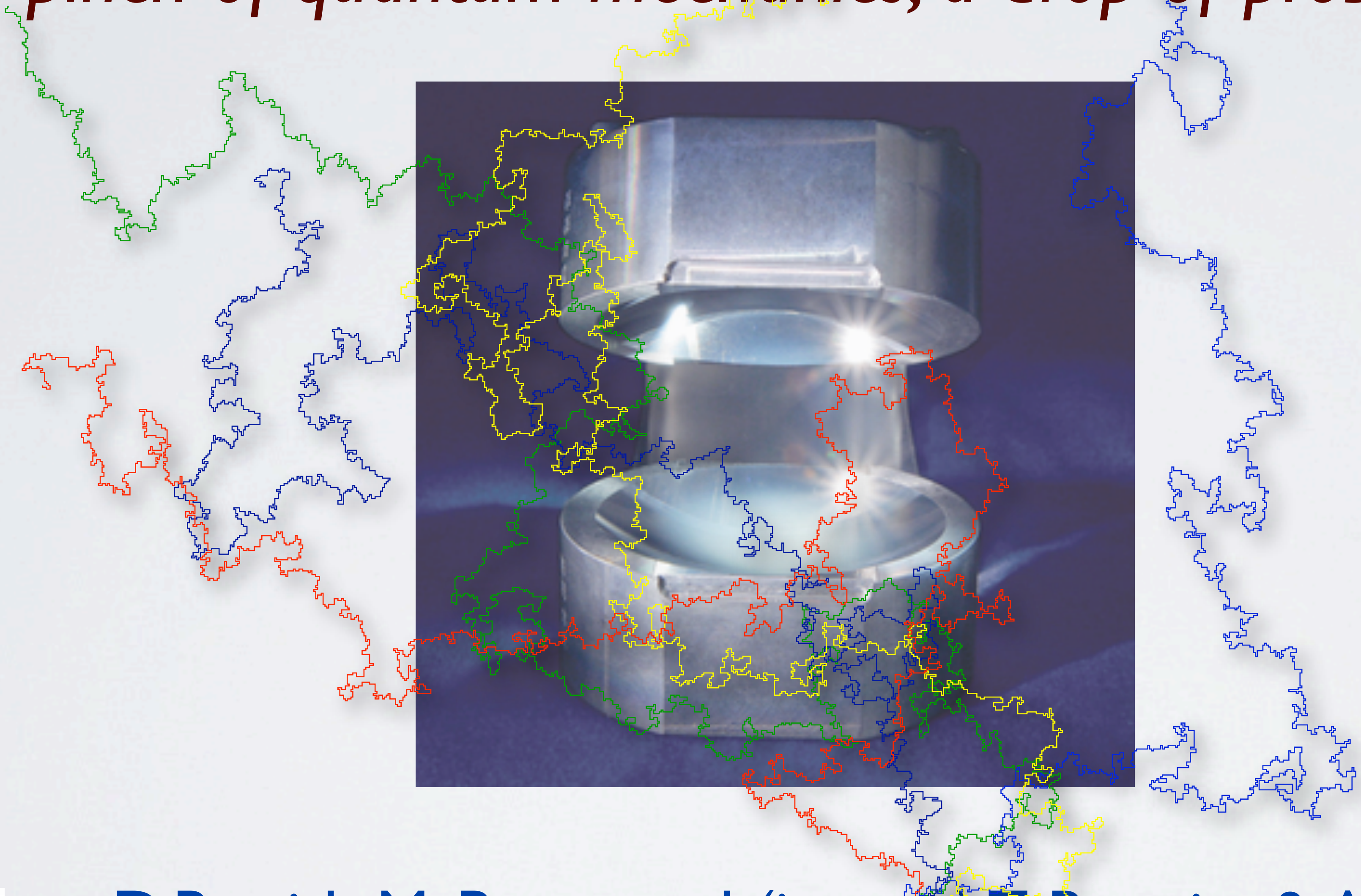
Real Time Imaging of Quantum and Thermal Fluctuations

(A pinch of quantum mechanics, a drop of probability, ...)



D.B. with M. Bauer, and (in part) T. Benoist & A. Tilloy.

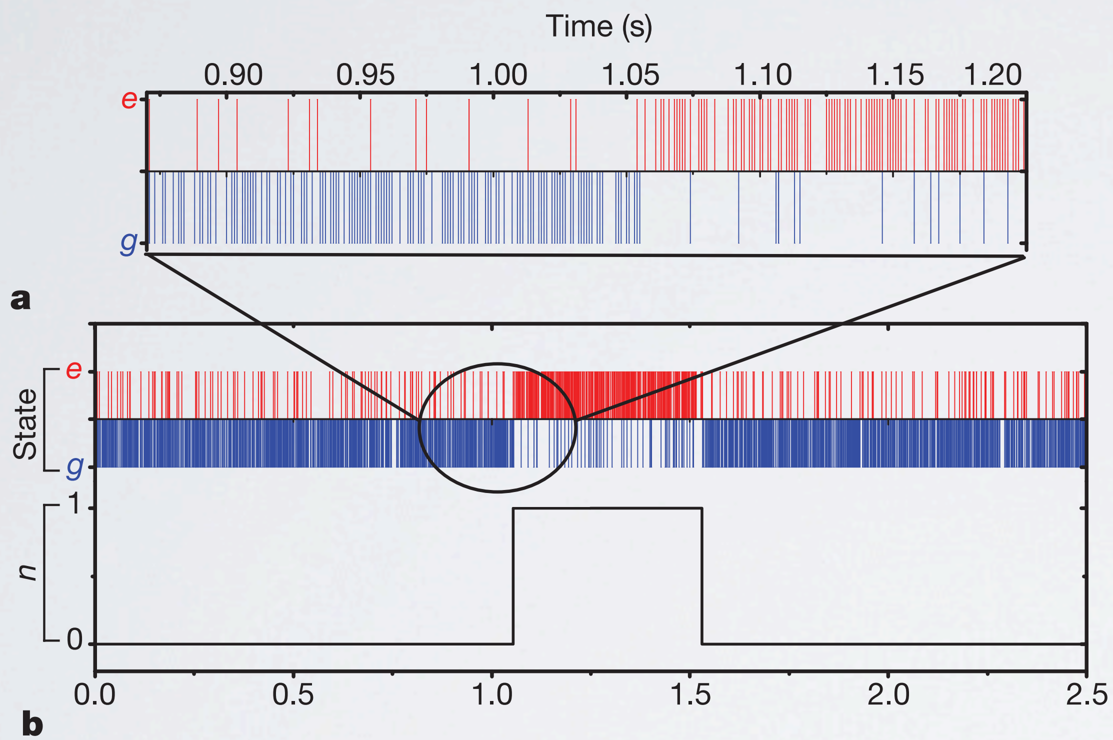
arXiv:1106.4953, arXiv:1210.0425, arXiv:1303.6658, to appear (hopefully soon...)

Kyoto, July 2013

Non-demolition measurements and Q-jumps

Quantum jumps of light recording the birth and death of a photon in a cavity

Sébastien Gleyzes[1], Stefan Kuhr[1]†, Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Ulrich Busk Hoff[1], Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

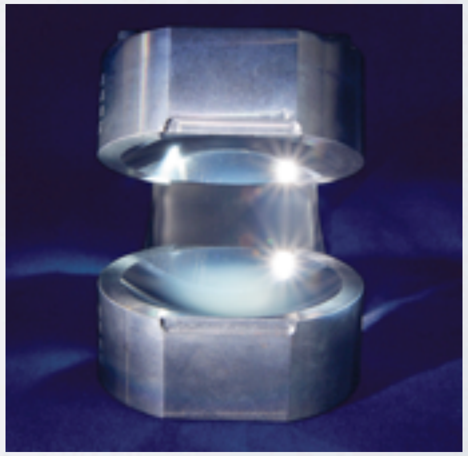


Creation-Annihilation of a thermal photon.

Figure 2 | Birth, life and death of a photon. a, QND detection of a single photon. Red and blue bars show the raw signal, a sequence of atoms detected in *e* or *g*, respectively (upper trace). The inset zooms into the region where the statistics of the detection events suddenly change, revealing the quantum jump from $|0\rangle$ to $|1\rangle$. The photon number inferred by a majority vote over

Courtesy of LKB-ENS.

-- How to measure photons without destroying them ?
-- How to record the cavity states ?
-- How to observe quantum jumps? Are they detector dependent ?
-- What determines the Q-jump dynamics ?

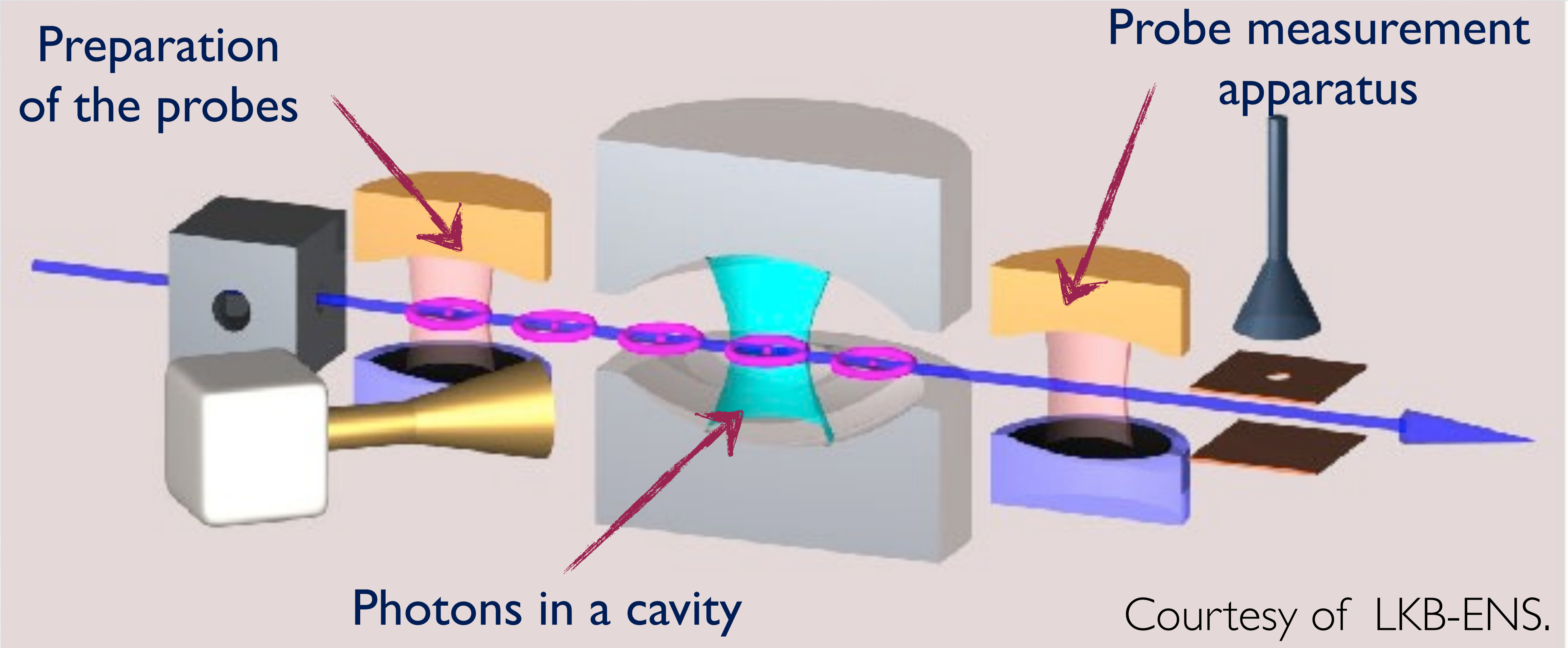


The photon system is probed indirectly via another quantum system.

Cavity QED experiments

-- Testing light/photon (the quantum system) with matter (the quantum probes).

System (S)= photons in a cavity.
Probes (P)= Rydberg atoms (two state systems)



-- Indirect measurements:

Direct (Von Neumann) measurements on an auxiliary system (the probes).

No direct observation of the cavity (the system).

-- Probe like gyroscope (spin half system).

-- Interaction like rotation of the gyroscope (with an angle depending on the number of photons)

$$U = \exp[i\theta\,\sigma^z\,N_{\rm photon}]$$

Progressive field-state collapse and quantum non-demolition photon counting

Christine Guerlin[1], Julien Bernu[1], Samuel Deléglise[1], Clément Sayrin[1], Sébastien Gleyzes[1], Stefan Kuhr[1]†, Michel Brune[1], Jean-Michel Raimond[1] & Serge Haroche[1,2]

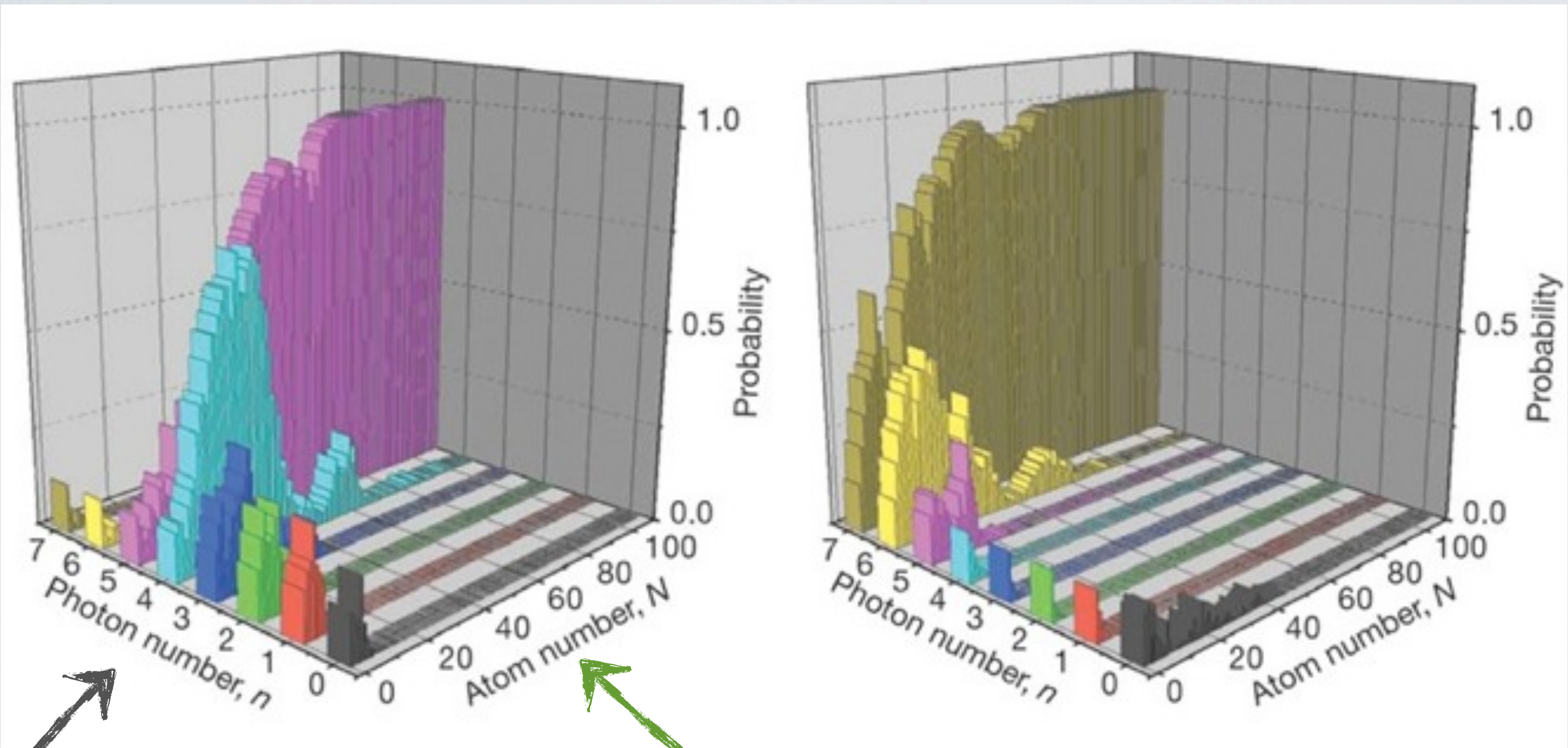


Figure 2 | Progressive collapse of field into photon number state.

[redacted] **c**, Photon number probabilities plotted versus photon and atom numbers n and N. The histograms evolve, as N increases from 0 to 110, from a flat distribution into $n = 5$ and $n = 7$ peaks.

Courtesy of LKB-ENS.

P.d.f. of the photon numbers

Number of indirect measurements

--- *Why does the p.d.f. change after each indirect measurement ?*
--- *How does it evolve? why does it become peaked (collapsed) ?*
--- *How does it represent the cavity state ?*
--- *What does continuous-in-time quantum measurement mean ?*
(Here: a `discrete version' of time continuous measurement)

Outline:

Part I:

Classical probability theory with a bit of quantum mechanics, (or the reverse).

-- A view on (quantum) noise.
(repeated interactions)

-- Repeated (quantum) measurements, Bayes' law and collapses.
(via the martingale convergence theorem).

-- Pointer states, exchangeability and de Finetti's theorem.
(objective or subjective probabilities)

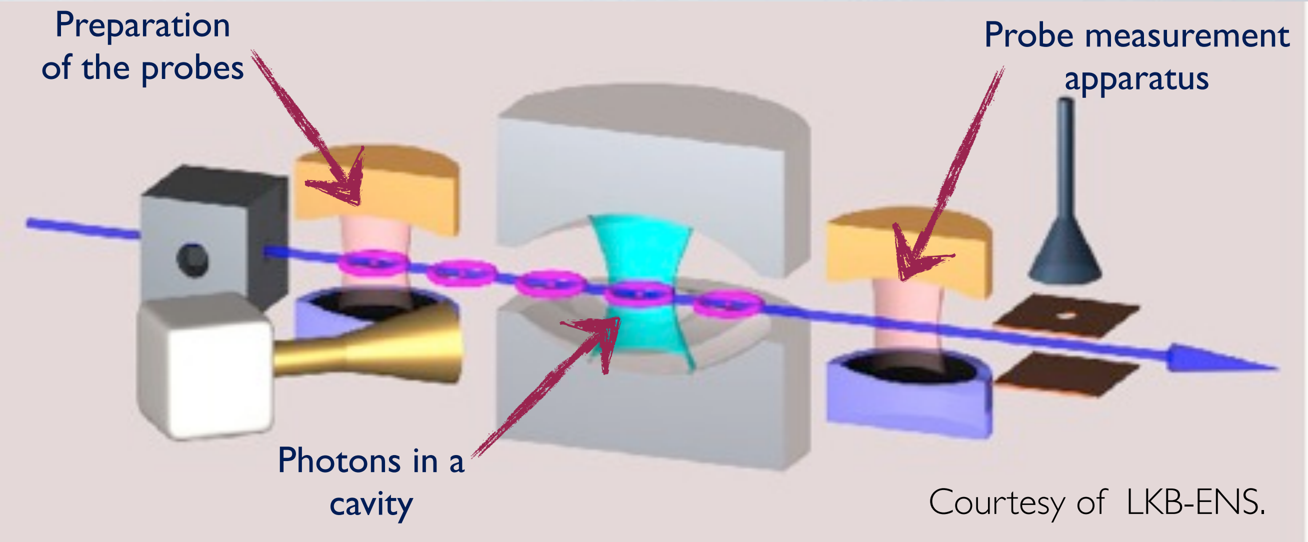
Part II:

Making real «virtual» fluctuations

-- Time continuous measurement and Q-jumps.
(Bi-stability (multi-stability) and Q-jumps)

-- Real time imaging of thermal and quantum fluctuations.
(observing quantum fluctuations continuously in time.)

Quantum non-demolition measurement and random bayesian updating.



-- Repeated cycles of interaction plus probe measurement:

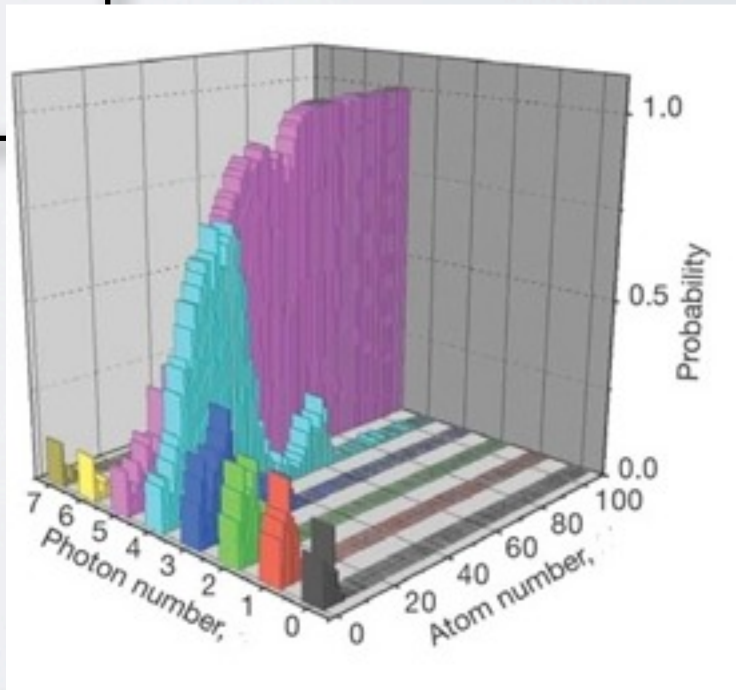
Partial gain of information at each iteration, because of system-probe entanglement.

-- Probe measurements give values + or - ; $\omega = (+,+,-,+,\cdots) = (\epsilon_1, \epsilon_2, \epsilon_2, \cdots)$

Recursion for the photon number p.d.f. from data of sequences

$\longrightarrow$ up-dating of the trial/estimated p.d.f. at each step, using an *a priori* model for the output conditional probabilities

$$P_{\text{estimated}}(N_{\text{photon}}|{}^{\text{probe}}_{\text{state}}) = \frac{P_{\text{a priori}}({}^{\text{probe}}_{\text{state}}|N_{\text{photon}})\ P_{\text{trial}}(N_{\text{photon}})}{P_{\text{normalisation}}({}^{\text{probe}}_{\text{state}})}$$



-- Bayesian approach (encoded into Q-mechanics) :

And the updating is random (because of Q-mechanics)

-- A two step analysis : -- What happens during one cycle ?

-- What happens for an iteration of cycles ?

Quantum mechanics implies «classical» Bayes' rules

Or what happens during one interaction + measurement cycle?

-- Preparation:

-- probe: $|\phi\rangle$

-- system: $|\psi\rangle = \sum_\alpha C(\alpha)|\alpha\rangle \quad , \quad Q_0(\alpha) = |C(\alpha)|^2$

-- Interaction: A delicate point : we suppose that there is a basis of system states *preserved* by the probe-system interaction, i.e.:

$U\,|\alpha\rangle \otimes |\phi\rangle = |\alpha\rangle \otimes U_\alpha|\phi\rangle$ for U the evolution operator of the probe-system interaction

This will be related to the existence of *pointer states*, to exchangeability,...

-- After interaction: -- probe + system : $\sum_\alpha C(\alpha)\ |\alpha\rangle \otimes U_\alpha|\phi\rangle$

-- After probe measurement: If output probe measurement is $|i\rangle$

-- probe + system : $\propto \Big(\sum_\alpha C(\alpha)\langle i|U_\alpha|\phi\rangle\ |\alpha\rangle\Big) \otimes |i\rangle$

-- New state distribution :

... and new system state.

$$Q_{\rm new}(\alpha) = \frac{p(i|\alpha)\,Q_0(\alpha)}{Z(i)}$$

with $p(i|\alpha) = |\langle i|U_\alpha|\alpha\rangle|^2$

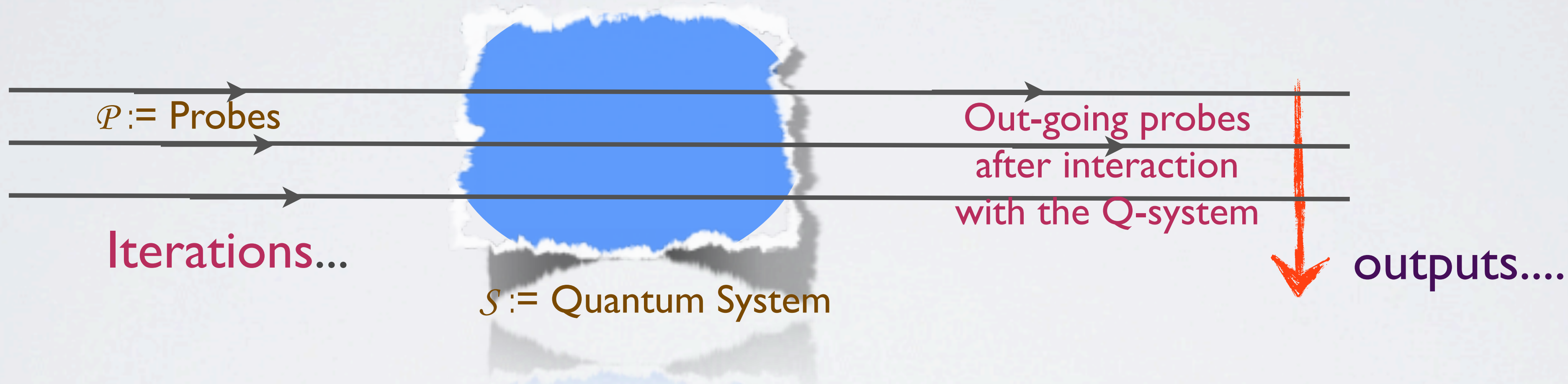
i.e. Bayes' rules

Evolution of the probability distributions:

Random Bayesian up-dating...

Pick a basis *a* of states of the Q-system.
Start with a probability distribution (initial system state): $Q_0(\alpha), \quad \sum_\alpha Q_0(\alpha) = 1,$

Let *i* be the output measurements on the probes.
Data (probe-system interaction) are probabilities to measure *i* conditioned on the Q-system to be in state *a*. $p(i|\alpha), \quad \sum_i p(i|\alpha) = 1$



Let $Q_{n-1}(a)$ be the probability distribution of the Q-system after (n-1) cycles,
The output of the n-th probe measurement is i_n with probability : $\pi_n(i)$
Then,

$$Q_n(\alpha) = \frac{1}{Z_n}\, p(i_n|\alpha)\, Q_{n-1}(\alpha), \quad \text{with} \quad Z_n = \sum_\alpha p(i_n|\alpha)\, Q_{n-1}(\alpha)$$

Collapse of the p.d.f. $Q_n(\alpha)$ as $n \to \infty$ *(A classical statement...)*

$$Q_n(\alpha) = \frac{p(i_n|\alpha)\,Q_{n-1}(\alpha)}{Z_n}, \quad \text{with proba} \quad \pi_n := Z_n = \sum_\alpha p(i_n|\alpha)\,Q_{n-1}(\alpha)$$

Claim:

* Peaked distributions are stable (stability of the pointer states):

  $Q_n(\alpha) = \delta_{\alpha;\gamma}$ are solutions. Then, outputs i_n are i.i.d. with probability: $p(i_n|\gamma)$

* Probability distributions converge a.s. (and in L1) towards peaked distributions (collapse of the wave function):

$$\lim_{n\to\infty} Q_n(\alpha) = \delta_{\alpha,\gamma_\omega}$$
$$\mathbb{P}\mathrm{rob}[\gamma_\omega = \beta] = Q_0(\beta)$$

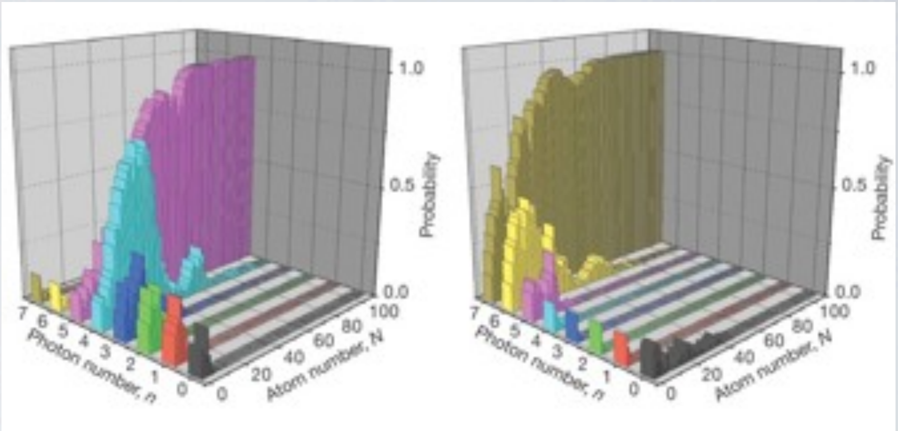
with a realisation dependent target γ_ω

(Von Neumann rules for quantum measurements)

* The convergence is exponential : $$Q_n(\alpha) \simeq \exp[-n\,S(\gamma_\omega|\alpha)] \quad (\alpha \neq \gamma_\omega)$$

with $S(\gamma|\alpha) = -\sum_i p(i|\gamma)\log\frac{p(i|\alpha)}{p(i|\gamma)}$ a relative entropy.

Mesoscopic collapses :



→ Quantum to classical transition.

Mesoscopic measurements.

"Collapse is nothing more than the updating of that calculational device on the basis of additional experience". D. Mermin, arXiv:1301.6551 (posterior to these works)

... A Bayesian point of view.

... and some robustness or universality in quantum measurements

What about if we don't record the probe outputs?

-- statistical ensemble of peaked distribution = diagonal density matrix

→ Progressive decoherence:

$$\langle\alpha|\rho_n|\alpha\rangle = Q_0(\alpha) = \text{const.}$$
$$\langle\alpha|\rho_n|\beta\rangle = \left(\langle U_\alpha^\dagger U_\beta\rangle\right)^n \langle\alpha|\rho_0|\beta\rangle \to 0$$

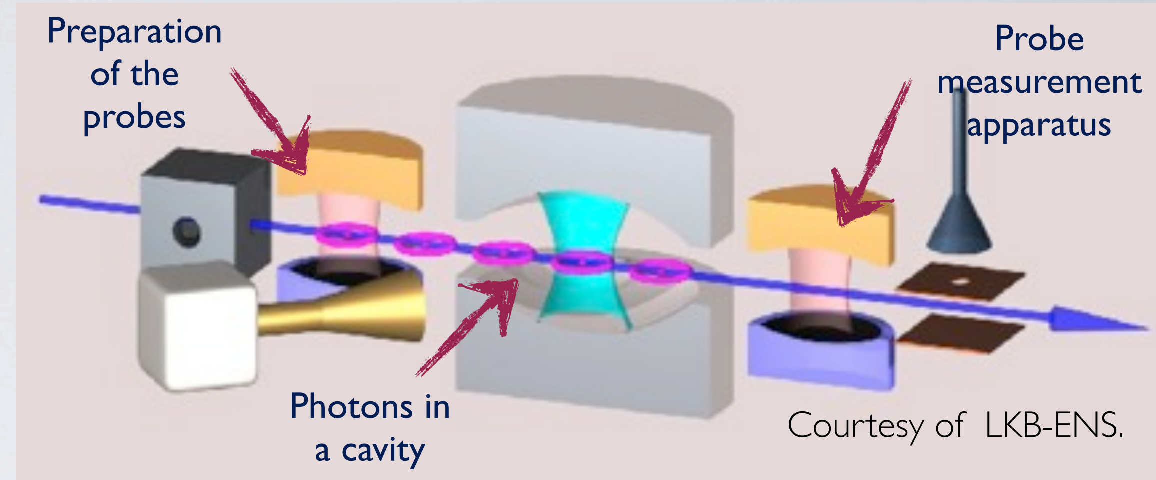
Elements of a proof :

-- $Q_n(\alpha)$ are bounded martingales, i.e. $\mathbb{E}[Q_n(\alpha)|\mathcal{F}_{n-1}] = Q_{n-1}(\alpha)$ as such they converge a.s. and in $\mathbb{L}^1$

-- Asymptotically, the outputs are i.i.d. with asymptotic frequencies : $N_n(i) \simeq_{n\to\infty} n\, p(i|\gamma_\omega)$

-- The limit is independent of the initial trial distribution. (Important for the experiment).

Quantum noise and repeated quantum interaction.



e.g. as in cavity QED experiments....

-- Hilbert space: $\mathcal{H} = \mathcal{H}_s \otimes \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_n \otimes \cdots$

-- Algebras of observable: $\mathcal{B}_n := \mathcal{A}_s \otimes \mathcal{A}_1 \otimes \cdots \otimes \mathcal{A}_n \otimes \mathbb{I}$

-- Filtration: $\mathcal{A}_s \subset \mathcal{B}_n \subset \mathcal{B}_m, \quad \text{for } n < m$

-- Gain of information : by testing output observable on the n-th first probes, but a probabilistic gain because of Q.M.

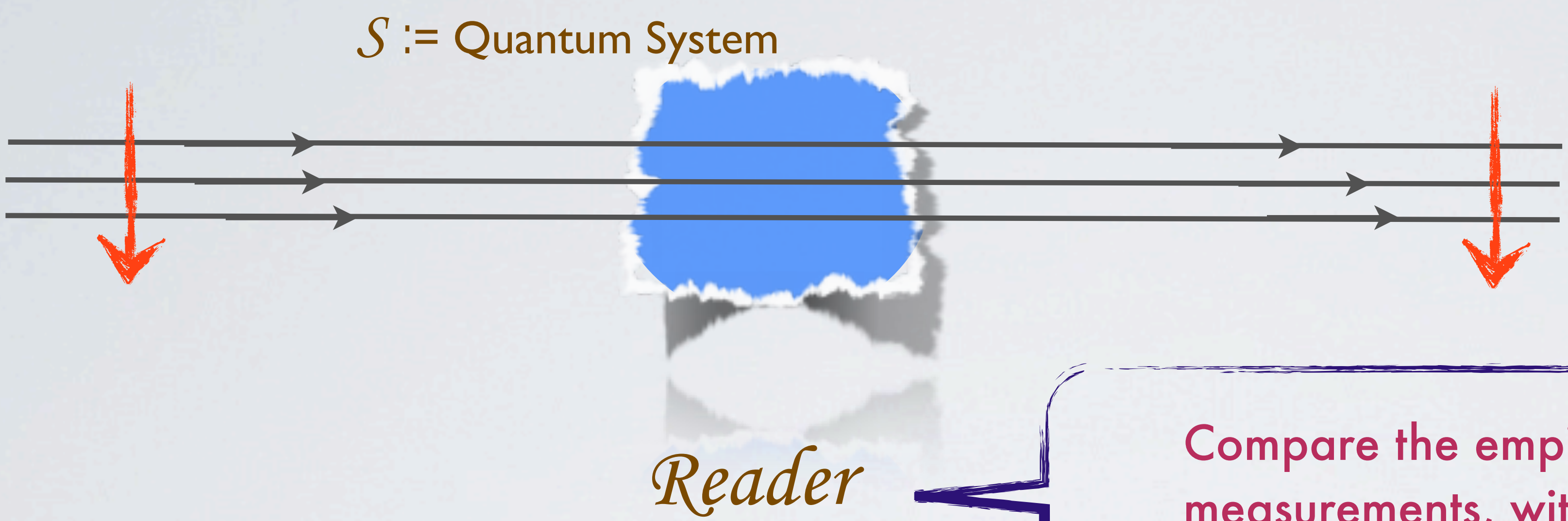
-- Measure some observable on the (n-th) output probes (not on the Q-system):

$\longrightarrow$ The quantum filtration is reduced to a classical filtration. (Quantum Trajectories)
(classical random process, the events are the out-put measurements)

Macroscopic measurement apparatus:

Measure whether the system is in state a,
i.e. measure observable with eigenstates a.

Data of the apparatus: the p.d.f. $p(i|\alpha)$ on $\mathcal{I}$, for all α.



For each infinite cycle, the apparatus provide the infinite sequence,

$$(i_1, \cdots, i_n, \cdots)$$

i.e. the frequencies $N(i|\alpha)$

Compare the empirical histogram of the output measurements, with the given distributions $p(i\,|\,a)$

⟶ *Target state = result of the measure*

Partial collapse for mesoscopic measurements.

But also «classical Bayesian measurement apparatus».

Generalizations :

-- with different probes, probe measurements, randomly chosen, etc..
-- continuous in time description, continuous measurements, etc....

⟶ **Applications**: e.g. control and state manipulations......

Time continuous measurement and Q-jumps:

Making real Bohr's «virtual» quantum jumps

-- As a model for discrete repeated measurement but with short time interval

-- hamiltonian evolution (T=0): $\rho \to U_{\mathrm{hamilton}}\, \rho\, U^\dagger_{\mathrm{hamilton}}$

-- measurement (POVM) : $\rho \to (F_i\, \rho\, F_i^\dagger)/\pi_i, \ \text{with}\ F_i := \langle i | U_{\mathrm{meas.}} | \psi \rangle$

-- If time duration of «probe+system interaction cycles» is small:

$$d\rho = i[H, \rho]\, dt + (d\rho)_{\mathrm{meas}}$$

Random time continuous measurements,
(randomness due to (random) output probe measurements)

-- If $\langle i | \psi \rangle \neq 0$ (a condition on probe data), these are diffusive like equations (Belavkin's eqs.)

For spin 1/2 probes : $\text{discrete}\ (+,+,-,+,-,\cdots) \to B_t := \text{brownian motion.}$

$$(d\rho)_{\mathrm{meas}} = L_{\mathrm{meas}}(\rho)\, dt + D_{\mathrm{meas}}(\rho)\, dB_t \qquad \text{(in law)}$$

For a Q-bit system : $|0\rangle,\ |1\rangle$

$\tau_{\mathrm{collapse}} \simeq \gamma^{-2}$

$$dQ_t = \gamma\, Q_t(1 - Q_t)\, dB_t, \ \text{for}\ Q_t = \langle 0 | \rho_t | 0 \rangle.$$

-- Non-demolition measurement : H and the measured observable commute.

Time continuous measurement and Q-jumps (II):

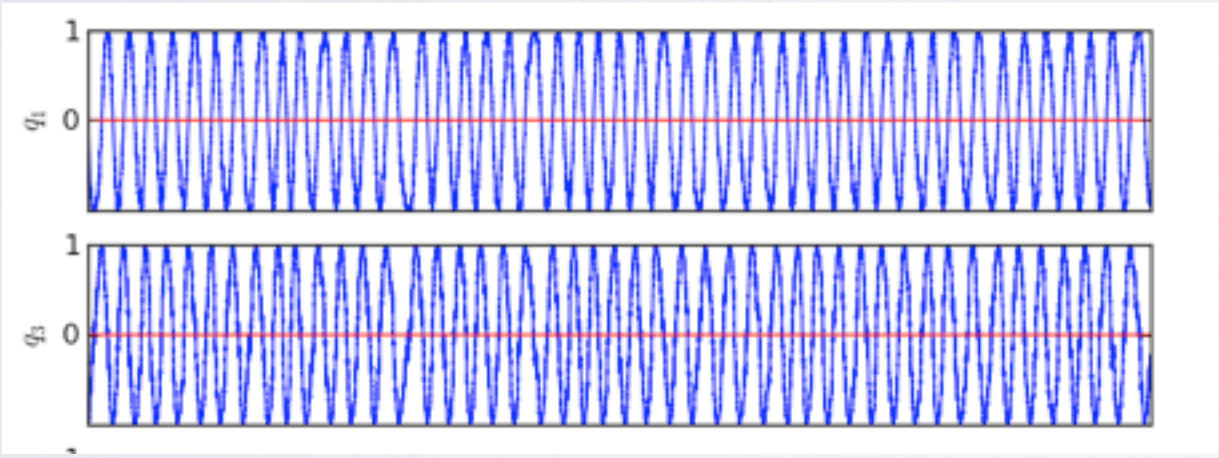
-- What happens if the H and the measured observable do not commute?
A system (spin half) under continuous measurement.

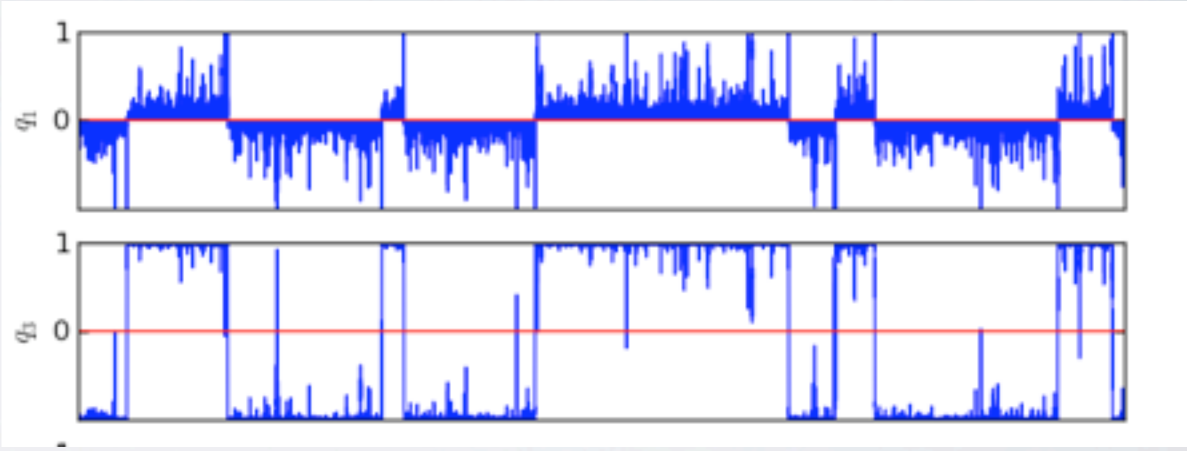
Take $H = \omega_0\,\sigma^2$ and measure $S^z = \sigma^3$

With: $\rho = \frac{1}{2}\big(1 + \cos\theta\sigma^3 + \sin\theta\sigma^1\big)$

$$d\theta_t = -(\omega_0 + \gamma^2 \sin 2\theta_t)dt - 2\gamma \sin\theta_t\, dB_t$$

$\gamma^2 \ll \omega_0$ Slightly deformed Rabi oscillations (measurement does take place)





$\gamma^2 \gg \omega_0, \quad \tau_{\text{collpase}} \ll \tau_{\text{evolution}}$

Q-jumps between the two eigen-states

$\tau_{\text{flip}} \simeq \tau^2_{\text{evolution}}/\tau_{\text{collapse}}$

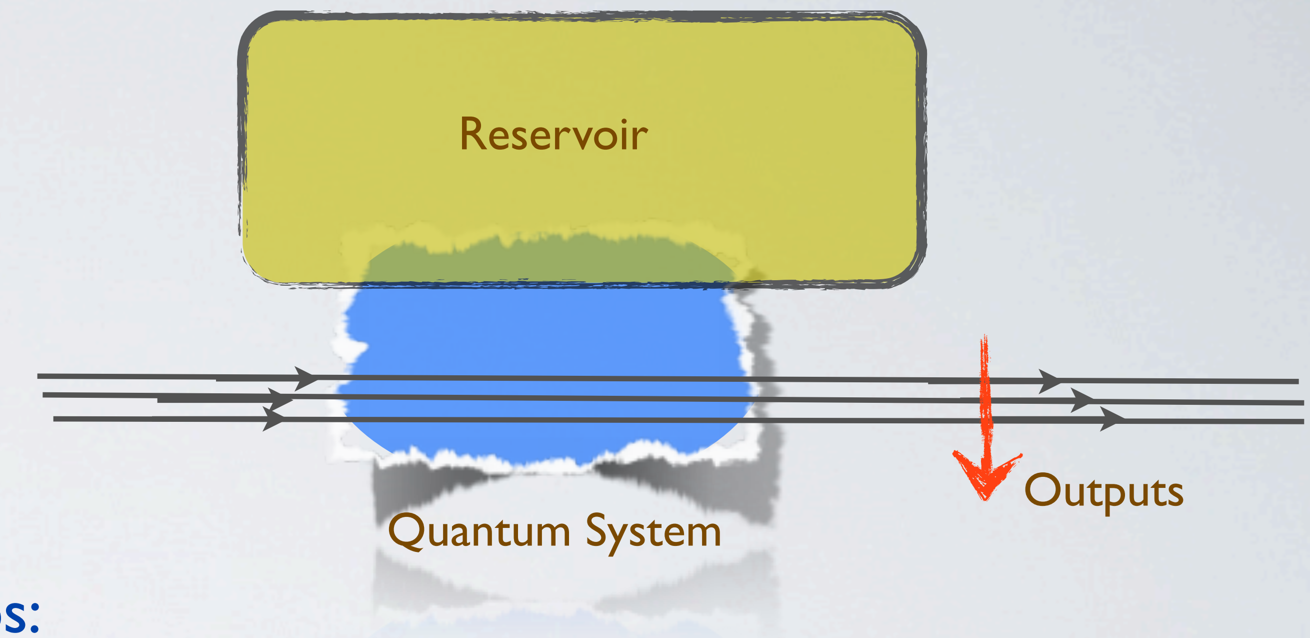
-- Technically: Kramers like transition for a two well potential random process.

Conclusion

-- Measuring an observable commuting with H : (progressive) collapse.

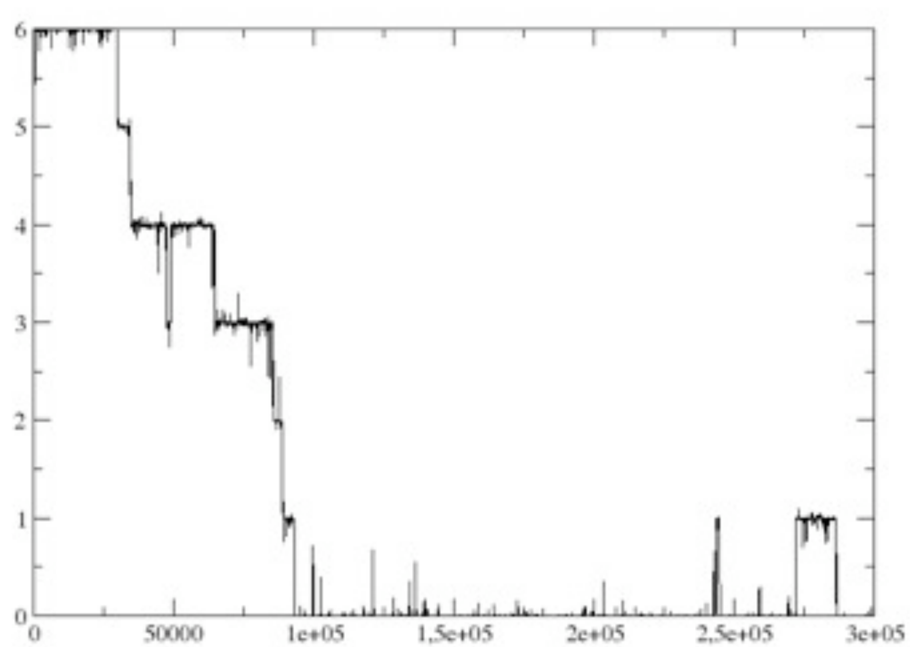
-- Measuring an observable not commuting with H : Q-jumps.

Real Time Imaging of Quantum and Thermal Fluctuations.



-- For system in contact
with a thermal reservoir
and under continuous measurements.

-- Quantum/Thermal fluctuations and Q-jumps:



Energy cascade and Q-jumps

What are the quantum trajectories ?

-- Evolution under thermal contact:

$$\rho_n \to \tilde{\rho}_n := M_{\mathrm{therm}}[\rho_n] := \sum_k B_k\, \rho_n\, B_k^\dagger$$

-- Recursive indirect measurements:

$$\tilde{\rho}_n \to \rho_{n+1} := M_{\mathrm{meas}}^{i_n}[\tilde{\rho}_n] := F_{i_n} \tilde{\rho}_n F_{i_n}^\dagger / \pi_{i_n}$$

with probability $\pi_{i_n} := \mathrm{Tr}(F_{i_n} \tilde{\rho}_n F_{i_n}^\dagger)$

Real Time Imaging of Quantum and Thermal Fluctuations (II).

-- Time continuous formulation:

$$d\rho = (d\rho)_{\mathrm{therm.}} + (d\rho)_{\mathrm{mes.}}$$

Deterministic thermal evolution (Lindblad)

Random time continuous measurements

Two time scales :

$\tau_{\mathrm{collapse}} \ll \tau_{\mathrm{therm.}}$

-- For two states systems (with spin half probes):

$$\rho = Q\,|0\rangle\langle 0| + (1-Q)\,|1\rangle\langle 1| \qquad (\gamma^2 \gg \lambda)$$

$$dQ_t = \lambda\,[p - Q_t]\,dt + \gamma\, Q_t(1-Q_t)\,dB_t.$$

* Waiting times :

$T_1 \simeq \tau_{\mathrm{therm}}/p, \quad T_0 \simeq \tau_{\mathrm{therm}}/(1-p), \quad T_0/T_1 = e^{\beta},$

and exponentially distributed.

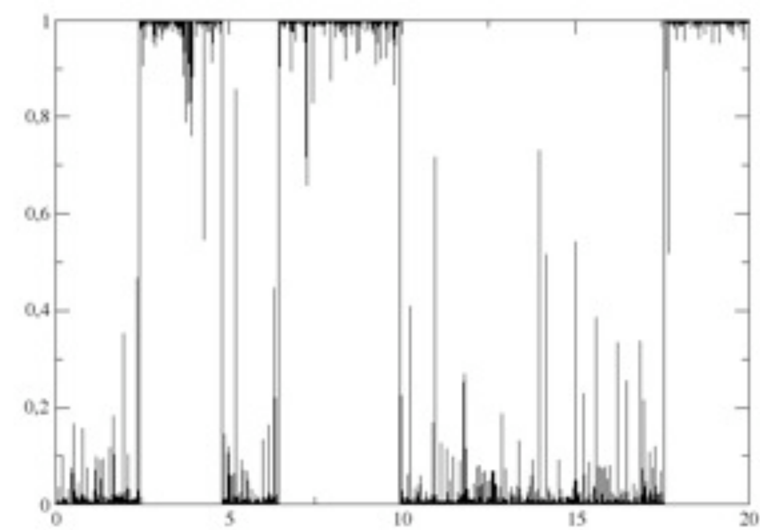
* Jump times :

$\tau_{\mathrm{jump}} \simeq \tau_{\mathrm{collapse}} \log(\tau_{\mathrm{therm}}/\tau_{\mathrm{collapse}})$

controlled by the measurement process,

* Stationary measure:

Close to Gibbs but not quite.



Q-trajectories and Q-jumps

Generalisations with many states and arbitrary probes **(OK)**, and **Applications**...... with more thermal bath (off-equilibrium).

Understanding repeated QND measurements
and mesoscopic measurements :
An interesting exercise in probability/Q.M. theory,
with some (probable) quantum applications...

Thank you.