CFT PROBES OF BULK GEOMETRY
& CAUSAL HOLOGRAPHIC INFORMATION

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Based on:

VH 1203.1044,
VH, H.Maxfield 1210.XXXX,
VH & M.Rangamani 1204.1698,
VH, M.Rangamani, E.Tonni 1210.XXXX

YKIS "From Gravity to Strong Coupling Physics," YITP
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AdS/CFT correspondence

String theory ($\ni$ gravity) $\Longleftrightarrow$ gauge theory (CFT)

"in bulk" asymp. AdS $\times$ K *"on boundary"*

Key aspects:

- ✶ Gravitational theory maps to non-gravitational one!
- ✶ *Holographic*: gauge theory lives in fewer dimensions.
- ✶ Strong/weak coupling duality.

Invaluable tool to:

- ~ Use gravity on AdS to learn about strongly coupled field theory
- ~ Use the gauge theory to define & study quantum gravity in AdS

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Pre-requisite: Understand the AdS/CFT 'dictionary'...

Motivation

To understand the AdS/CFT dictionary;
esp. how does spacetime (gravity) emerge?

- Most QG questions rest on bulk locality (& its breakdown)...
- Given a specific bulk location, what quantities in the CFT should we examine in order to learn about the physics at that location?
  - How deep into the bulk can various CFT probes see?
- esp.: can convenient CFT probes see into a black hole?
- Given full knowledge of physics ($\rho_{\mathcal{A}}$) in a certain boundary region $\mathcal{A}$,
  - in what region of the bulk does it determine the bulk geometry?
  - in what region of the bulk is it sensitive to the bulk geometry?

OUTLINE

- Motivation & Background
- Features of Extremal Surfaces
- Probing Horizons
- Causal Holographic Information
- Summary & Future directions

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Probes of bulk geometry

The bulk metric can be extracted using various CFT probes (which are described by geometrical quantities in the bulk):

Examples:

CFT probe	bulk quantity
✳ expectation values of local gauge-invariant operators	asymptotic fall-off of corresponding conjugate field
✳ correlation functions of local gauge-invariant operators	in WKB approx., proper length of corresponding geodesic
✳ Wilson loop exp. vals.	area of string worldsheet
✳ entanglement entropy	vol of extremal co-dim.2 surface

Bulk-cone singularities

e.g. bi-local CFT probes (not reliant on analytic continuation)
bulk-cone singularities:

[VH, Liu, Rangamani]

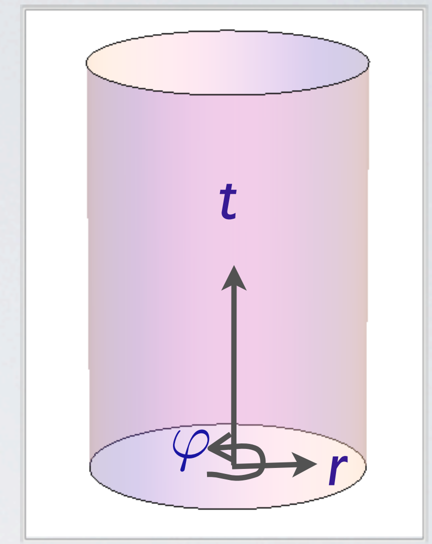
* Green's functions on curved bulk spacetime are singular at null-separated points
* Boundary correlation functions $\langle\Phi(x)\Phi(y)\rangle$ inherit these singularities
* Hence $\langle\Phi(x)\Phi(y)\rangle \to \infty$ when x and y are null-separated

(either along boundary or through the bulk)

* The set of bulk-cone singularities in the CFT directly give the endpoints of bulk null geodesics.
* One can use this information to learn about the bulk geometry

Bulk-cone singularities

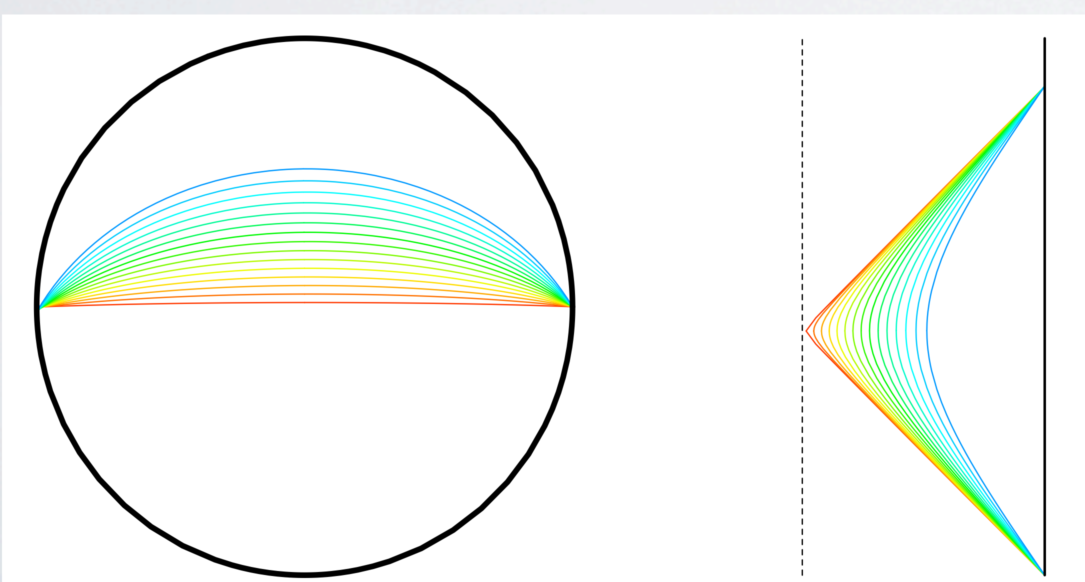
* Consider asymp.(global) AdS bulk, and study projection of null geodesics in $(r\,,t)$ and $(r\,,\varphi)$
* geodesic endpoints clearly distinguish between different bulk geometries:



Null geodesics in AdS:

const. t

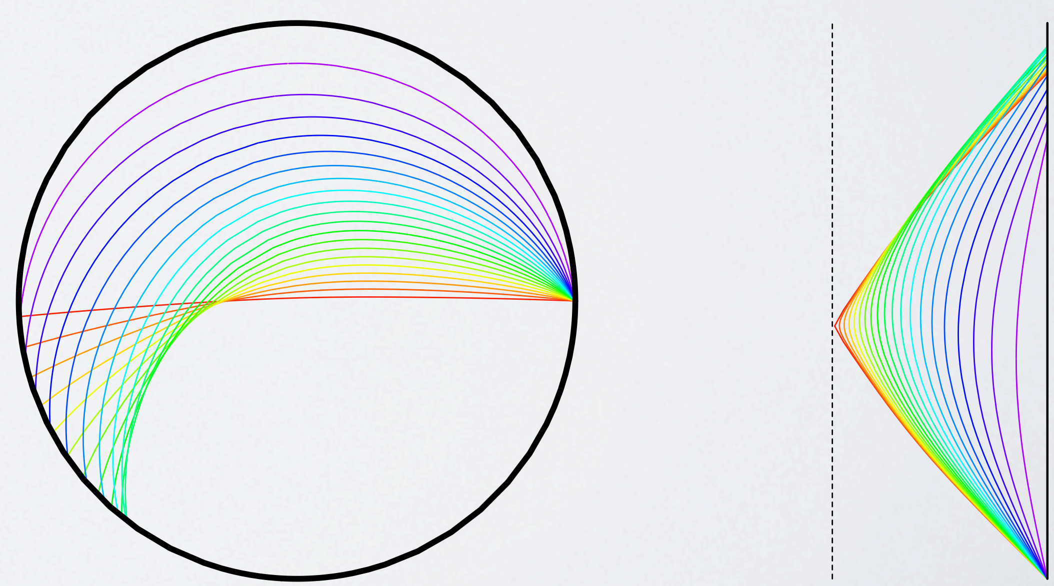
(r , t) plane



cf. Null geodesics in AdS 'star' geometry:

const. t

(r , t) plane



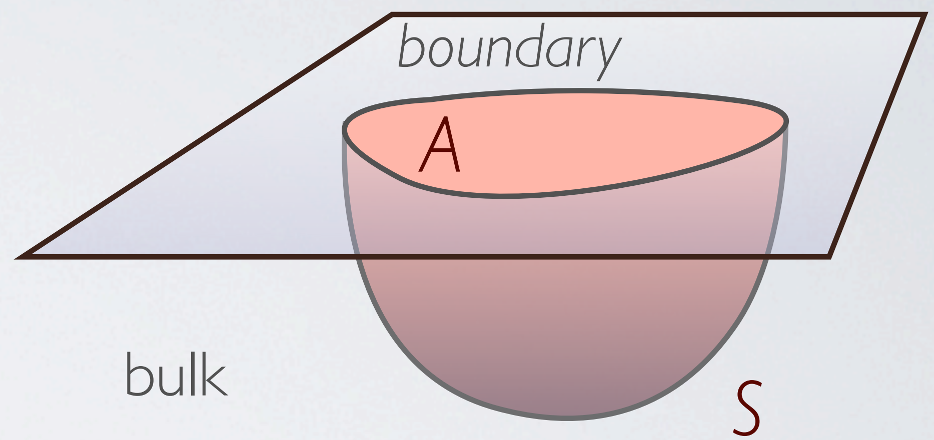
Holographic entanglement entropy

Proposal [Ryu & Takayanagi] for static configurations (at fixed t):

- Entanglement entropy of region $\mathcal{A}$ is

$$\mathcal{S}_{\mathcal{A}} = -\mathrm{Tr}\, \rho_{\mathcal{A}} \log \rho_{\mathcal{A}}$$

- In the bulk this is captured by area of minimal co-dimension 2 bulk surface S anchored on $\partial\mathcal{A}$.



In time-dependent situations, prescription must be covariantised:

- minimal surface → extremal surface
- equivalently, S is the surface with zero null expansions; cf. light sheet construction [Bousso]

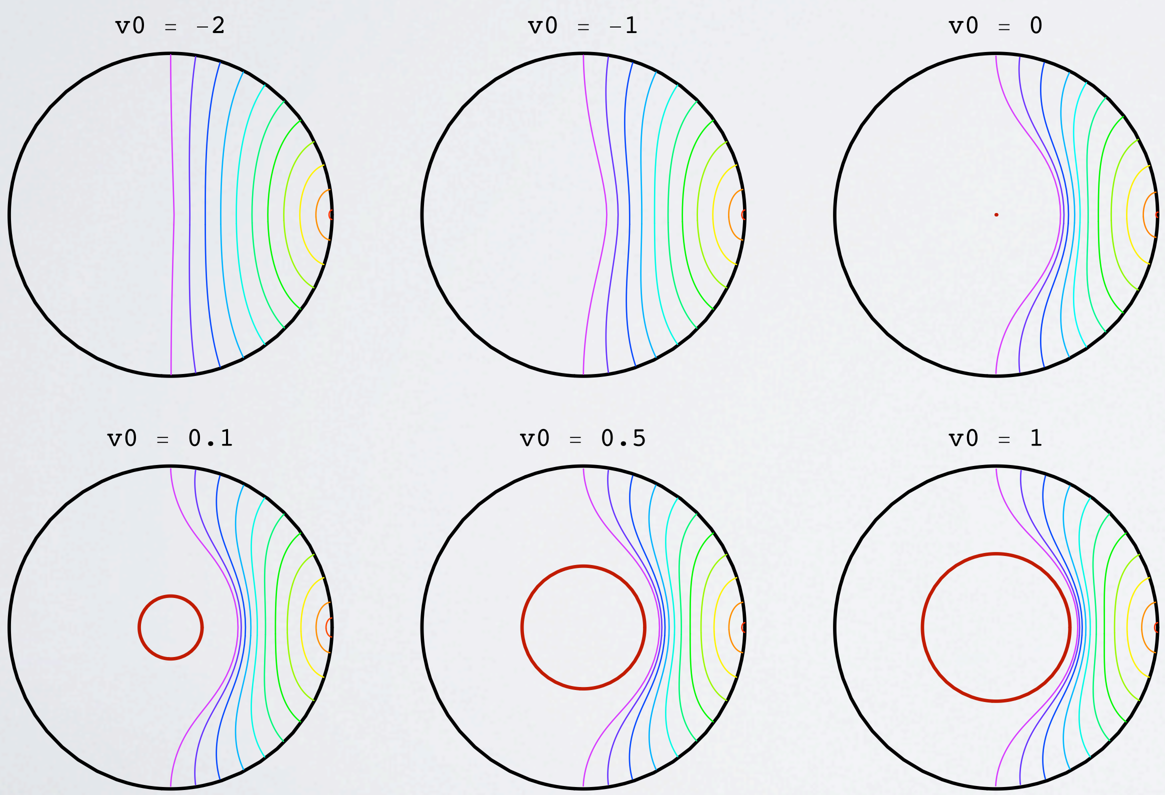
[VH, Rangamani, Takayanagi]

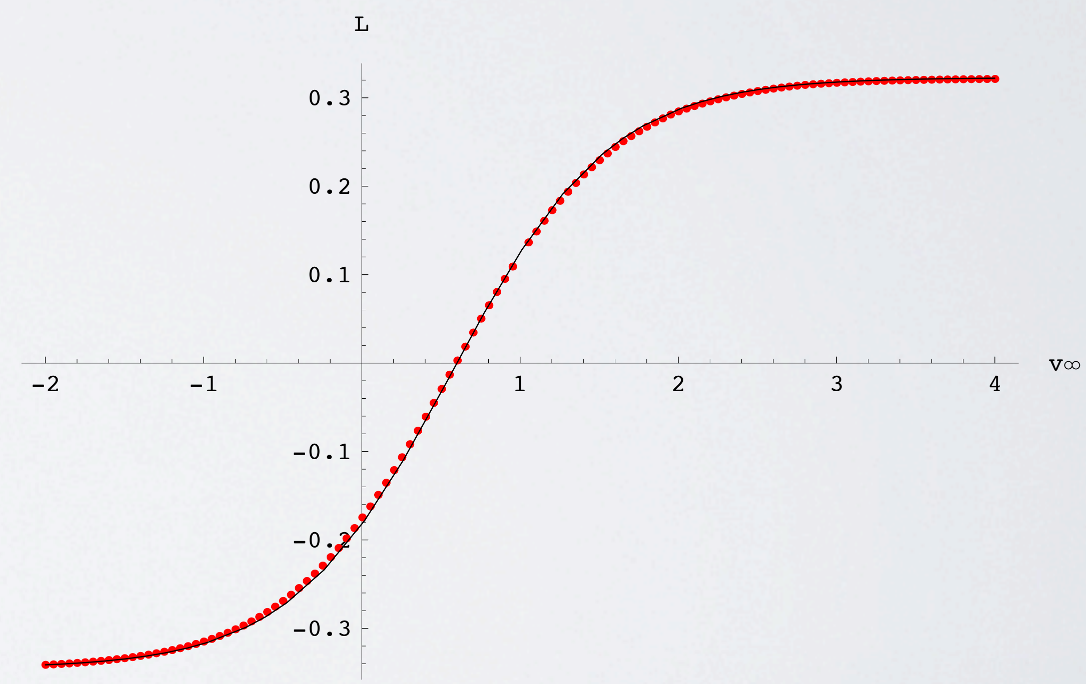
Holographic entanglement entropy

Entanglement entropy growth during thermalisation:
Bulk geometry = collapsing black hole (in 3-d):

behaviour of extremal surfaces
at times v_0 during collapse

corresponding entanglement entropy:





[VH, Rangamani, Takayanagi]

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Probe geodesics

- For simplicity, focus on static, spher. sym., asymp.(global) AdS

$$ds^2 = -f(r)\,dt^2 + h(r)\,dr^2 + r^2\,d\Omega^2$$

- Probe geodesics = bulk geodesics with both endpoints anchored on the (same) AdS bdy
- Can only be spacelike or null (timelike geods don't reach bdy)
- Consider how deep into the bulk these can probe (= $r_{\min}$) and the regularized proper length (= L_{reg}), for a given angular ($\Delta\varphi$) and temporal (Δt) separation of the endpoints:
- What part of the bulk is accessible to probe geodesics?
- Which are optimal geodesics for probing the bulk?

Probe geodesics

- geodesics w/ energy E and ang.mom. L have radial potential

$$\dot{r}^2 + V_{\text{eff}}(r) = 0\ , \qquad V_{\text{eff}}(r) = \frac{1}{h(r)}\left[-\kappa - \frac{E^2}{f(r)} + \frac{L^2}{r^2}\right]$$

- for turning point $r_{\min}$, the endpoints are separated by

$$\Delta t \equiv 2\int_{r_{\min}}^{\infty} \frac{E}{f(r)}\, g(r)\, dr \qquad \text{and} \qquad \Delta\varphi \equiv 2\int_{r_{\min}}^{\infty} \frac{L}{r^2}\, g(r)\, dr\ ,$$

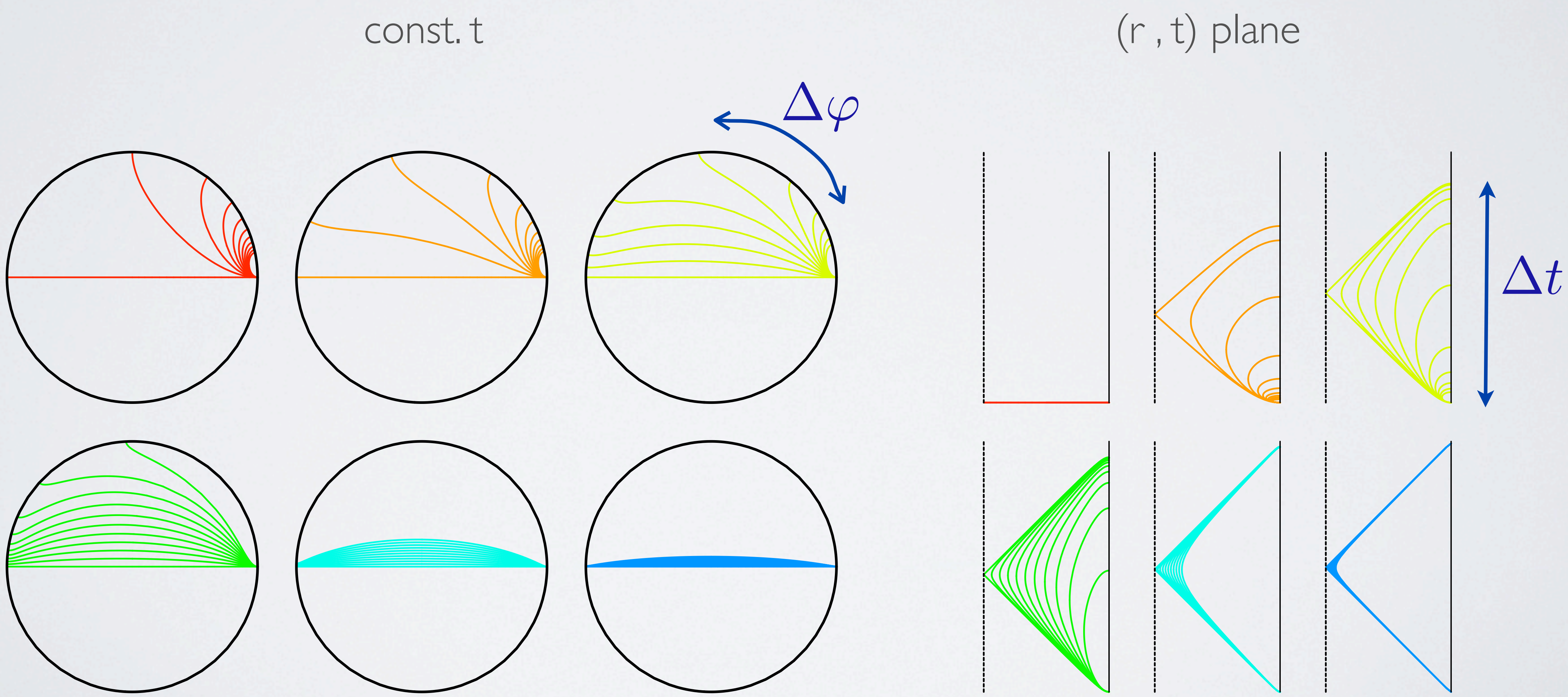
where

$$g(r) \equiv \sqrt{\frac{h(r)}{\kappa + \frac{E^2}{f(r)} - \frac{L^2}{r^2}}} = \frac{1}{\sqrt{-V_{\text{eff}}(r)}} = \frac{1}{|\dot{r}(r)|}$$

- with proper length $\mathcal{L}_R = 2\displaystyle\int_{r_{\min}}^{R} g(r)\, dr$

Probe geodesics in AdS

- Distinct E in distinct panes, denoted by color-coding
- Distinct L are distinct curves in each pane



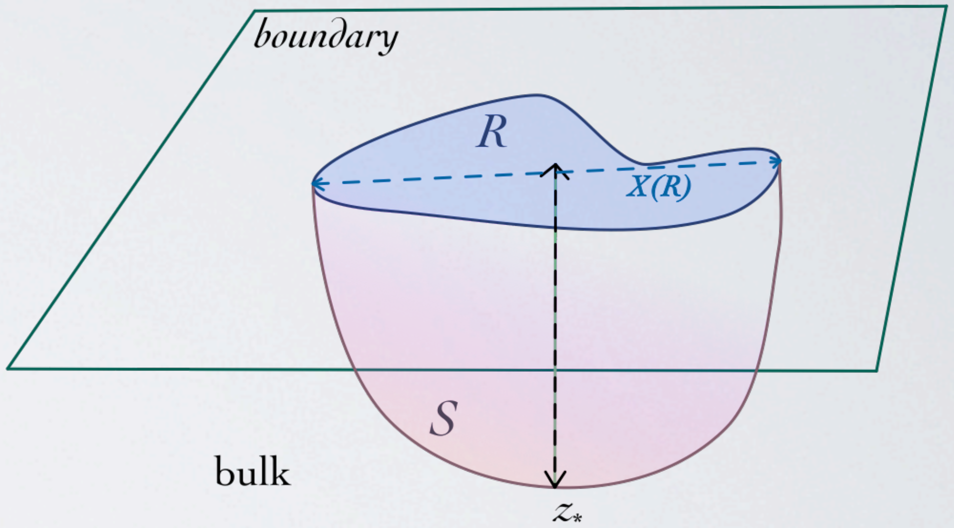
Results for probe geodesics

- In causally trivial spacetime, all of the bulk is accessible to spacelike as well as null geodesics
- In presence of a black hole, probe geodesics cannot penetrate the horizon (cf. part II)
- Spacelike geodesics probe deeper than null ones for fixed parameters E & L
- The `optimal' geodesics for probing bulk are the $E=0$ spacelike ones, as these minimize $r_{\min}$ at fixed $\Delta\varphi$
- (However, they have larger L_{reg})

Extremal surfaces

Simplified context: Focus on extremal surfaces S anchored on bdy n-dim region R in static planar asymp. (Poincare) AdS_{d+1}

Parameters we can dial:



- bulk geometry (specified by 2 fns of 1 variable):

$$ds^2 = \frac{1}{z^2}\left[-g(z)\,dt^2 + k(z)\,dx_i\,dx^i + dz^2\right]$$

- shape (*1* fn of *d-1* variables) & extent X (*1* real number) of bdy region R
- dimensionality n (=*1*,2,...,*d-1*) of the surface S

Key feature of S:

- Bulk depth reached z_*
  - (Note that z_* is geometrically well-defined.)

Results for extremal surfaces

Preview:

- Higher-dimensional surfaces probe deeper
  i.e. z_* increases with *n* for fixed extent *X(R)*
- Surfaces anchored on *R* = ball reach deepest
  compared to differently-shaped *R* with same extent or area
- Surfaces in pure AdS reach deeper
  for fixed *R*, compared asymp.AdS geometry

Results for extremal surfaces

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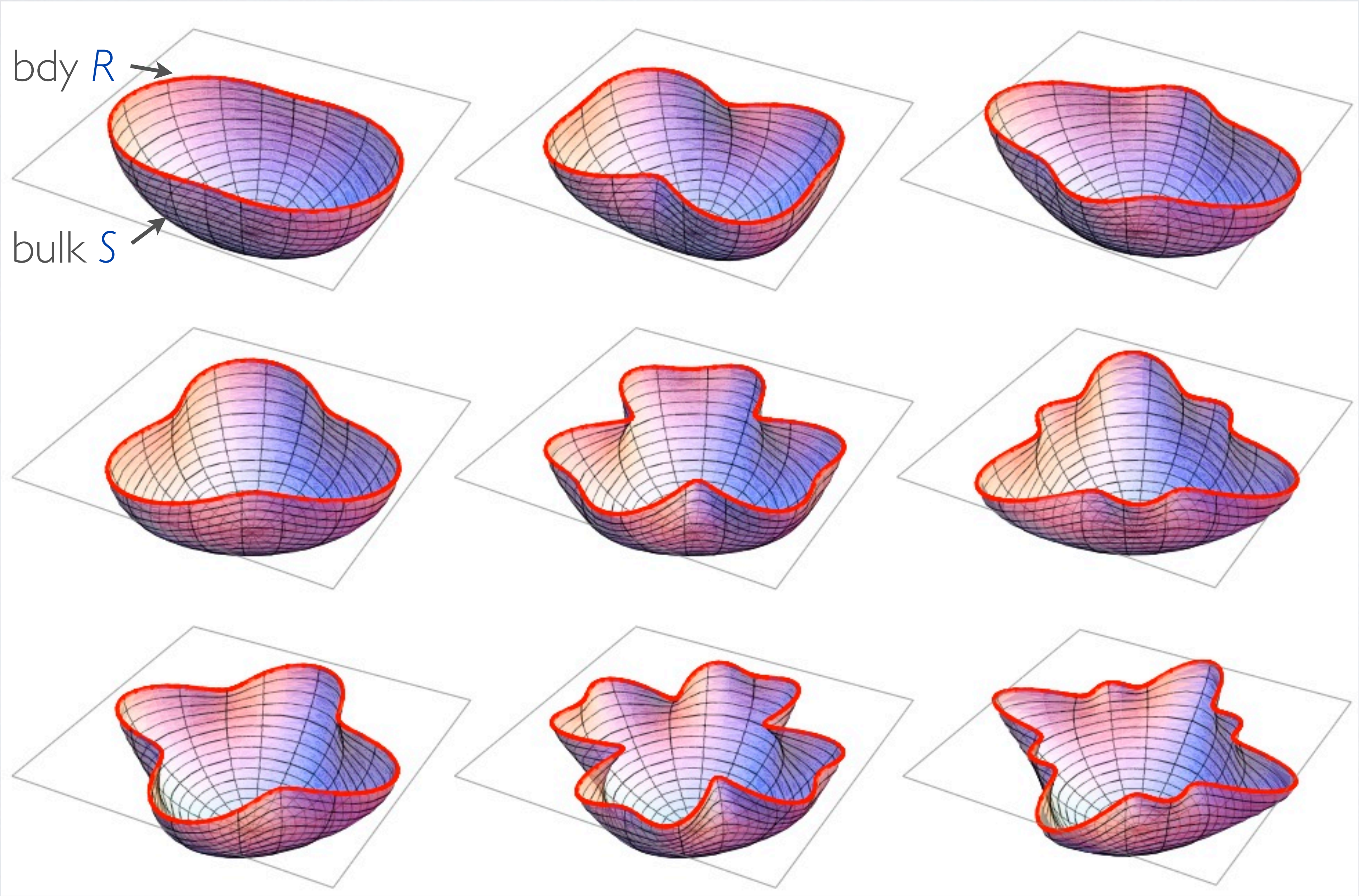
  compared to differently-shaped R with same extent or area
- Surfaces in pure AdS reach deeper

  for fixed R, compared asymp.AdS geometry

Results for extremal surfaces

- Consider R with fixed dimensionality n and `area' $A(R)$
- What shape of R maximizes z_* , i.e. when does S reach deepest?



Results for extremal surfaces

Surfaces anchored on R=ball reach deepest:

- Linearize around the hemisphere $\rho(\theta,\phi)=\rho_0$ in pure AdS:

$$ds^2 = \frac{1}{\rho^2\,\cos^2\theta}\left[-dt^2 + d\rho^2 + \rho^2\,\left(d\theta^2 + \sin^2\theta\, d\phi^2\right) + \sum_{j=3}^{d-1} d\tilde{y}_j^2\right]$$

- To 2nd order,

$$\rho(\theta,\phi) = \rho_0 + \epsilon\,\underline{\rho_1(\theta,\phi)} + \epsilon^2\,\underline{\rho_2(\theta,\phi)} + \mathcal{O}(\epsilon^3)$$

$$\rho_1(\theta,\phi) = \tan^{\ell}(\theta/2)\,(1+\ell\cos\theta)\,\cos\ell\phi\ .$$

$$\rho_2(\theta,\phi) = \frac{1}{4\,\rho_0}\,\tan^{2\ell}(\theta/2)\,\{(1+\ell\cos\theta)^2 + \left[\mu\,(1+2\,\ell\cos\theta)+\ell^2\,\cos^2\theta\right]\,\cos 2\ell\phi\}$$

- At fixed $\rho(\theta=0,\phi)=\rho_0$, the area $A(R) = \pi\,\rho_0^2\left(1+\frac{\epsilon^2}{\rho_0^2}+\mathcal{O}(\epsilon^4)\right)$ increases as R is perturbed from ball
- Hence *R*=round ball ⇒ *S* has greatest reach for fixed *A(R)*
- Confirmed numerically at non-linear level as well.

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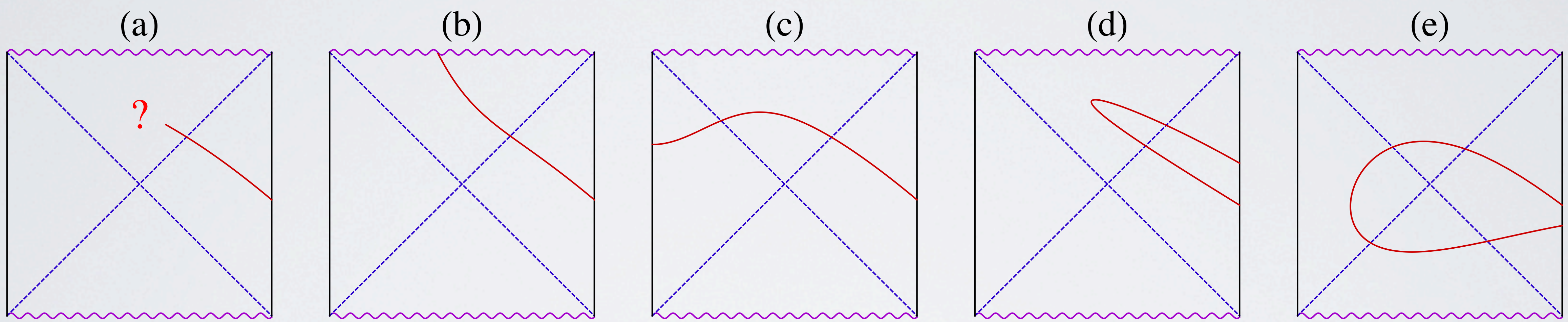
Probe geods can't penetrate horizons in static bulk

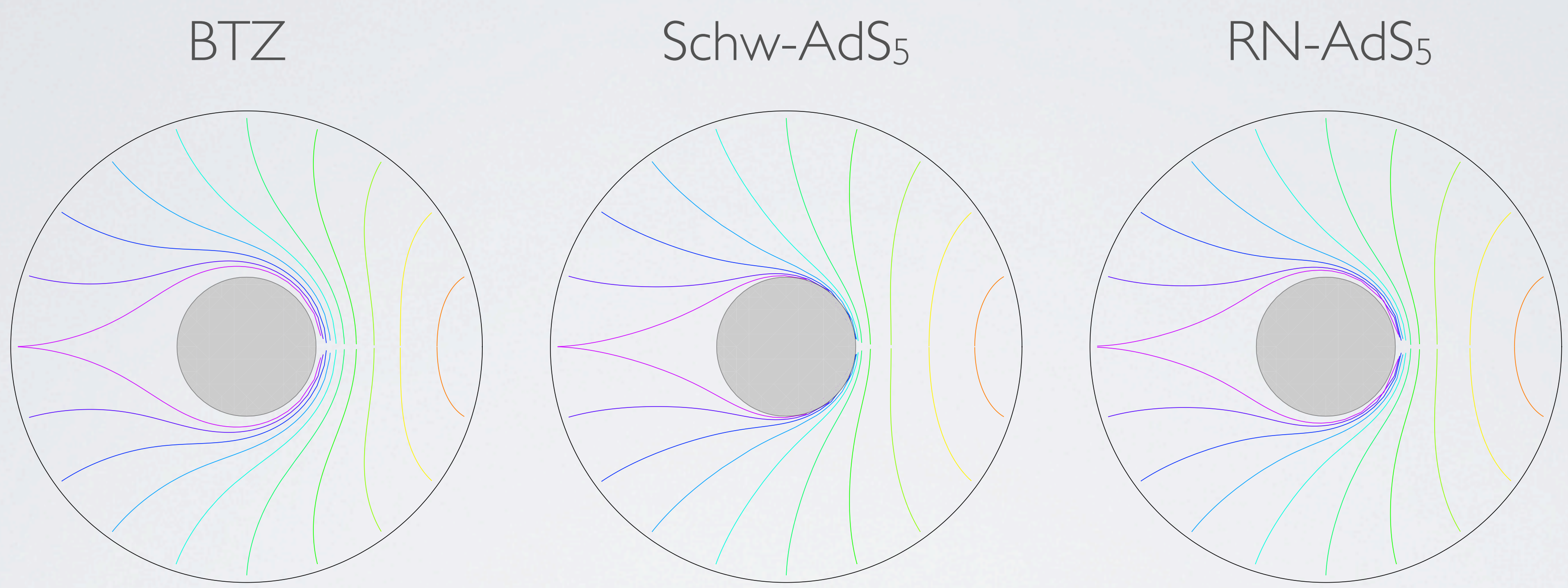
Consider a geod. crossing the horizon; what can happen?



- (b) & (c) are allowed, but don't correspond to probe geods
- (d) is disallowed by energy conservation:
  - Ingoing coords: $ds^2 = -f(r)\,dv^2 + 2\sqrt{f(r)\,h(r)}\,dv\,dr + r^2\,d\Omega^2$
  - conserved energy along geods: $E = f(r)\,\dot{v} - \sqrt{f(r)\,h(r)}\,\dot{r}$
- (e) is disallowed by assumption of reaching horizon from bdy

Probe geods can't penetrate horizons in static bulk

Eg:



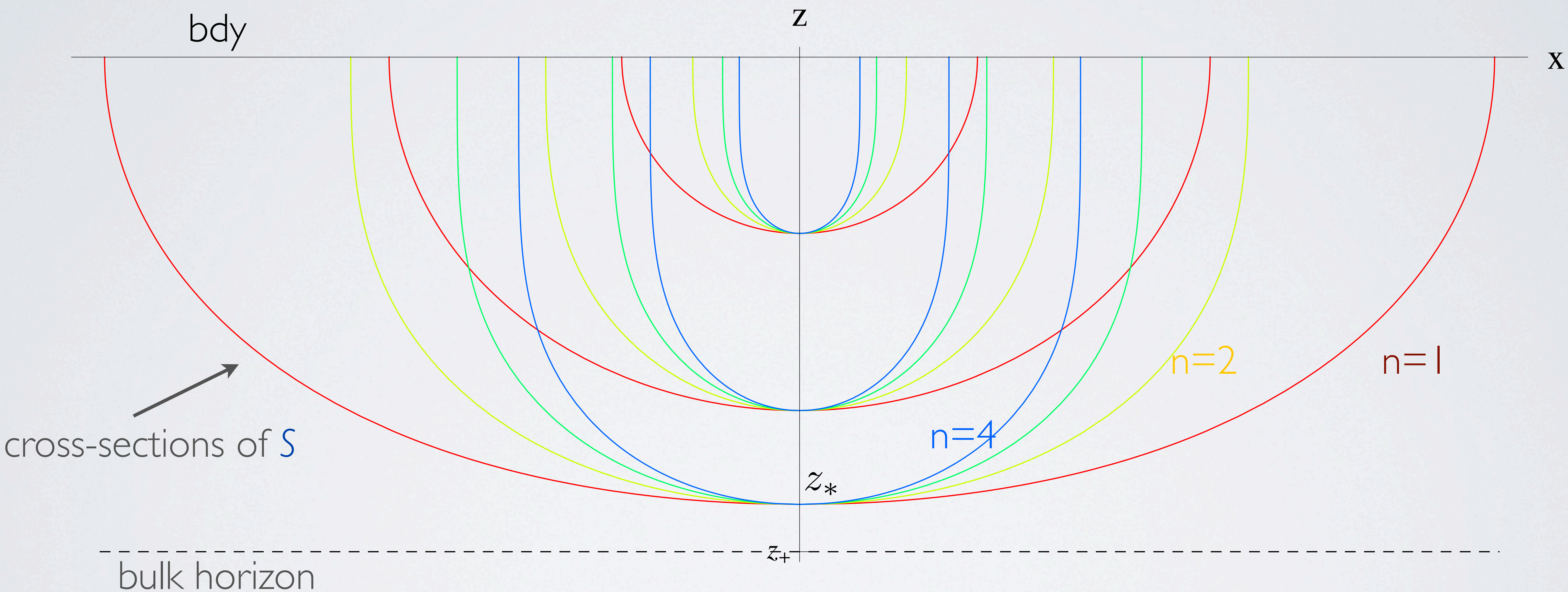
- spacelike geodesics can penetrate arb. close to horizon
- however as $r_{\min} \to r_+$, $\Delta\varphi \to \infty$ and $\mathcal{L}_{\text{reg}} \to \infty$
- limiting $\Delta\varphi = 2\pi$, for $r_+ = 0.5$

$$\frac{r_{\min} - r_+}{r_+} = \begin{cases} 9.03 \times 10^{-2} & \text{for BTZ} \\ 1.85 \times 10^{-3} & \text{for SAdS}_5 \\ 6.54 \times 10^{-2} & \text{for RNAdS}_5 \end{cases}$$

- null geodesics can only reach a finite distance from horizon

Extremal surfaces can't penetrate horizons in static bulk

- Consider extent *X(R)* for *n*-strip in Schw-AdS5 at fixed z_*



- We see extremal surfaces are repelled by the horizon

Extremal surfaces can't penetrate horizons in static bulk

Completely general proof, for any n, R, & bulk geom:

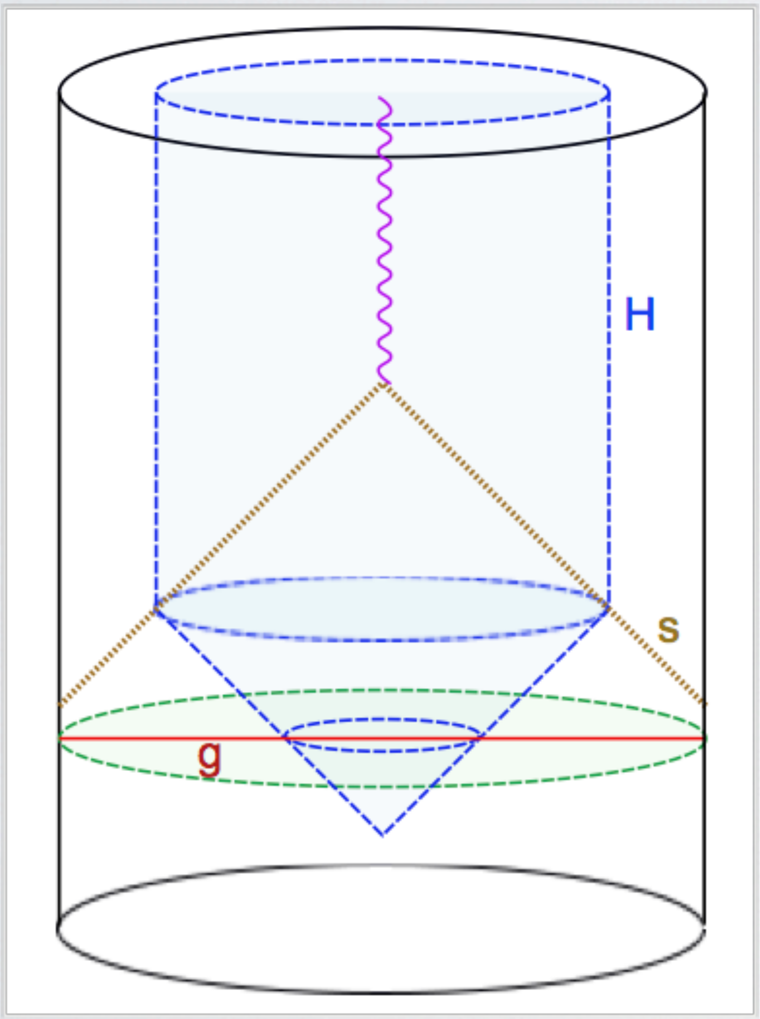
- Consider extremal surface S param. by $z(x^1, \ldots, x^n)$

  in bulk spacetime $ds^2 = \frac{1}{z^2}\left[-f(z)\, dt^2 + dx_i\, dx^i + h(z)\, dz^2\right]$
- At horizon, $f \to 0$ and $h \to \pm\infty$
- Lagrangian: $\mathcal{L}(z\,,\, z_{,1}\,,\, \ldots\,,\, z_{,n}\,;\, x^1, \ldots, x^n) = \sqrt{G} = \frac{\sqrt{1 + h(z)\left(z_{,1}^2 + \ldots + z_{,n}^2\right)}}{z^n}$
- EOM: $\sum_i z_{,ii}\left(1 + h(z) \sum_j z_{,j}^2\right) - h(z) \sum_{i,j} z_{,ij}\, z_{,i}\, z_{,j} + \sum_i z_{,i}^2 \left(\frac{n}{z} + \frac{h'(z)}{2\, h(z)}\right) + \frac{n}{z\, h(z)} = 0$
- Around the turning point, $z = z_*$: $\sum_i z_{,ii} + \frac{n}{z\, h(z)} = 0$
- In order for z to be maximum, $z_{,ii}(z_*) < 0$, which forces $h(z_*) > 0$
- Hence turning point z_* must be outside the horizon.

Extremal surfaces can penetrate horizons in time-evolving bulk

Gedanken-experiment to demonstrate that causality does not pose a fundamental obstacle to extracting information via CFT:

[VH]



* uses the teleological nature of event horizon & non-local nature of AdS/CFT:
  * Measure bulk event by spacelike CFT probe (precursor), e.g. geodesic **g**
  * Afterwards, collapse a shell **s**,
  * such that the resulting event horizon ***H*** encompasses the measured event.
* Then **g** is a probe geodesic which penetrates the event horizon
  * seen explicitly for geods in Vaidya-AdS

[VH, Maxfield]

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- Causal Holographic Information (CHI)
    - Construction of causal wedge and CHI
    - Properties of CHI for stationary configurations
    - Behavior of CHI in dynamical settings
- Summary & Future directions

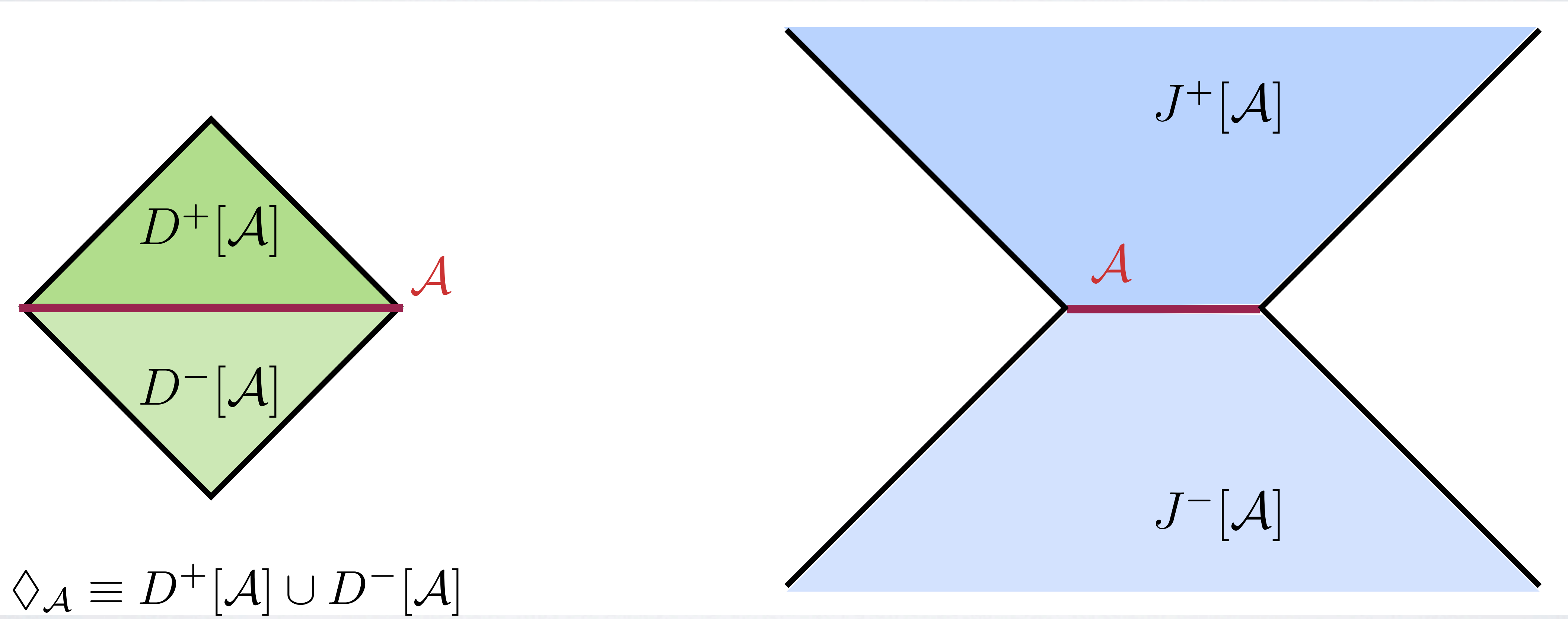
Bulk dual to a bdy region $\mathcal{A}$?

What is the most natural bulk region
associated to a given region $\mathcal{A}$ on the bdy?

- 'natural': try to be minimalistic, use only bulk causality
- Take $\mathcal{A}$ to be d-1 dimensional spatial region on bdy of asymp. AdS_{d+1} bulk spacetime.
- The unique minimal construction gives a bulk causal wedge associated with $\mathcal{A}$, and a corresponding d-1 dimensional bulk surface $\Xi_{\mathcal{A}}$
- Using geometrical information, we can associate a number $\chi_{\mathcal{A}}$ to $\mathcal{A}$, corresponding to area of $\Xi_{\mathcal{A}}$

Causal construction

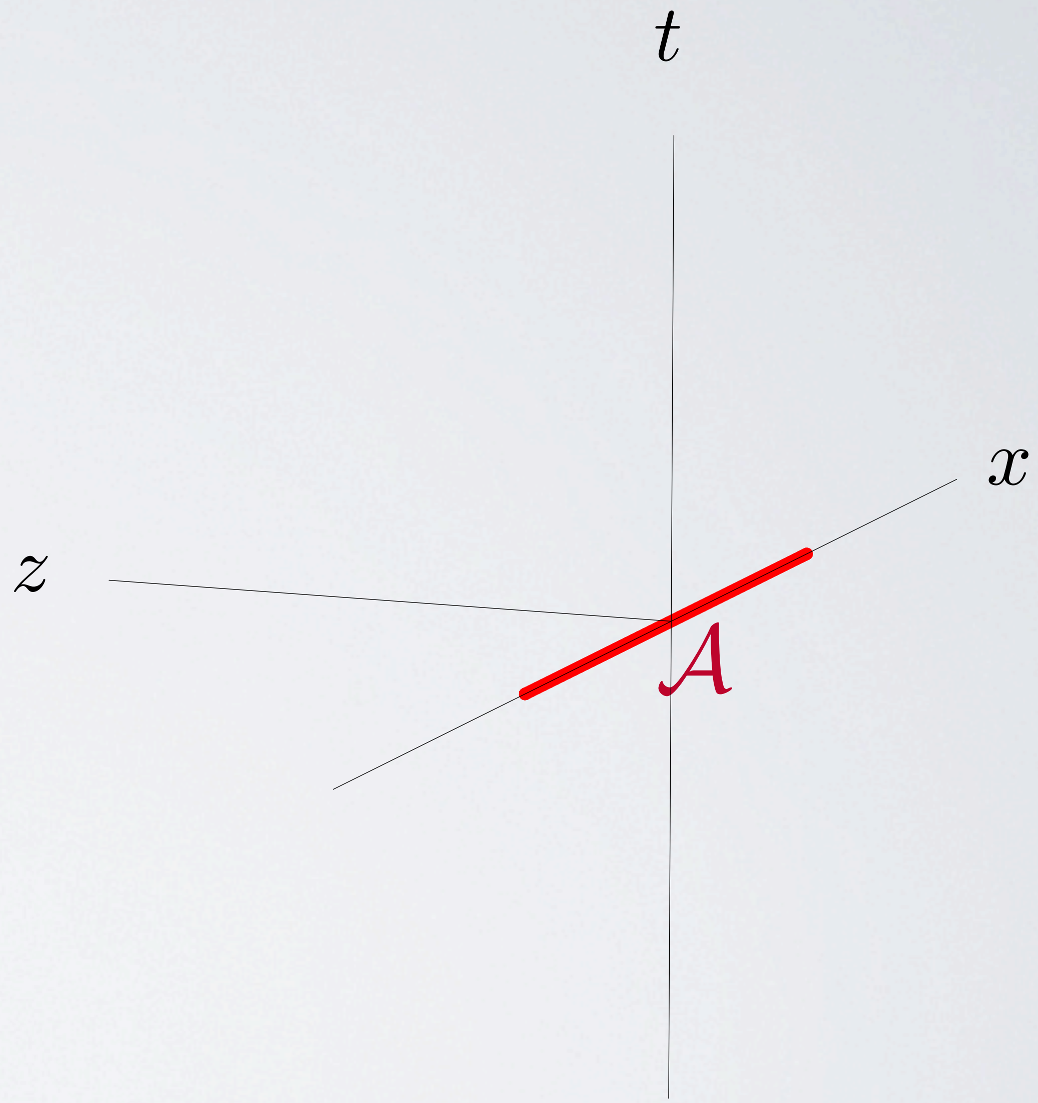
- domain of dependence $D^{\pm}[\mathcal{A}]$ = region which must influence or be influenced by events in $\mathcal{A}$
- domain of influence $I^{\pm}[\mathcal{A}]$ = region which can influence or be influenced by events in $\mathcal{A}$



- Given $\rho_{\mathcal{A}}$, we can determine observables in the entire $\Diamond_{\mathcal{A}}$

Causal construction

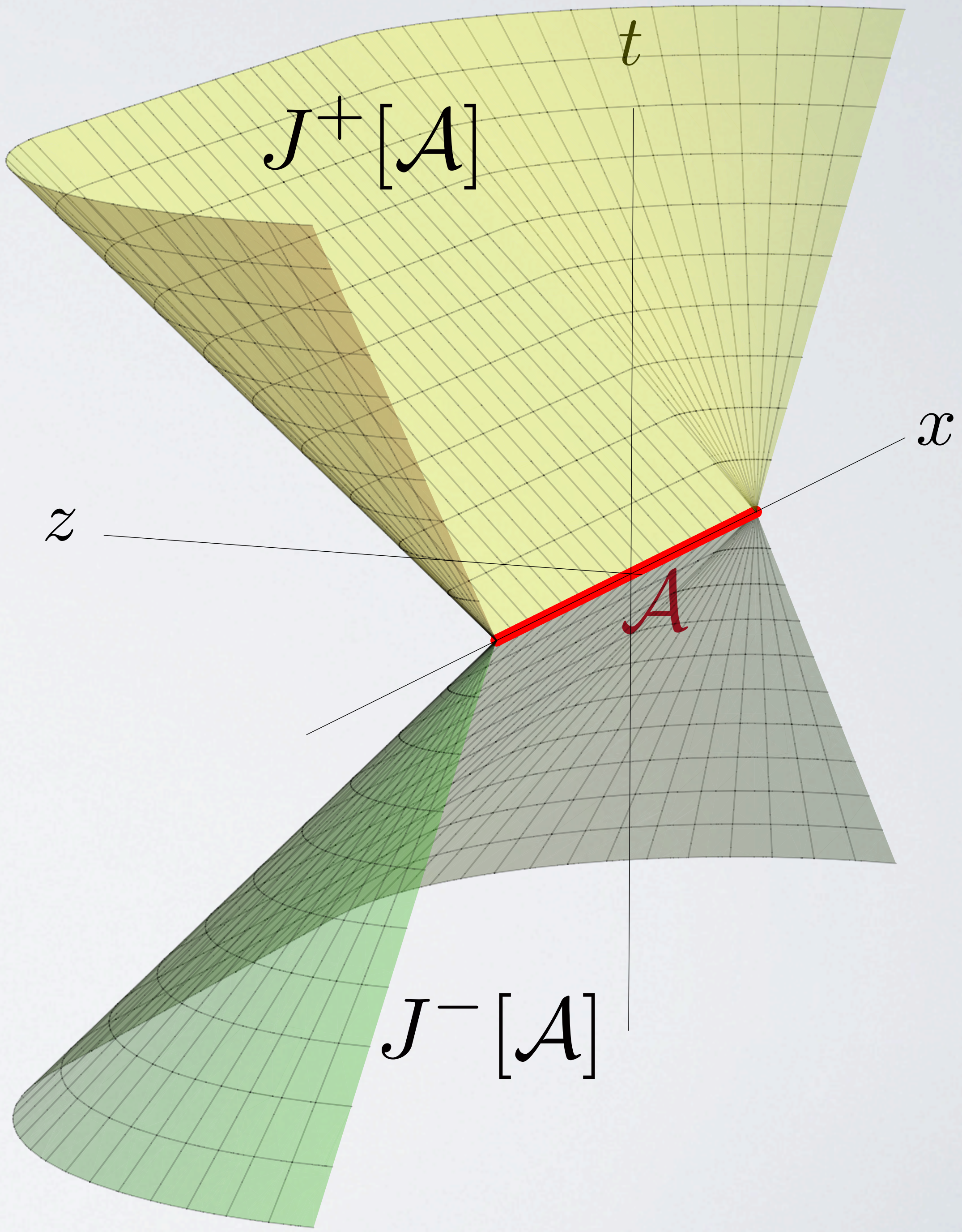
- Consider a bdy region $\mathcal{A}$



Causal construction

- Consider a bdy region $\mathcal{A}$
- Its bulk domains of influence extend arb. deep into the bulk and have trivial intersection

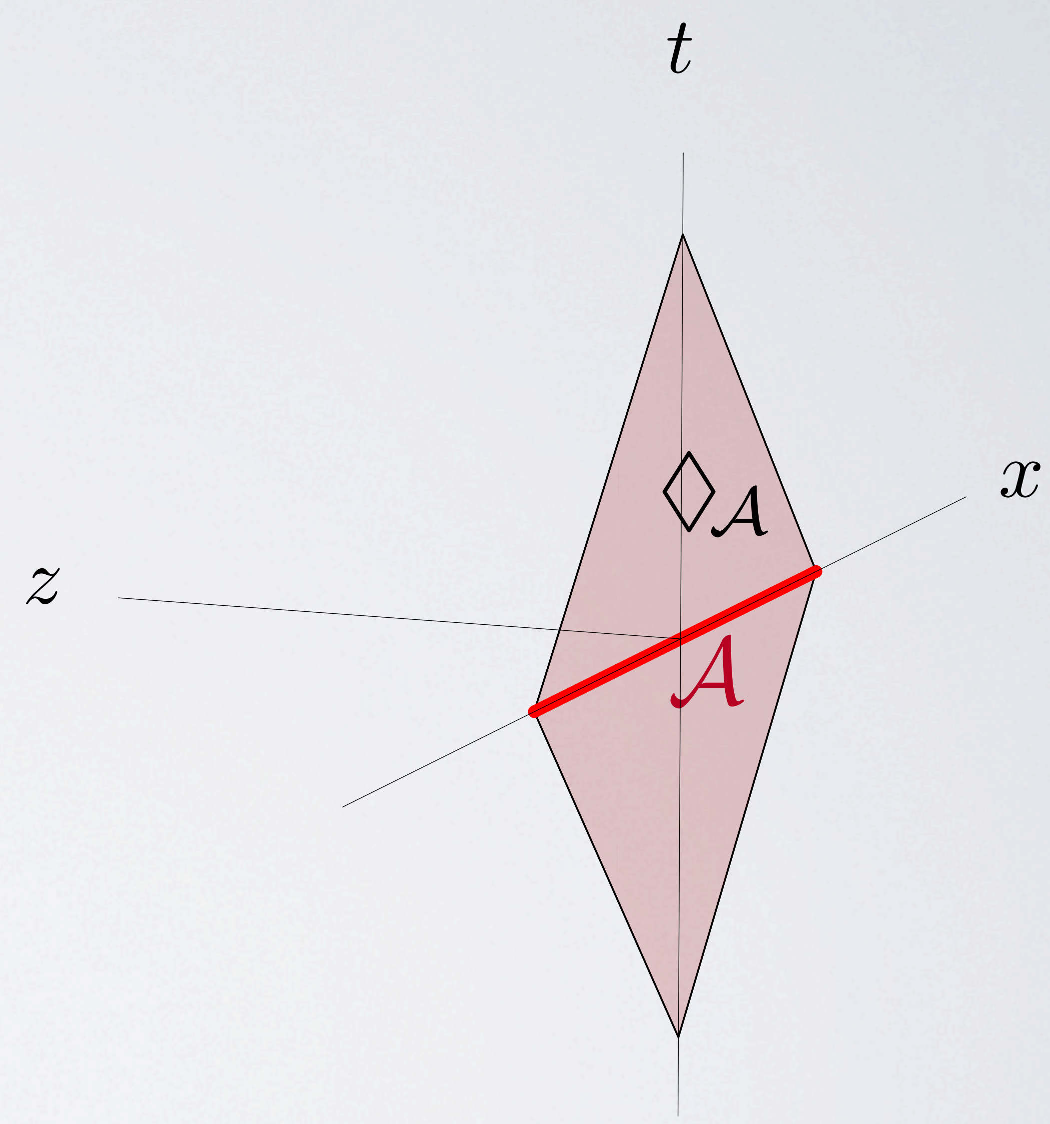
(and bulk domain of dependence of $\mathcal{A}$ is just the region $\mathcal{A}$ itself).



Causal construction

- Consider a bdy region $\mathcal{A}$
- Its bulk domains of influence extend arb. deep into the bulk and have trivial intersection
- Consider a bdy domain of dependence of $\mathcal{A}$, denoted $\Diamond_{\mathcal{A}}$

(observables in the entire region $\Diamond_{\mathcal{A}}$ can be determined solely from the initial conditions specified on $\mathcal{A}$)



Causal construction

- Consider a bdy region $\mathcal{A}$
- Its bulk domains of influence extend arb. deep into the bulk and have trivial intersection
- Consider a bdy domain of dependence of $\mathcal{A}$, denoted $\Diamond_{\mathcal{A}}$
- Its bulk domains of influence extend arb. deep, but their intersection doesn't

t

$J^{+}[\Diamond_{\mathcal{A}}]$

x

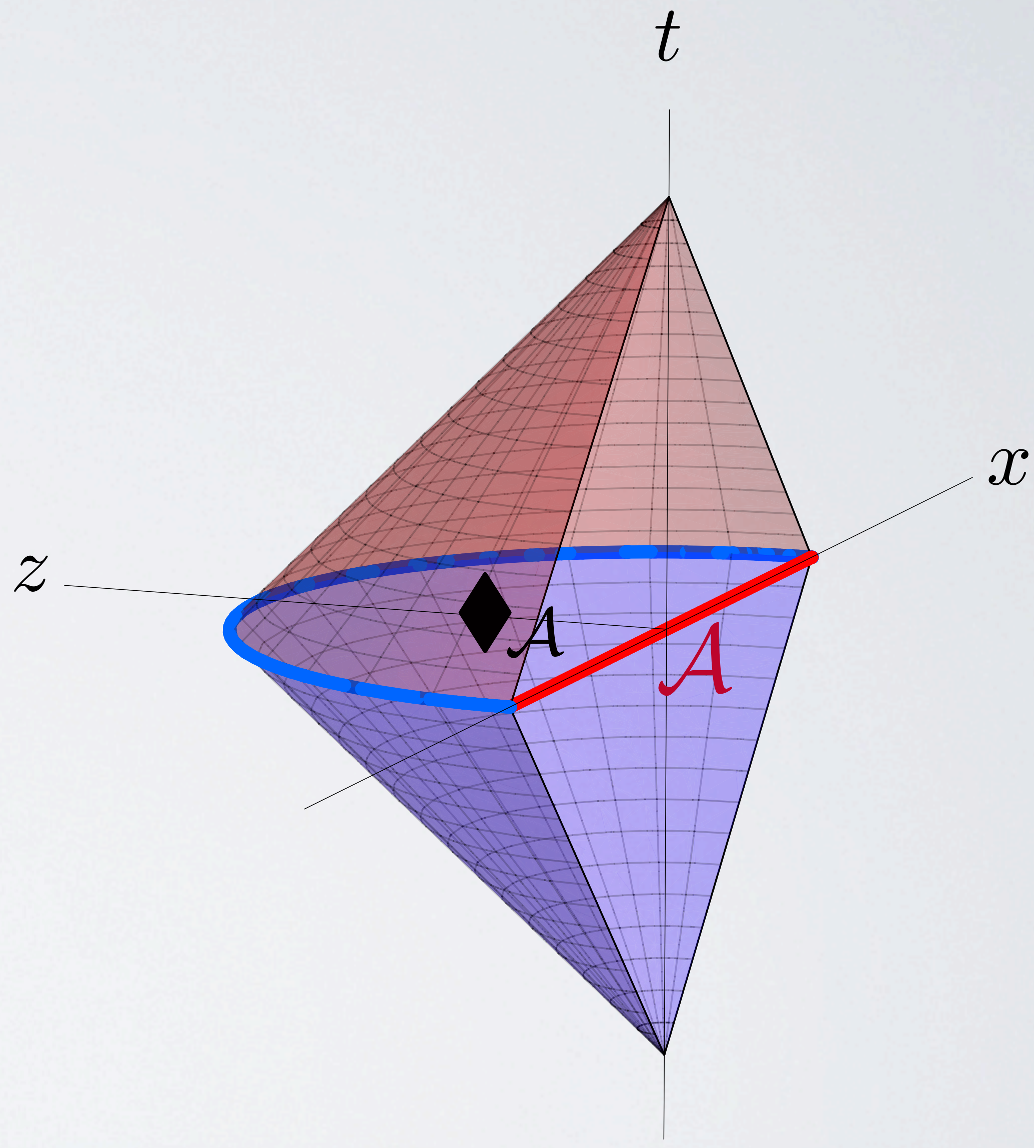
z

$\mathcal{A}$

$J^{-}[\Diamond_{\mathcal{A}}]$

Causal construction

- Consider a bdy region $\mathcal{A}$
- Its bulk domains of influence extend arb. deep into the bulk and have trivial intersection
- Consider a bdy domain of dependence of $\mathcal{A}$, denoted $\Diamond_{\mathcal{A}}$
- Its bulk domains of influence extend arb. deep, but their intersection doesn't
- This defines for us the bulk causal wedge of $\mathcal{A}$, denoted $\blacklozenge_{\mathcal{A}}$



Causal construction

- Bulk causal wedge $\blacklozenge_{\mathcal{A}}$

$$\blacklozenge_{\mathcal{A}} \equiv J^{-}[\lozenge_{\mathcal{A}}] \cap J^{+}[\lozenge_{\mathcal{A}}]$$

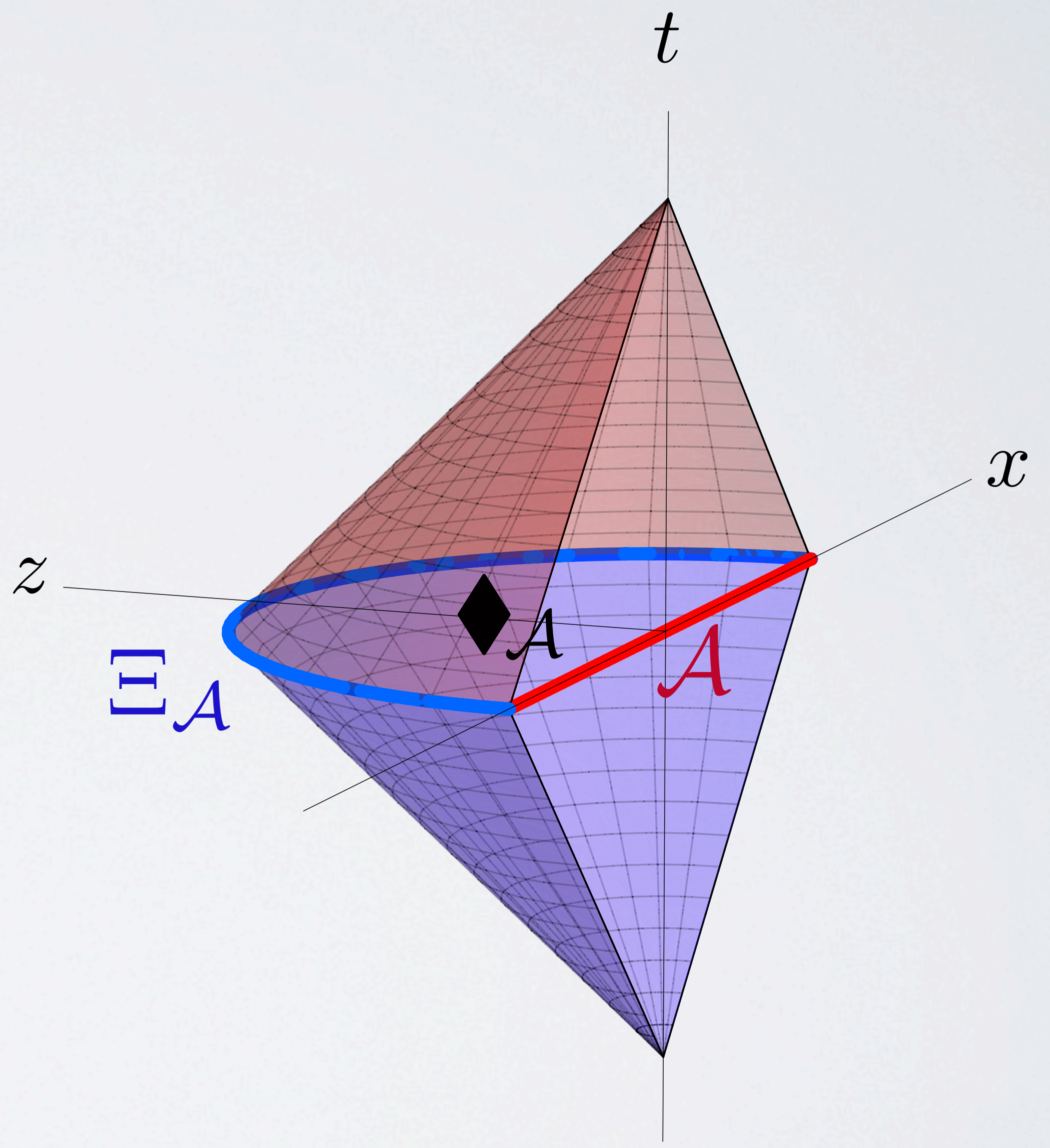
= { bulk causal curves which begin and end on $\lozenge_{\mathcal{A}}$ }

- Causal information surface $\Xi_{\mathcal{A}}$

$$\Xi_{\mathcal{A}} \equiv \partial_{+}(\blacklozenge_{\mathcal{A}}) \cap \partial_{-}(\blacklozenge_{\mathcal{A}})$$

- Causal holographic information $\chi_{\mathcal{A}}$

$$\chi_{\mathcal{A}} \equiv \frac{\text{Area}(\Xi_{\mathcal{A}})}{4\,G_N}$$

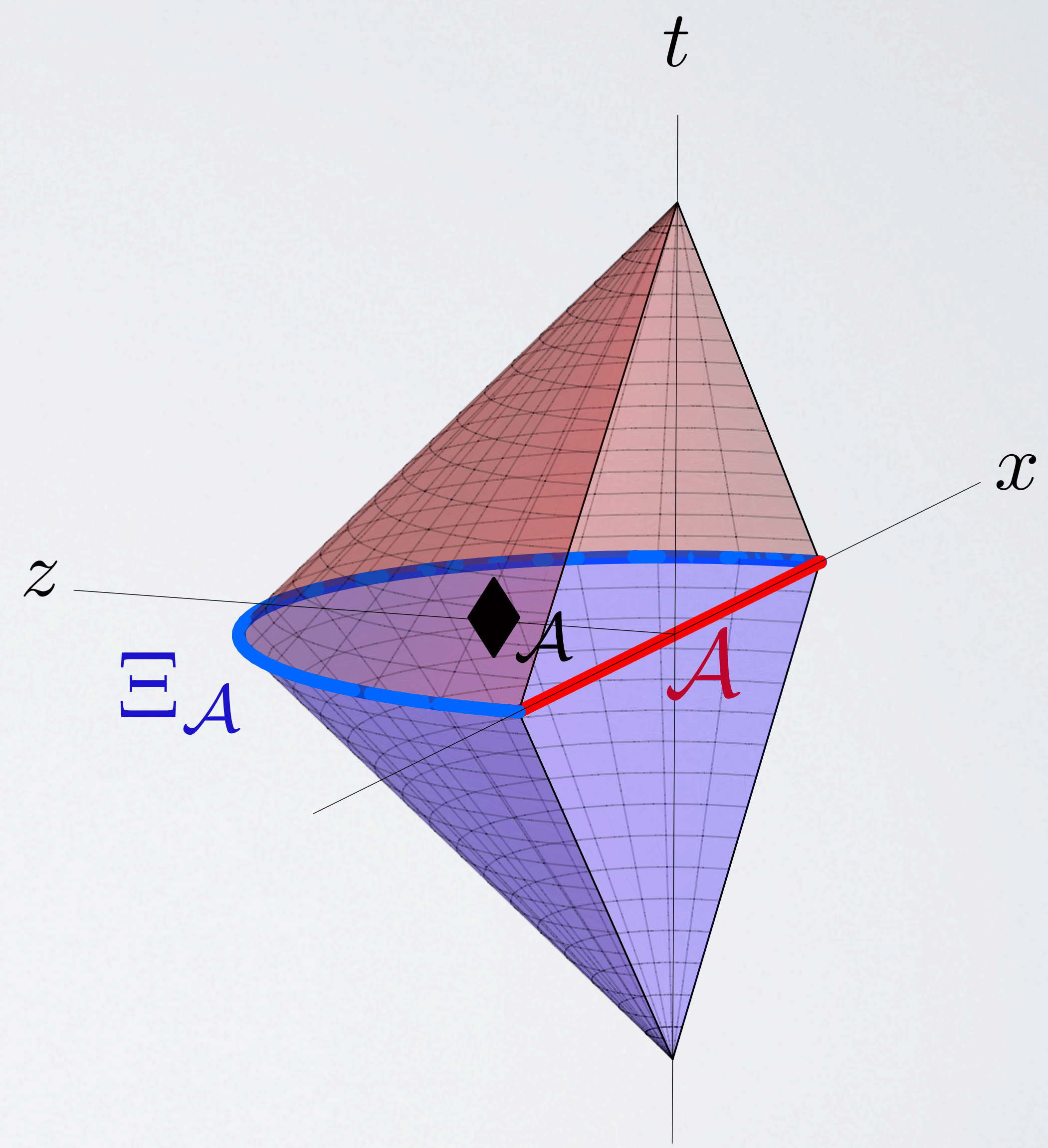


Main question:

What is the CFT interpretation of $\Xi_{\mathcal{A}}$ and $\chi_{\mathcal{A}}$?

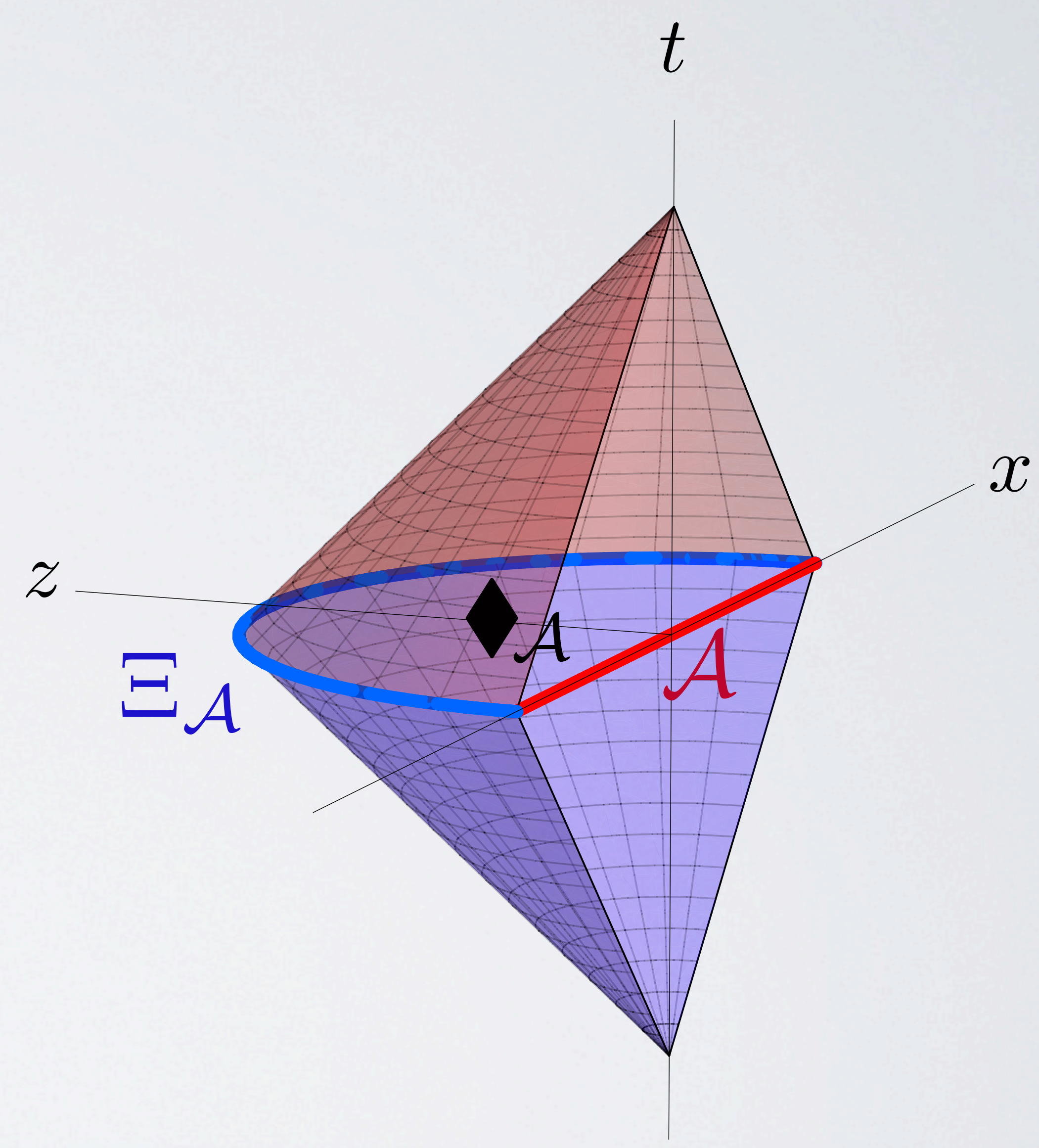
Gather hints

by considering geometrical properties and behavior of $\Xi_{\mathcal{A}}$ ...



General properties of $\Xi_{\mathcal{A}}$:

- Causal information surface $\Xi_{\mathcal{A}}$ is a *d-1* dimensional spacelike bulk surface which:
    - is anchored on $\partial\mathcal{A}$
    - lies within (on boundary of) $\blacklozenge_{\mathcal{A}}$
    - reaches deepest into the bulk from among surfaces in $\blacklozenge_{\mathcal{A}}$
    - is a minimal-area surface among surfaces on $\partial(\blacklozenge_{\mathcal{A}})$ anchored on $\partial\mathcal{A}$
- However, $\Xi_{\mathcal{A}}$ is in general not an extremal surface $\mathfrak{E}_{\mathcal{A}}$ in the bulk.

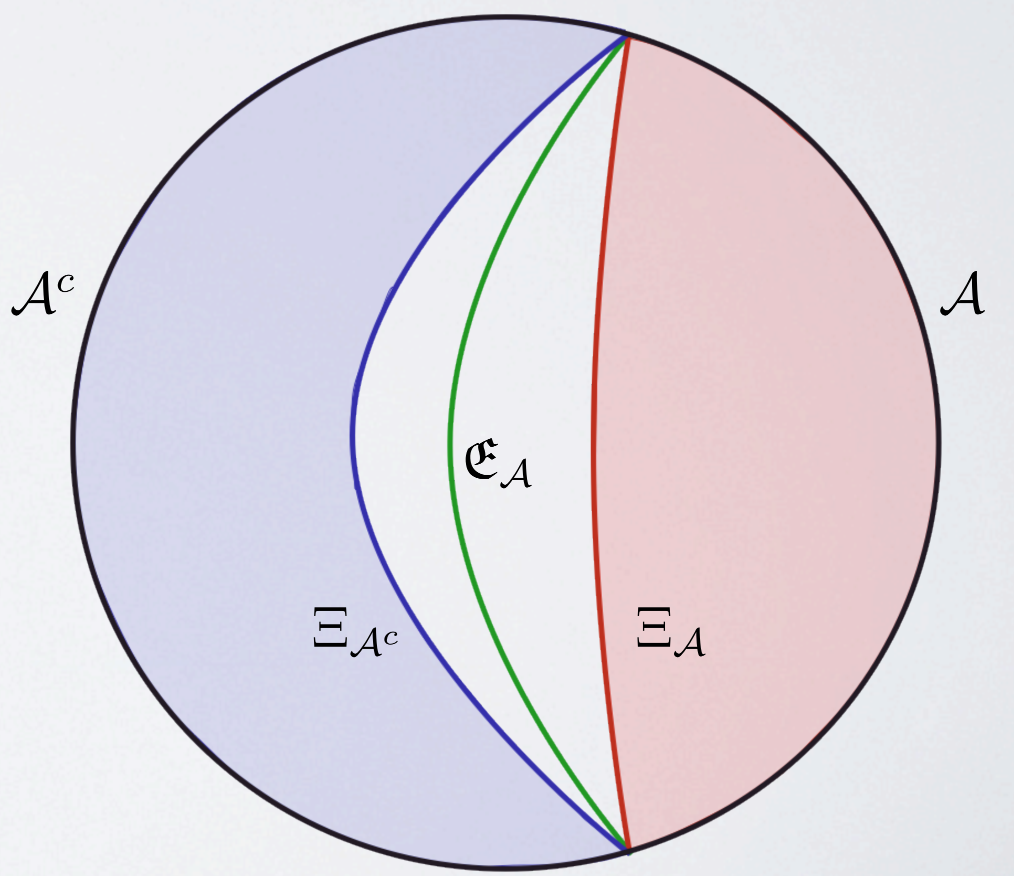


General properties of $\Xi_{\mathcal{A}}$:

- In general $\Xi_{\mathcal{A}}$ does not penetrate as far into the bulk as the bulk extremal surface $\mathfrak{E}_{\mathcal{A}}$ associated with $\mathcal{A}$
  - Justification 1: explicit calculation e.g. A=infinite strip in d>2 dim:

$$z_{\Xi}^* = \frac{w}{2} \; , \qquad z_{\mathfrak{E}}^* = \frac{\Gamma\left(\frac{1}{2(d-1)}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{d}{2(d-1)}\right)} \, \frac{w}{2}$$

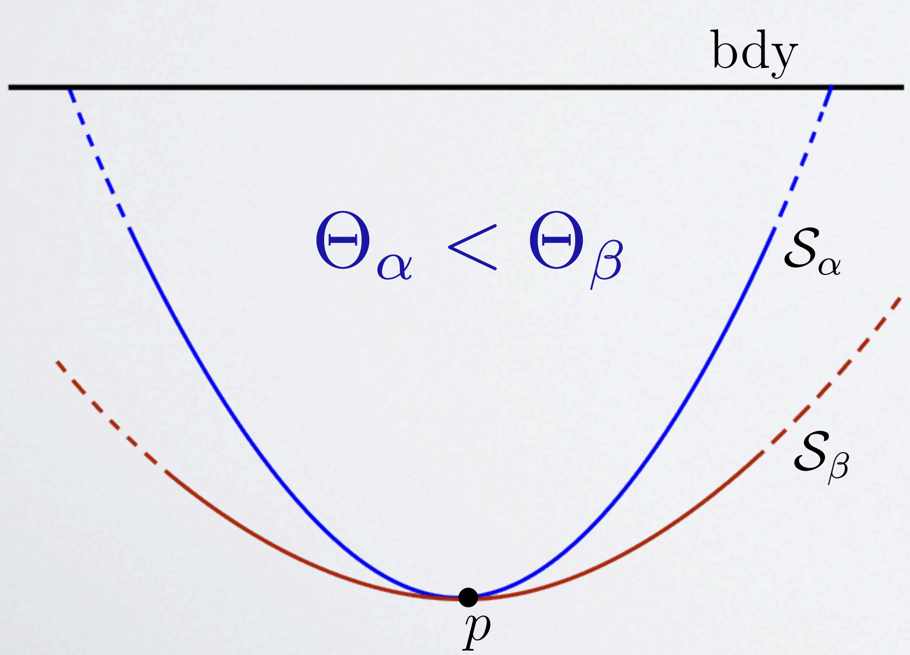
  - Justification 2: general argument: e.g. A=disk on bdy of global AdS and bdy state = pure: $S_{\mathcal{A}} = S_{\mathcal{A}^c}$
  - causal wedge differs for $\mathcal{A}$ and $\mathcal{A}^c$; and reach furthest in pure AdS, wherein $\Xi = \mathfrak{E}$, so in general Ξ recedes towards the bdy...

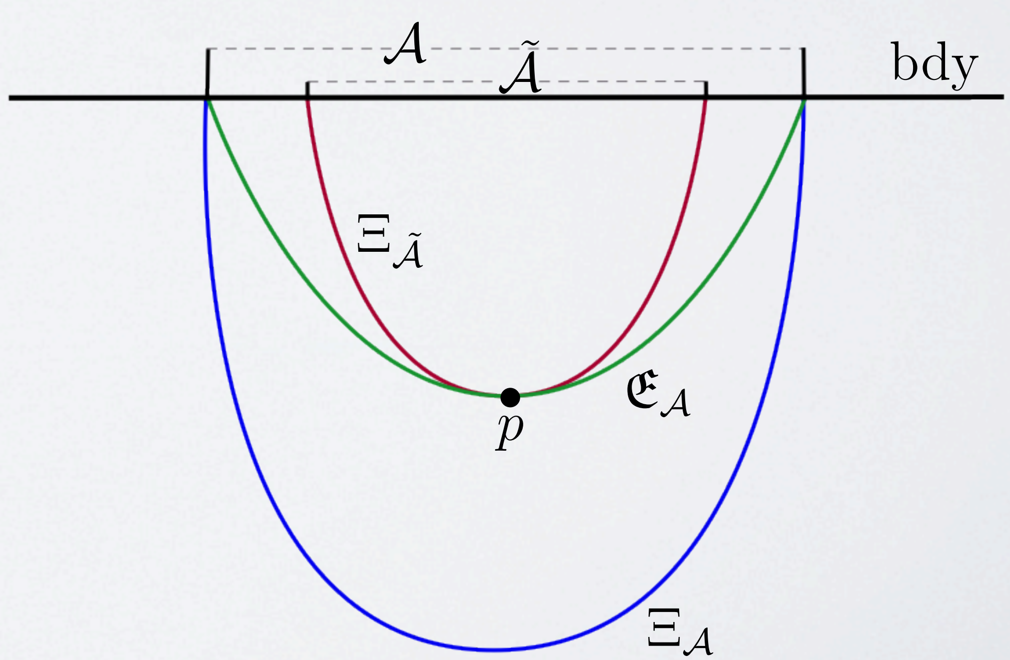


General properties of $\Xi_{\mathcal{A}}$:

- In general $\Xi_{\mathcal{A}}$ does not penetrate as far into the bulk as the bulk extremal surface $\mathfrak{E}_{\mathcal{A}}$ associated with $\mathcal{A}$

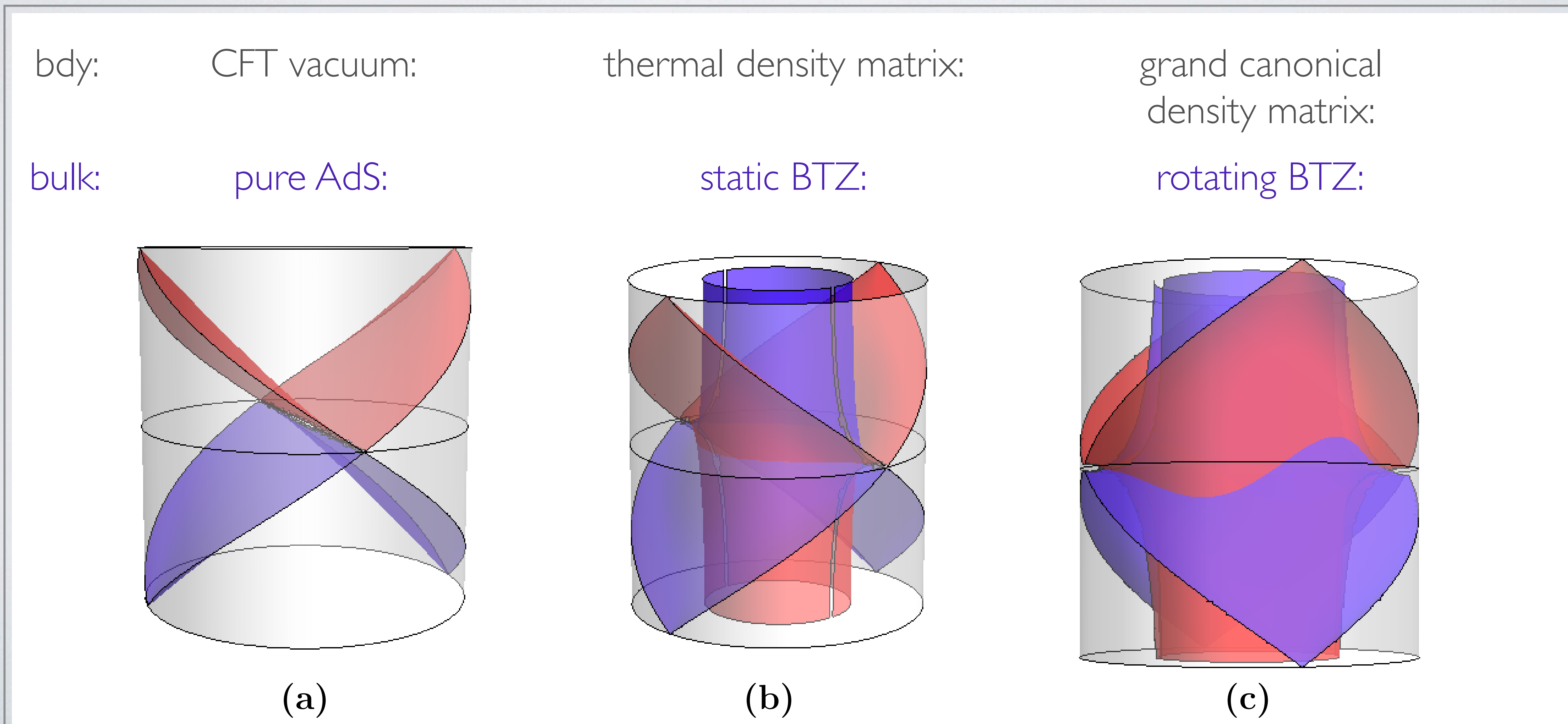
- Justification 3: general argument based on expansion of null generators: By construction, $\Theta_{\Xi} \geq 0$ while $\Theta_{\mathfrak{E}} = 0$
- Proof by contradiction: suppose $\mathfrak{E}_{\mathcal{A}}$ lay closer to bdy than $\Xi_{\mathcal{A}}$. Then tangent to $\mathfrak{E}_{\mathcal{A}}$, there is a surface $\Xi_{\tilde{\mathcal{A}}}$ for some smaller region $\tilde{\mathcal{A}}$. But for such configuration, $\Theta_{\Xi_{\tilde{\mathcal{A}}}} < 0$, which is a contradiction.





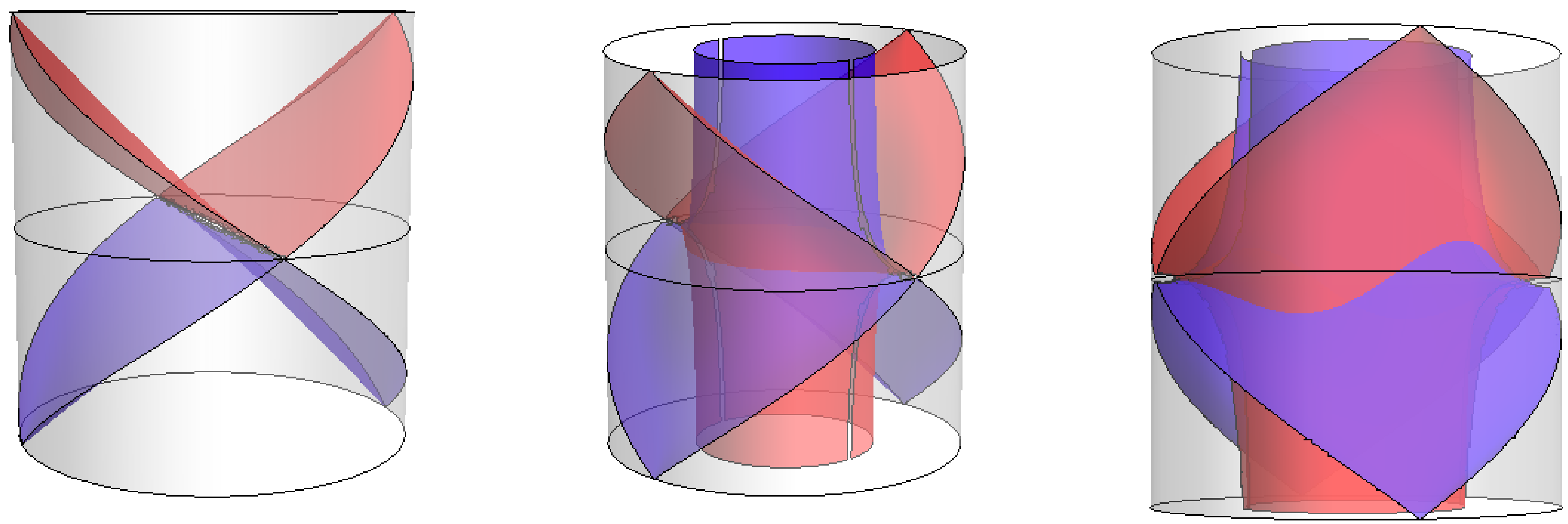
Cases when $\Xi_{\mathcal{A}}$ and $\mathfrak{E}_{\mathcal{A}}$ coincide:

- However, in all cases where one is able to compute entanglement entropy in QFT from first principles, independently of coupling, the surfaces $\mathfrak{E}_{\mathcal{A}}$ and $\Xi_{\mathcal{A}}$ agree!
- = When EE can be related to thermal entropy... cf. [Myers et.al.]



Cases when $\Xi_{\mathcal{A}}$ and $\mathfrak{E}_{\mathcal{A}}$ coincide:

bdy:	CFT vacuum:	thermal density matrix:	grand canonical density matrix:
bulk:	pure AdS:	static BTZ:	rotating BTZ:



(a) (b) (c)

$$\text{(a).}\qquad S_{\mathcal{A}} = \chi_{\mathcal{A}} = \frac{c_{\text{eff}}}{3}\,\log\left(\frac{2\varphi_0}{\varepsilon}\right)$$

$$\text{(b).}\qquad S_{\mathcal{A}} = \chi_{\mathcal{A}} = \frac{c_{\text{eff}}}{3}\,\log\left[\frac{\beta}{\pi\,\varepsilon}\,\sinh\left(\frac{2\pi\,\varphi_0}{\beta}\right)\right]$$

$$\text{(c).}\qquad S_{\mathcal{A}} = \chi_{\mathcal{A}} = \frac{c_{\text{eff}}}{6}\,\log\left[\frac{\beta_+\,\beta_-}{\pi^2\,\varepsilon^2}\,\sinh\left(\frac{2\pi\,\varphi_0}{\beta_+}\right)\sinh\left(\frac{2\pi\,\varphi_0}{\beta_-}\right)\right]$$

General properties of $\chi_{\mathcal{A}}$:

- The Causal Holographic Information $\chi_{\mathcal{A}}$
  - in special* cases, coincides with Entanglement entropy $\mathcal{S}_{\mathcal{A}}$

$$\chi_{\mathcal{A}} \equiv \frac{\text{Area}(\Xi_{\mathcal{A}})}{4\,G_N} \quad = \quad \mathcal{S}_{\mathcal{A}} \equiv -\text{Tr}\left(\rho_{\mathcal{A}}\,\log\rho_{\mathcal{A}}\right) = \frac{\text{Area}(\mathfrak{E}_{\mathcal{A}})}{4\,G_N}$$

  - but in general diverges more strongly than entanglement entropy
    e.g. for d=4, $\mathcal{A}$ = strip of width w, w/ IR regulator L & UV regulator ε ,

$$S_{\mathcal{A}} = c_{\text{eff}}\,L^2\left(\frac{1}{\varepsilon^2} - \frac{0.32}{w^2}\right) \;, \qquad \chi_{\mathcal{A}} = c_{\text{eff}}\,L^2\left(\frac{1}{\varepsilon^2} - \frac{2}{w^2} + \frac{4}{w^2}\log\left(\frac{w}{\varepsilon}\right)\right)$$

  - hence provides a bound on entanglement entropy $S_{\mathcal{A}} \leq \chi_{\mathcal{A}}$
  - unlike entanglement entropy, always varies smoothly with size of $\mathcal{A}$

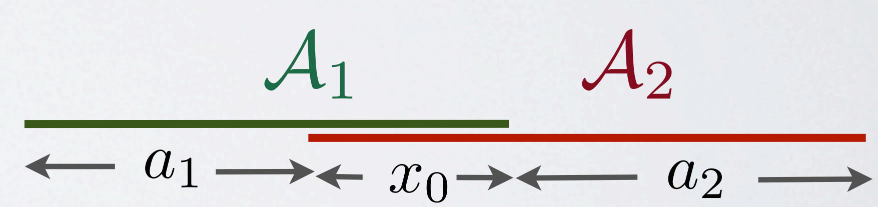
General properties of $\chi_{\mathcal{A}}$:

- The Causal Holographic Information $\chi_{\mathcal{A}}$
  - unlike entanglement entropy, does NOT satisfy strong subadditivity

$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \geq S_{\mathcal{A}_1 \cup \mathcal{A}_2} + S_{\mathcal{A}_1 \cap \mathcal{A}_2}$$
$$S_{\mathcal{A}_1} + S_{\mathcal{A}_2} \geq S_{\mathcal{A}_1 \backslash \mathcal{A}_2} + S_{\mathcal{A}_2 \backslash \mathcal{A}_1}$$

Geometric proof in static bulk and support in time-dep bulk [Headrick et.al.]
But counter-examples for $\chi_{\mathcal{A}}$:

  - explicit counter-example: 2 strips in d=4:



SS requires

$$F(a_1 + x_0) + F(a_2 + x_0) - F(a_1 + a_2 + x_0) - F(x_0) > 0\,, \qquad F(x) = \frac{1}{x^2} \log\left(\frac{x}{\tilde{\varepsilon}}\right)$$

but this can be violated - e.g. by $x_0 = a_1 = a_2$

Toy model for dynamics:

Vaidya-AdS spacetime, describing a null shell in AdS:

$$ds^2 = -f(r,v)\, dv^2 + 2\, dv\, dr + r^2\, d\Omega^2$$

where $f(r,v) = r^2 + 1 - \vartheta(v)\, m(r)$

with $m(r) = \begin{cases} r_+^2 + 1 & , \quad \text{in AdS}_3 \\ \frac{r_+^2}{r^2}\,(r_+^2+1) & , \quad \text{in AdS}_5 \end{cases}$

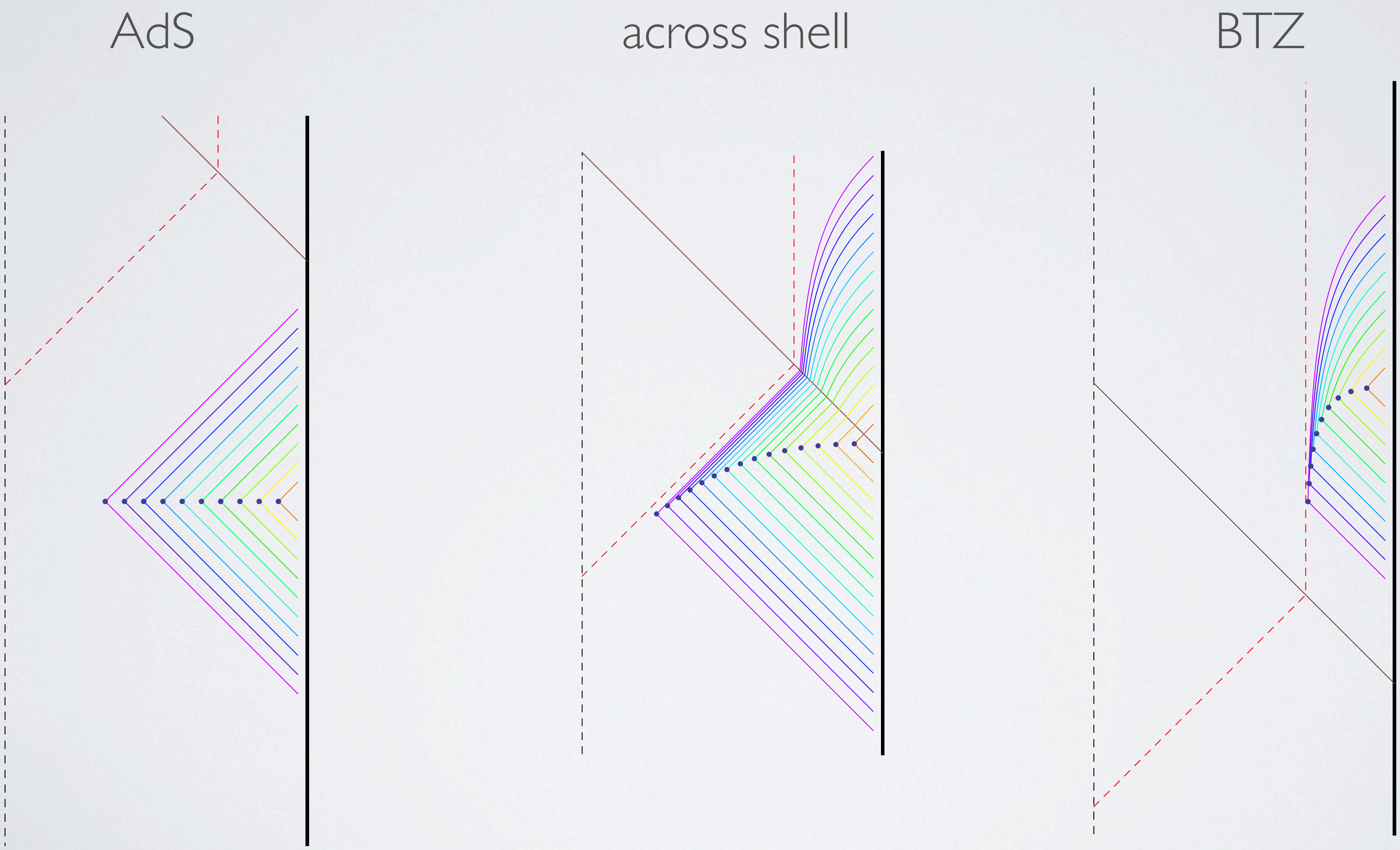
and $\vartheta(v) = \begin{cases} 0 & , \quad \text{for } v < 0 \\ 1 & , \quad \text{for } v \geq 0 \end{cases}$ → pure AdS / → Schw-AdS (or BTZ)

we can think of this as $\delta \to 0$ limit of smooth shell with thickness δ :

$$\vartheta(v) = \frac{1}{2}\left(\tanh\frac{v}{\delta} + 1\right)$$

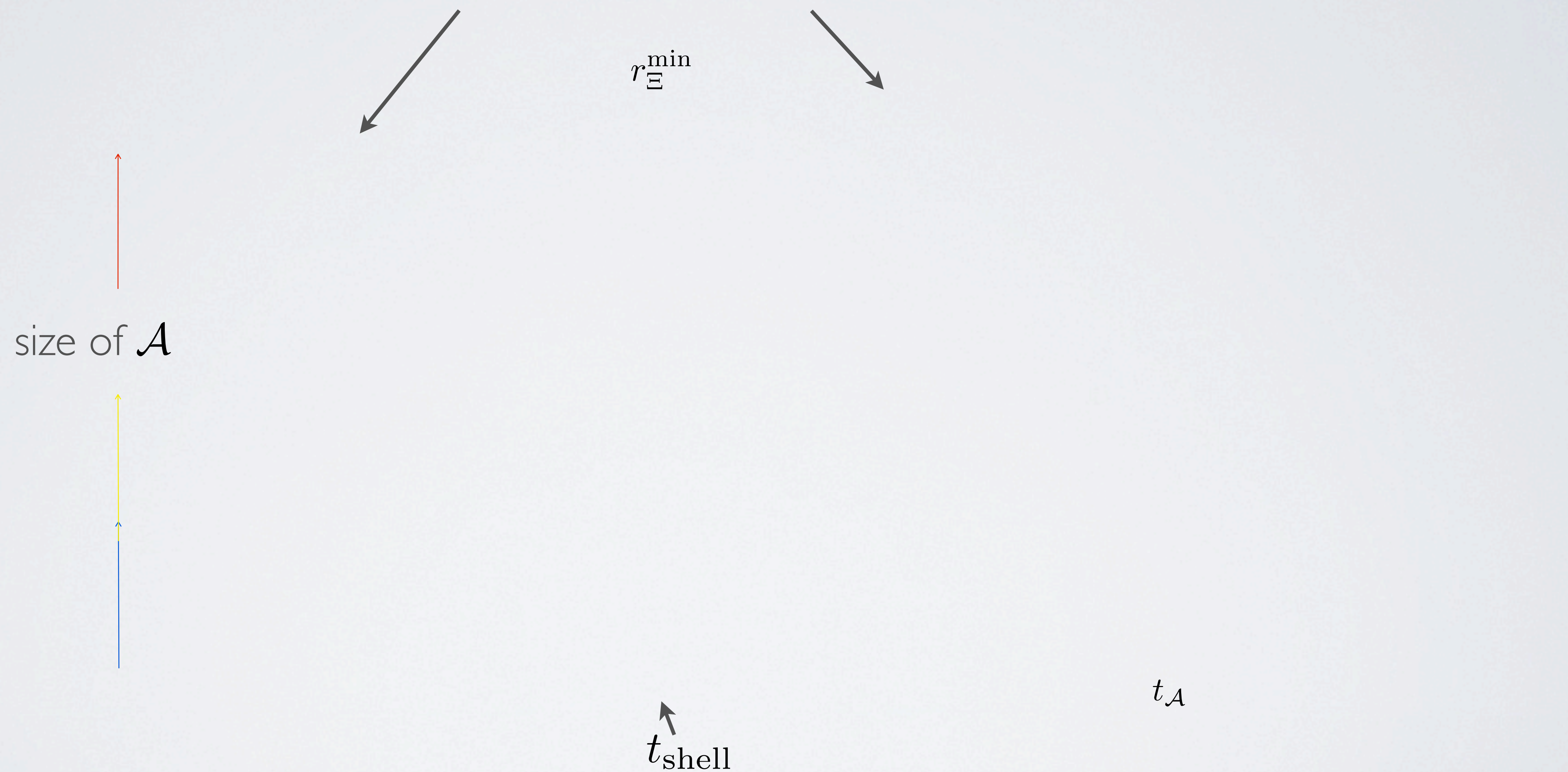
Causal wedge profile in Vaidya:

For fixed size of $\mathcal{A}$, causal wedge profile changes in time:



Quasi-teleological nature of $\chi_{\mathcal{A}}$:

For fixed size of $\mathcal{A}$, deepest reach of $\Xi_{\mathcal{A}}$ monotonically increases from AdS value to BTZ value:



Similarly for $\chi_{\mathcal{A}}$: Note that it starts increasing before $t_{\mathcal{A}} = t_{\text{shell}}$

Cf. deepest reach of $\Xi_{\mathcal{A}}$ vs. $\mathfrak{E}_{\mathcal{A}}$:

Unlike $\Xi_{\mathcal{A}}$, extremal surface $\mathfrak{E}_{\mathcal{A}}$ depends only on spatial info; starts increasing only at $t_{\mathcal{A}} = t_{\text{shell}}$:

$\Xi_{\mathcal{A}}$

$\mathfrak{E}_{\mathcal{A}}$

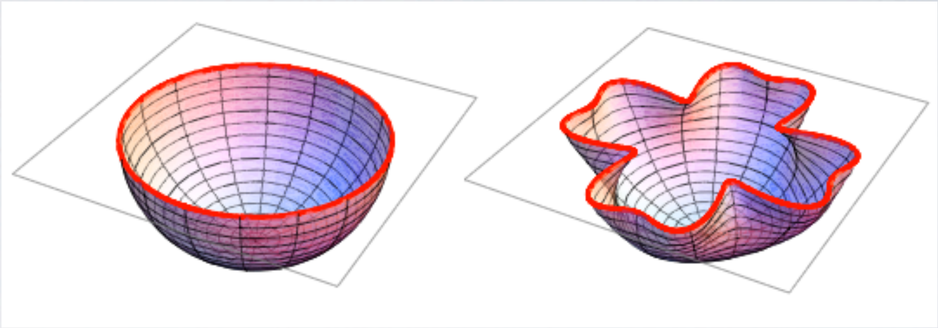
t_{shell}

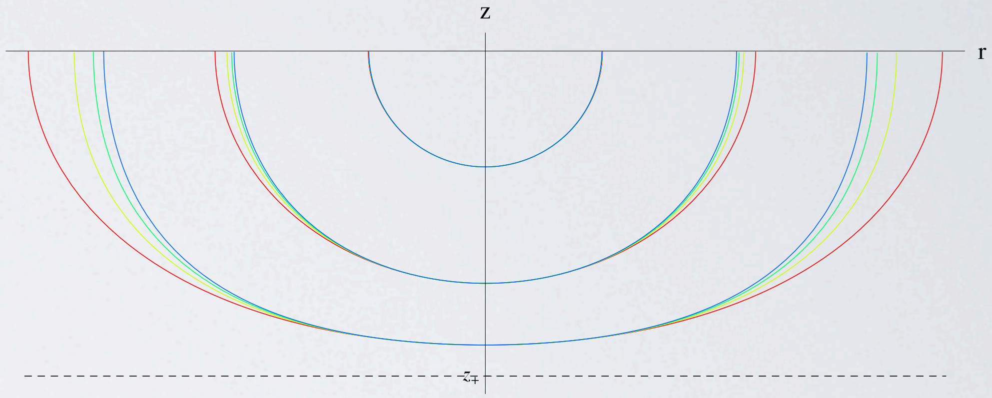
OUTLINE

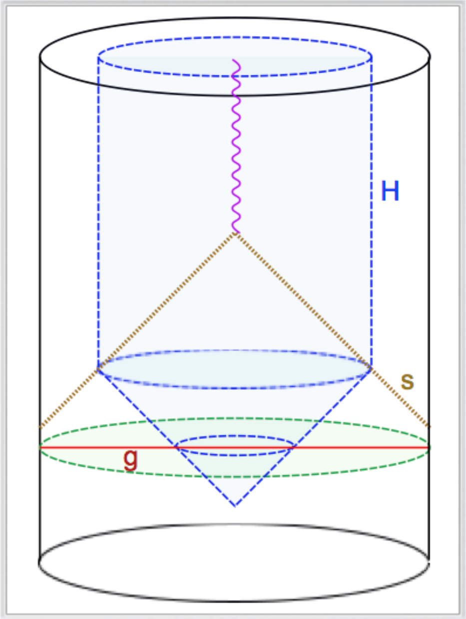
- Motivation & Background
- Features of Extremal Surfaces
- Probing Horizons
- Causal Holographic Information
- Summary & Future directions

Summary for extremal surfaces

- Spacelike geodesics reach deeper than null geodesics (at fixed spatial separation of endpoints).
- Higher-dimensional extremal surfaces reach deeper (at fixed extent of bounding region).
- Extremal surfaces anchored on sphere reach deepest (at fixed extent or volume of bounding region).
- Extremal surfaces of *any* dimension, anchored on *any* region, in *any* static planar black hole spacetime, cannot penetrate the horizon.
- Extremal surfaces can penetrate horizon of dynamically evolving black hole.







Summary for CHI

- The causal wedge $\blacklozenge_{\mathcal{A}}$
  - is the most natural (minimal nontrivial) bulk spacetime region related to $\mathcal{A}$
  - corresponds to bulk region most easily reconstructed from $\rho_{\mathcal{A}}$
  - cannot penetrate event horizon of a black hole

- The causal holographic information $\chi_{\mathcal{A}}$
  - coincides with entanglement entropy $S_{\mathcal{A}}$ in certain special cases (when DoFs in $\mathcal{A}$ are maximally entangled with those outside)
  - in general provides an upper bound on entanglement entropy
  - monotonically increases during thermalization
  - behaves quasi-teleologically, but only on light-crossing timescales
  - remains smooth as a function of time and the size of $\mathcal{A}$

Conjectured meaning of $\chi_{\mathcal{A}}$:

- We conjecture that $\chi_{\mathcal{A}}$ characterizes the amount of information contained in $\mathcal{A}$ which can be used to reconstruct the bulk geometry (entirely in $\blacklozenge_{\mathcal{A}}$ but possibly further)...
  - cons. set of local bulk `observers' starting & ending on bdy inside $\lozenge_{\mathcal{A}}$
  - these have access to full $\blacklozenge_{\mathcal{A}}$, but the info contained can be reduced:
    - bulk evolution: suffices to consider just Cauchy slice for $\blacklozenge_{\mathcal{A}}$
    - holography: suffices to consider just screen: natural region associated to $\mathcal{A}$ $= \Xi_{\mathcal{A}}$
  - hence natural to identify $\chi_{\mathcal{A}}$ with amount of info contained in $\mathcal{A}$
- This has entropy-like behavior, however, it does *not* correspond to a Von Neumann entropy:
  - e.g. it violates strong subadditivity.
- However, it provides a bound on Entanglement entropy;
  - and coincides in special, maximally-entangled, cases.

Future directions

Most important questions still remain:

- What is the direct boundary interpretation/construction of the causal holographic surface $\Xi_{\mathcal{A}}$ and 'information' $\chi_{\mathcal{A}}$?
- What bulk region can we fully reconstruct?

  (& What is the most efficient reconstruction method?)
- e.g. suppose we know $\{\chi_{\mathcal{Q}}\}$ for all sub-regions $\mathcal{Q} \in \mathcal{A}$; does this provide sufficient info to recover bulk metric in $\blacklozenge_{\mathcal{A}}$?
- What is the bulk dual of the reduced density matrix $\rho_{\mathcal{A}}$?
- Given a bulk location, how do we extract the geometry there from the CFT?

  (& How deep / late into BHs can various probes see?)
- How does the CFT encode bulk locality and causality?