Based on GMPTWY, '11
CMSY, '12.

Cast of Play.

3d CS-VM.

U(N) or SU(N) gauge group.

CS A_μ + □ matter field

boson ϕ or fermion ψ.

non-susic.

$$S = kCS(A) + \int |D_\mu\phi|^2 + \frac{f(\lambda)}{N^2}(|\phi|^2)^3$$

or $+\int \bar\psi \not{D} \psi$.

Now consider N_f flavors of □ scalars AND fermions.

- not (yet) supersymmetric.

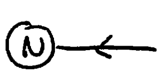
SUSY vector models obtained by dbl + triple trace deformations + possibly gauging flavor currents.

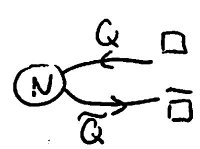
e.g. $\mathcal{N}=2$ CS-VM w/ □ chiral

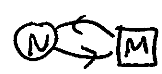
$$S = CS(A) + \int D\sigma + \bar\chi\chi + (\text{minimal coupling of } \mathcal{N}=2 \text{ chiral matter})$$

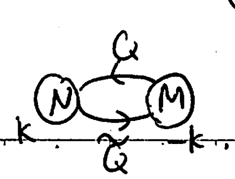
$\int$out $D, \sigma \Rightarrow \frac{1}{k}(\bar\phi\phi)(\bar\psi\psi)$, $\frac{1}{k^2}(\bar\phi\phi)^3$

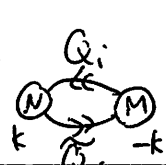
$\int$out $\bar\chi, \chi \Rightarrow \frac{1}{k}(\phi\bar\psi)(\bar\phi\psi)$



$\mathcal{N}=3$.  $+ W = \frac{1}{k}(Q\tilde{Q})^2$.



$\mathcal{N}=4$  $W=0$.

$\mathcal{N}=6$  $W = \frac{1}{k}Tr(Q_i\tilde{Q}_i Q_j\tilde{Q}_j) - \frac{1}{k}Tr(\tilde{Q}_i Q_i \tilde{Q}_j Q_j)$.

4d Vasiliev theory.

x^μ, Y, Z.

$\mathcal{A}(x|Y,Z)$, $B(x|Y,Z)$.

$$d\mathcal{A} + \mathcal{A} * \mathcal{A} = f_*(B*K)\,dz^2 + c.c.$$

$$dB + \mathcal{A}*B - B*\pi(\mathcal{A}) = 0.$$

$f(X) = Xe^{i\theta(X)}$, $\theta(X) = \theta_0 + \theta_2 X^2 + \dots$ real, even.

AdS$_4$ vacuum

$\mathcal{A} = W+S$, $W = W_0 = \omega_0 + e_0$

$S=0$, $B=0$.

Perturbation theory around AdS$_4$

$\Rightarrow$ interacting HS fields.

SUSY generalization. "n-extended Vasiliev" (a fake)

ψ_i, $i=1,\dots,n$. $\Gamma = i^{\frac{n(n-1)}{2}}\psi_1\cdots\psi_n$.

$\mathcal{A}(x|Y,Z,\psi)$, $B(x|Y,Z,\psi)$

$R = K\bar{K}\Gamma$, $[R,W] = \{R,S\} = [R,B] = 0 = [R,\epsilon]$

reality condition: $\iota(\mathcal{A})^* = -\mathcal{A}$,

$\iota(B)^* = \Gamma K*B*K = \bar{K}*B*\bar{K}\Gamma$.

ι: involution on $*$-algebra, reverses order.

$$d\mathcal{A} + \mathcal{A}*\mathcal{A} = f_*(B*K)\,dz^2 + \bar{f}_*(B*\bar{K}\Gamma)\,d\bar{z}^2.$$

- Most susies & all HS sym will be broken by bdry condition for generic θ_0.

Spectrum and correlators.

VM.

single trace operators.

$\bar{\phi}_i \phi_i$, $\quad \bar{\phi}_i \partial^s \phi_i$

$\bar{\psi}_i \psi_i$, $\quad \bar{\psi}_0 \gamma_\mu \psi$, $\quad \bar{\psi} \gamma_{(\mu_1} \partial_{\mu_2} \cdots \partial_{\mu_s)} \psi \cdots$

- $\Delta = s + 1 + \mathcal{O}(\frac{1}{N})$ @ FINITE λ.

correlators.

Consider CS-scalar-VM as example.

$\tilde{N} \sim N$.

$\langle J_s J_{s'} \rangle = \tilde{N} \delta_{ss'}$

$$\langle J_{s_1} J_{s_2} J_{s_3} \rangle = \tilde{N} \left[\frac{1}{1+\tilde{\lambda}^2} (FB) + \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} (FF) + \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} (odd) \right] + \mathcal{O}(N^0)$$

for $s_i \neq 0$.

$\tilde{N}, \tilde{\lambda}$ related to N, λ.

$$\langle J_0 J_{s_1} J_{s_2} \rangle = \tilde{N} \left[\frac{1}{\sqrt{1+\tilde{\lambda}^2}} (FB) + \frac{\tilde{\lambda}}{\sqrt{1+\tilde{\lambda}^2}} (odd) \right]$$

HS sym breaking:

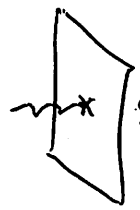
$$\partial \cdot J = \frac{f(\lambda)}{N} \Sigma JJ + \frac{g(\lambda)}{N^2} \Sigma JJJ.$$

HS GT in AdS_4.

minimal bosonic. $\quad s = 0, 2, 4, \ldots$

non-min $\quad s = 0, 1, 2, 3, \ldots$

SUSY. include half-integer spins.



given boundary source $J_{i_1 \cdots i_s}$

solve bulk spin-s field $\varphi_{\mu_1 \cdots \mu_s}$.

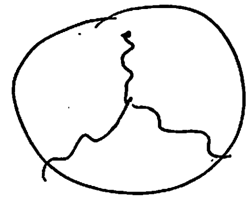
$\varphi_{i_1 \cdots i_s} \to z^{s+1} \delta(\vec{x} - \vec{x}_0) \varepsilon_{i_1 \cdots i_s}$

translate into A_μ, B master fields

Then solve $\tilde{D}_0 B^{(2)} = -A^{(1)} * B^{(1)} + B^{(1)} * \pi(A^{(1)})$

extract 2^{nd} order φ from Weyl tensor $\; B^{(2)}(x|Y; z=0)$

bdy value $\propto$ 3-pt fn.



$\langle J_0 J_{s_1} J_{s_2} \rangle$

$= \tilde{N} \left(\cos\theta_0 \, (FB) + \sin\theta_0 \, (odd) \right).$

$$\boxed{\tilde{\lambda} = \tan\theta_0}$$

- structure of master fields.

$A = W + S$, $\quad W = W_0 + \hat{W}$, $\quad$ set $\Omega^{(x|Y)} = \hat{W}\big|_{Z=0}$, $\; C = B\big|_{Z=0}$.

$$\begin{array}{ccccccccc}
\Omega^{(0,2s-2)} \cdots & \leftarrow & \Omega^{(s-2,s)} & \leftarrow & \Omega^{(s-1,s-1)} & \rightarrow & \Omega^{(s,s-2)} & \rightarrow \cdots & \Omega^{(2s-2,0)} \\
\downarrow & & & & \| & & & & \downarrow \\
C^{(0,2s)} & & & & \text{metric-like} & & & & C^{(2s,0)} \\
\downarrow & & & & \text{spin-s field} & & & & \downarrow \\
C^{(1,2s+1)} & & & & & & & & C^{(2s+1,1)} \\
\downarrow & & & & & & & & \downarrow \\
\vdots & & & & & & & & \vdots
\end{array}$$

spin-s Weyl tensor

No.

Date

Symmetry & breaking.

symmetries of AdS_4 sol'n: $\quad \epsilon(x|Y,z,\psi)$ that obey.

$$d_z \epsilon = 0 \Rightarrow \epsilon = \epsilon(x|Y,\psi).$$

$$D_0 \epsilon(x|Y,\psi) = 0, \qquad W_0 = L^{-1} * dL.$$

$$\Rightarrow \epsilon = L^{-1} * \epsilon_0(Y,\psi) * L.$$

e.g. SUSY generators.

$$\epsilon(x|Y,\psi) = \Lambda^\alpha(x|\psi)\, y_\alpha + \bar\Lambda^{\dot\alpha}(x|\psi)\, \bar y_{\dot\alpha}$$

$$\Lambda(x|\psi) = z^{-\frac{1}{2}}\big(\underset{\substack{\uparrow \\ Q}}{\Lambda_0(\psi)} + \vec{x}\cdot\vec{\sigma}\,\sigma^z \Lambda_-(\psi)\big) + z^{\frac{1}{2}} \Lambda_-(\psi).$$

S

$$\delta_\epsilon B = \underset{\substack{\uparrow \\ \text{mixes fields of different spins.}}}{-\epsilon * B} + B * \pi(\epsilon).$$

consider spin-s fields sourced by $J^{(s)}$ in AdS_4,

perform spin-s' ~~gauge tran~~ global sym transf.

$$\delta_\epsilon \underset{\substack{\uparrow \\ \text{scalar}}}{B^{(0)}} \neq 0. \quad \cancel{= a z + b z^2 + \cdots}$$

$$= \cos\theta_0 (\cdots) z + \sin\theta_0 (\cdots) z^2 + \cdots$$

preserved by: $\Delta = 1$ b.c. if $\theta_0 = 0$.

$\Delta = 2$ b.c. if $\theta_0 = \frac{\pi}{2}$.

broken ~~for~~ either b.c. if θ_0 generic.

general $\mathcal{N}$-extended Vasiliev
$U(2^{\frac{n}{2}-1})\times U(2^{\frac{n}{2}-1})$
bosonic spin-1 gauge fields, ...

$\mathcal{N}=3$ b.c. for $\mathcal{N}=4$ Vasiliev theory.

$\psi_1, \dots, \psi_4$.

SUSY generator.

$$\epsilon \leftrightarrow \Lambda_0(\psi) = \eta\psi_1,\ \eta\psi_2,\ \eta\psi_3.$$

out of $2^3=8$ fermionic sym.
(monomials in ψ_i)

b.c.

scalar.

$$B^{(0)}(\vec{x},z|\psi) = (e^{i\gamma}+\Gamma e^{-i\gamma})\tilde{f}_1(\psi)z + (e^{i\gamma}-\Gamma e^{-i\gamma})\tilde{f}_2(\psi)z^2 + O(z^3)$$

fermion

$$B^{(\frac{1}{2})}(\vec{x},z|Y,\psi) = z^{\frac{3}{2}}\left[e^{i\alpha}(\chi y) - \Gamma e^{-i\alpha}(\bar{\chi}\bar{y})\right] + \dots$$

$$\chi = \sigma^z\bar{\chi}.$$

spin-1 gauge field

$$B^{(1)}(\vec{x},z|Y,\psi) = z^2\left[e^{i\beta}(yFy) + \Gamma e^{-i\beta}(\bar{y}\bar{F}\bar{y})\right] + \dots$$

$$F = -\sigma^z\bar{F}\sigma^z.$$

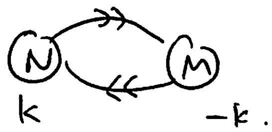
Characterized by: γ, α, β. — linear operators on fn of ψ.

$\mathcal{N}=3$ b.c. — $\beta=\theta_0$, $\alpha=\theta_0(1-P_{\psi_1\psi_2\psi_3}) - \theta_0 P_{\psi_1\psi_2\psi_3}$, $\gamma=\theta_0$.

$P_{\psi_i\psi_4}\tilde{f}_{1,2} = \tilde{f}_{1,2}$. $i=1,2,3$

vs $\mathcal{N}=4$ b.c. $\beta=\theta_0(1-P_\Gamma)$, $\alpha=\theta_0 P_{\psi_i}$, $\gamma=\theta_0 P_1$, $P_\Gamma\tilde{f}_{1,2}=0$.

ABJ triality



$\longleftrightarrow$ IIA on $AdS_4 \times \mathbb{CP}^3$ w/ $\int_{\mathbb{CP}^1} B = \frac{N-M}{k} + \frac{1}{2}$.

$\updownarrow$

$\mathcal{N}=6$ Vasiliev theory. $U(M)$ CP factor

w/ $\theta_0 = \frac{\pi}{2}\lambda$.

and b.c.

$$\beta = \theta_0 (1 - P_\Gamma) - \theta_0 P_\Gamma ,$$

$$\alpha = \theta_0 (1 - P_{\psi_i \Gamma}) - \theta_0 P_{\psi_i \Gamma}, \quad i = 1, \dots, 6.$$

$$\gamma = \theta_0 P_{1, \psi_i \psi_j}$$

$$P_{\Gamma, \psi_i \psi_j \Gamma} \widetilde{f}_{1,2} = 0 .$$

- preserve global SUSY gen. by

$$\Lambda_0 = \eta \psi_i, \quad i = 1, \dots, 6 .$$

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Date

Comments

- HS/VM duality, as we understand it, involves only local gauge invariant operators on the CFT side.

  Putting VM on 3-manifolds of nontrivial topology requires extension of Vasiliev's system

  (Also clear in AdS_3/CFT_2 HS - W_N-min model duality)

- Holography is not mysterious, bulk locality is.

- Embedding into string theory?

  2 constructions.

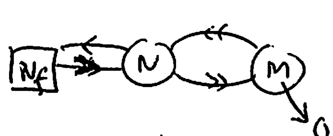
  - ABJ $\quad U(N)_k \times U(M)_{-k} \longrightarrow$ IIA on $AdS_4 \times \mathbb{CP}^3$.

    $\big\downarrow N \gg M.$ $\quad$?

    $\mathcal{N}=6$ $U(M)$ - Vasiliev w/ $\mathcal{N}=6$ b.c.

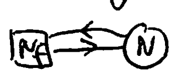
    single string $\longleftrightarrow$ multi- HS-particles.

  - Gaiotto - Jafferis



$\longrightarrow$ IIA on $AdS_4 \times \mathbb{CP}^3$ w/ D6 on $AdS_4 \times \mathbb{RP}^3$

$\Downarrow$



$\mathcal{N}=3$ CS-VM

$\downarrow$

$\mathcal{N}=4$ $U(N_f)$ - Vasiliev w/ $\mathcal{N}=3$ b.c.

OSFT // Vasiliev ?!