Nucleon spectral function in nuclear medium from QCD sum rules

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Outline

• Introduction
• Nucleon QCD sum rules
• Nucleon QCD sum rules in nuclear medium
• Conclusion
Mass spectrum of the nucleons

Positive parity
- N(1440)
- N(1535)
- N(1650)

Negative parity
- N(1650)
- N(1535)

• It is predicted that Chiral symmetry breaking cause these difference.

• The mass difference between nucleon ground state and N(1535) is about 600 MeV.

When chiral symmetry is restored, the mass spectrum will change.

To investigate these properties from QCD, non perturbative method is needed.

Analysis of QCD sum rule in nuclear medium
Nucleon QCD sum rules

\[ \Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\bar{\eta}(0)] | 0 \rangle d^4x \]

\( \eta(x) \) : interpolating field, which has the same quantum number as the hadron of interest.
Nucleon QCD sum rules

\[ \Pi(q) \equiv i \int e^{iqx} \langle 0 | T[\eta(x)\bar{\eta}(0)] | 0 \rangle d^4x \]

\[ = \int_0^\infty \frac{1}{\pi} \frac{\text{Im} \Pi(t)}{t-q^2} dt = \int_0^\infty \frac{\rho(t)}{t-q^2} dt \]

is calculated by the operator product expansion (OPE)

\[ \Pi(q) = q \Pi_1(q^2) + \Pi_2(q^2) \]

\[ = q \sum_i C_i(q^2) \langle 0 | O_i | 0 \rangle + \sum_j C'_j(q^2) \langle 0 | O'_j | 0 \rangle \]

\[ = q \left( C_0(q^2) + C_4(q^2) \left( \frac{\alpha_s}{\pi} G^2 \right) + C_6(q^2) \langle \bar{q}q \rangle^2 + \cdots \right) \]

\[ + C_3(q^2) \langle \bar{q}q \rangle + C_5(q^2) \langle \bar{q}g \sigma \cdot G q \rangle + \cdots \]

\[ \Pi(q) \text{ calculated by OPE is related to the hadronic spectral function} \]

\[ C_i(q^2) : \text{Coefficient} \quad \langle 0 | O_i | 0 \rangle : \text{Condensate} \]

Chiral condensate
Nucleon QCD sum rules

\[ \Pi_{OPE}(q^2) = \int_0^\infty \frac{\rho(t)}{t - q^2} dt \]

\[ G_{OPE}(x) = \int_0^\infty K(x, \omega) \rho(\omega) d\omega \]

Borel sum rule: \[ K(M_B, \omega) = \exp\left(-\frac{\omega^2}{M_B^2}\right) \]

Parameter: \( M_B \) (Borel mass)

Gaussian sum rule: \[ K(S, \tau, \omega) = \frac{1}{\sqrt{4\pi \tau}} \exp\left(-\frac{(\omega^2 - s)^2}{4\tau}\right) \]

Parameter: \( S, \tau \)

Parity projection

\[ G_{\pm}(x) = \left[ C_0(x) + C_4(x)\frac{\alpha_s}{\pi} G^2 + C_6(x)\langle qq\rangle^2 + \cdots \right] \]

\[ \pm \left[ C_3(x)\langle qq\rangle + C_5(x)\langle qg\sigma \cdot Gq \rangle + \cdots \right] \]

Nucleon QCD sum rules


\[ G_{OPE}(s, \tau) = \int_0^\infty K(s, \tau, \omega) \rho(\omega) \, d\omega \]

\[ K(s, \tau, \omega) d\omega = \frac{1}{\sqrt{4\pi \tau}} \omega e^{-\frac{(\omega^2 - s)^2}{4\tau}} d\omega \]

\[ \tau = 1.5 \text{GeV}^4 \]

The contribution of the chiral condensate term is dominant.

Perturbative term is also dominant.
Nucleon QCD sum rules

\[ G_{OPE}(s, \tau) = \int_0^\infty K(s, \tau, \omega) \rho(\omega) \, d\omega \]

\[ K(s, \tau, \omega) \, d\omega = \frac{1}{\sqrt{4\pi \tau}} \omega \, e^{-\frac{(\omega^2 - s)^2}{4\tau}} \, d\omega \]

\[ \tau = 1.5 \text{GeV}^4 \]

The contribution of the chiral condensate term is dominant.
In both positive and negative parity, the peaks are found. In the negative parity analysis, the peak correspond to the $N(1535)$ or (and) $N(1650)$. 
Nucleon QCD sum rules in the nuclear matter

\[ \Pi(q) = i \int d^4 x e^{i q x} \langle 0 | T[\eta(x)\overline{\eta}(0)] | 0 \rangle \]

Vacuum

\[ \Pi(q) = i \int d^4 x e^{i q x} \langle \Psi_0 | T[\eta(x)\overline{\eta}(0)] | \Psi_0 \rangle \]

Nuclear medium

Application of this analysis to the spectral function in nuclear matter

Condensate: \[ \langle 0 | O_i | 0 \rangle \]

For example, \[ \langle 0 | \Psi_0 | O_i | \Psi_0 \rangle \]

\[ \Psi_0 \]: Nuclear matter ground state

Chiral condensate: \[ \langle \overline{q} q \rangle_0 \]

\[ \langle \overline{q} q \rangle_{\rho_N} = \langle \overline{q} q \rangle_0 + \frac{\sigma_N}{2m_q} \rho_N + \cdots \]

\[ \langle q^\dagger q \rangle_{\rho_N} \]
Nucleon QCD sum rules in the nuclear matter

Positive parity OPE data: \( + C_1 \langle \overline{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N} \)

Negative parity OPE data: \( - C_1 \langle \overline{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N} \)

n_0: nuclear matter density
Nucleon QCD sum rules in the nuclear matter

Positive parity OPE data: \[ C_1 \langle \bar{q} q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N} \]

Negative parity OPE data: \[ - C_1 \langle \bar{q} q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N} \]

n_0: nuclear matter density

Positive parity OPE data
- \langle \bar{q} q \rangle_{\rho_N}
- \langle q^\dagger q \rangle_{\rho_N}

Negative parity OPE data
- \langle \bar{q} q \rangle_{\rho_N}
- \langle q^\dagger q \rangle_{\rho_N}

Vacuum
Nucleon QCD sum rules in the nuclear matter

\[ n = n_0 \]
\[ n = 0.75n_0 \]
\[ n = 0.5n_0 \]

Positive parity OPE data:
\[ + C_1 \langle \bar{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N} \]

Negative parity OPE data:
\[ - C_1 \langle \bar{q}q \rangle_{\rho_N} + C_2 \langle q^\dagger q \rangle_{\rho_N} \]

The peak position (\( E = \sqrt{q^2 + M^2 + \Sigma^v} \)) is hardly sifted.
Nucleon QCD sum rules in the nuclear matter

Investigation of $M_{0\pm}^*$ and $\Sigma_{0\pm}^v$

$$\Pi(q) = \frac{i}{d^4 x e^{iqx}} \langle \Psi_0 | T[\eta(x)\bar{\eta}(0)] | \Psi_0 \rangle$$

$$= q\Pi_1(q^2, q \cdot u) + \Pi_2(q^2, q \cdot u) + \gamma \Pi_u(q^2, q \cdot u)$$

$$= \sum_n \left[ \lambda_{n+}^2 \frac{q + M_{n+}^* - \gamma \Sigma_{n+}^v}{(q_0 - E_{n+} + i\epsilon)(q_0 + \overline{E}_{n+} - i\epsilon)} + \lambda_{n-}^2 \frac{q - M_{n-}^* - \gamma \Sigma_{n-}^v}{(q_0 - E_{n-} + i\epsilon)(q_0 + \overline{E}_{n-} - i\epsilon)} \right]$$

$$E = \sqrt{q^2 + M_{n+}^* + \Sigma_{n+}^v}$$

$q_0 \Pi_1 \quad \sum |\lambda_+|^2 \frac{E^+}{2\sqrt{M_+^* + q^2}} \frac{1}{q_0 - E^+ + i\epsilon}$

$\Pi_u \quad \sum |\lambda_+|^2 \frac{-\Sigma_+^v}{2\sqrt{M_+^* + q^2}} \frac{1}{q_0 - E^+ + i\epsilon}$

$q_0 \Pi_1 \quad \sum |\lambda_-|^2 \frac{E^-}{2\sqrt{M_-^* + q^2}} \frac{1}{q_0 - E^- + i\epsilon}$

$\Pi_u \quad \sum |\lambda_-|^2 \frac{-\Sigma_-^v}{2\sqrt{M_-^* + q^2}} \frac{1}{q_0 - E^- + i\epsilon}$

After fitting OPE side and Phenomenological side, $M_{0\pm}^*$ and $\Sigma_{0\pm}^v$ can be obtained.
Nucleon QCD sum rules in the nuclear matter

\[ n_0: \text{nuclear matter density} \]

<table>
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<th></th>
<th>Vacuum</th>
<th>( n=0.25n_0 )</th>
<th>( n=0.5n_0 )</th>
<th>( n=0.75n_0 )</th>
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<td>( M_{0^+}^* )</td>
<td>930</td>
<td>850</td>
<td>710</td>
<td>470</td>
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<td>270</td>
<td>500</td>
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<td>( M_{0^-}^* )</td>
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<td>1630</td>
<td>1650</td>
<td>1680</td>
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<tr>
<td>( \Sigma_{0^-}^v )</td>
<td>0</td>
<td>0</td>
<td>-20</td>
<td>-50</td>
</tr>
</tbody>
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Positive parity

Negative parity
Conclusion

- We analyze the nucleon spectral function by using QCD sum rules with MEM.
- We construct the parity projected sum rule using phase-rotated Gaussian kernel with $\alpha_s$ correction.
- It is found that, in this sum rule, chiral condensate term is dominant and continuum contributions is reduced.
- The information of not only the ground state but also the negative parity excited state is extracted.
- We investigate the effective masses and the vector self-energies in the nuclear medium.