Ferminoic Basis in Integrable Models: Profile and Prospect

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In this talk we are concerned with an old topic from integrable quantum field theory in two dimensions: to describe *the space of local fields* and their *vacuum expectation values (VEVs)*, or one point functions.

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To study the correlator of some field $\Phi(x)$ at short distances, one can apply the *operator product expansion*,

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where $\{A_i(0)\}$ is a complete set of local fields in the theory. The coefficients $C^i_{\Phi\,\Phi}(x)$ are local data accessible by perturbation theory. In contrast, the VEVs $\langle A_i(0)\rangle$ are global data which encode all non-perturbative information. For the characterization of correlation functions, it is necessary to know *all of them*.

Our main example is the sine-Gordon (sG) model

$$\mathcal{L}_{\mathrm{s}G} = \frac{1}{16\pi}(\partial_\mu \varphi)^2 - \frac{\boldsymbol{\mu}^2}{\sin \pi\beta^2}(e^{-i\beta\varphi} + e^{i\beta\varphi})\,.$$

It is a perturbation of a CFT of massless bosons.
In CFT, the space of fields is a Verma module spanned by a

primary field $\quad e^{ia\varphi}$

and their descendants,

$$\partial^{m_1}\varphi \cdots \partial^{m_K}\varphi\, \bar{\partial}^{n_1}\varphi \cdots \bar{\partial}^{n_L}\varphi \cdot e^{ia\varphi}\,.$$

In the sG model we consider local fields of this form. Among them, VEV has been known for the primary field and for the first non-trivial descendant.

Known results about VEV

Primary field (LZ 1997)

$$\langle e^{ia\varphi}\rangle = [\Gamma(\nu)\boldsymbol{\mu}]^{\frac{\nu\alpha}{2(1-\nu)}}$$
$$\times \exp\Bigl(\int_0^\infty \Bigl(\frac{\sinh^2(\nu\alpha t)}{2\sinh(1-\nu)t\sinh t\cosh\nu t} - \frac{\nu^2\alpha^2}{2(1-\nu)}e^{-2t}\Bigr)\frac{dt}{t}\Bigr).$$

First non-trivial descendant (FFLZZ 1998)

$$\frac{\langle L_{-2}\bar{L}_{-2}e^{ia\varphi}\rangle}{\langle e^{ia\varphi}\rangle} = -\frac{(\Gamma(\nu)\boldsymbol{\mu})^{4/\nu}}{(1-\nu)^2}\frac{\gamma(-\frac{1}{2}+\frac{\alpha}{2}+\frac{1}{2\nu})}{\gamma(\frac{1}{2}+\frac{\alpha}{2}-\frac{1}{2\nu})}\frac{\gamma(\frac{\alpha}{2}-\frac{1}{2\nu})}{\gamma(\frac{\alpha}{2}+\frac{1}{2\nu})}$$

where

$$\gamma(x) = \frac{\Gamma(x)}{\Gamma(1-x)}, \quad \nu = 1-\beta^2, \quad \nu\alpha = 2\beta a\,.$$

The goal of this talk is to explain that there is a conjectural basis better suited for the systematic description of VEVs.

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The second is a set of fermions commuting with IM,

$$\boldsymbol{\beta}^*_p, \boldsymbol{\gamma}^*_p, \bar{\boldsymbol{\beta}}^*_p, \bar{\boldsymbol{\gamma}}^*_p \quad (p = 1, 3, 5, \cdots).$$

The basis in question is given by

$$i_K \bar{i}_{\bar{K}} \boldsymbol{\beta}^*_{J^+} \boldsymbol{\gamma}^*_{J^-} \bar{\boldsymbol{\beta}}^*_{\bar{J}^+} \bar{\boldsymbol{\gamma}}^*_{\bar{J}^-} e^{ia\varphi} \quad (\sharp J^+ = \sharp J^-, \, \sharp \bar{J}^+ = \sharp \bar{J}^-)$$

where

$$\boldsymbol{\beta}^*_{J^+} = \boldsymbol{\beta}^*_{j_1} \cdots \boldsymbol{\beta}^*_{j_k} \quad (J^+ = \{j_1, \cdots, j_k\}, \; j_1 < \cdots < j_k), \text{ etc..}$$

The VEV of the basis elements are given by

$$\frac{\langle \boldsymbol{\beta}^*_{J^+}\boldsymbol{\gamma}^*_{J^-}\bar{\boldsymbol{\beta}}^*_{\bar{J}^+}\bar{\boldsymbol{\gamma}}^*_{\bar{J}^-}\, e^{ia\varphi}\rangle}{\langle e^{ia\varphi}\rangle} = \boldsymbol{\mu}^{2\nu^{-1}(|J^+|+|J^-|)}\delta_{J^+,\bar{J}^-}\delta_{J^-,\bar{J}^+}$$
$$\times \prod_{p\in J^+}\frac{i}{\nu}\cot\frac{\pi}{2\nu}(p+\nu\alpha)\prod_{r\in J^-}\frac{i}{\nu}\cot\frac{\pi}{2\nu}(r-\nu\alpha)\,,$$

where $|I| = \sum_{p\in I} p$.

The IM do not contribute to VEV: $\langle [I_p, \mathcal{O}]\rangle = \langle [\bar{I}_p, \mathcal{O}]\rangle = 0$.

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They are not to be confused with the fermions of the massive Thirring model.
These fermions seem to appear totally out of the blue, but they appeared already in the literature (Babelon et al. 1997, will comment later.)

Plan of the talk

1. 6 vertex model and expectation values (Existence theorem)
2. Field theory limit (Conjectures)
3. Relation to previous works: Form factors, Reflection equation
4. Summary

Joint work with

T.Miwa, F.Smirnov,
H.Boos, Y.Takeyama (in part)

Lattice regularization: 6 vertex model

Consider a six vertex model on an infinite cylinder,

$$q = e^{\pi i \nu} \quad (0 < \nu < \frac{1}{2}).$$

On one row, we allow background fields $q^{\kappa\sigma_j^3}$ (for all j), $q^{\alpha\sigma_j^3}$ (for $j \le 0$) and a local dislocation $\mathcal{O}$. One can think of

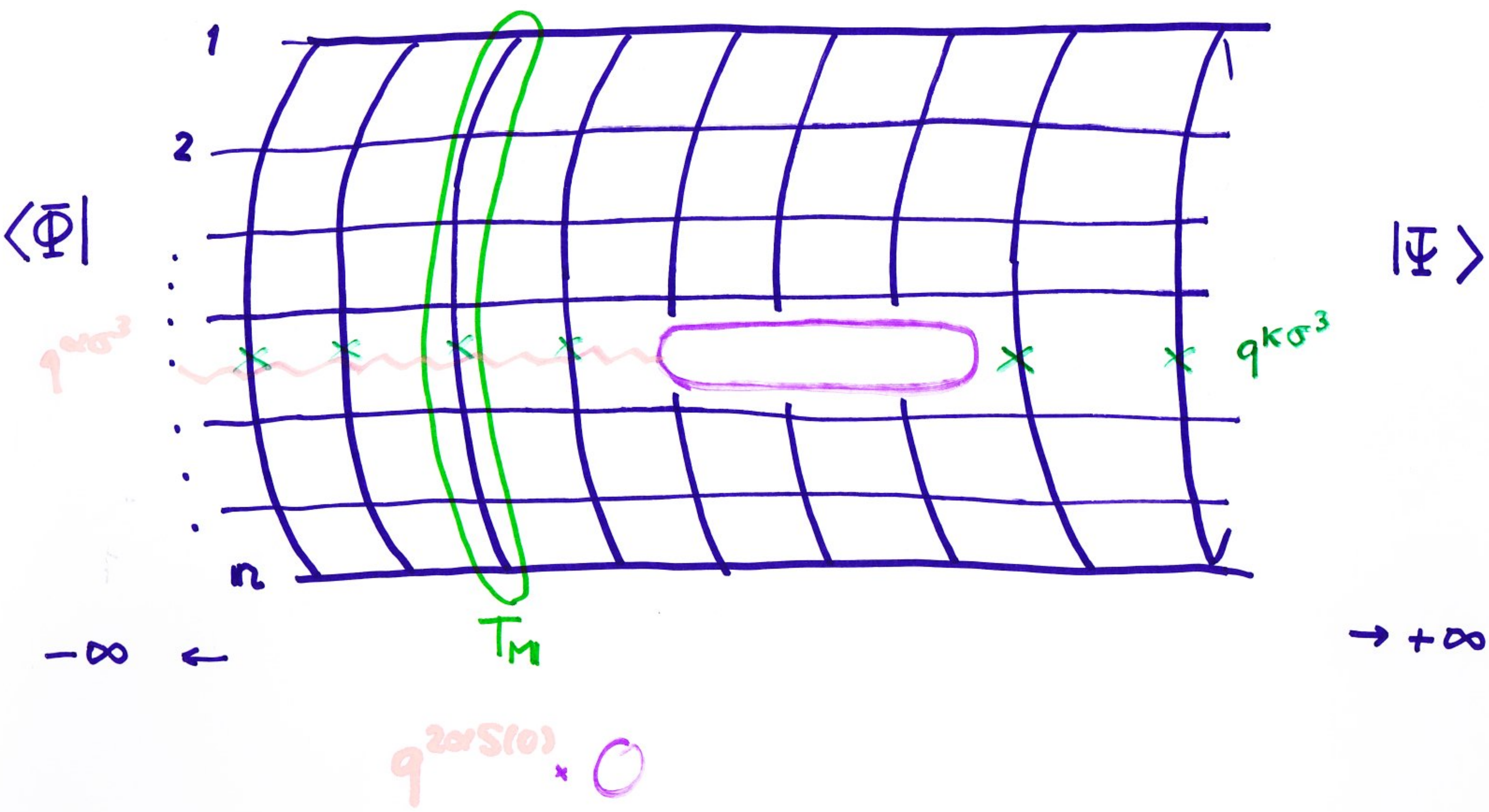
$$q^{2\alpha S(0)}, \quad S(0) = \frac{1}{2} \sum_{j=-\infty}^{0} \sigma_j^3 ,$$

as a lattice analog of the primary field $e^{ia\varphi}$.
We take the following set as an analog of the space of local fields.

$$\mathcal{W}^{(\alpha)} = \bigoplus_{s \in \mathbb{Z}} \mathcal{W}_{\alpha - s, s},$$

$$\mathcal{W}_{\alpha - s, s} = \{q^{2(\alpha - s)S(0)}\mathcal{O} \mid \mathcal{O}\text{: local, spin } s\}$$

6 vertex model on a cylinder



Lattice analogs of VEV (more generally, of matrix elements) is the ratio of partition functions with/without dislocation,

$$Z\Big\{q^{2\alpha S(0)}\mathcal{O}\Big\} = \frac{\langle\Phi|\mathrm{Tr}_{[K,L]}\left\{\mathcal{T}_{[K,L],\mathbf{M}}(1)q^{2\kappa S_{[K,L]}+2\alpha S_{[K,0]}}\mathcal{O}\right\}|\Psi\rangle}{\langle\Phi|\mathrm{Tr}_{[K,L]}\left\{\mathcal{T}_{[K,L],\mathbf{M}}(1)q^{2\kappa S_{[K,L]}+2\alpha S_{[K,0]}}\right\}|\Psi\rangle},$$

where $K \ll 0 \ll L$,

$$\mathcal{T}_{[K,L],\mathbf{M}} = \prod_{j=K}^{\overset{\curvearrowright}{L}} \mathcal{T}_{j,\mathbf{M}}(1)\,, \quad \mathcal{T}_{j,\mathbf{M}}(\zeta) = \prod_{\mathbf{m}=\mathbf{1}}^{\overset{\curvearrowleft}{\mathbf{n}}} L_{j,\mathbf{m}}(\zeta)$$

is the monodromy matrix, and Φ, Ψ are eigen(co)vector of the transfer matrix in the vertical direction

$$T_{\mathbf{M}}(\zeta,\kappa) = \mathrm{Tr}_j\Big[\mathcal{T}_{j,\mathbf{M}}(\zeta)q^{\kappa\sigma_j^3}\Big].$$

Fermions on the lattice

One can construct fermions acting on $\mathcal{W}^{(\alpha)}$ (BJMST 2007–2009)

$$\mathbf{b}(\zeta),\ \mathbf{c}(\zeta),\ \mathbf{b}^*(\zeta),\ \mathbf{c}^*(\zeta)\,,$$

$$\mathbf{b}(\zeta)=\sum_{p=1}^{\infty}\mathbf{b}_p(\zeta^2-1)^{-p}\,,\quad \mathbf{b}^*(\zeta)=\sum_{p=1}^{\infty}\mathbf{b}^*_p(\zeta^2-1)^{p-1}\,, etc.$$

such that

- They commute with integrals of motion: $\mathbf{t}^*_1,\mathbf{t}^*_2,\cdots$
- The following is a basis of $\mathcal{W}^{(\alpha)}$:

$$(\mathbf{t}^*_1)^p\mathbf{t}^*_{i_1}\cdots\mathbf{t}^*_{i_r}\mathbf{b}^*_{j_1}\cdots\mathbf{b}^*_{j_s}\mathbf{c}^*_{k_1}\cdots\mathbf{c}^*_{k_t}(q^{2\alpha S(0)})$$
$$(i_1\geq\cdots\geq i_r\geq 2,\ j_1>\cdots>j_s\geq 1,$$
$$k_1>\cdots>k_t\geq 1,\ p\in\mathbb{Z},\ r,s,t\geq 0)\,.$$

The main formula

The following is the key result (JMS 2009).

$$Z\left\{t^*(\eta_1)\cdots t^*(\eta_s)\mathbf{b}^*(\zeta_1)\cdots\mathbf{b}^*(\zeta_r)\mathbf{c}^*(\xi_r)\cdots\mathbf{c}^*(\xi_1)(q^{2\alpha S(0)})\right\}$$
$$=\prod_{i=1}^{s}2\rho(\eta_i)\cdot\det\left(\omega(\zeta_j,\xi_k)\right)\,,$$

where

$$\rho(\eta)=\frac{T_\Phi(\eta)}{T_\Psi(\eta)}\,,\qquad T_{\mathbf{M}}(\eta)|\Psi\rangle=T_\Psi(\eta)|\Psi\rangle\,,$$

and $\omega(\zeta,\xi)$ is defined through linear and non-linear integral equations of Thermodynamic Bethe Ansatz type.

TBA data

- auxiliary function characterizing $|\kappa\rangle$

$$\log \mathfrak{a}(\zeta,\kappa) = -2\pi i\nu\kappa + \log\frac{a(\zeta)}{d(\zeta)} - \int_\gamma K_0(\zeta/\xi)\log(1+\mathfrak{a}(\xi,\kappa))\frac{d\xi^2}{\xi^2}\,,$$

where γ encircles the Bethe roots clockwise, and

$$a(\zeta) = (1-q\zeta^2)^{\mathbf{n}}\,,\quad d(\zeta) = (1-q^{-1}\zeta^2)^{\mathbf{n}}\,,$$

$$K_\alpha(\zeta) = \Delta_\zeta\psi(\zeta,\alpha)\,,\quad \psi(\zeta,\alpha) = \frac{\zeta^\alpha}{\zeta^2-1}\,,$$

$$\Delta_\zeta f(\zeta) = f(q\zeta) - f(q^{-1}\zeta)\,.$$

- resolvent

$$\mathcal{R}_{\rm dress} - \mathcal{R}_{\rm dress}\star K_\alpha = K_\alpha\,,$$

$$f\star g(\zeta,\xi) = \int_\gamma f(\zeta,\eta)g(\eta,\xi)\frac{1}{1+\mathfrak{a}(\eta,\kappa)}\frac{1}{\rho(\eta)}\frac{d\eta^2}{\eta^2}\,.$$

- formula for ω is

$$\frac{1}{4}\omega(\zeta,\xi) = f_{\rm left} \star (I + \mathcal{R}_{\rm dress}) \star f_{\rm right}(\zeta,\xi) - \omega_0(\zeta,\xi)\,,$$

where

$$f_{\rm left}(\zeta,\xi) = \frac{1}{2\pi i}\delta^-_\zeta \psi(\zeta/\xi,\alpha)\,, \quad f_{\rm right}(\zeta,\xi) = \delta^-_\xi \psi(\zeta/\xi,\alpha)\,,$$
$$\delta^-_\zeta f(\zeta) = f(q\zeta) - \rho(\zeta) f(\zeta)\,,$$
$$\omega_0(\zeta,\xi) = -\delta^-_\zeta \delta^-_\xi \Delta^{-1}_\zeta \psi(\zeta/\xi,\alpha)\,.$$

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- The result holds in a very general setting (arbitrary spins in rows, finite temperature, ...)

Field theory limit

Let us explain the continuous limit to CFT

$$\epsilon = \frac{2\pi R}{\mathbf{n}} \to 0\,, \quad \zeta = (C\epsilon)^{\nu}\lambda \quad (\lambda \text{ fixed})\,.$$

In this limit, the transfer matrix $T_{\mathbf{M}}(\zeta,\kappa)$ tends to that of a chiral CFT on a cylinder with the central charge $c = 1 + 6Q^2$, where

$$Q = b + b^{-1}, \quad b = i\beta.$$

It is hard to control the limit of the fermionic basis at the level of operators. We define them in the 'weak sense', i.e., take the limit of matrix elements.

In the definition of $\omega(\zeta,\xi)$, let us choose Φ,Ψ to be the ground states of $T_{\mathbf{M}}(\zeta,\kappa')$, $T_{\mathbf{M}}(\zeta,\kappa)$ and take the scaling limit

$$\omega^{\mathrm{sc}}(\lambda,\mu)=\lim\frac{1}{4}\omega\Big((C\epsilon)^{\nu}\lambda,(C\epsilon)^{\nu}\mu\Big)\,.$$

We postulate that each coefficient of the asymptotic expansion

$$\omega^{\mathrm{sc}}(\lambda,\mu)\simeq\sum_{p,r:odd>0}\omega_{p,r}\,\lambda^{-\frac{p}{\nu}}\mu^{-\frac{r}{\nu}}\quad(\lambda^2,\mu^2\to\infty)\,.$$

is a three-point function with the insertion of primary fields at $z=0,\infty$, and some descendant of $e^{ia\varphi}$ at $z=1$.

We then *define* this descendant of $e^{ia\varphi}$ to be $\boldsymbol{\beta}_p^*\boldsymbol{\gamma}_r^* e^{ia\varphi}$:

$$\frac{\langle 1-\kappa' | \boldsymbol{\beta}_p^*\boldsymbol{\gamma}_r^* e^{ia\varphi} | 1+\kappa \rangle}{\langle 1-\kappa' | e^{ia\varphi} | 1+\kappa \rangle} = \omega_{p,r}\,.$$

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Following the same logic in the sine-Gordon case, we arrive at the formulas for VEV of the fermionic basis.

Form factors

In a massive theory with asymptotic particles, giving a local field $\mathcal{O}$ is the same as giving the tower of matrix elements

$$f_{\mathcal{O}} = (f_{\mathcal{O},n})_{n=0}^{\infty}, \quad f_{\mathcal{O},n}(\theta_1, \cdots, \theta_n) = \langle \theta_1, \cdots, \theta_n | \mathcal{O} | \mathrm{vac} \rangle \,.$$

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For the (restricted) sine-Gordon model, Babelon, Bernard and Smirnov 1997 have shown that the space of all towers have the same size as the space of local fields in CFT. For this purpose they devised certain fermions which act on the space of towers preserving the form factor axioms.

An important part of the form factor axioms is the qKZ equation. Its solutions are constructed by representation theory of quantum affine algebras. In the sine-Gordon model the relevant q shift is $\sqrt{-1}$, at which point the Chevalley generators of the quantum affine algebra become fermions (Tarasov 2000). This is the origin of the 'combinatorial fermions' of Babelon et al.

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We have compared our fermions with those of Babelon et al.. It turns out that they just coincide.

Reflection equation

The VEV for the first non-trivial descendant

$$H(a)=\frac{\langle(\partial\varphi)^2(\bar\partial\varphi)^2e^{ia\varphi}\rangle}{\langle e^{ia\varphi}\rangle}$$

was found by Fateev et al. 1999, making use of the so-called Liouville reflection equation for the sinh-Gordon (shG) model. Roughly speaking the argument goes as follows.
The shG model

$$\mathcal{L}_{\mathrm{sh}G}=\frac{1}{16\pi}(\partial_\mu\varphi)^2+\frac{\boldsymbol{\mu}^2}{\sin\pi b^2}(e^{b\varphi}+e^{-b\varphi})$$

can be viewed as a perturbation either of the Liouville theory, or of the Gaussian theory.

Correspondingly we have two bases of the space of fields modulo IM: the descendants by the Virasoro algebra

$$L_{-N_1}\cdots L_{-N_k}\bar{L}_{-\bar{N}_1}\cdots\bar{L}_{-\bar{N}_l}e^{a\varphi}$$

or descendants by the Heisenberg algebra

$$(\partial^{m_1}\varphi)\cdots(\partial^{m_k}\varphi)(\bar{\partial}^{n_1}\varphi)\cdots(\bar{\partial}^{n_l}\varphi)e^{a\varphi}.$$

If $V(a)$, $H(a)$ are the matrix of normalized VEV, we have the obvious symmetries

$$V(a)=V(Q-a),\quad H(a)=H(-a)\,.$$

Introducing the transition matrix by $V(a)=U(a)H(a)$ we obtain

$$V(Q+a)=S(a)V(a),\quad S(a)=U(-a)U(a)^{-1}\,.$$

For the first non-trivial descendant, the matrix $S(a)$ is a scalar, and this equation was solved under the assumption on analyticity.

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$$a \to -a, \quad a \to Q - a,$$

the fermions transform as

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So the matrix $W(a)$ of the normalized VEV for the fermionic basis satisfies

$$W(-a) = JW(a)\,, \quad W(Q-a) = JW(a)\,,$$

where J is the permutation matrix exchanging $\boldsymbol{\beta}^*$ and $\boldsymbol{\gamma}^*$.

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- Such a basis exists on the lattice.
- Field theory limit is defined as 'weak limit'. Obscure, but the picture seems natural and self-consistent.

There are more questions than answers. Among others,

- The present construction relies too much on the specific nature of spin 1/2. Give a conceptual explanation.
- Asymptotic analysis of the function ω, esp. in the presence of IM. Is there a connection to spectral problems of ODE and PDE?
- Study OPE in the fermionic basis.
- $\cdots$

THANK YOU VERY MUCH FOR YOUR ATTENTION