Aspects Gauge-Strings Duality and some applications

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Kyoto, October 2012

Outline

1. Summarize different aspects of some work done since 2009. It deals with string duals to field theories with N_c colors and N_f flavors (both large); with theories with a strange 'walking' dynamics and the relation with quiver gauge theories.
2. Towards the end, I will present some recent work applying the previous ideas to Cosmology.
3. This is work done with: Hoyos, Papadimitriou, Piai, Ramallo, Conde, Gaillard, Martelli, Elander, Caceres, Pando-Zayas, Tasinato, Halter, Erdmenger, in different collaborations.
4. I will try to take an 'orthogonal' approach to the one I presented in the papers. Hopefully the idea will be clear, but I may leave some loose ends with this way of presenting the topic.

The Maldacena Conjecture, AdS/CFT, Gauge-Strings Duality, is perhaps the most useful (in Physics) of all the known dualities. This audience needs no motivation nor summary of the general ideas.

Just to set our minds, this talk is in the framework of four-dimensional $\mathcal{N} = 1$ Supersymmetric gauge theories (with different chiral matter fields). In the context of the duality, this leads to geometries related to the conifold.

Remember that to construct geometries dual to different $N = 1$ theories, physicists have proceeded in various ways (in descending order of conceptual clarity):

- Deforming $N = 4$ SYM with marginal, relevant operators or by giving VEV's
- Placing D3 branes on the tip of CY_3 cones
- Wrapping D_{3+p} branes on small and compact manifolds so that the low energy effective theory has $N = 1$ in $d = 3 + 1$.

One goal of this talk is to comment on the connections between the procedures/models above.

Flows from $\mathcal{N}=4$ SYM

(I)

$\mathcal{N}=4$ SYM $+\ \mathcal{O}_1$

$+\ \langle \mathcal{O}_2 \rangle$

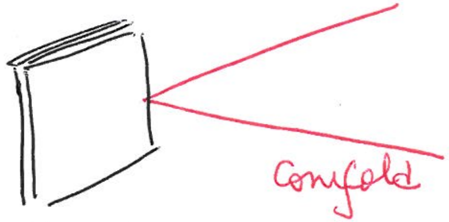
flows

$\mathcal{N}=1$ YM $+\ \sum \mathcal{O}_i$

$\langle \mathcal{O}_i \rangle$.

Flows from $\mathcal{N}=1$ SCFT

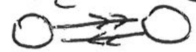
(II)



conifold

N_c D3 branes

N_c N_c

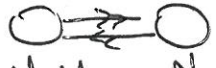


$+\ W$

+

M D_5 branes

$\oplus$ deformation / resolution of conifold

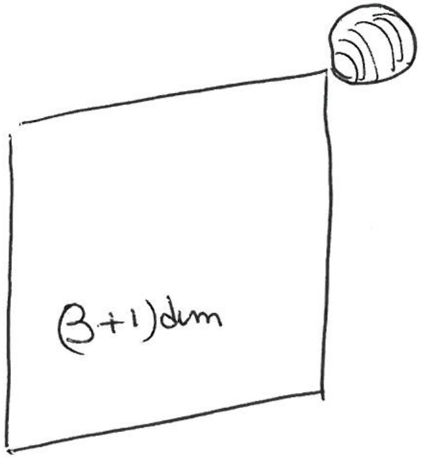


N_c+M N_c

$+\ W$.

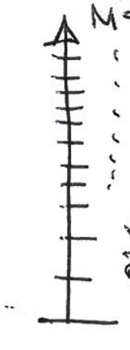
Wrapped branes

(III)



S^p

(3+1) dim



M^2

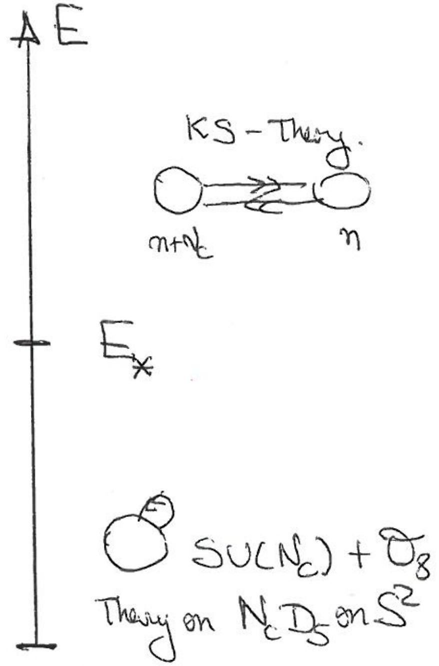
ψ_α

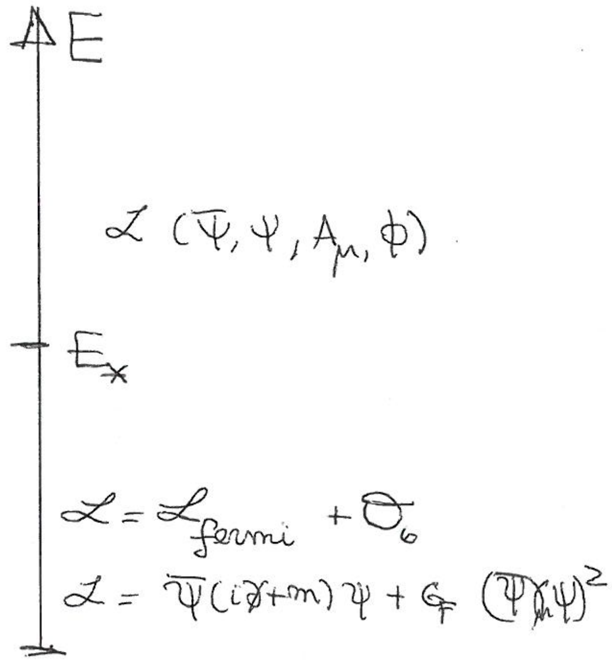
Φ_r

(A_μ, λ)

Summary of results of this seminar I

- The Klebanov-Strassler and the wrapped five branes set-ups are related via a *Brout-Englert/Higgs/Guralnik-Hagen-Kibble* -like mechanism. In the following, I will *unfairly* refer to the mechanism of gauge symmetry breaking as "higgsing".
- The theory on the five branes acts like an 'effective theory' for the KS-quiver





An analogy: consider the 'Fermi-like' Theory

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + G_F(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi).$$

that can be written as,

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m + g\gamma^\mu A_\mu)\psi - \frac{F_{\mu\nu}^2}{4} + M^2 A_\mu^2.$$

or, using a complex scalar field Φ as,

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m + g\gamma^\mu A_\mu)\psi - \frac{F_{\mu\nu}^2}{4} - \frac{|D_\mu\Phi|^2}{2} - \frac{\lambda}{4}(\Phi^2 - v^2)^2.$$

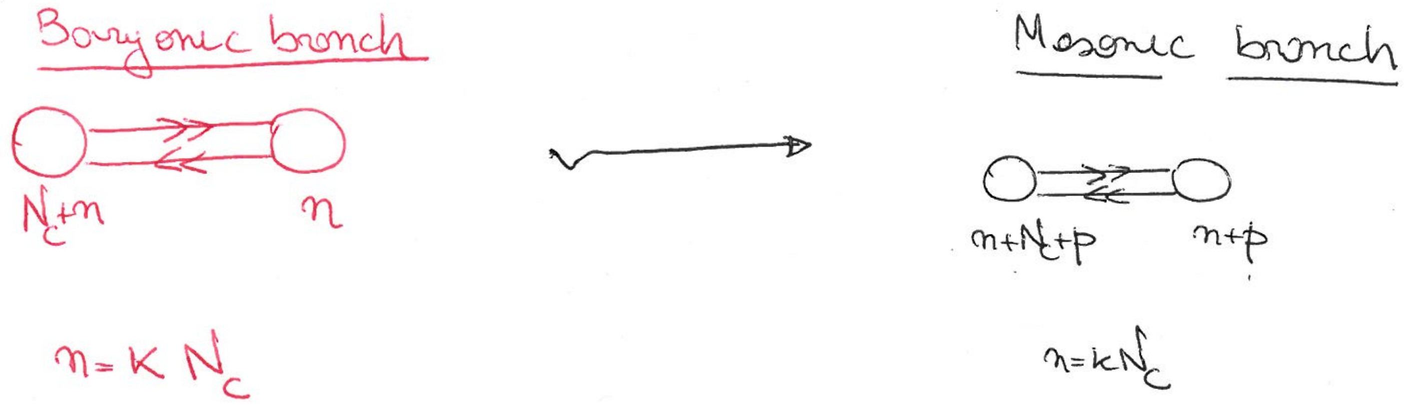
where

$$G_F = \frac{g^2}{M^2}, \quad M^2 = e^2 v^2.$$

The coupling of the irrelevant operator G_F is inversely related to a VEV.

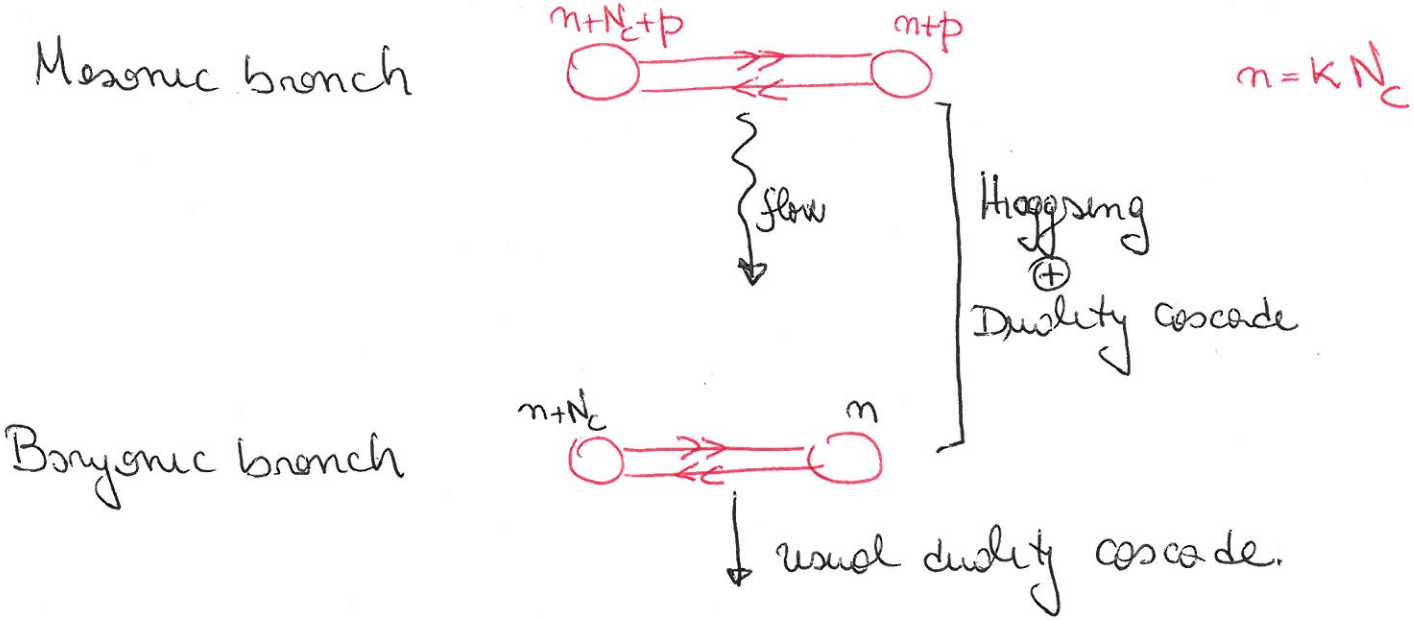
Summary of results of this seminar II

- The Klebanov-Strassler theory can be put in a *mesonic branch* by the addition of D3 and D5 sources in the dual background.
- This is nicely realizing some ideas by Aharony (2000), where he proposed that one can move in the moduli space of a field theory-giving VEV's- by 'mirroring' the VEV distribution by placing source D-branes accordingly



Summary of results of this seminar III

- One can construct string backgrounds that interpolate between a *mesonic branch* and a *baryonic branch* by the addition of D3 and D5 sources with a suitable *profile* in the dual background.
- The dynamics (decreasing the number of degrees of freedom towards the IR) is a superposition of the usual Seiberg cascade and a succession of 'higgsings' of the gauge groups.



PERTURBATIVE ASPECTS ($\hbar \to 0$).

These first few transparencies owe a lot to the works of Dymarsky, Klebanov and Seiberg (2005), Dorey and Andrews (2007 x 2) and Maldacena and Martelli (2009).

Let me start by discussing **perturbative** aspects of three field theories.

One of them is $\mathcal{N}=1^*$ Yang-Mills, this is the deformation of $\mathcal{N}=4$ SYM once masses for the three scalar multiplets are added.

$$L=\int d^4\theta \Phi_i^\dagger e^V \Phi_i + \int d^2\theta W_\alpha W_\alpha + \mathcal{W},$$

$$\mathcal{W}=i\sqrt{2}\Phi_1[\Phi_2,\Phi_3]+\eta(\Phi_1^2+\Phi_2^2+\Phi_3^2)$$

one can study this QFT around different vacua, compute the perturbative spectrum and degeneracies.

Dorey and Andrews did this for a particular higgs vacuum $\Phi_i=J_i^{(N_c)}$, Also for $\Phi_i=1_p\times J_i^{(q)}$.

They got a spectrum and a Lagrangian for the small fluctuations around this vacuum.

The interesting bit of their papers is that they also studied another QFT apparently very different. It was the 'twisted compactification' of 6d Yang-Mills with 16 SUSY's (the theory on D5 branes at low energy), so that it preserves only four supercharges.

By twisted compactification I will mean a special version of Kaluza-Klein, such that after compactification on curved manifolds, there is some amount of SUSY preserved.

These twisted compactifications can be done (and were done) in a variety of examples. The one that will occupy us here is the case when we KK the N_c-D5 6-d theory on S^2 preserving only four SUSY's; $\mathcal{N} = 1$ in four dimensions.

The theory obtained was carefully studied by Dorey and Andrews. Its spectrum, degeneracies and Lagrangian, coincide exactly with the expansion of $\mathcal{N} = 1^*$ SYM around the vacuum mentioned above.

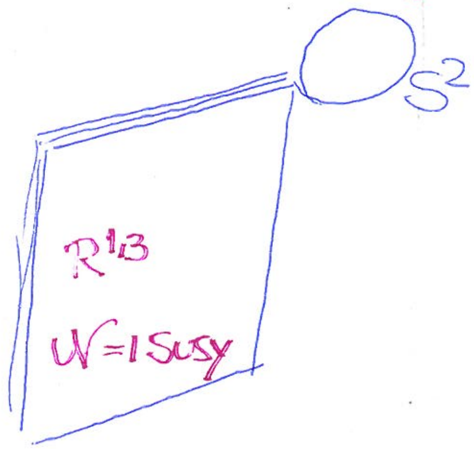
In other words, the four dimensional $N = 1^*$ 'deconstructs' the six dimensional theory.

$d=4$ $\mathcal{N}=1^*$ SYM

SU(N)

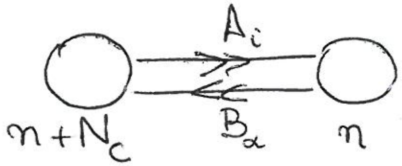
in a Higgs vacuum $\rightsquigarrow$

d=6 Field Theory.



N_c D_5 branes

The third theory I would like to discuss is represented by a quiver with two nodes $SU(N_c + n) \times SU(n)$ gauge groups and two sets of two bifundamental fields A_i, B_α with $i, \alpha = 1, 2$.



$$\oplus \; W = \kappa\, \epsilon_{ij}\, \epsilon_{\alpha\beta}\; A_\alpha^{al} B_i^{lb} A_\beta^{bm} B_j^{ma}$$

Once a quartic superpotential is added $W_{tree} \sim ABAB$ this is the field theory describing the Klebanov-Strassler system. The evolution of this field theory towards lower energies involves interesting non-perturbative dynamics, that I will only discuss upon request.

Let me just focus for the moment on the perturbative dynamics of the quiver. By studying its D and F-term equations one can compute the different (perturbative) vacua

[A: is $(n, n+N_c)$ matrix ; B is $(n+N_c, n)$ matrix]

$$AA^\dagger - B^\dagger B = \frac{U}{n} 1_n \quad A^\dagger A - BB^\dagger = \frac{U}{n+N_c} 1_{n+N_c},$$
$$U = Tr(AA^\dagger - B^\dagger B).$$

One can find different solutions to these eqs for the matrices A, B.
Some solutions are such that that both A, B are nonzero. Others that either A or B are zero.
The first set of solutions are called 'mesonic'-the 'meson' field $\mathcal{M} = AB$ is nonzero. The second set are called 'baryonic', because the 'baryon' field $\mathcal{B} \sim A^{power}$ is turned on.
Of the solutions above, the mesonic ones are the most generic. On the other hand, baryonic solutions only exist if the ranks of the gauge groups are in a relation such that $n = kN_c$.
Let me give some technical details about these mesonic and baryonic *classical* solutions.

Some details about Mesonic and Baryonic *classical* solutions

In the mesonic solutions, both fields A_i, B_j are non-vanishing. The meson field $\mathcal{M}_{ij,\alpha,\beta} = A^a_{i,\alpha} B^a_{j,\beta}$ is non-zero.

The baryonic solutions are such that the fields A_i are non-zero, while B_j vanish (viceversa for anti-baryonic ones).

<u>Mesonic</u>

$$B_\alpha = \begin{array}{c} \overset{p}{\longleftrightarrow} \quad \overset{M}{\longleftrightarrow} \\ \left[\begin{array}{cccc|c} B^{11}_\alpha & & 0 & & \\ & B^{22}_\alpha & & & 0 \\ & & B^{33}_\alpha & & \\ 0 & & & B^{pp}_\alpha & \end{array}\right] \end{array} \updownarrow p \quad ; \quad A_\alpha = \begin{array}{c} \overset{p}{\longleftrightarrow} \\ \left[\begin{array}{cccc} A^{11}_\alpha & & 0 & \\ & A^{22}_\alpha & & \\ 0 & & \ddots & A^{pp}_\alpha \\ \hline & 0 & & \end{array}\right] \end{array} \begin{array}{l} \updownarrow p \\ \updownarrow M \end{array}$$

<u>with</u> $\displaystyle\sum_{\alpha=1}^{2} |A_\alpha|^2 - |B_\alpha|^2 = \frac{u\,\mathbb{1}_p}{p} \equiv 0 \quad \underline{(u=0)}$

<u>Baryonic</u> (only if $M+p=(k+1)M \rightsquigarrow p=kM$ $k\in\mathbb{N}$).

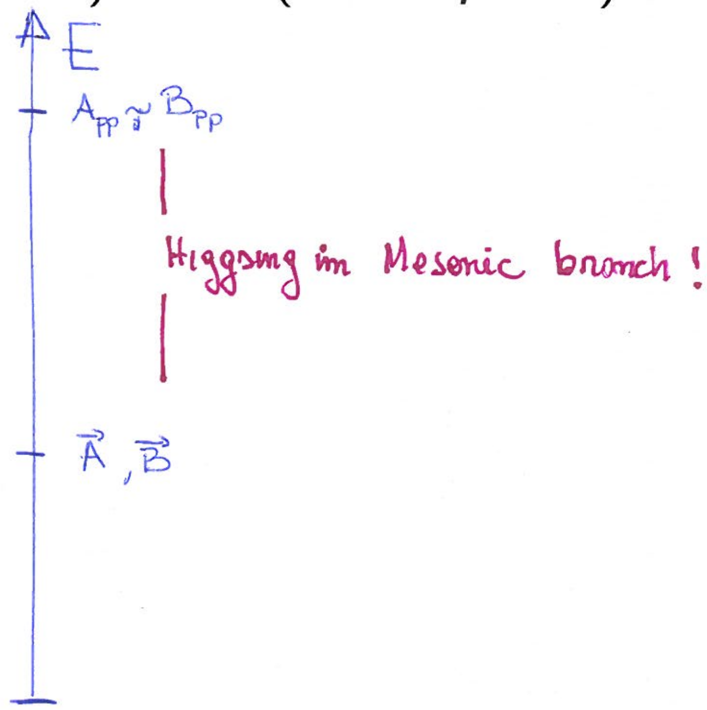
$$A_{\alpha=1} = \begin{array}{c} \overset{p}{\longrightarrow} \\ \left[\begin{array}{cccc} \sqrt{p} & & & 0 \\ & \sqrt{p-1} & & \\ & & \sqrt{p-2} & \\ 0 & & & \sqrt{1} \\ \hline & 0 & & \end{array}\right] \end{array} \begin{array}{l} \updownarrow p \\ \updownarrow M \end{array} \quad ; \quad A_{\alpha=2} = \begin{array}{c} \overset{p}{\longrightarrow} \\ \left[\begin{array}{ccccc} 0 & 1 & & & 0 \\ 0 & 0 & \sqrt{2} & & \\ 0 & 0 & 0 & \sqrt{3} & \\ & 0 & & \ddots & \sqrt{p} \\ \hline & & 0 & & \end{array}\right] \end{array} \begin{array}{l} \updownarrow p \\ \updownarrow M \end{array}$$

with $B_\alpha \equiv 0$; $\begin{pmatrix} \text{anti-baryon} \\ B \leftrightarrow A \end{pmatrix}$

SOMETHING INTERESTING I

If we start from a mesonic solution and separate the eigenvalues, such that for example A_{pp}, B_{pp} are very large compared to the others, we can think of integrating them out, hence

$$SU((k+1)M+p) \times SU(kM+p) \to$$
$$SU((k+1)M+p-1) \times SU(kM+p-1) \times U(1).$$

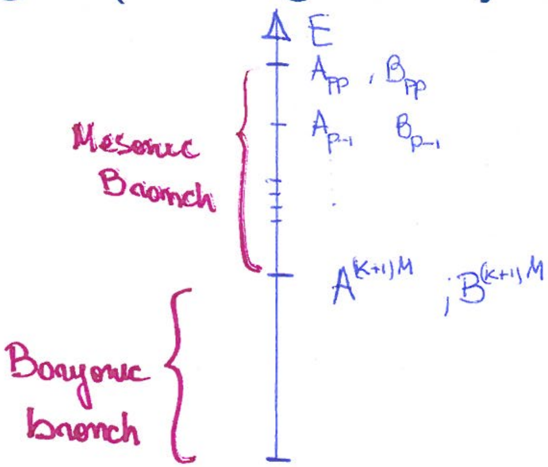


SOMETHING INTERESTING II

One may think of separating as many eigenvalues and integrating them out so that we end-up with the numerology of a baryonic branch

$$SU((k+1)M+p) \times SU(kM+p) \to$$
$$SU((k+1)M+p-1) \times SU(kM+p-1) \times U(1) \to .... \to$$
$$SU((k+1)M) \times SU(kM) \times U(1)^p.$$

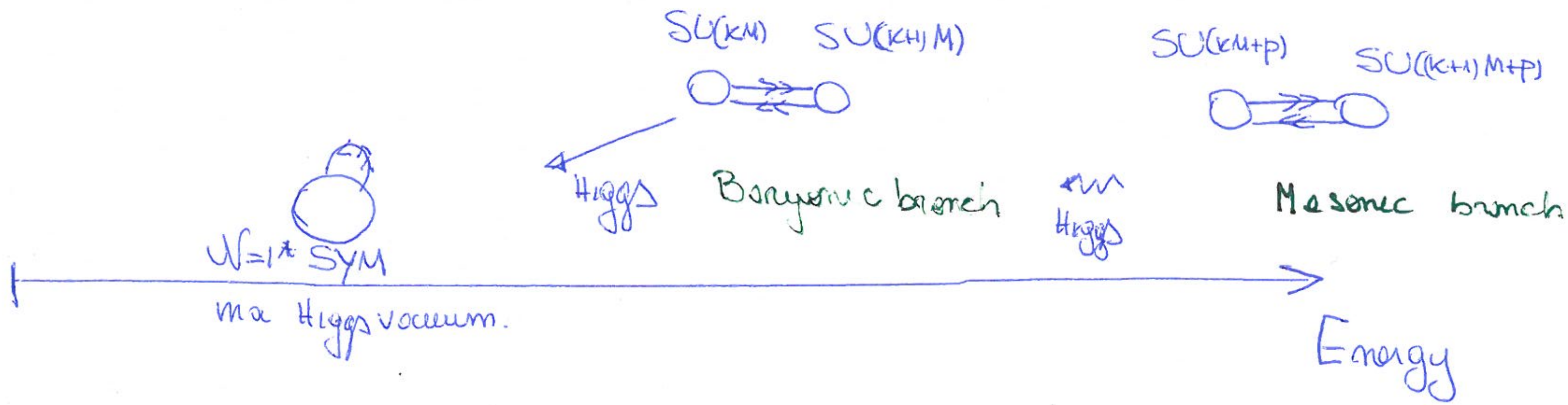
The subtle point here is that in this case, the baryonic symmetry, mixes with the $U(1)'s$. Hence the baryon symmetry is actually gauged (not a global symmetry).



SOMETHING INTERESTING III

If one expands the quiver theory around one of these **perturbative** baryonic vacua, one gets a spectrum of fluctuations and degeneracies closely related to the one described by Dorey and Andrews for the theory on the D5 branes after twisting. This was done by Maldacena and Martelli (2009).

This tells us that there is a relation between the KS-quiver QFT and the QFT on D5 branes compactified on S^2 preserving minimal SUSY. This also suggest some relation between $N = 1^*$ SYM (at least in a particular vacuum) and the KS quiver.



NON-PERTURBATIVE ASPECTS ($\hbar \to \infty$).

We will now focus on the connection between these field theories, once the strong coupling dynamics sets-in. We will do so, using the (trustable/nonsingular) string duals.

In order to discuss the connection between the field theories on twisted-compactified five branes and the KS-theory, I will start by describing the Type IIB string dual to the twisted five brane theory

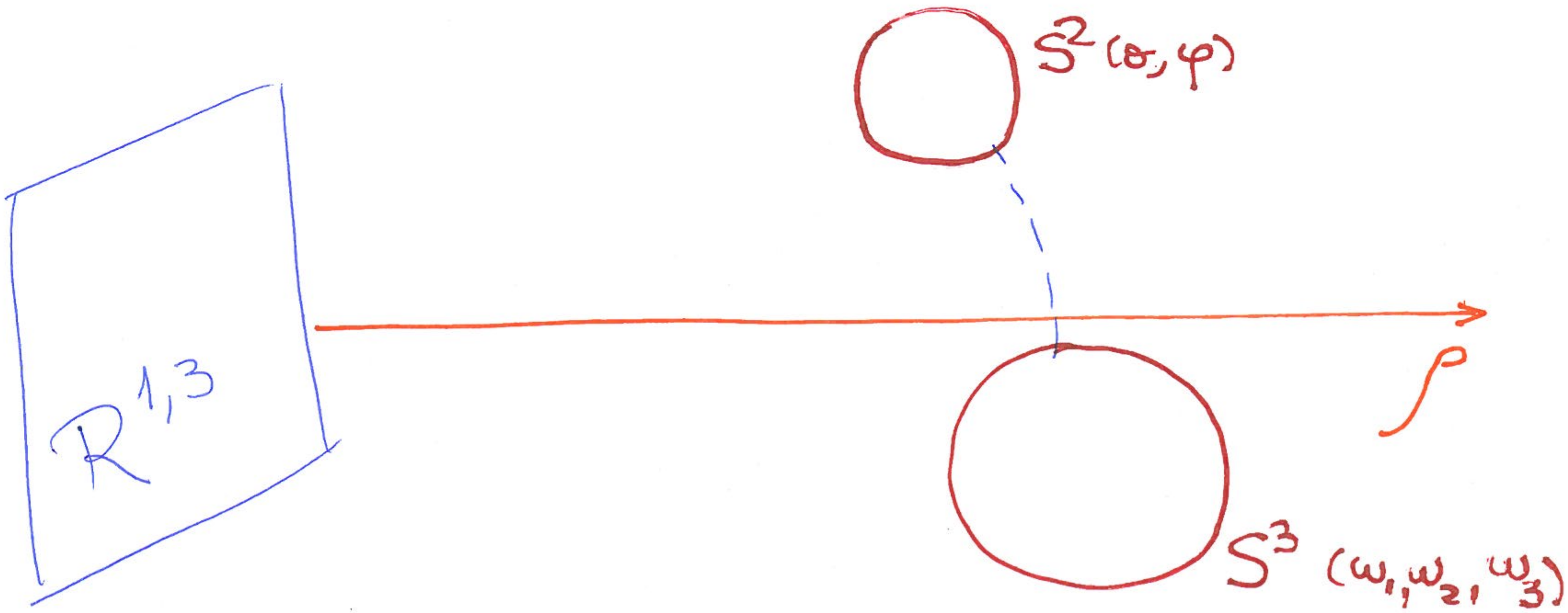
To do so, let me describe the background(s) proposed as dual to the theories on N_c D5 branes. I will focus on the metrics. There are fluxes and scalars turned-on. I have some technical back-up transparencies if needed.

$$ds^2 = \alpha' g_s e^{\Phi(\rho)/2}\Big[(\alpha' g_s)^{-1} dx_{1,3}^2 + ds_6^2\Big],$$

$$ds_6^2 = e^{2k(\rho)} d\rho^2 + e^{2h(\rho)}(d\theta^2 + \sin^2\theta d\phi^2) + \frac{e^{2k(\rho)}}{4}(\tilde{\omega}_3 + \cos\theta d\phi)^2$$
$$+ \frac{e^{2g(\rho)}}{4}\left((\tilde{\omega}_1 + a(\rho)d\theta)^2 + (\tilde{\omega}_2 - a(\rho)\sin\theta d\phi)^2\right),$$

There is also a RR three form and a dilaton, $\Phi(\rho)$ that acts as a 'warp factor'.

Space looks like



Imposing that the system preserves some fraction of SUSY, implies that one can write a set of first order equations (and a constraint) that are equivalent to all the second order eqs (Einstein, Maxwell, Bianchi, dilaton)

This BPS eqs. for the functions $[h, g, k, \Phi, a, (b)]$ are : first order, non-linear and coupled. there is a very non-trivial change of variable that decouples them

$$[\Phi, h, g, k, a, b] \to [P, Q, Y, \phi, \sigma]$$

The change is involved. It is not your first guess. Let me avoid writing it here.

Reserving technical details of this change of variables for the back-up transparencies, let me state that everything boils into just solving one second order non-linear equation for a function $P(\rho)$.

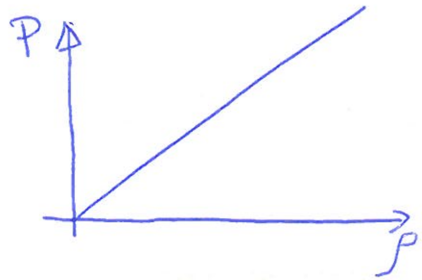
$$P'' + P'\left[\frac{P' - Q'}{P + Q} + \frac{P' + Q'}{P - Q} - 4\coth(2\rho)\right] = 0$$

with $Q = N_c(2\rho\coth(2\rho) - 1)$.

So, whatever solution you manage to find to the eq. above will-if non-singular- faithfully describe different aspects of the non-perturbative dynamics of the theory on the compactified five branes.

There are some exact solutions and also some semi-analytical solutions. I would like to discuss special ones that will play a role in the following.

An exact solution is $P = 2N_c\rho$. This will play almost no role in what follows.



The solutions can be found in semi-analytic way: series expansions near the UV and near the IR and a simple numerical interpolation

$$P = e^{4\rho/3}\Big[c_+ + \frac{e^{-8\rho/3}N_c^2}{c_+}\left(4\rho^2 - 4\rho + \frac{13}{4}\right) + e^{-4\rho}\left(c_- - \frac{8c_+}{3}\rho\right) + O(e^{-16\rho/3})\Big].$$

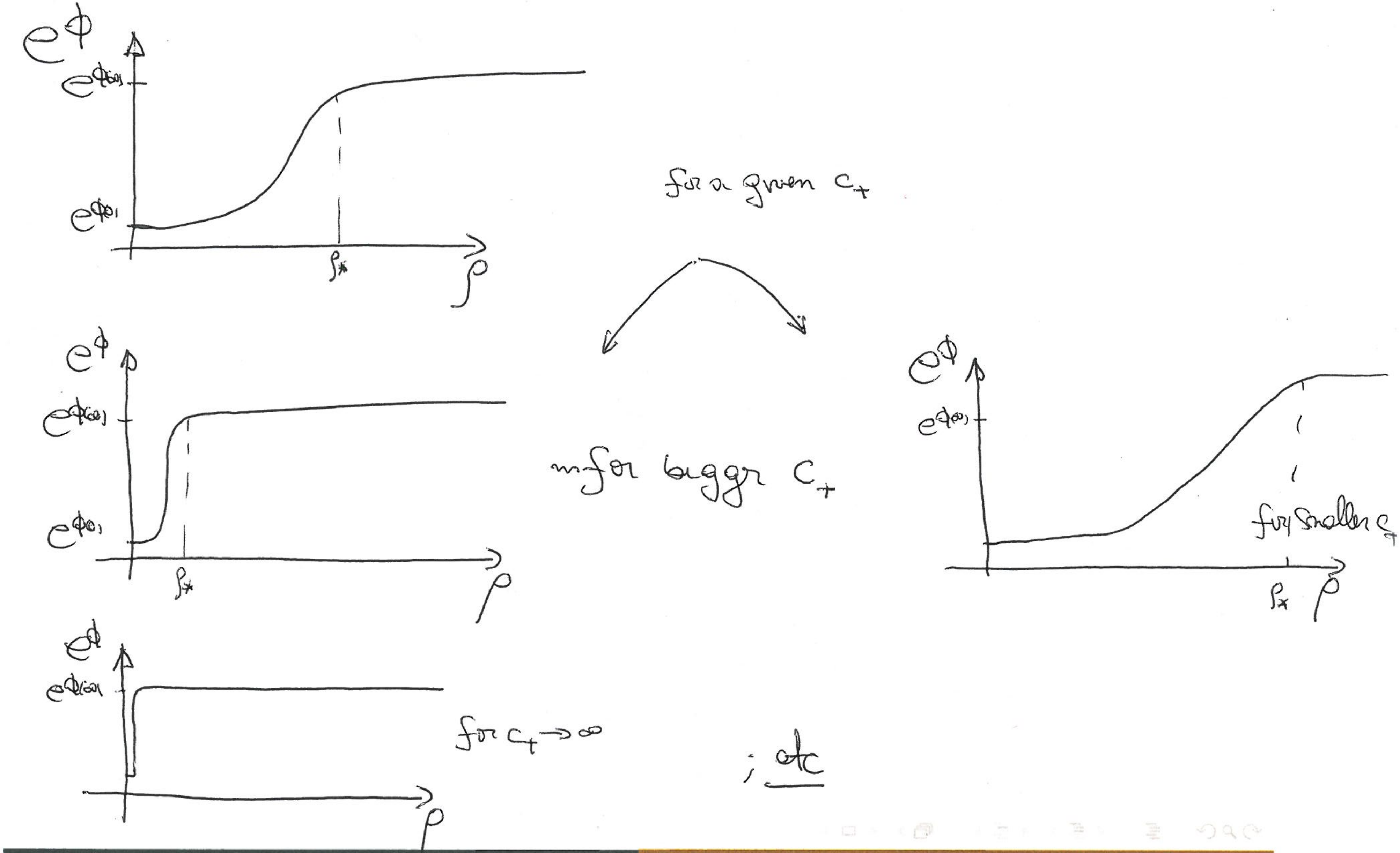
With two integration constants, $c_+ > 0$ and c_-. Regarding the IR expansion,

$$P = h_1\rho + \frac{4h_1}{15}\left(1 - \frac{4N_c^2}{h_1^2}\right)\rho^3 + O(\rho^5),$$

where $h_1 > 2N_c$ is an arbitrary constant (there is another integration constant, $P_0 = P(0)$, taken to zero here, to avoid singularities).

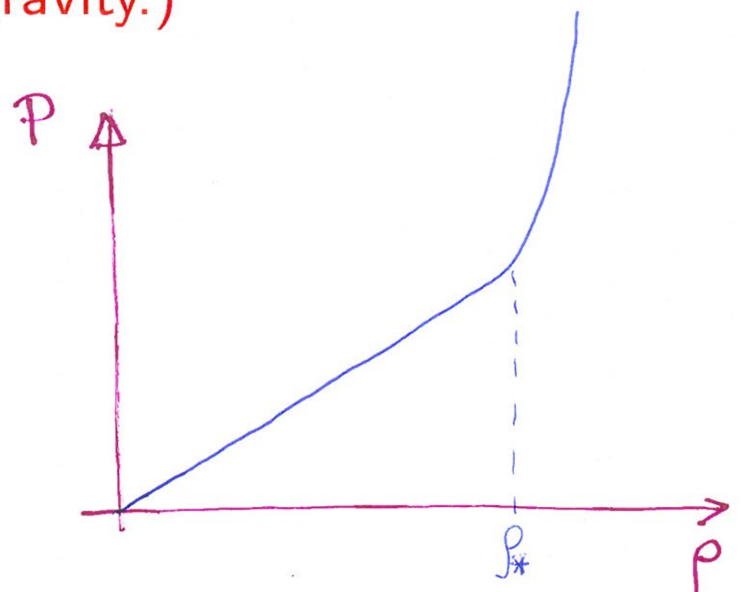
This produces a set of functions $[h, g, k, \Phi, a, b]$. Let me focus on the dilaton (the warp factor of the metric).

The dilaton looks like.

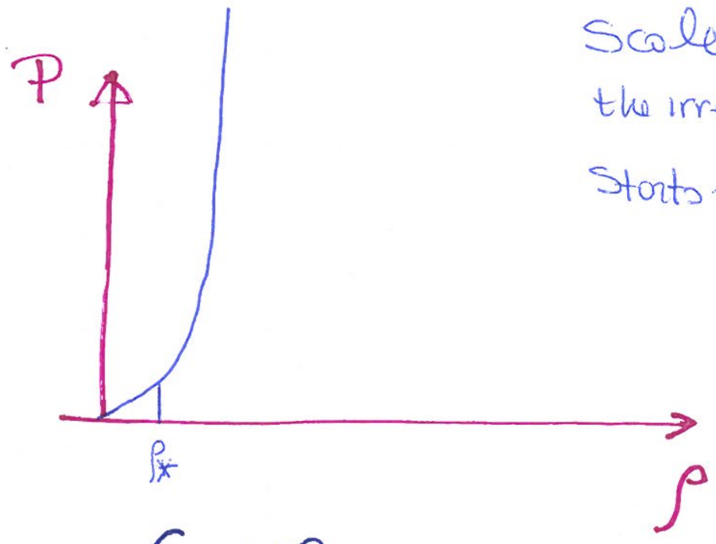


Let me leave these solutions there. Of course, various aspects of the dual field theory (beta functions, anomalies, spectrum, Wilson loops behavior, etc) were explored in all the cases.

I want to point out just one field theory fact. For these solutions with asymptotically stabilized dilaton, one can see that the UV of the field theory is dominated by an irrelevant operator of dimension eight (much like 'keeping the 1" in the warp factor of the D3 branes. In other words, these field theories are typically coupled to gravity.)



$C_+ \to 0$ $\quad h_1 \to 2N_c$



Scale ρ_*: where the irrelevant $\underline{O_8}$ starts to dominate

$C_+ \to \infty$

$h_1 \to 2N_c +$ large number

A SOLUTION GENERATING TECHNIQUE

The relation between D5 branes and the Klebanov-Strassler system

In the following let me focus the attention on a *'solution generating technique'*.

This is just some set of operations such that *'create' a new solution*. You need not solve again the eqs of motion.

There are two quantities in backgrounds like the ones we discussed above, that encode all the SUSY information. These are called the complex three form Ω_3 and the (non)-Kahler two form J_2.

They satisfy various relations. For example, the BPS eqs of the system, can be written as differentials of J, Ω. Again, I leave this for the back-up transparencies.

Then, one can consider the effect on these backgrounds, of a rescaling of J, Ω-this is actually the 'solution generating',

$$J_{new} = \cos^2 \xi(\rho) J_{old}, \quad \Omega_{new} = \cos^3 \xi(\rho) \Omega_{old}, \quad \sin \xi(\rho) = \kappa e^{\Phi}.$$

integration constant

The first two rescalings 'are' the solution generating technique. The expression for $\sin \xi$ is equivalent to imposing *not* to generate D7 charge

Scaling the J_2, Ω_3, actually changes the metric, but most interestingly turns on some fluxes. Under the conditions above, it generates H_3, F_5 aside from the already existent F_3, Φ. I leave details of this to be given-upon request-in the back-up slides at the end

The newly generated solutions have a metric

$$ds^2 = \hat{h}(\rho)^{-1/2} dx_{1,3}^2 + \hat{h}(\rho)^{1/2} ds_6^2,$$
$$ds_6^2 = e^{2k(\rho)} d\rho^2 + e^{2h(\rho)}(d\theta^2 + \sin^2\theta d\phi^2) + \frac{e^{2k(\rho)}}{4}(\tilde{\omega}_3 + \cos\theta d\phi)^2$$
$$+ \frac{e^{2g(\rho)}}{4}\left[(\tilde{\omega}_1 + a(\rho)d\theta)^2 + (\tilde{\omega}_2 - a(\rho)\sin\theta d\phi)^2\right],$$

and

$$\hat{h}(\rho) = 1 - \kappa^2 e^{2\Phi(\rho)}$$

The number κ is the integration constant mentioned above. This precisely the kind of metrics that appear as duals to Klebanov-Strassler quivers!

If one looks at the fluxes, one finds that indeed, the system generated coincides with the so called 'Baryonic Branch of KS", a beautiful paper by Butti, Graña, Minasian, Petrini and Zaffaroni.

Let us analyze the UV asymptotics of the generated backgrounds to get results for the dual QFT.

The procedure is the usual. We start by reducing the system to five dimensions (this is the so called Papadopoulos-Tseytlin system).

Then, we have five dimensional fields, that are actually combination of the ten-dimensional ones $[h, g, k, \hat{h}, \Phi, a, b]_{d=10} \rightarrow [\Phi, h_1, \tilde{g}, a, b, h_2, x, p]_{d=5}$. Details in technical transparencies.

Then we expand the 5d-fields for large values of ρ, changing variables to $z = e^{-2\rho/3}$

some generic field goes like $\varphi \sim z^{\Delta}$, indicating either a VEV for a $< O_\Delta >$ or the deformation of the Lagrangian : $L' = L + O_\Delta$

$$L = L_{IIB} \rightarrow$$

$$L_5 = \sqrt{g_5}\Big[R - \frac{1}{2}G^{ab}\partial\Phi_a\partial\Phi_b - V(\Phi_i)\Big]$$

Expanding these fields, one finds that the BPS-eqs. imply that only some particular deformations and VEV's that are allowed by SUSY. Two (quasi)-marginal operators associated with the dilaton and the field h_1. They correspond in the QFT to the two gauge couplings of the quiver. Two operators of dimension three, getting VEV's. They are asociated with the functions $a(\rho)$, $b(\rho)$ and QFT gauginos' VEV.

More interestingly, associated with the integration constant c_-, an operator of dimension six is getting a VEV. Associated with the combination of fields $\left[e^{2\tilde{g}} + a^2 - 1\right]$ is the VEV of a dimension two operator $< O_2 >$.

$<O_3>, <\tilde{O}_3> \; ; <O_6>, <O_2>$

Even more interestingly:

a deformation by a dimension-eight operator O_8 is associated with the warp factor $\hat{h}$ and $x - p$. A noticeable point-when doing the expansions- is that the coefficient of the dimension-eight operator is inverse with the coeficient of the VEV of $< O_2 >$

$$e^{2\tilde{g}} + a^2 - 1 \sim \frac{N_c}{c_+} z^2 (3 \log z + 1) \ldots.$$

$$x - p \sim (\frac{c_+}{N_c})^2 \frac{\log z + \ldots}{z^4}$$

$$\hat{h} \sim 1 - \kappa^2 e^{2\Phi(\infty)} + \underbrace{\alpha z^4 + \ldots}_{\text{"}AdS_5\text{-like"}}$$

Remind that....

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + G_F(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi) \to^{UV}$$
$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m + g\gamma^\mu A_\mu)\psi - \frac{F^2_{\mu\nu}}{4} - \frac{|D_\mu \Phi|^2}{2} - \frac{\lambda}{4}(\Phi^2 - v^2)^2.$$

where

$$G_F = \frac{g^2}{M^2}, \quad M^2 = e^2 v^2.$$

The coupling of the irrelevant operator G_F is inversely related to a VEV.

$$G_f \sim \frac{1}{v^2}.$$

This should resonate with what we discussed. One of the many interesting Physics points here is this:let us look at the warp factor

$$\hat{h} = 1 - \kappa^2 e^{2\Phi}.$$

given the asymptotics of Φ, one can choose the number $\kappa = e^{-\Phi(\infty)}$ such that the '1' is cancelled asymptotically. This UV-completes the theory on the twisted D5 branes (it gives asymptotics of KS $\hat{h} \sim z^4 \log z$) and decouples gravity from the QFT.

The *un-higgsing* of a gauge group, gives place to a quiver with the precise matter content (choice of κ), such that all irrelevant operators are absent.

The analogies with the Fermi-like theory are more or less clear. There one could also UV complete with the 'wrong gauge group and matter content' so that we would still be in the presence of irrelevant operators. That is not what one usually does.

Let us move into applications of this set of ideas and techniques.

Of course, once one develops this solution generating technique it can be applied to a variety of examples.

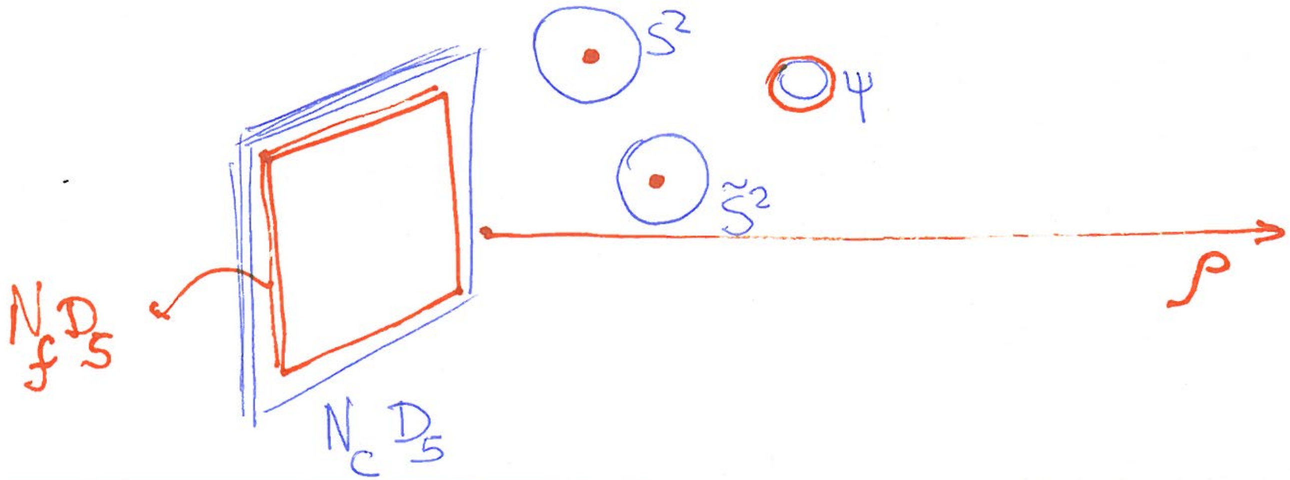
There is one example I would like to pick on, because it has lots of interesting Physics.

It will also give us the connection to the 'mesonic'branch of KS from the perspective discussed above.

Suppose that we consider situations where we add to Type IIB some sources (D branes).
Depending on technical subtleties, this can be often be identified with the addition of flavors (fundamentals) in the dual QFT. The number of fundamentals and the number of adjoints is comparable $N_f \sim N_c$. Hence one looks for backgrounds solutions of

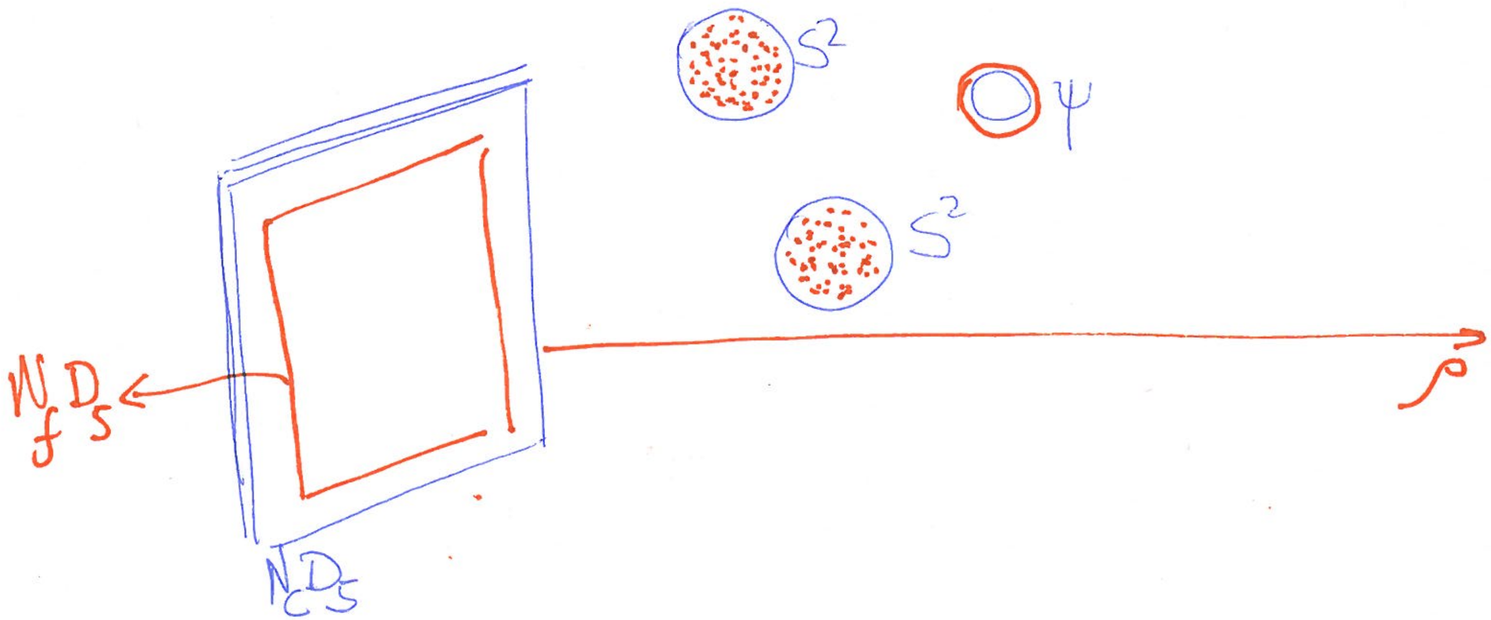
$$S = S_{IIB/A} + S_{BIWZ},$$

To be concrete, let me focus on the case of the D5 branes. We will add a set of N_f D5 branes that will extend over the Minkowski directions-to preserve $SO(1,3)$ and also on the radial direction and the R-symmetry direction.



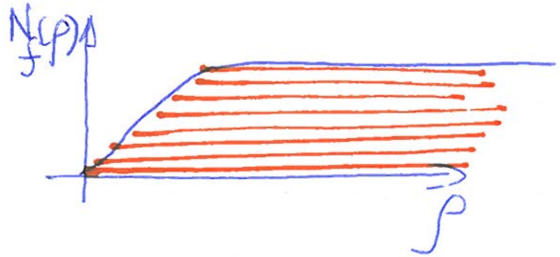
One can then cranck the machine and obtain a set of BPS equations. They will be: non-linear, coupled and *partial* differential equations.

To avoid difficulties, one can 'smear' the D5 branes on the manifold Σ_4. We will do so that only the Fourier zero mode on Σ_4 is kept. In field theoretical words, you are adding the flavor dynamics so that the global symmetries of the new theory are the same as those of the unflavored theory.



Still, one has the possibility of distributing the branes on the radial direction according to a given profile (technically subtle to determine).

One can see the dual view of this 'profiling' by thinking that in the QFT, there is a coupling between the quark multiplets and the adjoints



$$W \sim \tilde{Q} \Phi Q,$$

that implies that a Higgs mechanism (energy dependent) is at play.

$$W \sim <\Phi> \tilde{Q} Q = M(E) \tilde{Q} Q.$$

What one gains is that singularities due to N_f branes coming together are resolved.

So, using a similar change of variables as the one discussed above, now in the presence of sources with profile $N_f(\rho) = N_f S(\rho)$ one finds that the whole system (after smearing) just boils to solving one non-linear second order differential equation

$$(P'' + N_f S') + (P' + N_f S)\Big[\frac{P' - Q' + 2N_f S}{P + Q} + \frac{P' + Q' + 2N_f S}{P - Q} - \\ -4\coth(2\rho)\Big] = 0,$$

where $Q(\rho)$ is

$$Q(\rho) = N_c(2\rho\coth(2\rho) - 1) - N_f \coth(2\rho)\int^{\rho} S[x]\tanh^2(2x)dx,$$

Non-singular solutions to this equation, will produce trustable backgrounds encoding the non-perturbative aspects of $N = 1$ field theories with flavors.
Various solutions were found and non-perturbative aspects of field theories studied. Let me not focus on them in the interest of time.

But, here again, I want to briefly discuss the qualitative behavior of some solutions

$$S(\rho) = N_f \tanh^4(2\rho),$$

$$P(\rho \to 0) \sim h_1\rho + 4\frac{h_1^2 - 4N_c^2}{15h_1}\rho^3 +$$

$$\frac{16}{1575h_1^3}(3h_1^4 - 4h_1^2N_c^2 - 32N_c^4 - 450h_1^3N_f)\rho^5 + O(\rho^7),$$

$$P(\rho \to \infty) \sim c_+ e^{4\rho/3} + \frac{9N_f}{8} +$$

$$\frac{e^{-4\rho/3}}{c_+}\left((2N_c)^2(\rho^2 - \rho + \frac{13}{16} - \frac{81N_f^2}{64})\right) + O(\rho e^{-8\rho/3})$$

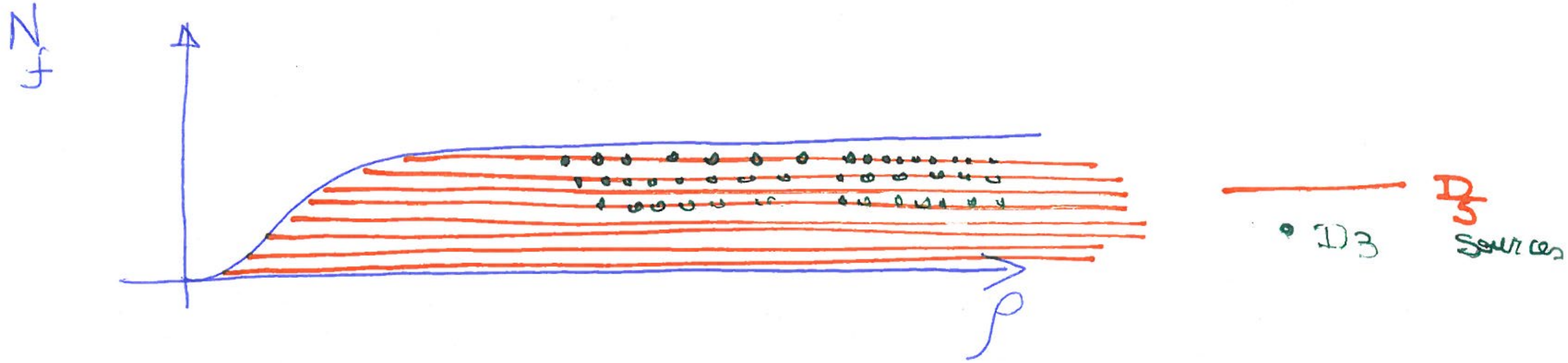
~~: effect of sources

It is almost immediate to act on these solutions with the 'solutions generating technique' previously developed.

So, rescaling as before J, Ω and imposing the absence of D7 brane charge $\sin\xi(\rho) = \kappa e^{\Phi(\rho)}$, one finds new backgrounds, resembling the KS system, but now with the presence of these sources.

An interesting effect is that due to the presence of a NS B_2 field, the source D5 branes acquire an induced D3 charge.

$$\hat{h}(\rho \to \infty) \sim 1 - \kappa^2 e^{-2\Phi(\infty)} + \frac{N_f}{c_+} e^{-4\rho/3} +$$
$$+ \frac{e^{-8\rho/3}}{c_+^2} \left((2N_c)^2 (\rho^2 - \rho + \frac{13}{16} - \frac{81 N_f^2}{64}) \right) + \ldots. \quad (1)$$



One can even write, in the limit of $c_+ \to \infty$ and exact and analytic solution for any $S(\rho)$. This generalizes the Klebanov-Strassler solution in the presence of sources.

$$\hat{h} = \hat{h}_{KS} + \frac{4}{\epsilon^{8/3}}\nu \int_\rho^\infty dx \frac{S(x)\,(-4x+\sinh(4x))^{1/3}}{(-4x+\sinh(4x))^{2/3}}$$

The function

$$G(\rho) = \int_\rho^\infty dx \frac{1}{(-4x+\sinh(4x))^{2/3}},$$

indicates that the solution above is just the solution to the Laplace equation (for the function $\hat{h}$) in the presence of fluxes F_5, H_3, F_3 and D3 sources. So, we have the KS geometry being deformed by a distribution of source D3-branes,

$$n_f \sim \nu\, S(\rho)\,(\sinh(4\rho) - 4\rho)^{1/3} \ ,$$

that are supersymmetric when placed on the deformed conifold. Hence, this exact and analytical solution could have been written without going over all this effort, just assuming this strange distribution of D3 sources

We have made various checks (beta functions, anomalies, central charges, Wilson loops, etc.)
Indicating that indeed, this is what these sources do to the KS quiver. One can see, though this is technical, that the field theory is on a mesonic branch now and is making a transtition to a baryonic branch-with the baryonic symmetry gauged. With the exact and analytic solutions describing the dual to this dynamics, the solutions confirm after calculation the picture advocated

This is a good point to recall some ideas by Aharony (2000), where he proposed that one can move in the moduli space of a field theory-giving VEV's- by 'mirroring' the VEV distribution by placing source D-branes accordingly

Hence, everytime we cross a source (D3 in this case) we are higgsing the gauge groups. We have then (in the middle region of the geometry) a coexistence of two phenomena, the higgsing and the usual duality cascade.

Presently, I am exploring some applications of this kind of backgrounds.

OTHER APPLICATIONS—VERY BRIEFLY!

Leave aside the sources for the moment. One may consider solutions, where the function $P(\rho)$ behaves as

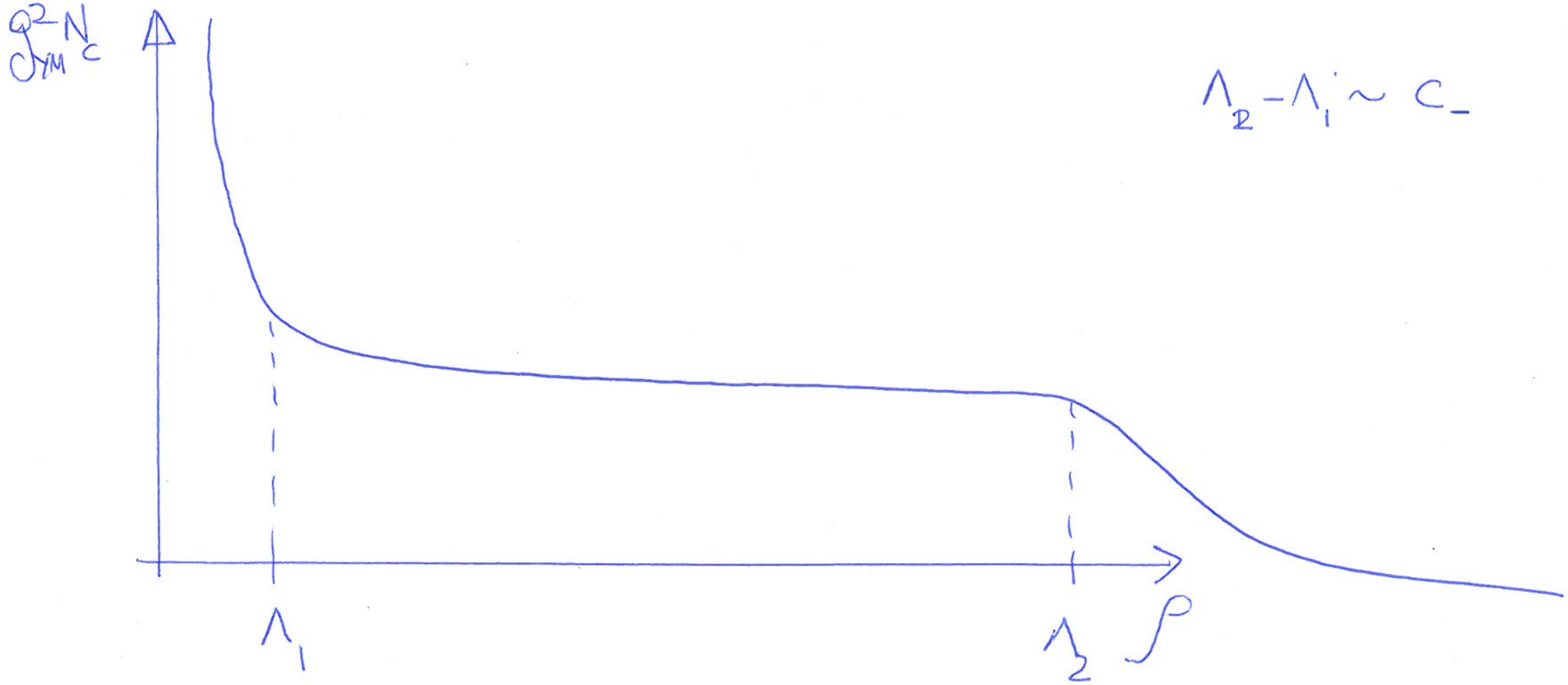
$$P(\rho \to \infty) = c_+ e^{4\rho/3} + 4\frac{N_c^2}{c_+}\left(\rho^2 - \rho + \frac{13}{16}\right) e^{-4\rho/3}$$
$$+ \left(-\frac{8}{3}c_+\rho - \frac{3c_-}{64c_+^2}\right) e^{-8\rho/3} + \ldots.$$
$$P(\rho \to 0) = c_0 + k_3 c_0 \rho^3 + \frac{4}{5}k_3 c_0 \rho^5 - k_3^2 c_0 \rho^6 +$$
$$+ \frac{16(2c_0^2 k_3 - 5k_3 N_c^2)}{105 c_0}\rho^7 + \ldots.$$

where $c_0, k_3, c_\pm$ are integration constants.

One can see that a solution like this is dual to a QFT with two *independent and tunable* scales.

It can be shown that a suitable defined gauge coupling is almost constant between these two scales.

This was used to make a simple model of 'walking dynamics' (with the hope of getting some hint on walking technicolor)



Physically, the two more interesting results derived from this background are

- The glueball-mass spectrum of the dual QFT presents an anomalously light scalar excitation. The longer the 'walking region' the lightest this scalar.
- For ranges of separation in the two relevant scales, one gets a first order phase transition (at zero Temperature) for the Energy/Separation between two external sources (Wilson loop).

These are in principle, measurable effects in a suitable QFT.

YET ANOTHER APPLICATION—VERY BRIEFLY!

One may consider 'translating' the slow running (walking) of the previous coupling constant, into the 'slow motion' of an object under the influence of a potential.

This may have some application in Cosmology, basically, the idea is to construct a little model of brane inflation in a KS-like throat.

The things turn out to work quite nicely. Almost no fine tuning is needed to find an inflationary trajectory satisfying all the experimental constraints. The set-up is flexible enough to be able to say some solid things about the so-called eta-problem.

Brief summary

Let me just say that there is a nice interplay between different backgrounds dual to field theories with minimal SUSY in four dimensions.

This is the strong-coupling version of some perturbative relations between apparently different field theories.

The relation extends with nice subtleties when you add flavors to the initial QFT's.

Or when you consider subtle variations of the dynamics, like the addition of operators that dominate the dynamics at different scales (making it a theory with various tunable scales). Something of certain phenomenological interest..

This little landscape of dual geometries to N=1 SUSY models may be enlarged by getting inspirations from perturbative connections between the field theories.