[With ESCOBEDO, GROMOV, SEVER] [OTHER REFS IN THE BB]

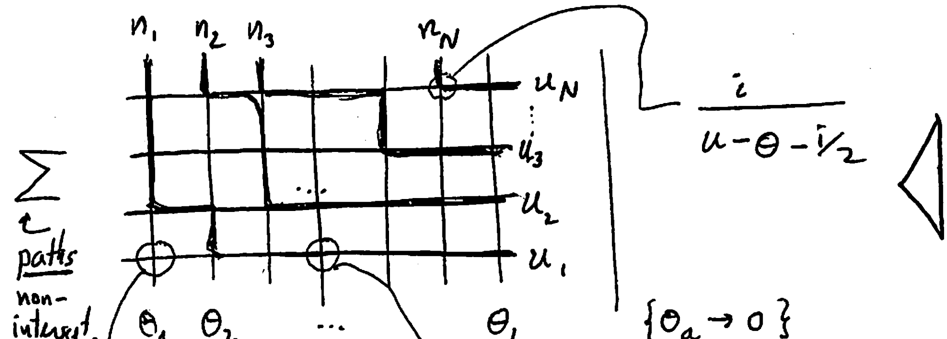
$\mathcal{N}=4$ SYM in the Planar limit

$$\mathcal{O}(x) = \sum_{1 \leq n_1 < \dots < n_N \leq L} \psi(n_1, \dots, n_N)\, \mathrm{Tr}[Z \dots Z \underset{n_1}{X} Z \dots Z \underset{n_2}{X} Z \dots]$$

from resolving the tree level mixing using 1 loop mixing matrix

ALGEBRAIC BETHE ANSATZ

$$\psi(n_1 \dots n_N) = [\dots] \sum_{\substack{\text{paths} \\ \text{non-intersect.}}}$$



Where

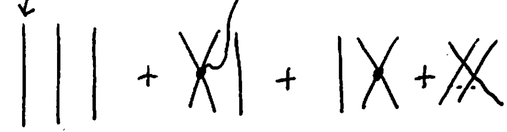
for 2 loop mixing, i.e. 1 loop structure constants

$$[\dots] = 1 + \frac{g^2}{2} \sum_{a=1}^{L} \left(\partial_{\theta_a} - \partial_{\theta_{a+1}}\right)^2 - g^2\, \Gamma\left[\, \underset{\mathbb{1}}{||||\dots||} - \underset{\mathbb{P}_{L,1}}{||||\dots||} \,\right]$$

1 loop states, i.e. tree level C_{123}

COORDINATE BA

$e^{ip_1 n_2}$ $\quad S(p_1, p_2)$, $p_j = p(u_j)$, $u = \frac{1}{2}\cot\frac{p}{2}$

$\psi(n_1, n_2, n_3) =$  $+ \dots$

$$1 + g^2 \delta_{n_1+1, n_2}\, C^{(2)}_{\bullet\bullet}(p_1, p_2) + \dots + g^2 \delta_{n_1+1, n_2, n_3-1}\, C^{(2)}_{\bullet\bullet\bullet}(p_1, p_2, p_3)$$

2 loops

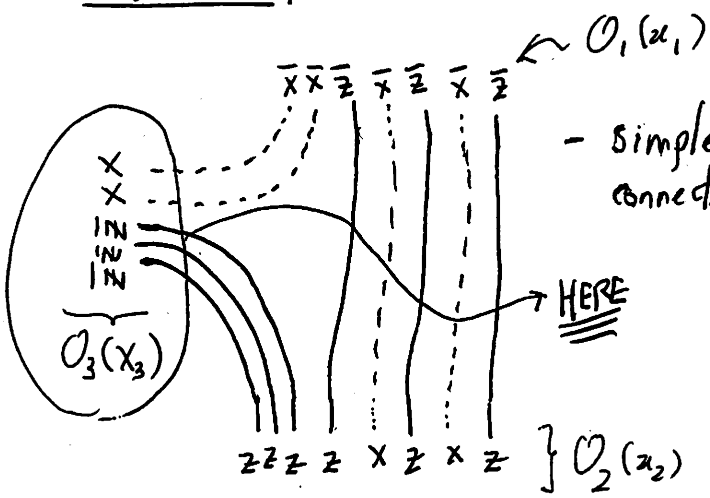
2ptc contact term

3ptc contact term etc.

$C^{(2)}_{\bullet\bullet\bullet\bullet}(p_1,\dots,p_4)$ already takes one full page!

now we have $\mathcal{O}(x)$. We should glue them into C_{123}.

SU(2) SETUP



- simplest C_{123} where everyone connects to every-one
- mixing w/ double trace suppressed

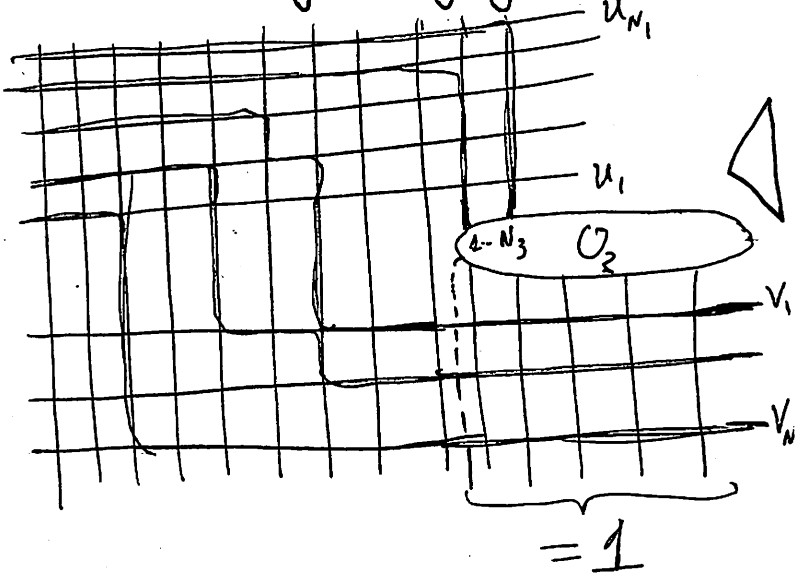
$$\text{let } \mathcal{O}_3 = \frac{1}{\sqrt{\binom{L_3}{N_3}}}\Big[\mathrm{tr}\,(\bar{Z}^{L_3-N_3}X^{N_3}) + \text{permutations}\Big]$$

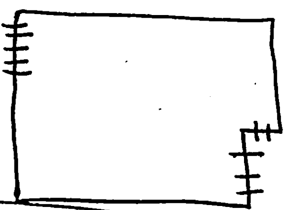
vacuum descendant $(S_-)^{N_3}\cdot Z^{L_3}$

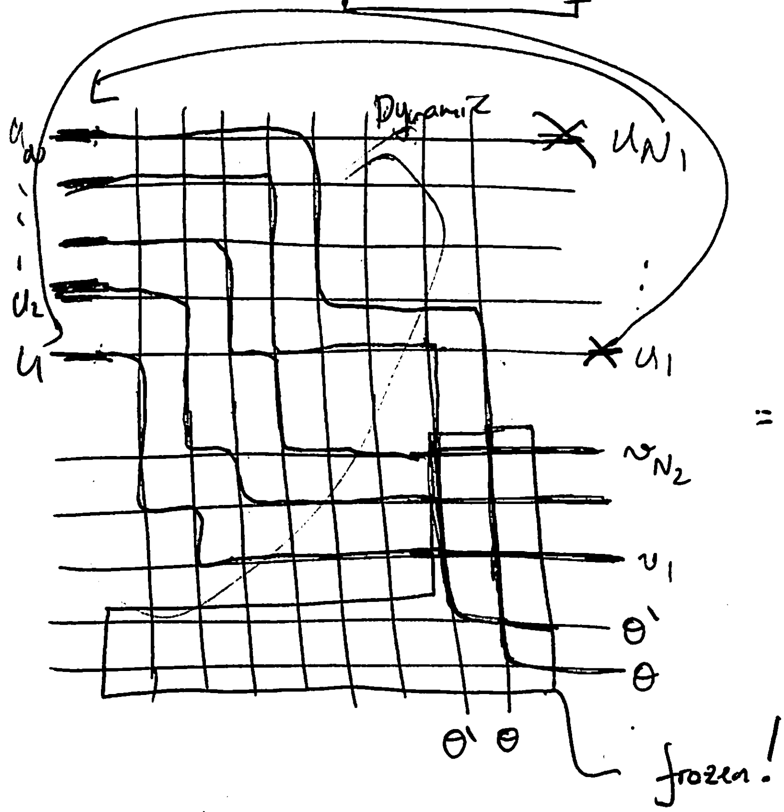
BPS state

in the planar limit only this guy matters

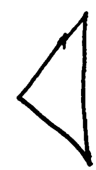
$$C_{123} = \frac{1}{\sqrt{\binom{L_3}{N_3}}}\;\frac{1}{\underbrace{\sqrt{\langle 1|1\rangle\langle 2|2\rangle}}_{\text{spin chain norms}}}\cdot$$



claim :  $= \langle u_1 \dots u_{N_1} | v_1 \dots v_{N_2} ; \theta_{L_2 - N_3 + 1} \cdots \theta_{L_2} \rangle$



= frozen stuff x what we want.



So [OMAR]

2 norms + 1 numerator

$$C_{123} = F(\text{three } \langle u_i | v_i \rangle)$$

like $|\uparrow \dots \uparrow\rangle = z \dots z$

$$|u_i\rangle = B(u_1) \dots B(u_N) \, |\text{vac}\rangle$$

$$= B^+(u_1) \dots B^+(u_N) \underbrace{(S_-)^{2N} \, |\text{vac}\rangle}_{\text{"charged vacuum"}}$$

So [Kostov, Matsuo]

$$C_{123} = C_{123}\Big[\, \langle \text{CHARGED VAC} \mid z_1 \dots z_M \rangle \,\Big] \qquad M = 2N$$

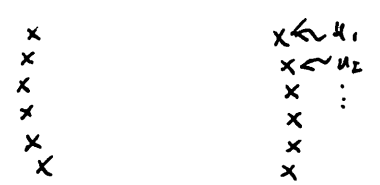
$$= \det_{1\le k,j\le M}\left[\, u_k^{\,j-1} + (u_k+i)^{j-1} \overbrace{\left(\frac{u_k+i/2}{u_k-i/2}\right)^{L}}^{V(u_k)\,=\,e^{ip_k L}} \,\right]$$

$$C_{1\text{loop}} = \left[1 + \frac{g^2}{2}\sum(\partial_{\theta_a}-\partial_{\theta_{a+1}})^2\right] C_{\text{tree}}(\{u\},\{\theta\}) \;!$$

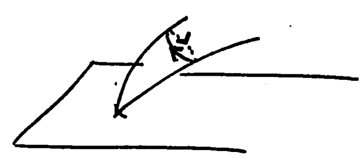
AdS/CFT

↑ loops + boundary + mixed terms cancel! @ the end $u+i/2 \to x(u+i/2)$ etc.

Weak coupling

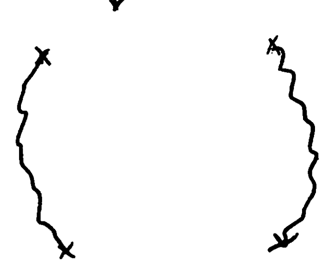


strong coupling



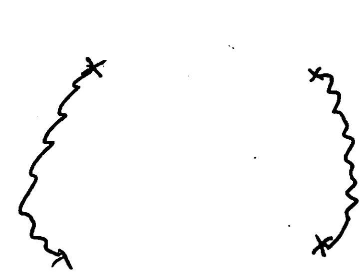
monodromy $= 2\cos p_{su(2)}(u)$

$\downarrow N \to \infty$



identified!
KMMZ

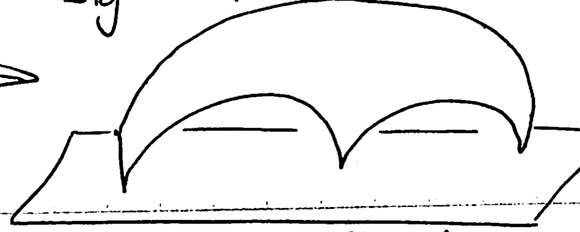
$p(u) =$



E.g.

@ weak ← us, I. Kostov, ...
& strong ← Kozcaz

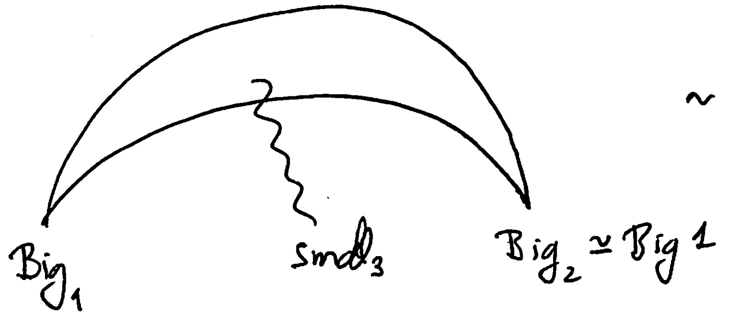
Big recent advances on



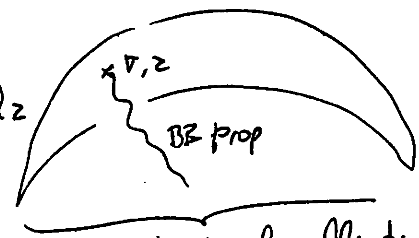
Cand 4pt.

but no total result yet!

better understood



$\sim \int d\sigma dz$



bunch of elliptic functions $sn(\ldots), dn(\ldots)$

<u>STRONG</u>

$$[\alpha + q(1-2\alpha)]/[3\alpha(1-\alpha)] + O\left(\frac{\lambda}{J^2}\right)$$

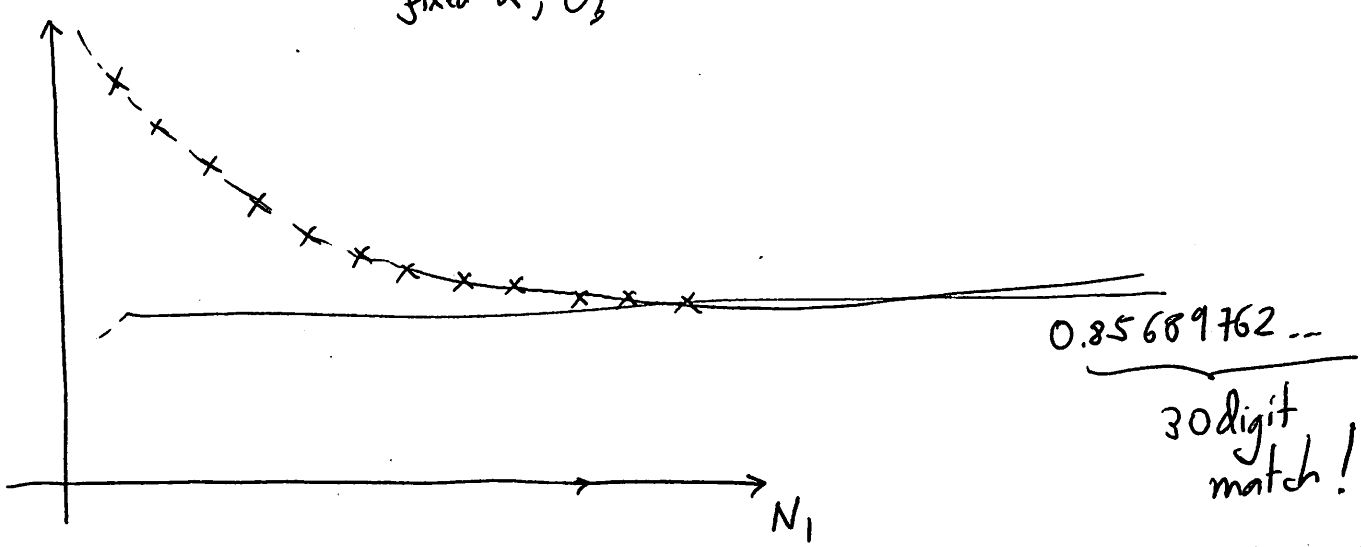
= folded string $N/L = 1/4 = \alpha$
small $O_3 = \mathrm{tr}\, \bar{z}^2 x^2$

$$\text{where} \quad \alpha = 1 - \mathbb{E}(q)/\mathbb{K}(q)$$

$$= 0.85689762\ldots + O\left(\frac{\lambda}{J^2}\right)$$

<u>weak coupling</u>

fixed α, O_3



No.

DATE

Next

- All loop guess à la spectrum, e.g. all loop BDS
- more sectors $\longrightarrow$ SL(2) (bootstrap, data available)
  $\longrightarrow$ SU(3) (nested)
- coherent states, more analytic limits, more contact with spectrum and strong.

weak coupling = AWESOME LAB.