# On M5 Branes

Kimyeong Lee
KIAS

YKIS 2012
From Gravity to Strong Coupling Physics
Yukawa Institute for Theoretical Physics  Oct 2012

Ho-Ung Yee, KM [hep-th/0606150],

Bolognesi,  KM: On 1/4 BPS Junctions, [arXiv:1105.5073]

Hee-Cheol Kim, Seok Kim, E. Koh, KM, Sungjay Lee: dyonic instantons [arXiv:1110.2175]

Hee-Cheol Kim, K.M.   Supersymmetric M5 Brane Theories on R×CP$^2$ [arXiv:1210.0853]

# Outline

- 6d (2,0) M5 brane theories in $R^{1+5}$
- 1/4 BPS Junctions & 1/16 BPS Webs
  - New 11 Equations for 1/16 BPS Objects
  - Finite size BPS ‘t Hooft operators
  - Degenerate Limit and speed of light
- Dyonic Instantons Index and DLCQ on M5
- New Supersymmetric M5 Brane Theories on $R \times CP^2$
- Concluding Remarks

# 5d N=2 YM as the M5 brane theory

- Circle compactifiction of 6d (2,0) superconformal field theories
- coupling constant $1/g_{YM}^2 = 4\pi^2/R$
- instanton = quantum of KK modes of unit momentum
- complete by its own?
- symmetry: super Poincare symmetry + Sp(2)=SO(5) R-symmetry
- spatial SO(4) rotation: $J_{1L}$, $J_{2L}$
- $Sp(2)_R$=SO(5) R-symmetry: $J_{1R}$, $J_{2R}$
- Monopole strings in Coulomb phase
- Electric description: nonabelian gauge field $A^a_\mu$, $F = dA + A^2$
- Magnetic description: nonabelian $B^a_{\mu\nu}$, $H=dB+\ldots= *F$ ???

# Instanton particles in 5d

- KK modes of mass k/ R = a threshold bound states of k instantons
- abelian action in 5d:

$$H^2 + inH \wedge B, \text{ or } n^2 B^2 + iH \wedge B$$

- circle compactification and T-duality: D0 becomes D1 strings ending on D3 branes
- KK modes becomes the fields in the adjoint representation of the magnetic gauge group.
- The nonabelian version in 4d should be

$$\partial_\mu B_{\rho\sigma} \Rightarrow \partial_\mu B_{\rho\sigma} - i[A_\mu^{magnetic}, B_{\rho\sigma}]$$

# 6d (2,0) M5 Brane Theory

- 6d (2,0) superconformal field theory with symmetry OSp(2,6|2)= O(2,6) x Sp(4)
- 2-form tensor field B, spinor Ψ, scalar $\Phi_I$
- purely quantum (*H=H=dB): $\hbar=1$
- nonabelian ADE types: N-M5, NM5+OM5, Type IIB on $C^2/\Gamma$
- $N^3$ degrees of freedom
- $AdS_7 \times S^4$ correspondence

# Selfdual Strings in Coulomb phase

- Supersymmetric Transformation: $\Gamma^{012345}\epsilon = -\epsilon$, $\Gamma^{012345}\,\lambda = \lambda$
- $\delta\lambda \sim H_{\mu\nu\rho}\,\Gamma^{\mu\nu\rho}\,\epsilon + D_\mu\,\Phi_I\,\Gamma^\mu\,\rho^I\,\epsilon$, $H_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\alpha\beta\gamma}\,H^{\alpha\beta\gamma}/6$, $\epsilon^{012345}=1$

- 1/2 BPS selfdual strings along $x^5$ : $\Gamma^{05}\rho_5\,\epsilon = \epsilon$, $H_{05\mu} \sim {}^*H_{05\mu} \sim D_\mu\Phi_5$
- 1/4 BPS string junctions: $\Gamma^{04}\,\rho_4\,\epsilon = \epsilon$, $H_{04\mu} \sim {}^*H_{04\mu} \sim D_\mu\Phi_4$


- 1/16 BPS webs of strings:
- 5d Monopole string webs: Kapustin-Witten equation (Ho-Ung Yee, KL)
  - **$F_{ab} = \epsilon_{abcd}\,D_c\,\Phi_d - i[\Phi_a, \Phi_b]$, $D_a\,\Phi_a = 0$** 7 equations = octonions
  - $F_{a0} = D_a\,\Phi_5$ 4 equations: 7+4=11 equations
  - Gauss law $D_a^2\,\Phi_5 - [\Phi_a, [\Phi_a, \Phi_5]] = 0$

# Junctions and Webs in Coulomb Phase

- 6d generalization of Kapustin-Witten equations
- 1/16 BPS condition: $\Gamma^{0\mu} \rho_\mu \varepsilon = \varepsilon$, ($\mu$ =1,2,3,4,5 ):
- $SO(5)_{rot} \times SO(5)_R \Rightarrow SO(5)_{locking}$

- BPS equations: a generalization of KW
- $\mathbf{H_{0\mu\nu} = {}^*H_{0\mu\nu} = \partial_\mu \Phi_\nu - \partial_\nu \Phi_\mu}$ **,** $\mathbf{\partial_\mu \Phi_\mu=0}$
- 11 equations:
- dH=d*H=0 is the equation of motion d*H=0, d*d $\Phi_\mu$ =0

# N-Cubic Degrees of Freedom

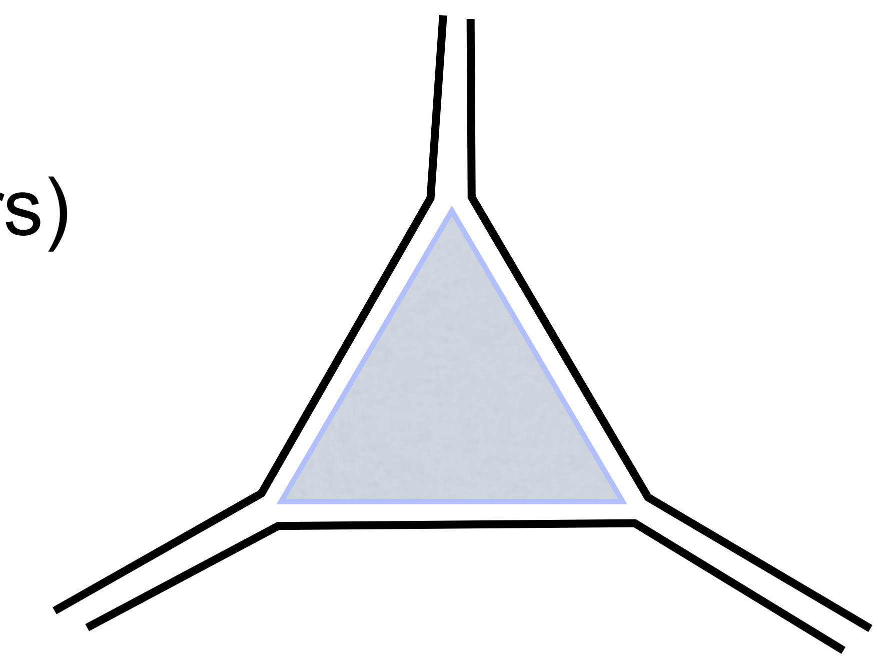
- Anomaly Coefficient: $C_G = h_G \times d_G /3$

| Group | $r_G$ | $d_G$ | $h_G$ | $c_G/3$ |
|---|---|---|---|---|
| $A_{N-1} = SU(N)$ | $N-1$ | $N^2-1$ | $N$ | $\frac{1}{3}N(N^2-1)$ |
| $D_N = SO(2N)$ | $N$ | $N(2N-1)$ | $2(N-1)$ | $\frac{2}{3}N(2N-1)(N-1)$ |
| $E_6$ | 6 | 78 | 12 | 312 |
| $E_7$ | 7 | 133 | 18 | 798 |
| $E_8$ | 8 | 248 | 30 | 2480 |

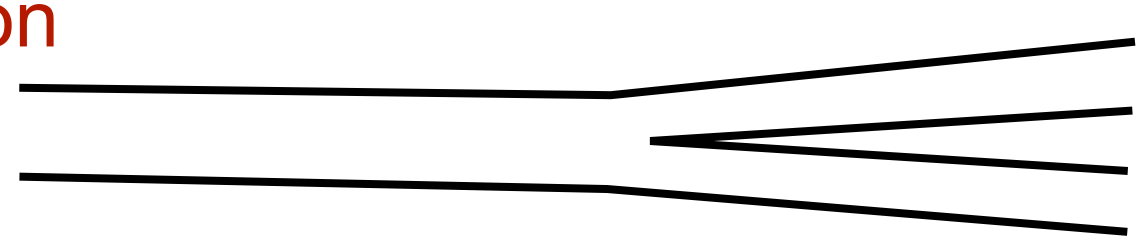
- 1/4 BPS objects in Coulomb phase
  - for ADE, dual Coxeter number=Coxeter number: h = (d-r)/r $\Rightarrow$ d=r(h+1)
  - selfdual strings with left and right moving waves: number of roots = d-r = hr
  - junctions and anti-junctions: su(3) roots imbedding = rh(h-2)/3 $\Leftarrow$ done by counting explicitly in Bolognesi &KM
  - total number of 1/4 BPS objects: rh(h+1)/3= hd/3=anomaly coefficient
  - More fundamental than 1/2 BPS selfdual strings?
- Finite temperature phase transition in Coulomb phase
  - beyond Hagedorn temperature, the webs of junctions could dominate the entropy

# (2,0) 1/16 BPS equation for Webs of selfdual strings

- 1/4 BPS objects are more fundamental than 1/2 BPS objects ?
- Symmetric phase: New formalism including junctions?
  - W-boson scattering => Nonabelian gauge theory…
  - Vertex or Junction Fields?
- Finite size BPS probes (tHooft operators)



- Degenerate limit: string+ momentum = S-dual of dyonic instanton



# Index of Dyonic Instantons in 5d N=2 SYM

- Index for BPS states with k instantons $Q = Q^{+}_{+}$ $\left.\begin{array}{l} SU(2)_{2R} \\ SU(2)_{1R} \end{array}\right\} \Rightarrow SU(2)_R$

$$I_k(\mu^i, \gamma_1, \gamma_2, \gamma_3) = \mathrm{Tr}_k \Big[ (-1)^F e^{-\beta Q^2} e^{-\mu^i \Pi_i} e^{-i\gamma_1(2J_{1L}) - i\gamma_2(2J_{2L}) - i\gamma_R(2J_R)} \Big]$$

adjoint hyper flavor

  - $\mu_i$ : chemical potential for $U(1)^N \subset U(N)_{\text{color}}$
  - $\gamma_1, \gamma_2, \gamma_R$ : chemical potential for $SU(2)_{1L}$, $SU(2)_{2L}$ , $SU(2)_R$
- calculate the index by the localization: $I(q, \mu^i, \gamma_{1,2,3}) = \sum_{k=0}^{\infty} q^k I_k$
- 5d *N=2** instanton partition function on $R^4 \times S^1$: $t \sim t + \beta$
- In $\beta \to 0$ and small chemical potential limit, the index becomes 4d Nekrasov instanton partition function :
- $a_i = \frac{\mu_i}{2}$ $\quad -\epsilon_1 = i\frac{\gamma_1 - \gamma_R}{2}$ $\quad \epsilon_2 = i\frac{\gamma_1 + \gamma_R}{2}, \quad m = i\frac{\gamma_2}{2}$ $\quad q = e^{2\pi i \tau}$

instanton fugacity

Scalar Vev

Omega deformation parameter

Adj hypermultiplet mass

# U(1) instantons

- D0's on a single D4
- As instantons are KK modes of (2,0) theory, one expects a unique threshold bound state for each instanton number k.
- U(1) index for k=1

$$I_{cm} = \frac{\sin\left(\frac{\gamma_1+\gamma_2}{2}\right)\sin\left(\frac{\gamma_1-\gamma_2}{2}\right)}{\sin\left(\frac{\gamma_1+\gamma_R}{2}\right)\sin\left(\frac{\gamma_1-\gamma_R}{2}\right)}$$

| | $SU(2)_{1L}$ | $SU(2)_{1R}$ | $SU(2)_{2L}$ | $SU(2)_{2R}$ |
|---|---|---|---|---|
| $B_2$ | 3 | 1 | 1 | 1 |
| $\phi_I$ | 1 | 1 | 2 | 2 |
| | 1 | 1 | 1 | 1 |
| $\lambda$ | 2 | 1 | 2 | 1 |
| | 2 | 1 | 1 | 2 |

- U(1) index [Iqbal-Kozcaz-Shabbir 10]

$$I_{U(1)}(q, e^{\gamma_i}) = PE\left[\frac{q}{1-q} I_{cm}(e^{\gamma_i})\right], \quad \text{Plythethytic exponential}$$

- Expand the single particle index in q

$$\sum_{k=0}^{\infty} q^k I_{cm} \quad \text{unique threshold bound state}$$

- Shown to be true for some U(N) for small number of k

# Nonrelativistic Superconformal index

- To get the index in symmetric phase, integrate over $\mu_i = i\, a_i$ with Haar measure
- DLCQ on null circle: Nonrelativistic superconformal symmetry
  - $P_-$ on the null circle = instanton number [Aharony-Berkooz-Seiberg 97]
  - Superalgebra: $2i\{Q,S\} = iD \mp (4J_{2R} + 2J_{1R}) \;\rightarrow\; iD \geq \pm(4J_{2R} + 2J_{1R})$
- Nonrelativistic superconformal index

$$I_{SC} = \mathrm{Tr}\left[(-1)^F e^{-\beta\{\hat{Q},\hat{S}\}} e^{-2i\gamma_R J_R} e^{-2i\gamma_1 J_{1L} - 2i\gamma_2 J_{2L}} e^{-i\alpha_i \Pi_i}\right]$$

- In the limit $\beta \rightarrow 0$, this superconformal index becomes our index.
- For single instanton with $t = e^{-i\gamma_R}$

$$I_{k=1} = \frac{e^{i\gamma_2} + e^{-i\gamma_2} - e^{i\gamma_1} - e^{-i\gamma_1}}{(1 - te^{i\gamma_1})(1 - te^{-i\gamma_1})}\left[t + \sum_{n=1}^{N-1}(e^{in\gamma_2} + e^{-in\gamma_2})t^{n+1} - \chi_{\frac{N-2}{2}}(\gamma_2)t^{N+1}\right]$$

- Large $N$

$$I_{N\rightarrow\infty,k=1} = \frac{e^{i\gamma_2} + e^{-i\gamma_2} - e^{i\gamma_1} - e^{-i\gamma_1}}{(1 - te^{i\gamma_1})(1 - te^{-i\gamma_1})}\,\frac{t - t^3}{(1 - te^{i\gamma_2})(1 - te^{-i\gamma_2})}$$

AdS7 x S4 calculation confirm it.

# 6d (2,0) Theory on RxS$^5$

Linander&Ohlsson, Kallen& Zabzine, Kim&Kim,Hosomich&Seong&Terashima, Kallen&Qui&Zabzine, Jafferis&Pufu,Kim&Kim&KM

- 6d (2,0) theory on R$^6$: radial quantization R x S$^5$: OSp(2,6|2): SO(2,6) x Sp(2)=SO(5)$_R$

$$S=\int_{R\times S^5} dt d\Omega_{S^5}\left\{-\frac{1}{12}H_{MNP}H^{MNP}-\frac{i}{2}\bar{\lambda}\Gamma^M\hat{\nabla}_M\lambda-\frac{1}{2}\partial_M\phi_I\partial^M\phi_I-\frac{2}{r^2}\phi_I\phi_I\right\}.$$

- S$^5$ = U(1) fiber over CP$^2$ : $ds^2_{S5} = ds^2_{CP2} + (dy+V)^2$, dV= 2J
- SO(6)=SU(4) ⊃ SU(3) x U(1)
- 32 Killing spinors = 24 (SU(3) triplet) + 8 (SU(3) singlet)
  - (I) $\varepsilon_+ \sim \exp(-it/2 + 3i\, y/2)\ldots$ : singlet
  - (II) $\varepsilon_+ \sim \exp(-it/2 - i\, y/2)\ldots$ : triplet
- Write down abelian theory on R x S$^5$ & Change the variables

$$\textbf{(I)}\quad \epsilon_{old}=e^{-\frac{3\rho_{45}}{2}y}\epsilon_{new},\ \lambda_{old}=e^{-\frac{3\rho_{45}}{2}y}\lambda_{new},\ (\phi_4+i\phi_5)_{old}=e^{+3iy}(\phi_4+i\phi_5)_{new}.$$

$$\textbf{(II)}\quad \epsilon_{old}=e^{+\frac{\rho_{45}}{2}y}\epsilon_{new},\ \lambda_{old}=e^{+\frac{\rho_{45}}{2}y}\lambda_{new},\ (\phi_4+i\phi_5)_{old}=e^{-iy}(\phi_4+i\phi_5)_{new}.$$

- Write down the theory in new variables

$$\textbf{(I)}\quad \partial_y\to\partial_y+3iR_2 \qquad\qquad \textbf{(II)}\quad \partial_y\to\partial_y-iR_2.$$

# $Z_k$ Modding & Dimensional Reduction to R x $CP^2$

- $Z_k$ modding of new variables

$$y \sim y + \frac{2\pi}{k}$$

- New variables to be independent of y: preserve some supersymmetries
  - (I) $\rho_{45}\, \varepsilon = -i\varepsilon$, 4 supersymmetries
  - (ii) $\rho_{45}\, \varepsilon = -i\varepsilon$, 12 supersymmetries
- Killing spinor equation on R x $CP^2$ $\partial_t \epsilon = \frac{i}{2}\gamma_0 \tilde{\epsilon},\ D_m \epsilon = -\frac{i}{2} J_{mn}\gamma^n \epsilon + \frac{i}{2}\gamma_m \tilde{\epsilon},$

$$(\mathbf{I})\quad \rho_{45}\epsilon_+ = -i\epsilon_+,\quad D_a = \nabla_a + \frac{3\rho_{45}}{2} V_a,\qquad \tilde{\epsilon} = -\left[3\rho_{45} + \frac{1}{2} J_{ab}\gamma^{ab}\right]\epsilon,$$
$$(\mathbf{II})\quad \rho_{45}\epsilon_+ = -i\epsilon_+,\quad D_a = \nabla_a - \frac{\rho_{45}}{2} V_a,\qquad \tilde{\epsilon} = \left[\rho_{45} - \frac{1}{2} J_{ab}\gamma^{ab}\right]\epsilon,$$

- Complete the supersymmetry with Abelian gauge field and its action
- Non-abelianize it
- Fix the coupling constant: instantons should represent KK modes

$$\frac{1}{g_{YM}^2} = \frac{k}{4\pi^2 r},$$

# 4 Supersymmetric Theory on R x CP$^2$

* Lagrangian

$$S_{\mathrm{I}} = \frac{k}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\ \mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta - \frac{i}{3}A_\rho A_\sigma A_\eta\Big)$$
$$-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I + \frac{1}{4}[\phi_I,\phi_J]^2 + \frac{i}{3}\epsilon_{abc}\phi_a[\phi_b,\phi_c] - 2\phi_a^2 - \frac{13}{2}\phi_i^2$$
$$-\frac{i}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda - \frac{i}{2}\bar{\lambda}\rho_I[\phi_I,\lambda] - \frac{1}{8}\bar{\lambda}\gamma^{mn}\lambda J_{mn} + \frac{3}{4}\bar{\lambda}\rho_{45}\lambda\Big], \qquad (2.22)$$

* Covariant derivative

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu], \\ D_\mu\phi_a &= \partial_\mu\phi_a - i[A_\mu,\phi_a], \\ D_\mu\phi_i &= \partial_\mu\phi_i - i[A_\mu,\phi_i] + 3V_\mu\epsilon_{ij}\phi_j, \\ D_\mu\lambda &= \Big[\partial_\mu\lambda + \frac{1}{4}\omega_\mu^{ab}\gamma^{ab} + \frac{3}{2}V_\mu\rho_{45}\Big]\lambda - i[A_\mu,\lambda]. \end{aligned}$$

* Supersymmetric Transformation

$$\begin{aligned} \delta A_\mu &= +i\bar{\lambda}\gamma_\mu\epsilon = -i\bar{\epsilon}\gamma_\mu\lambda, \\ \delta\phi_I &= -\bar{\lambda}\rho_I\epsilon = \bar{\epsilon}\rho_I\lambda, \\ \delta\lambda &= +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon - 3\epsilon_{ij}\phi_i\rho_j\epsilon - 2\phi_I\rho_I\tilde{\epsilon}. \end{aligned}$$

# 12 Supersymmetric Theory on R x CP$^2$

* Lagrangian

$$S_{\mathbf{II}} = \frac{k}{4\pi^2}\int_{\mathrm{R}\times\mathrm{CP}^2} d^5x\ \mathrm{tr}\Big[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma\eta}J_{\mu\nu}\Big(A_\rho\partial_\sigma A_\eta - \frac{i}{3}A_\rho A_\sigma A_\eta\Big)$$
$$-\frac{1}{2}D_\mu\phi_I D^\mu\phi_I + \frac{1}{4}[\phi_I,\phi_J]^2 + \frac{i}{3}\epsilon_{abc}\phi_a[\phi_b,\phi_c] - 2\phi_a^2 - \frac{5}{2}\phi_i^2$$
$$-\frac{i}{2}\bar\lambda\gamma^\mu D_\mu\lambda - \frac{i}{2}\bar\lambda\rho_I[\phi_I,\lambda] - \frac{1}{8}\bar\lambda\gamma^{mn}\lambda J_{mn} - \frac{1}{4}\bar\lambda\rho_{45}\lambda\Big], \qquad (2.25)$$

* Covariant derivative

$$\begin{aligned}
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu],\\
D_\mu\phi_a &= \partial_\mu\phi_a - i[A_\mu,\phi_a],\\
D_\mu\phi_i &= \partial_\mu\phi_i - i[A_\mu,\phi_i] - V_\mu\epsilon_{ij}\phi_j,\\
D_\mu\lambda &= \Big[\partial_\mu\lambda + \frac{1}{4}\omega_\mu^{ab}\gamma^{ab} - \frac{1}{2}V_\mu\rho_{45}\Big]\lambda - i[A_\mu,\lambda].
\end{aligned}$$

* Supersymmetric Transformation

$$\begin{aligned}
\delta A_\mu &= i\bar\lambda\gamma_\mu\epsilon = -i\bar\epsilon\gamma_\mu\lambda,\\
\delta\phi_I &= -\bar\lambda\rho_I\epsilon = \bar\epsilon\rho_I\lambda,\\
\delta\lambda &= +\frac{1}{2}F_{\mu\nu}\gamma^{\mu\nu}\epsilon + iD_\mu\phi_I\rho_I\gamma^\mu\epsilon - \frac{i}{2}[\phi_I,\phi_J]\rho_{IJ}\epsilon + \epsilon_{ij}\phi_i\rho_j\epsilon - 2\phi_I\rho_I\tilde\epsilon.
\end{aligned}$$

# Properties

- Hamiltonian on $CP^2$ = Hamiltonian on $S^5$ = Conformal Dimension D
- Instanton number

$$v = \frac{1}{16\pi^2} \int_{CP^2} d^4x \, \frac{1}{4} \mathrm{tr} F \wedge F$$

- KK-mode has mass k and so $v$ instantons should have action ks: the coupling constant

$$\frac{1}{g_{YM}^2} = \frac{k}{4\pi^2}$$

- 5-d Chern-Simons term: Linander and Ohlsson

$$ds^2_{R \times S^5} = ds^2_{R \times CP^2} + (dy + V)^2$$

$B, A$ are $2$ and $1$ forms in $R \times CP^2$

$$H = dB + F \wedge dy, \;\; F = dA$$

$$H = (dB - F \wedge V) + F \wedge (dy + V), \;\; F = dA$$

$$dB - F \wedge V = {}^*F \text{ in } R \times CP^2$$

$$d(dB - F \wedge V) = -2F \wedge J = d^*F$$

$$d^*F + 2J \wedge F = 0$$

- Myers term but no nontrivial vacuum

# Harmonic Analysis on $S^5$ & $CP^2$

Pope,Hosomich at.al.,Kim&Kim

- Scalar harmonics on R x $S^5$: $-\partial_t^2\Phi = (-\Delta_{S5} + 4)\Phi$

$$(-\nabla^2_{S^5}+4)Y^{\ell_1,\ell_2} = (\ell_1+\ell_2+2)^2 Y^{\ell_1,\ell_2}, \; -i\partial_y Y^{\ell_1,\ell_2} = (\ell_1-\ell_2)Y^{\ell_1,\ell_2}.$$

  - highest weight vector: $\ell_1 w_1 + \ell_2 w_2$ degeneracy: $(\ell_1+1)(\ell_2+1)(\ell_1+\ell_2+2)/2$
- On $CP^2$: y-independent mode for $\Phi_{1,2,3}$ : $(-\nabla^2_{\mathrm{CP}^2}+4)Y^{\ell,\ell} = 4(\ell+1)^2 Y^{\ell,\ell},$
  - conformal dimension: $\epsilon = 2\ell+2$ $2(\ell+1)^3$.
  - first KK mode: $Y^{0,k}$ $Y^{k,0}$: $\epsilon = k+2$, $(k+1)(k+2)/2$
  - higher KK modes: $\ell_1 - \ell_2 = kn$, n=1, -1,2, -2,...
- $\Phi_{4,5}$: $(-\nabla^2_{S^5}+4)Y^{\ell,\ell+3} = (-D^2_{CP^2}+13)Y^{\ell,\ell+3} = (\ell+5)^2 Y^{\ell,\ell+3}$
- Fermions: 5/2+….

$$\Psi_1 = Y^{l,l+3}\epsilon_+ , \quad \Psi_2 = \gamma^\tau\gamma^m D_m Y^{l,l+3}\epsilon_+ , \quad \Psi_3 = Y^{l,l}\epsilon_- , \quad \Psi_4 = \gamma^\tau\gamma^m D_m Y^{l,l}\epsilon_-,$$

- Vector bosons: 4+...

$$\mathcal{A}_\tau = Y^{l,l}, \quad \mathcal{A}^1_m = D_m Y^{l,l} , \quad \mathcal{A}^2_m = J_{mn}D^n Y^{l,l}, \quad \mathcal{A}^3_m = \epsilon_-^\dagger \gamma_m\gamma^n D_n Y^{l,l+3}\epsilon_+ .$$

# Superconformal Index

- choose Q & S to be one of four supercharges: SU(3) singlet

$$\{Q,S\} = \varepsilon - j_1 - j_2 - j_3 + 2R_1 + 2R_2 \equiv \Delta,$$

- (2,0) index

$$I(x, y_1, y_2, q) = \mathrm{tr}\left[(-1)^F x^{\varepsilon+R_1} y_1^{j_1-j_2} y_2^{j_2-j_3} q^j\right], \qquad x=e^{-\beta}, y_1=e^{-i\gamma_1}, y_2=e^{-i\gamma_2}.$$

- U(1) index: J. Bhattacharya, S. Bhattacharyya, S. Minwalla, S. Raju

$$I = \exp\left[\sum_{n=1}^{\infty} \frac{1}{n} f(x^n, y_i^n, q^n)\right],$$

$$f(x, y_1, y_2, q) = \frac{x + x^2q^3 - x^2q^2(1/y_1 + y_1/y_2 + y_2) + x^3q^3}{(1 - xqy_1)(1 - xqy_2/y_1)(1 - xq/y_2)}.$$

- q=0 limit= half index (16 susy): S. Bhattacharyya, S. Minwalla

$$I_{1/2-\mathrm{BPS}} = \prod_{m=1}^{N} \frac{1}{1 - x^m}.$$

# Path Integral

- Path integral $I(x, y_i, q) = \int_{S^1\times CP^2} \mathcal{D}\Psi e^{-S_I^E[\Psi]}$. $\quad I = \frac{1}{N!}\int \prod_{i=1}^{N}[\frac{d\alpha_i}{2\pi}]\prod_{i<j}^{N}\left[2\sin\left(\frac{\alpha_i-\alpha_j}{2}\right)\right]^2 \times I_{1-loop}$.

- Perturbative Contribution: split to hyper and vector multiplets
  - $\rho_{12}\, \varepsilon = -i\, \varepsilon$, $\rho_{12}\, \psi = -i\, \psi$, $\rho_{12}\, \chi = i\chi$
  - hyper: $\varphi_1 + i\, \varphi_2$, $\varphi_4 - i\varphi_5$, $\psi$
  - vector: $A_\mu$, $\chi$, $\varphi_3$

- hyper and vector contributions

$$\frac{\det_{H,f}}{\det_{H,b}} = \prod_{\alpha\in root}\frac{1}{\sin\left(\frac{\alpha-i\beta}{2}\right)} \sim \exp\left[\sum_{n=1}^{\infty}\sum_{i,j}\frac{1}{n}x^n e^{ni\alpha_{ij}}\right]. \qquad \frac{\det_{V,f}}{\det_{V,b}} = 1.$$

- 1-loop contribution= 1/2 index

$$\begin{aligned} I(x, y_1, y_2)_{k\to\infty} &= \frac{1}{N!}\int \prod_{i=1}^{N}[\frac{d\alpha_i}{2\pi}]\prod_{i<j}^{N}\left[2\sin\left(\frac{\alpha_i-\alpha_j}{2}\right)\right]^2 \exp\left[\sum_{n=1}^{\infty}\sum_{i,j}\frac{1}{n}x^n e^{ni\alpha_{ij}}\right] \\ &= \prod_{m=1}^{N}\frac{1}{1-x^m}. \qquad (4. \end{aligned}$$

# Supergravity

- AdS$_7$ x S$^4$ $ds^2 = R^2(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\Omega_5^2) + \frac{1}{4}R^2 d\Omega_4^2,$

  $F_4 \sim N\epsilon_4, \ R/\ell_p = 2(\pi N)^{1/3}.$

- Z$_k$ moddi $ds^2_{S^5} = ds^2_{CP^2} + (dy' + V)^2,$ $\quad y' = \frac{y}{k}, \ \chi' = \chi + \frac{3y}{k},$

  $ds^2_{S^4} = d\vartheta^2 + \sin^2\vartheta d\chi'^2 + \cos^2\vartheta ds_{S^2}.$

- Type IIA $ds^2_{11} = e^{-2\sigma/3} ds^2_{10} + e^{4\sigma/3}(dy + \mathcal{A})^2,$

  $F^4_{11} = e^{4\sigma/3} F^4_{10} + e^{\sigma/3} F^3_{10} \wedge dy.$

- 10-d me

$$ds^2_{10} = \frac{R^3}{2k}\Big[(-\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho ds^2_{CP^2}) + \frac{1}{4}(d\vartheta^2 + \cos^2\vartheta ds^2_{S^2})\Big]. \qquad (5.15)$$

The curvature scale of the type IIA theory is of order $\sqrt{R^3/2k} \sim \sqrt{N/k}$ which is large when 't Hooft coupling $\lambda = N/k$ is large.

- Fiber radius

$$e^{2\sigma/3} \sim \frac{N^{1/3}}{k} \sinh\rho$$

- M-region: k < N$^{1/3}$

# Conclusion

- Index of dyonic instantons should tell more about selfdual strings ending on multiple M5 branes, including the degenerated junctions

- New supersymmetric theories on M5 are found.

- We are working on the full Index calculation including instantons.

- More work to be done on M5