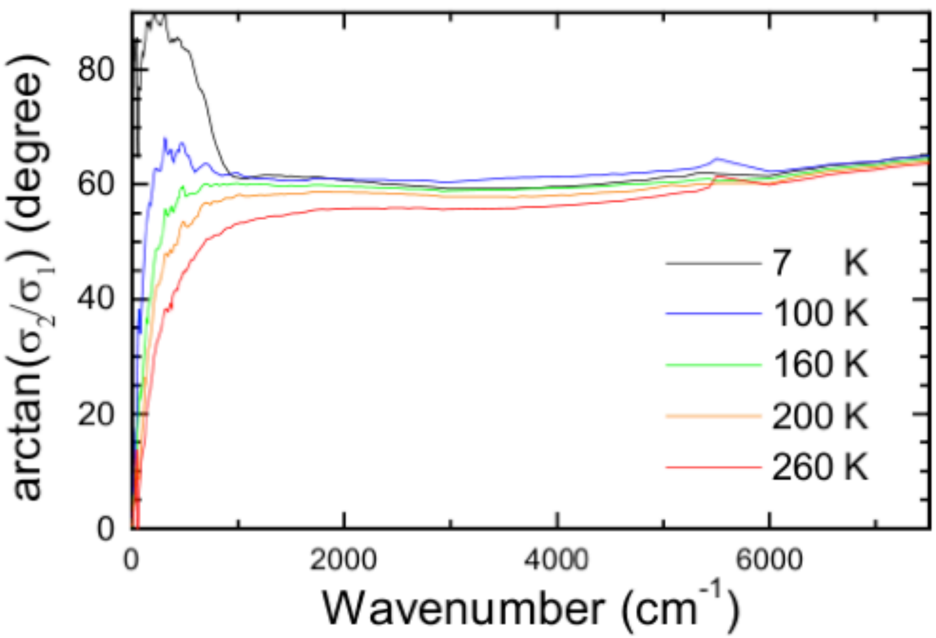
Conductivity at four different temperatures for ionic lattice (.033 < T/μ < .055)





Even the phase is similar





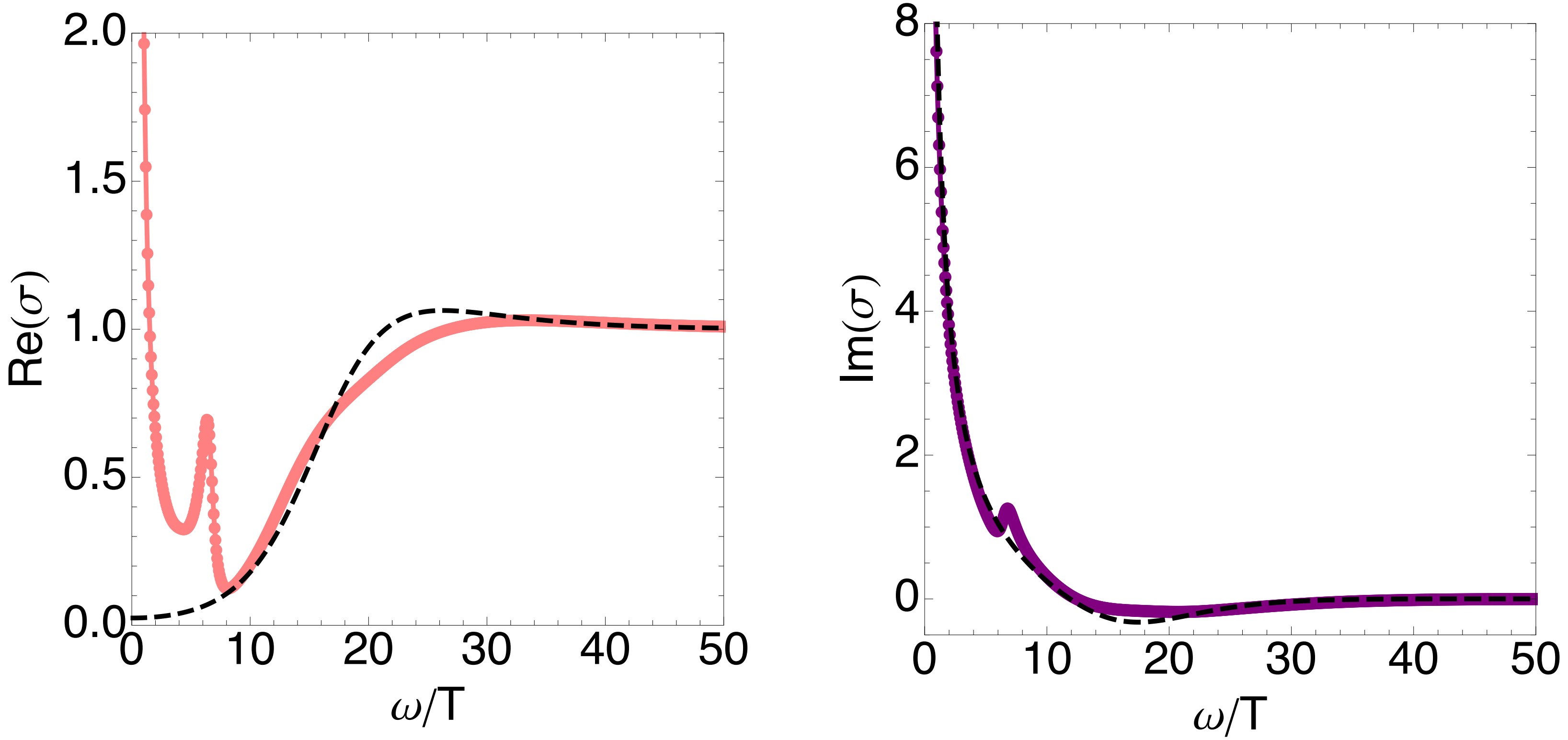
$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

Resonances

At larger frequencies, the optical conductivity has resonances. In the bulk, this is due to quasinormal modes of the charged black hole.

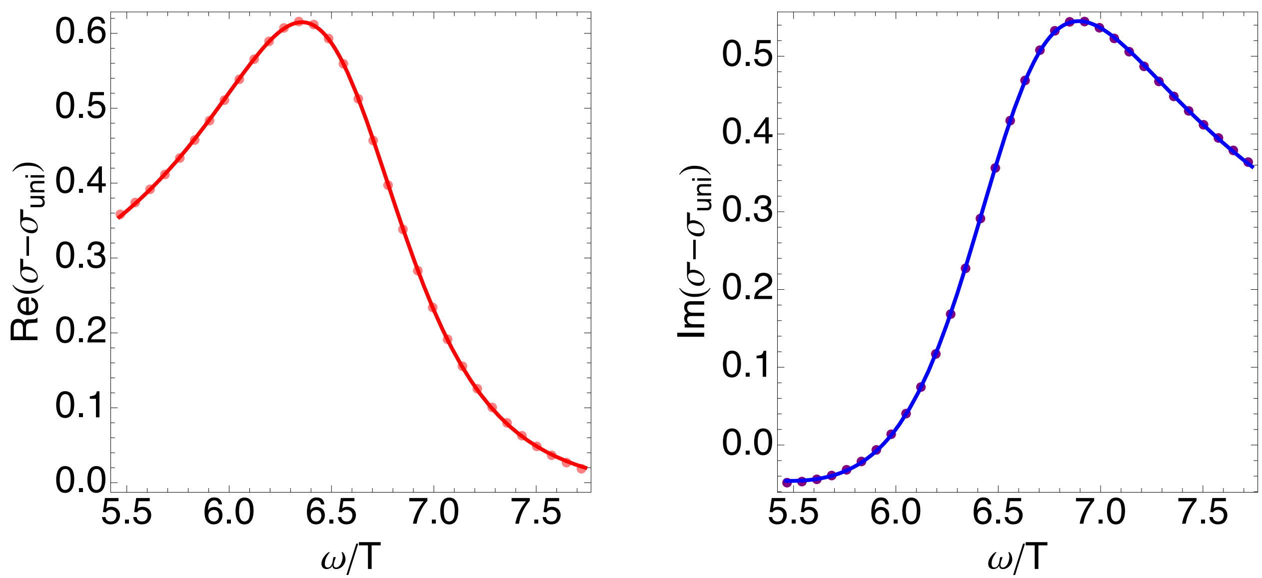
They do not arise in the homogeneous case, but are a generic feature of holographic lattices.

Example of a resonance for $k_0/\mu = 1/2$



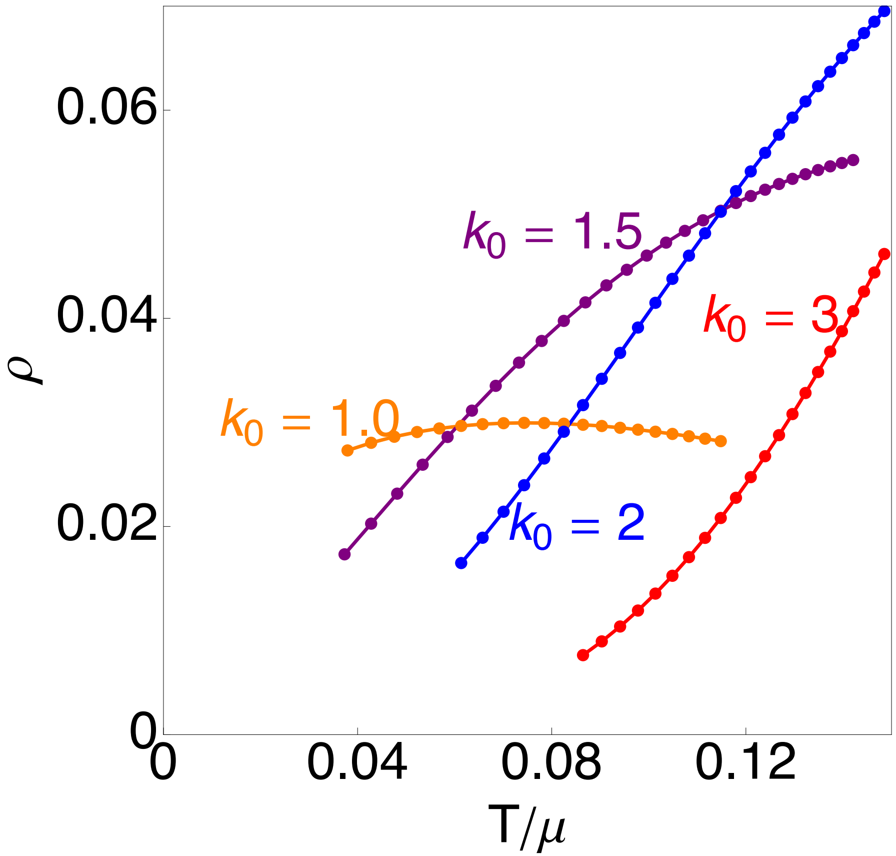
Resonance is fit by a pole in retarded Greens fn

$$\sigma(\omega) = \frac{G^R(\omega)}{i\omega} = \frac{1}{i\omega}\frac{a + b(\omega - \omega_0)}{\omega - \omega_0}$$



$$\omega_0/T = 6.6 - 0.64i$$

DC resistivity



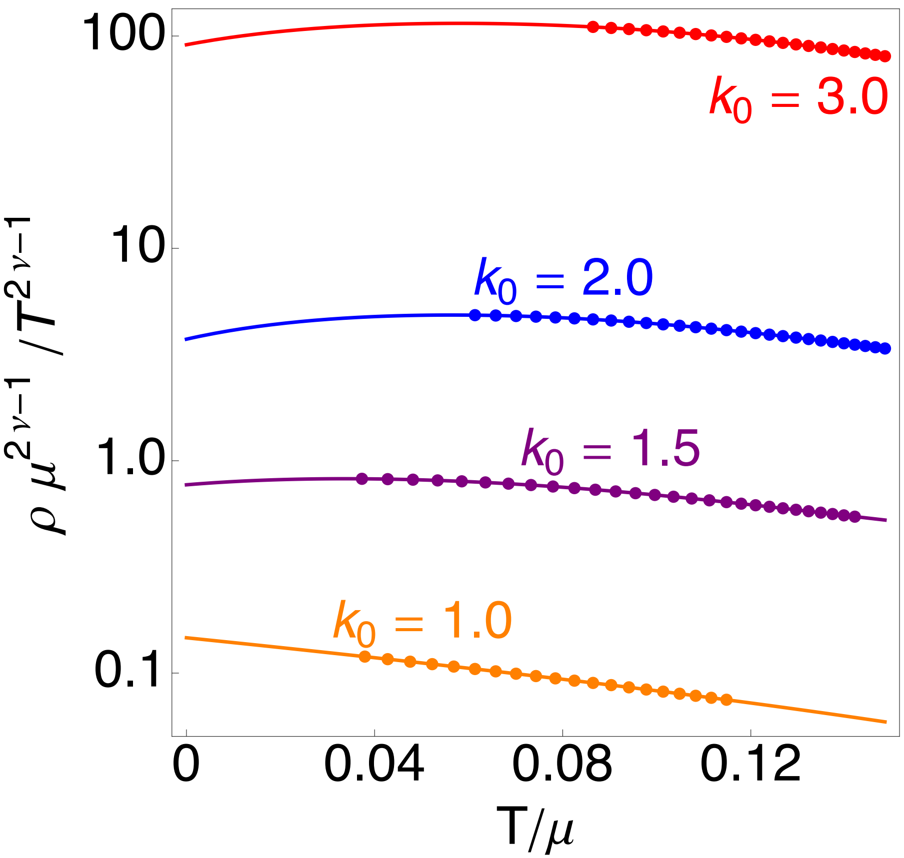
The DC resistivity $\rho = (K \tau)^{-1}$ depends on the lattice wavenumber k_0 as well as T.

Near horizon geometry of T = 0 black hole is $AdS_2 \times R^2$. Hartnoll and Hofman (2012) showed that at low T, ρ can be extracted from the two point function of the charge density evaluated at the lattice wavenumber:

$$\rho \propto T^{2\nu-1}$$

$$\nu = \frac{1}{2}\sqrt{5 + 2(k/\mu)^2 - 4\sqrt{1 + (k/\mu)^2}}$$

Our data is in good agreement with the Hartnoll-Hofman result (with k = $2k_0$)



One can always tune k/μ so that $\rho \propto T$
as observed in the cuprates.

While not a robust prediction of this model, it is not totally unjustified.

Tuning k/μ is equivalent to tuning the charge per lattice site which is analogous to doping.

Thermoelectric conductivity

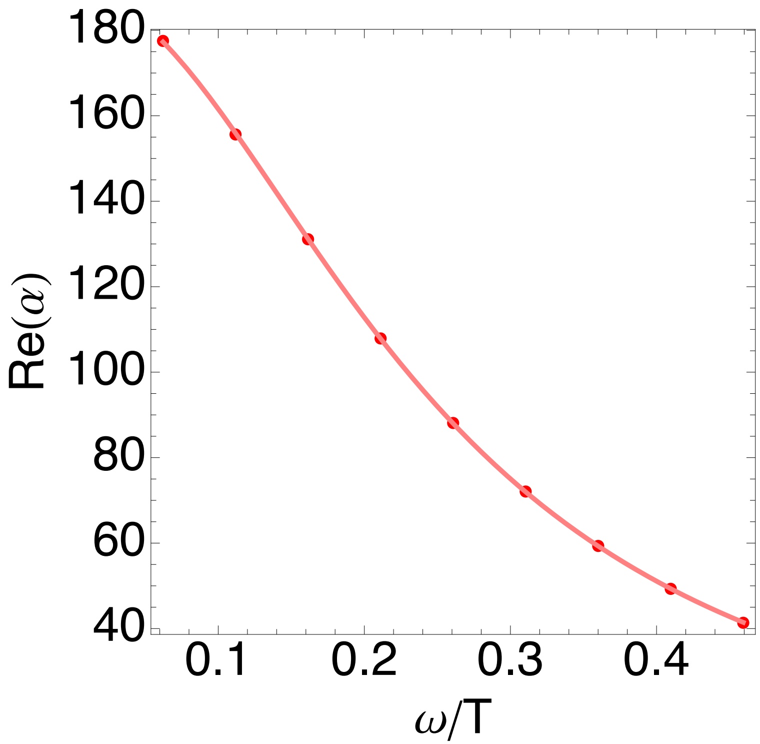
The heat current is $Q_x = T^t{}_x - \mu\, J_x$

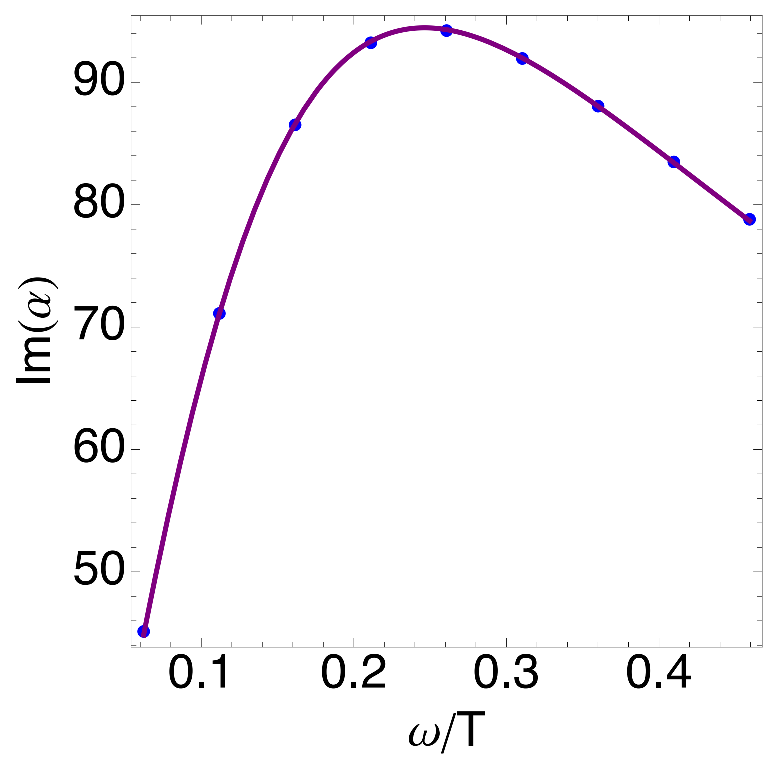
(Follows from 1st law: $TdS = dE - \mu\, dq$)

In general, applying a homogeneous electric field E_x produces not only an electric current J_x but also Q_x.

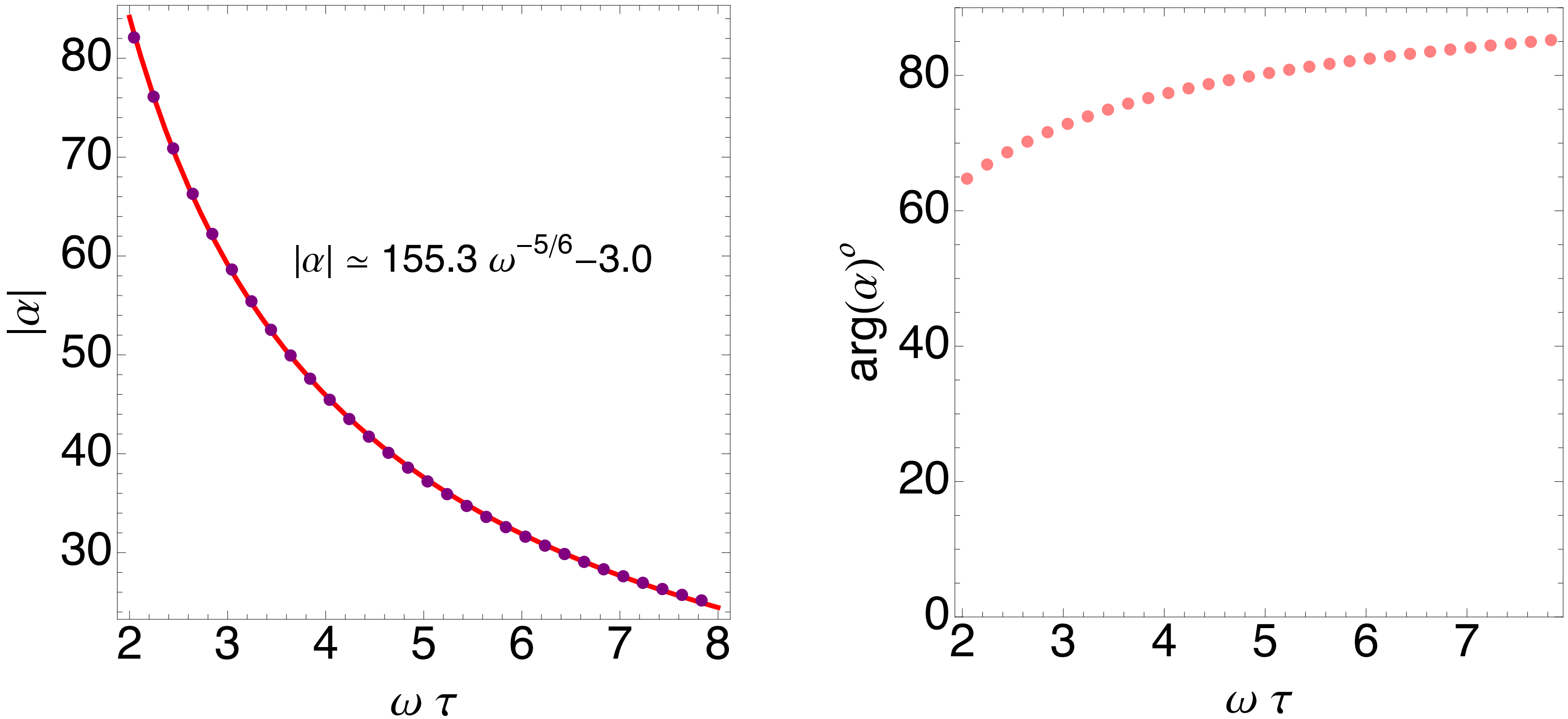
The thermoelectric coefficient is $\alpha = Q_x / TE_x$.

For small ω, α(ω) is given by the Drude form with the same relaxation time τ as σ(ω).





For 2 < ω τ < 8, we find $$|\alpha(\omega)| = \frac{\tilde{B}}{\omega^{5/6}} + \tilde{C}$$



For large frequency, we find

$$\alpha(\omega) = -\frac{\mu\sigma}{T} + i\frac{\tilde{\rho}}{\omega T}$$

which is the result is in the absence of a lattice (Hartnoll and Herzog, 2007).

The thermal conductivity is harder to compute numerically since one has to impose a temperature gradient. This is an O(1) contribution to $\delta g_{\mu\nu}$ making it hard to read off the $O(z^3)$ contribution needed for $T_{\mu\nu}$.

Five-dimensional holographic lattice

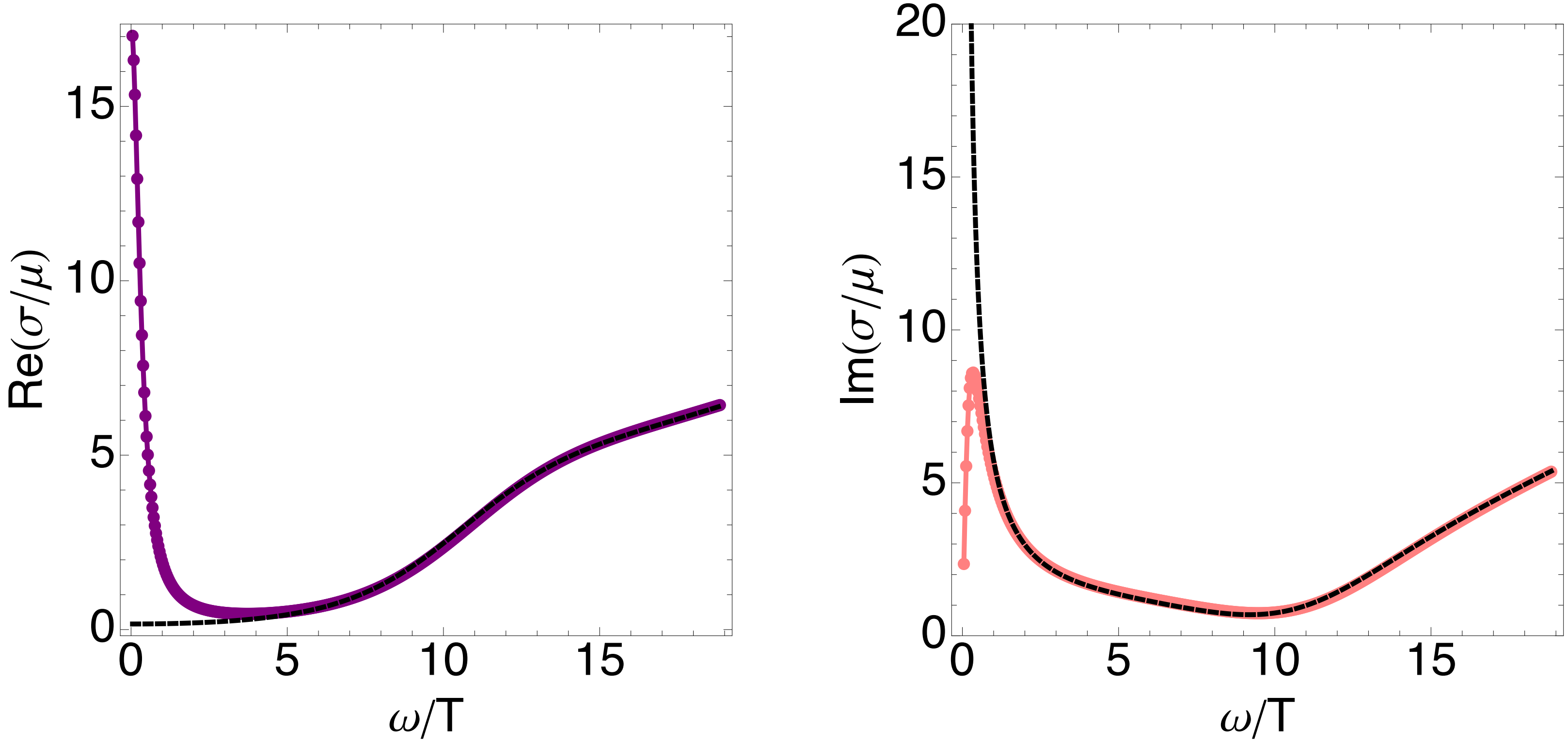
We use the same Einstein-Maxwell scalar action, but now in 5D with scalar mass $m^2 L^2 = -15/4$. The asymptotic behavior is

$$\Phi = z^{3/2}\phi_1 + z^{5/2}\phi_2 + \cdots$$

To impose a lattice, we again require

$$\phi_1 = \mathcal{A}_0 \cos(k_0 x)$$

Optical conductivity for a 3+1 system



At very low frequency, one again finds Drude behavior.

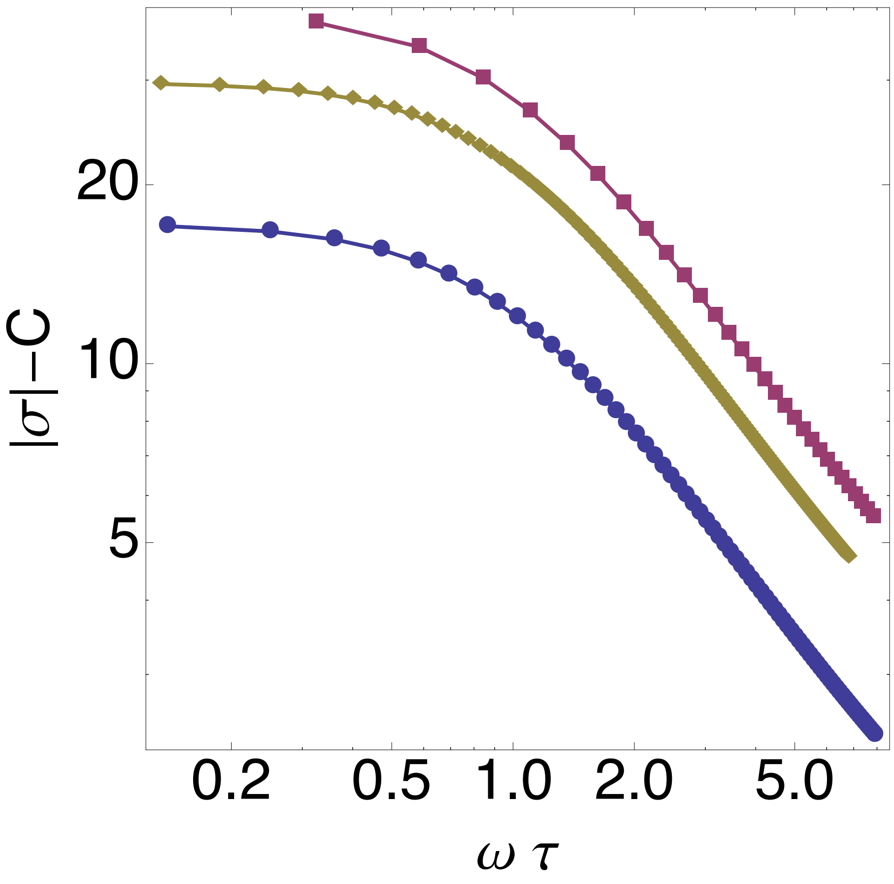
At intermediate frequency, there is a power law fall-off, but the exponent is different:

$$|\sigma| = \frac{B}{\omega^{\sqrt{3}/2}} + C$$

Remarkably, this again holds for $2 < \omega\tau < 8$!

It would be great to find a 3+1 analog of the cuprates to compare this to!

The exponent is again robust against changing the parameters in our model:



3+1 conductivity for k_0 = 1, 2, 3

Discussion

The power law behavior of transport coefficients at intermediate frequencies does not appear to be related to either the UV AdS_4 or the IR AdS_2 symmetries.

Could it be a result of a new scaling symmetry in some intermediate radius region of our holographic lattice?

This is hard to check since there is no preferred radial coordinate.

Since we are interested in homogeneous transport, is it sufficient to find a scaling symmetry of some homogeneous approximation to the bulk geometry?

No.

Expanding the perturbation equation in a Fourier series in x e.g. $\delta A(x) = \Sigma\, \delta A_n \cos(nk_0 x)$ yields

$$\mathcal{O}_0 \delta A_0 = \sum \mathcal{O}_n \delta A_n$$

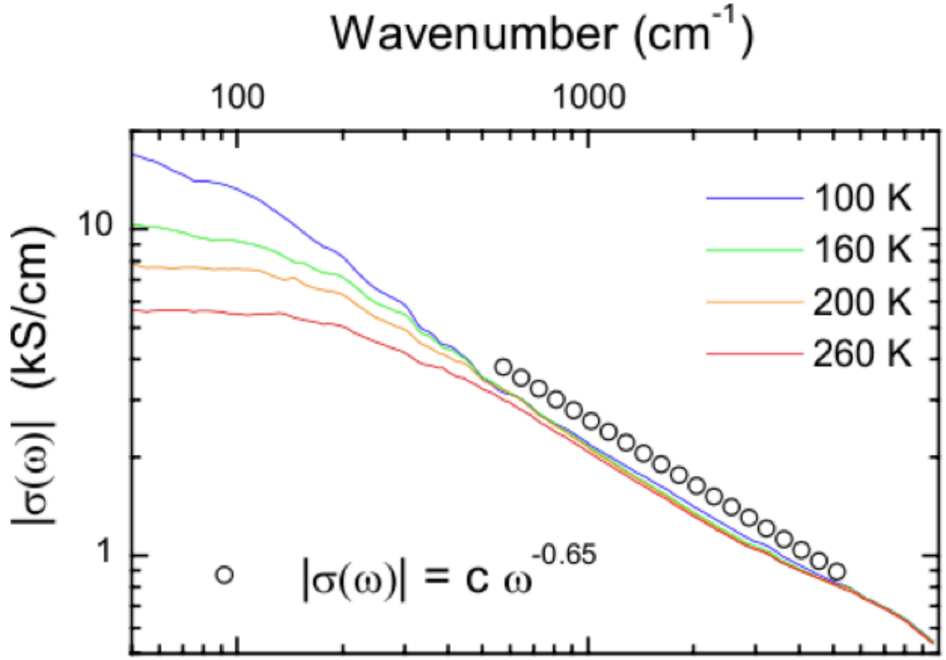
Differential operator with $\cos(nk_0\, x)$ dependence

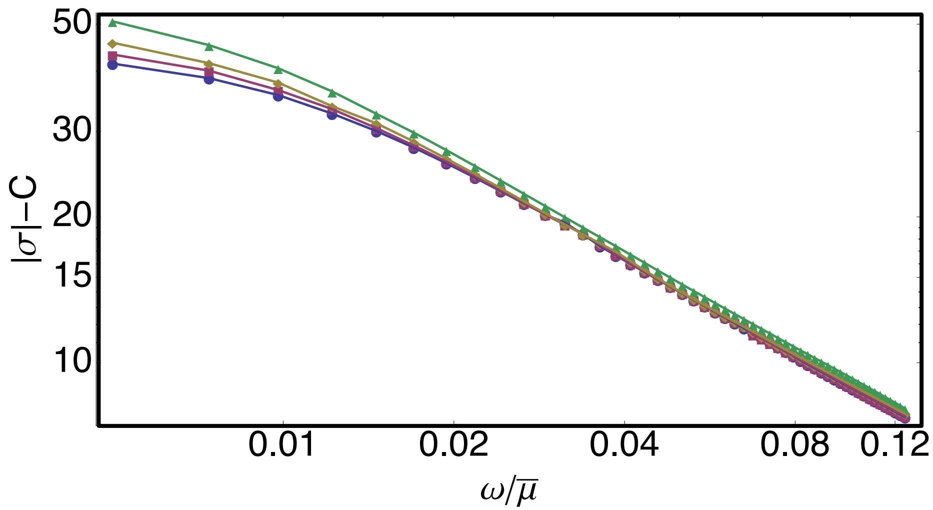
How can the power law exponent be independent of k_0, $\mathcal{A}_0$, T when the delta function should emerge when each vanishes?

The power law holds in the range $2 < \omega\tau < 8$. In the above limits, τ diverges.

Summary

- We have constructed holographic lattices in Einstein-Maxwell-(scalar) theory in 4D and 5D
- We perturbed the solutions and computed the optical and thermoelectric conductivity
- Simple Drude behavior at low frequencies
- Intermediate power law with exponent that agrees with the normal phase of the cuprates
- DC resistivity scales like a power of T which depends on k_0/μ.


Wavenumber (cm^{-1})
100
1000
10
1
|σ(ω)| (kS/cm)
100 K
160 K
200 K
260 K
|σ(ω)| = c ω$^{-0.65}$


50
30
20
15
10
|σ|–C
0.01
0.02
0.04
0.08
0.12
ω/μ̄

Why
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