A TALE OF TWO CFT's

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Based on : S. Giusto and R. R 1311.5536

S. Giusto and R. R 1211.1957

S Giusto, R. R, D. Turton 1108.6331

(see also 1007.2856 with W. Black

& 0912.2270 with F. Morales)

Exotic Structures of SPACETIME

YITP 20/3/14

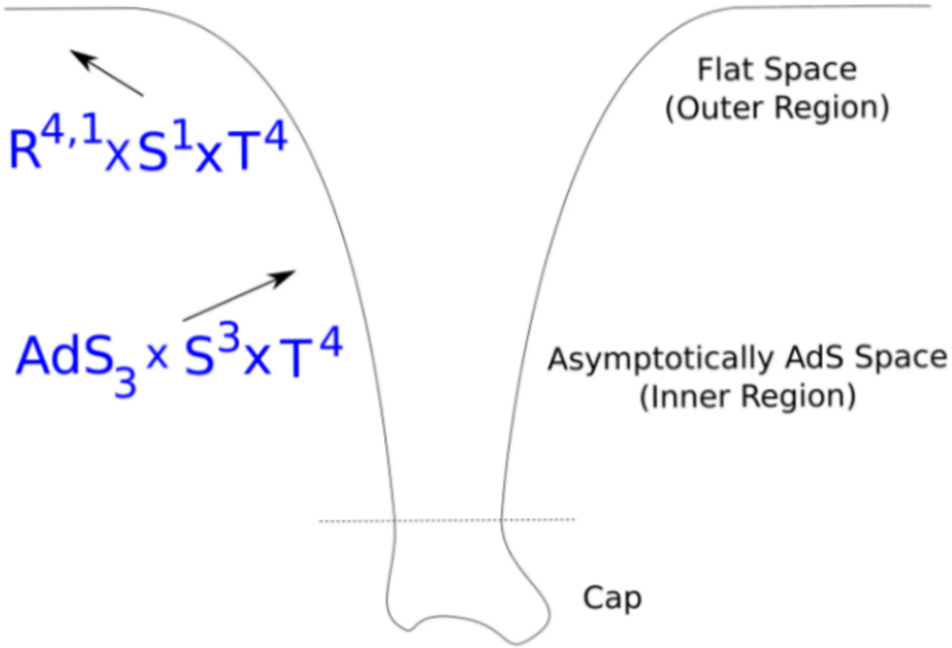
Introduction: the D1-D5-P system

- Paradigmatic example of BPS black hole in string theory (Strominger, Vafa)

Type II B

$$
\begin{array}{ccccccccc}
 & \mathbb{R} & \times & \mathbb{R}^4 & \times & S^1 & \times & T^4 \\
 & t & & x^i & & y & & z^a \\
D1: & - & & \circ & & - & & \circ \\
D5: & - & & \circ & & - & & - \\
P: & - & & \circ & & - & & \circ
\end{array}
$$

QUALITATIVE STRUCTURE of THE MICROSTATE GEOMETRIES



$\longrightarrow$ WORLD-SHEET CFT describing fundamental open/closed strings

$\longrightarrow$ DUAL (in the AdS/CFT sense) CFT describing type IIB in the decoupling limit

GENERAL ANSATZ for the 10D METRIC

$$ds^2 = -\frac{2\alpha}{\sqrt{Z_1 Z_2}}(dv+\beta)\left[du+\omega+\frac{\mathcal{F}}{2}(dv+\beta)\right] + \sqrt{Z_1 Z_2}\, ds_4^2 + \sqrt{\frac{Z_1}{Z_2}}\, ds_{T^4}^2$$

$\beta + \omega \rightarrow$ Angular momentum in R^4

$\beta - \omega \rightarrow$ K-K monopole dipole charge

$$v = \frac{t+y}{\sqrt{2}} \quad , \quad u = \frac{t-y}{\sqrt{2}}$$

Let us focus on the 2-charge geometries

- The (T^4-isometric) geometries are encoded in a profile in $\mathbb{R}^5$ $(F^i(v'), \mathcal{F}(v'))$. For instance:

$$Z_1 = 1 + \frac{Q_2}{L}\int_0^L \frac{dv'\,|\dot{\mathbf{F}}|^2}{|x - F|^2}$$

$\longrightarrow \sum_i (F^i)^2 + \mathcal{F}^2$

$\longrightarrow \sum (x^i - F^i)^2$

$$\Rightarrow \quad Q_1 = \frac{R}{2\pi}\int_0^L |\dot{\mathbf{F}}|^2 dv'$$

$\rightarrow$ A family of 2-charge geometries is

- Start from the D1-D5 geometry associated with the profile

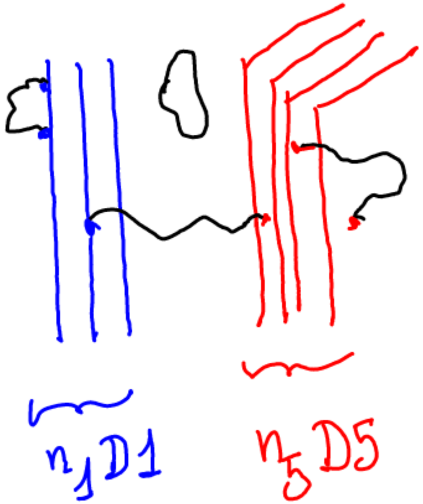
$$F^1 + iF^2 = a\, e^{\frac{2\pi i\, v'}{L}}\ ,\ \ F^3 = F^4 = 0\ ,\ \ \mathcal{F} = -b\sin\left(\frac{2\pi\, v'}{L}\right)$$

- A class of 3-charge geometries, in the near-horizon (AdS) limit can be generated by a change of variables ($\neq 0$ at the boundary)

World Sheet CFT

- How can we get information about the asymptotically flat part?

$\rightarrow$ Consider the microscopic description



- We have a fundamental CFT description (Polchinski)
- Three types of open string states ($\rightarrow$ D-branes)
- closed strings (gravitational sector)

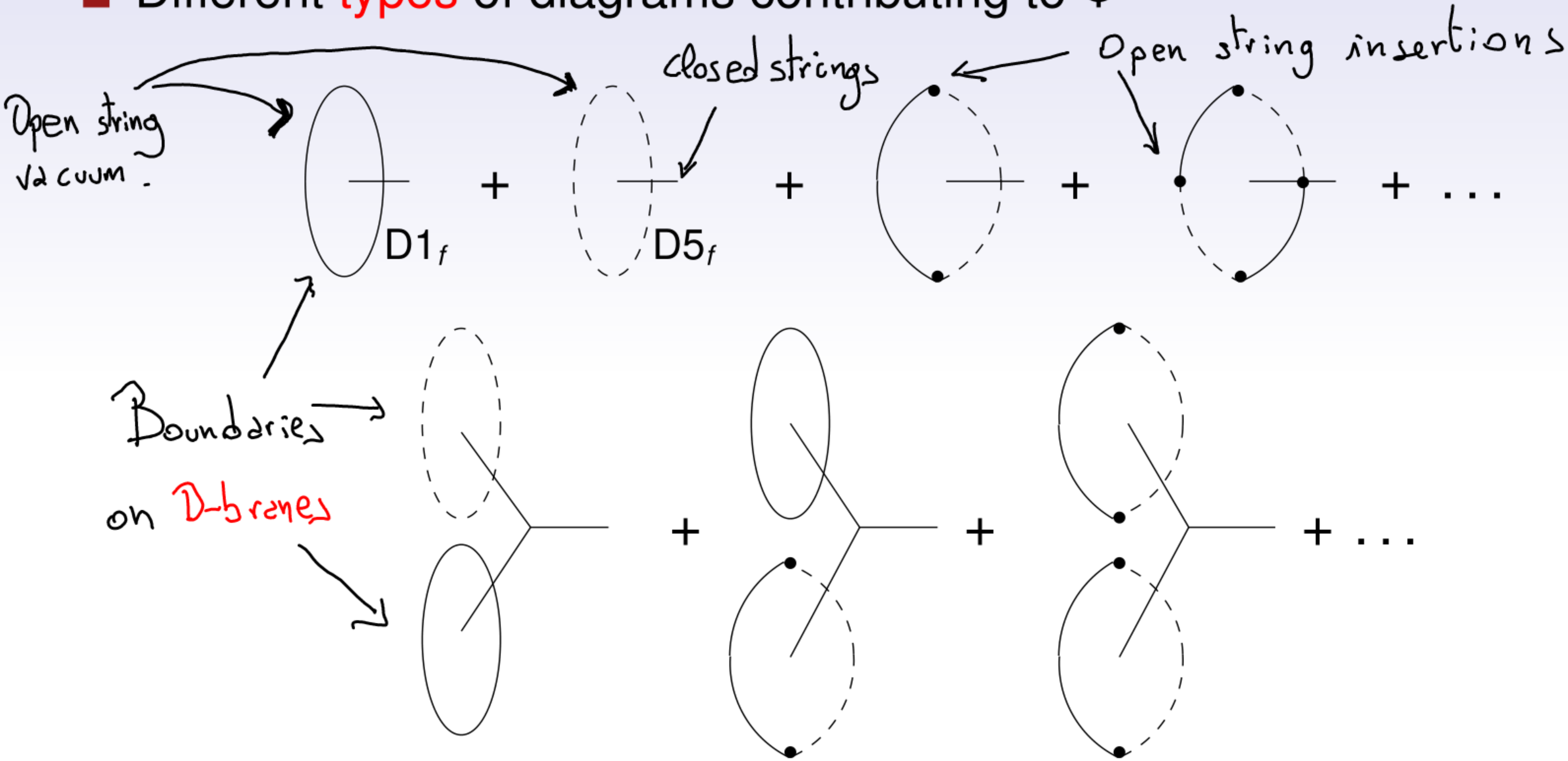
$$X^\mu(z,\bar{z}) \;,\; \psi^\mu(z) \;,\; \tilde{\psi}^\mu(\bar{z}) \;,\; [\, b, c, \beta, \gamma \;\&\; \tilde{b}, \dots \,]$$

$\rightarrow$ RNS formalism

- We can write all the closed and open states. When $g_s \neq 0$ these states interact and this describes the D-brane's backreaction

Diagrammatic interpretation

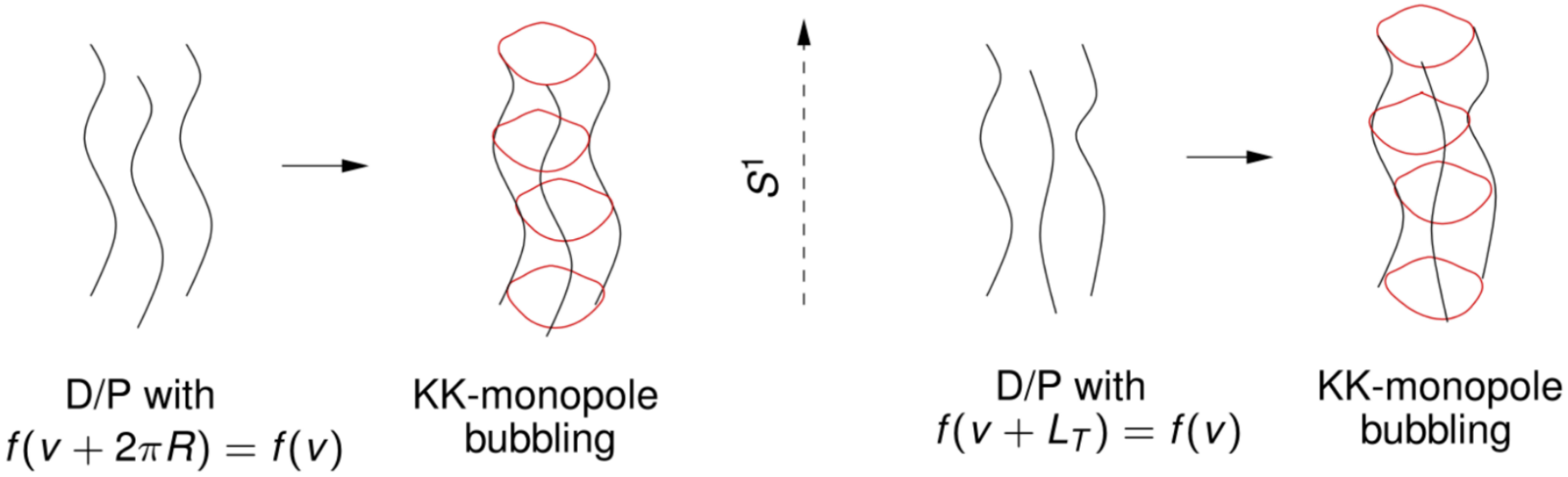
- Different types of diagrams contributing to Φ



- So far we used the information from the first three diagrams only

- We can treat the open strings that give rise to the D1 or D5 dipole charges EXACTLY
- The KK-monopole dipole charges can be described only perturbatively
- It's the natural setup to carry out the proposal sketched below

From 1203.1348 (Niehoff, Vasilakis and Warner)



- Notice that we are switching on dipoles in the opposite order than before!

String results: an example

$$\langle W_\Phi \rangle_{NS} \sim \; e^{-ik_i f^i(v)}\, k^l \Big[(\mathcal{G}^{uj} + \mathcal{G}^{ju})\, v_{ujl} + (\mathcal{G}^{vj} + \mathcal{G}^{jv})\, v_{vjl} - 2\,\mathcal{G}^{vv}\, \dot{f}^j\, v_{vjl}$$

$$- 2\,\eta^{uv}\,\mathcal{G}^{jv}\,|\dot{f}|^2\, v_{ujl} - 2\,\mathcal{G}^{uv}\,\dot{f}^j\, v_{ujl} + 2\,\eta^{uv}\,\mathcal{G}^{ij}\,\dot{f}^j\, v_{uil}\Big] \qquad (*)$$

$$+ \eta^{uv}\, k_v \Big[-2\,\mathcal{G}^{jv}\,\dot{f}^i\, v_{uji} + \mathcal{G}^{ij}\, v_{uij} \Big]$$

- Even when $k_v \to 0$, this solution has interesting new features
  - The B-field is non-trivial, so (*) is not part of the 5D STU model
  - From the complete list of $\langle W_\Phi \rangle_D$, we derived a generalization of the STU ansatz, which can be embedded in a $\mathcal{N} = 2$ 5D sugra with scalar manifold $SO(1,1) \otimes (SO(1,2)/SO(2))$
  - In the v-independent case, we have the full non-linear sugra equations and some explicit examples of solutions are known (a black ring, or the bubbling in 1202.1819 Vasilakis)
  - The solution is build on a 4D hyperkhaler base metric

- Open data: $f^\alpha(v)$ (D-brane profile) ; $\upsilon_{vij} = -\frac{1}{2}\epsilon_{ijkl}\,\upsilon_{vkl}$ $(F^i, \mathcal{F})$
- $\beta(x^i) \sim \upsilon_{vli}\,\frac{x^l}{r^4}\,dx^i$, which satisfies $d\beta = *_4 d\beta$ with respect to δ_{ij}

$$
\bar{\beta}^\alpha = \beta(x - f^\alpha(v)) \ , \quad \hat{\beta} = \sum_\alpha \bar{\beta}^\alpha . \tag{6}
$$

Similarly for the metric in the R^4 we have

δ_{ij}

$$
\hat{ds}_4^{\,2} \equiv ds_4^2 + \sum_\alpha (\bar{\beta}^\alpha_i\,\dot{f}^\alpha_j + \bar{\beta}^\alpha_j\,\dot{f}^\alpha_i - g^{(0)}_{ij}\,\bar{\beta}^\alpha_k\,\dot{f}^\alpha_k)\,dx^i dx^j \, . \tag{7}
$$

If we indicate with $J_0^{(A)}$ the complex structures of the flat R^4 then we have

$$
\begin{aligned}
\hat{J}^{(A)} &\equiv J_0^{(A)} + J_1^{(A)} \\
&= J_0^{(A)} - \sum_\alpha \frac{1}{2}\left[\dot{f}^{i\alpha}\left(\bar{\beta}^\alpha \wedge J_0^{(A)}\right)_{ijk} dx^j \wedge dx^k\right] \\
&= J_0^{(A)} - \sum_\alpha \left[\bar{\beta}^\alpha_i \dot{f}^{i\alpha} J_0^{(A)} - \bar{\beta}^\alpha \wedge J^{(A)}_{0\,ij}\,\dot{f}^{j\alpha} dx^i\right] \, ,
\end{aligned} \tag{8}
$$

Eqs. (6) and (8) solve the base equations at linear order in β ($D \equiv d - \beta \wedge \partial_v$)

$$
D\hat{\beta} = \star_4 D\hat{\beta} \, , \quad \hat{J}^{(A)} \wedge \hat{J}^{(B)} = -2\delta^{AB} \star_4 1 \, , \quad d\hat{J}^{(A)} = \partial_v(\hat{\beta} \wedge \hat{J}^{(A)}) \, . \tag{9}
$$

Conclusions

- CFT techniques provide a powerful tool to extract geometric data
- A generic supergravity ansatz leads to singular solutions. The CFT data guide us towards the "right" gravity configuration
- The WCFT approach can be used also in a 4D case
  (→ Connection to "non-geometric" background, Masaki 1209.6056)
- Extension to non-BPS, but extremal configurations
- Precision holography (match gravity and the dual CFT beyond the level of charges)
- Dynamical processes (scattering)

work in collaboration G. D'Appollonio, P. Di Vecchia, G. Veneziano