Large-distance asymptotic behavior of the temperature correlators in the one-dimensional Bose gas

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- thermodynamical Bethe ansatz
- Bose gases as continuum limits of lattice models:

  XXZ chain

  Uimin-Sutherland model
- path integral formalism for lattice model, quantum transfer matrix
- largest eigenvalue analysis + continuum limit $\rightarrow$ finite number of non-linear integral equations
- excited states, correlation lengths: crossover behaviour at finite T
- numerical results

Support by Volkswagen Foundation

Hamiltonian for n-component gas

$$H = -\frac{\hbar^2}{2m}\sum_{i=1}^{N}\frac{\partial^2}{\partial x_i^2} + g\sum_{1\le i\le j\le N}\delta(x_i - x_j) - \sum_{i=1}^{n}\mu_i N_i\,.$$

$g = \hbar^2 c/m$: coupling constant, μ_i: chemical potentials

$n = 1$ scalar Bethe ansatz — Lieb, Liniger 1963; McGuire 1964

$$e^{ik_jL} = -\prod_{l=1}^{N}\frac{k_j - k_l + \mathrm{i}c}{k_j - k_l - \mathrm{i}c}$$

$n = 2$ nested Bethe ansatz — Yang 1967; Sutherland 1968

$$e^{ik_jL} = -\prod_{l=1}^{N}\frac{k_j - k_l + \mathrm{i}c}{k_j - k_l - \mathrm{i}c}\prod_{\alpha=1}^{M}\frac{k_j - \lambda_a - \frac{\mathrm{i}c}{2}}{k_j - \lambda_a + \frac{\mathrm{i}c}{2}},$$

$$\prod_{l=1}^{N}\frac{\lambda_\alpha - k_l - \frac{\mathrm{i}c}{2}}{\lambda_\alpha - k_l + \frac{\mathrm{i}c}{2}} = -\prod_{\beta=1}^{M}\frac{\lambda_\alpha - \lambda_\beta - \mathrm{i}c}{\lambda_\alpha - \lambda_\beta + \mathrm{i}c},$$

systems are thermodynamically unstable for attractive interaction c

For repulsive interactions ($c > 0$):

$n = 1$ **no bound states**

TBA for one function: Yang, Yang (1969)

thermodynamical potential density

$$g = -T \int_{-\infty}^{\infty} \frac{dk}{2\pi} \ln \left[1 + e^{-\varepsilon(k)/T} \right]$$

with dressed energy $\varepsilon(k)$

$$\varepsilon(k)/T = (k^2 - \mu)/T - K * \ln \left[1 + e^{-\varepsilon(k)/T} \right], \qquad K(k) = \frac{1}{\pi} \frac{c}{k^2 + c^2}$$

N.b.: for nested case additional kernels appear

$$\kappa_j(k) = \frac{1}{\pi} \frac{jc/2}{k^2 + (jc/2)^2}, \quad \text{(special case } K = \kappa_2\text{)}$$

Thermodynamics: TBA for two-component Bose gas

$n=2$ ∞-many bound states Takahashi (1971)

thermodynamical potential

$$g=-T\int_{-\infty}^{\infty}\frac{dk}{2\pi}\ln\left[1+e^{-\varepsilon(k)/T}\right]$$

∞-many integral equations $\mu=(\mu_1+\mu_2)/2,\ \Omega=(\mu_1-\mu_2)/2$

$$\begin{aligned}
\varepsilon(k)/T &= (k^2-\mu-\Omega)/T-\kappa_2*\ln\left[1+e^{-\varepsilon(k)/T}\right]-\sum_{j=1}^{\infty}\kappa_j*\ln\left[1+e^{-\varepsilon_j(k)/T}\right]\\
\varepsilon_1(k)/T &= s*\ln\left[1+e^{-\varepsilon(k)/T}\right]+s*\ln\left[1+e^{\varepsilon_2(k)/T}\right], \qquad s(k)=\frac{1}{2c\cosh(\pi k/c)}\\
\varepsilon_j(k)/T &= s*\ln\left[1+e^{\varepsilon_{j-1}(k)/T}\right]+s*\ln\left[1+e^{\varepsilon_{j+1}(k)/T}\right], \qquad (j>1),
\end{aligned}$$

with asymptotic condition

$$\lim_{j\to\infty}\frac{\varepsilon_j(k)}{j}=2\Omega$$

Gu, Li, Ying, Zhao (2002)

Density-density correlations: single component Bose gas

Seel, Bhattacharyya, Göhmann, AK 2007: multiple integrals in continuum limit of XXZ

Seel, Göhmann, AK 2008: Fredholm determinant for $c \to \infty$, explicit results

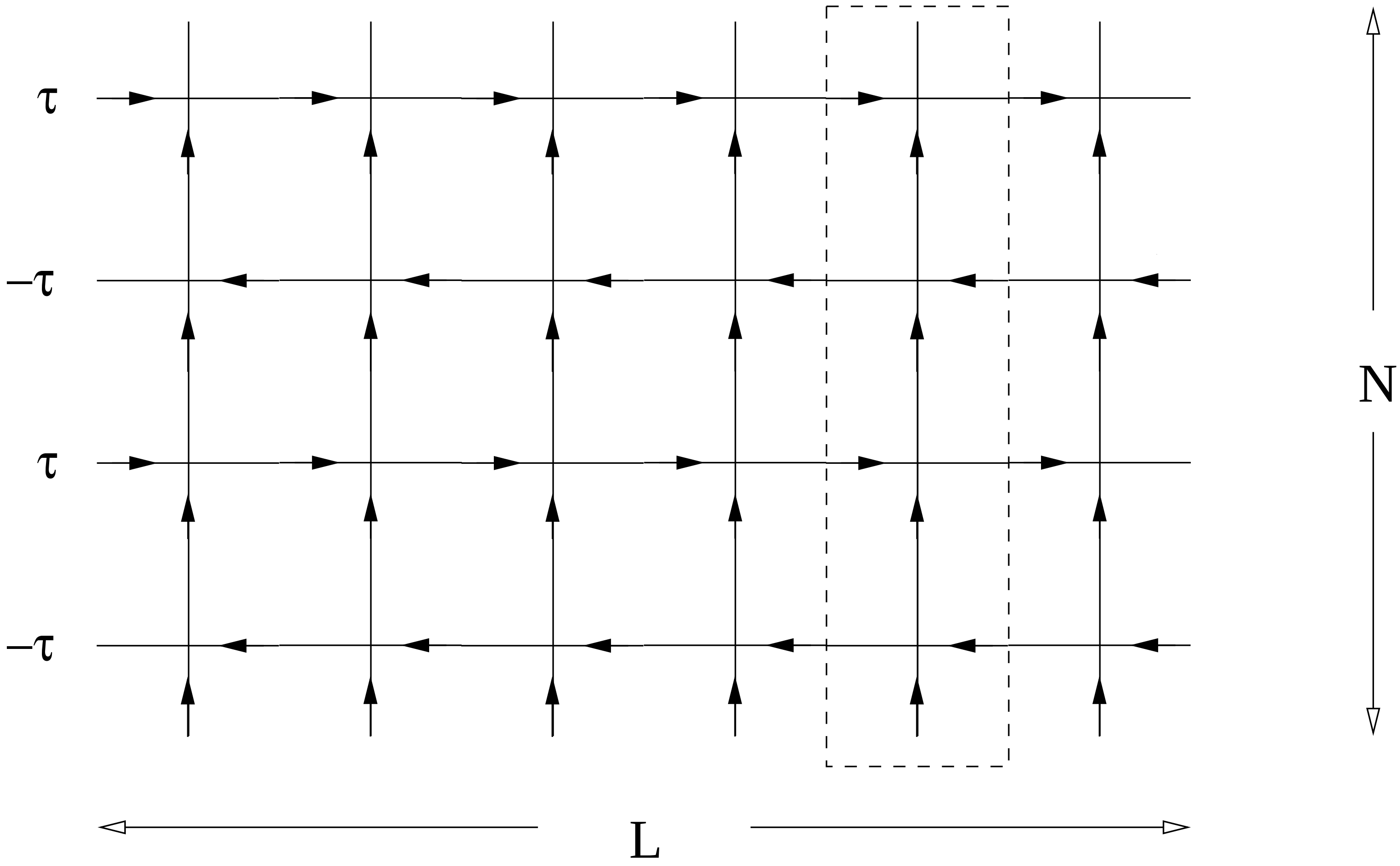
Kozlowski, Maillet, Slavnov 2010:
multiple integral formula for density-density correlations directly for continuum Bose gas
multiple integrals, resummation, asymptotic expansion $\simeq$ sum over all states of QTM

Here: finite number of equations for multi-component case
by analysis of leading eigenvalues of quantum transfer matrix
different set(s) of non-linear integral equations

Problem: quantum transfer matrix does not exist for continuum models

Quantum transfer matrix for lattice models

$\text{Tr}e^{-\beta H} = \text{Tr}\, T_{QTM}^{L} =$



$\tau = \beta/N$

free energy from largest eigenvalue of column-to-column transfer matrix (quantum transfer matrix, QTM)

Continuum limit of the spin-1/2 Heisenberg I

Continuum limit of the spin-1/2 XXZ chain close to the ferromagnetic XXX point: one- and two-particle data.

One-particle energy and momentum (with scales J and $1/\delta$ =1/lattice constant) and two-particle scattering:

$$E_m(v) = \frac{2J\mathrm{sh}^2\eta}{\mathrm{sh}(v+\eta/2)\mathrm{sh}(v-\eta/2)} + h\,, \quad P_m(v) = -\frac{\mathrm{i}}{\delta}\ln\frac{\mathrm{sh}(v-\eta/2)}{\mathrm{sh}(v+\eta/2)}, \qquad S(v_j,v_l) = \frac{\mathrm{sh}(v_j-v_l-\eta)}{\mathrm{sh}(v_j-v_l+\eta)}$$

For small v and ε where $\eta = \mathrm{i}\gamma = \mathrm{i}(\pi-\varepsilon)$:

$$E_m(v) = 2J\delta^2\left[\left(\frac{\varepsilon v}{\delta}\right)^2 - \left(\frac{\varepsilon}{\delta}\right)^2\right] + h, \qquad P_m(v) = \frac{\varepsilon}{\delta}v, \qquad S(v_j,v_l) = \frac{\frac{\varepsilon}{\delta}v_j - \frac{\varepsilon}{\delta}v_l + \mathrm{i}\frac{\varepsilon^2}{\delta}}{\frac{\varepsilon}{\delta}v_j - \frac{\varepsilon}{\delta}v_l - \mathrm{i}\frac{\varepsilon^2}{\delta}}$$

Now we demand for the continuum limit ($\delta = l/L \to 0$) the finiteness of

$$\frac{\varepsilon}{\delta}v = k, \qquad 2J\delta^2 = 1\left(=\frac{1}{2m_B}\right), \qquad \frac{\varepsilon^2}{\delta} = c, \qquad \left(\frac{\varepsilon}{\delta}\right)^2 - h = \mu$$

Yields one and two-particle data of single component Bose gas.

Continuum limit of the spin-1/2 Heisenberg II

Continuum limit of the spin-1/2 XXZ chain close to the ferromagnetic XXX point

XXZ chain (5 parameters)		Bose gas (4 parameters)
lattice constant δ		
number of lattice sites L	$L = \ell/\delta$	physical length ℓ
interaction strength $J > 0$	$J = 1/(4\delta^2 m_B)$	particle mass $m_B (= 1/2)$
magnetic field $h > 0$	$h = c/\delta - \mu$	chemical potential μ
anisotropy $\Delta = \varepsilon^2/2 - 1$	$\varepsilon^2 = c\delta$	repulsion strength c

(Seel, Bhattacharyya, Göhmann, AK (2007))

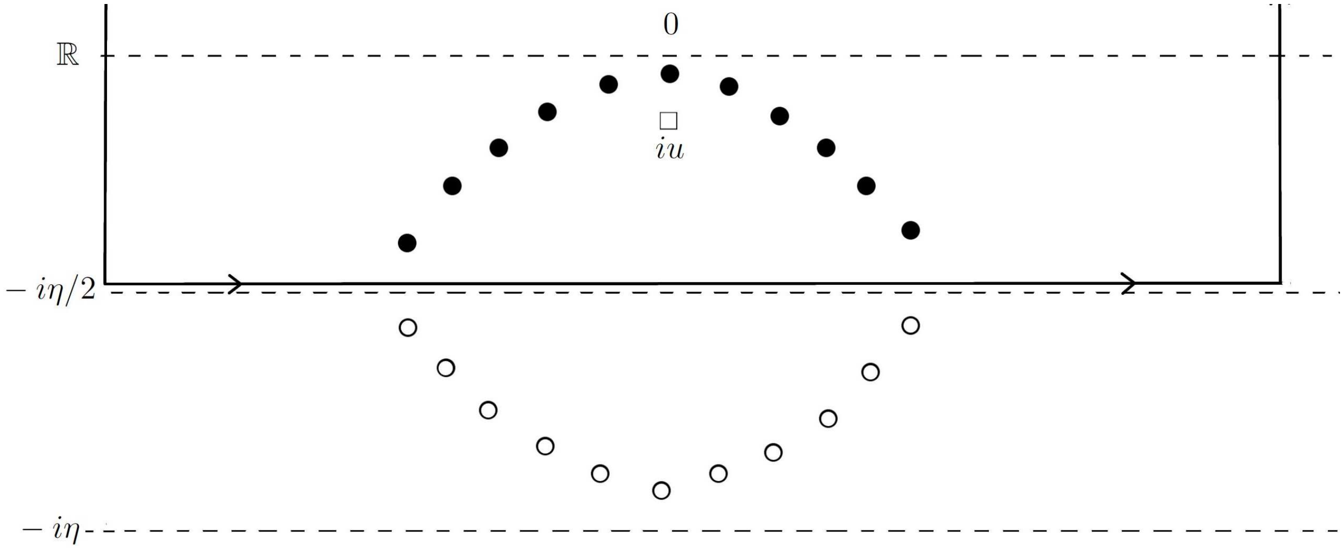
Thermodynamical potential by auxiliary function $\mathfrak{a}(v)$ from non-linear integral equation

$$\ln \mathfrak{a}(v) = -\frac{h}{T} + \frac{2J\mathrm{sh}^2(\mathrm{i}\varepsilon)}{T\mathrm{sh}v\,\mathrm{sh}(v-\mathrm{i}\varepsilon)} + \int_C \frac{dw}{2\pi\mathrm{i}} \frac{\mathrm{sh}(2\mathrm{i}\varepsilon)}{\mathrm{sh}(v-w+\mathrm{i}\varepsilon)\mathrm{sh}(v-w-\mathrm{i}\varepsilon)} \ln(1+\mathfrak{a}(w))$$

$$\frac{F_{XXZ}}{L} = \frac{h}{2} - T\int_C \frac{dw}{2\pi\mathrm{i}} \frac{\mathrm{sh}(\mathrm{i}\varepsilon)}{\mathrm{sh}w\,\mathrm{sh}(w-\mathrm{i}\varepsilon)} \ln(1+\mathfrak{a}(w))$$

continuum limit: drop line Im $v = \varepsilon/2$, keep line Im $v = -(\pi-\varepsilon)/2$

continuum limit: drop line Im $v = \varepsilon/2$, keep line Im $v = -(\pi - \varepsilon)/2$



Define $\varepsilon := -T \log \mathfrak{a}$ on axis Im$v = -(\pi - \varepsilon)/2$, NLIE

$$\frac{\varepsilon(k)}{T} = \frac{k^2 - \mu}{T} - \frac{1}{2\pi} \int_{\mathbb{R}} K(k - k') \log\left(1 + e^{-\varepsilon(k')/T}\right) dk' .$$

Seel, Bhattacharyya, Göhmann, AK 2007: free energy and correlation functions!

Seel, Göhmann, AK 2008: Fredholm determinant

1-component Bose gas: thermodynamics

NLIEs for the excited states with particle and hole excitations

$$\frac{\varepsilon_x(k)}{T} = \frac{k^2-\mu}{T} + \mathrm{i}\sum_{j=1}^{h}\theta(k-k_j^+) - \mathrm{i}\sum_{j=1}^{p}\theta(k-k_j^-) - \frac{1}{2\pi}\int_{\mathbb{R}} K(k-k')\log\left(1+e^{-\varepsilon_x(k')/T}\right)dk'$$

where $\theta(k) = i\log\left(\frac{ic+k}{ic-k}\right)$, $\theta'(k) = K(k)$, and $k_j^{\pm}$ have to satisfy the subsidiary conditions

$$1+e^{-\varepsilon_x(k_j^{\pm})/T} = 0.$$

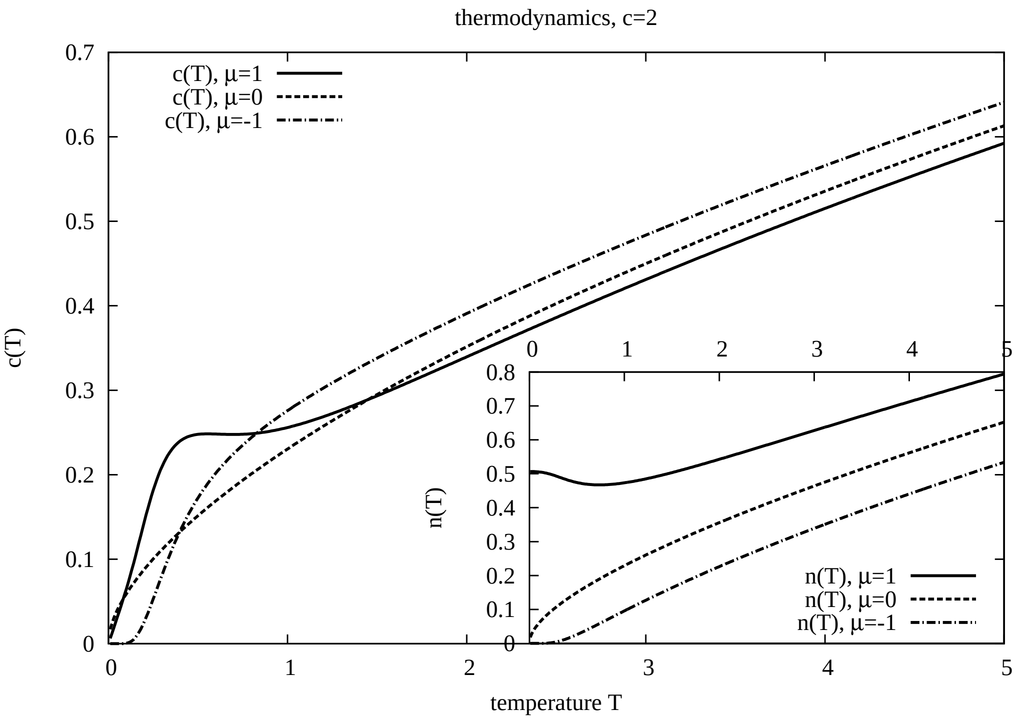
Then the correlation length is given by

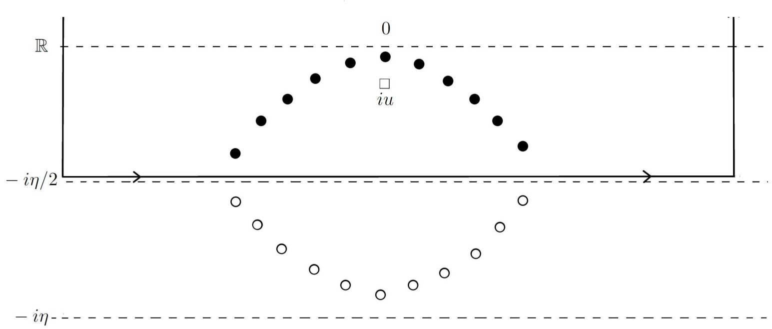
$$\frac{1}{\xi} = \frac{1}{2\pi}\int_{\mathbb{R}}\log\left(\frac{1+e^{-\varepsilon(k)/T}}{1+e^{-\varepsilon_x(k)/T}}\right)dk - \mathrm{i}\sum_{j=1}^{h}k_j^+ + \mathrm{i}\sum_{j=1}^{p}k_j^-$$

(see also Kozlowski, Maillet, Slavnov 2011 for density-density correlation)

1-component Bose gas: thermodynamics

Specific heat and particle density in grand-canonical ensemble

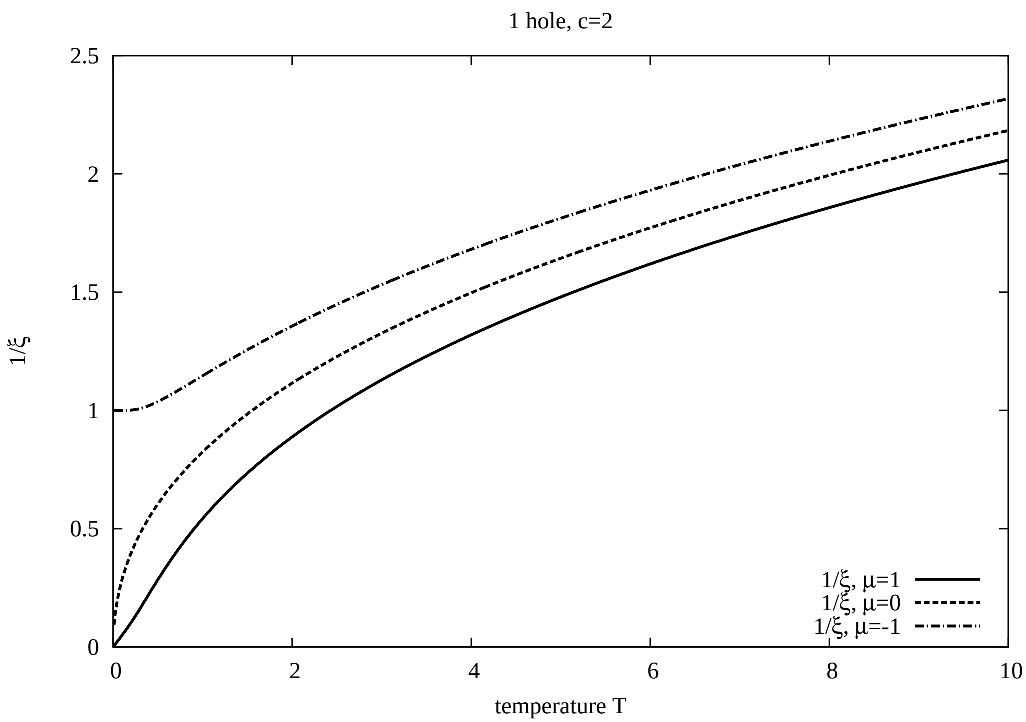


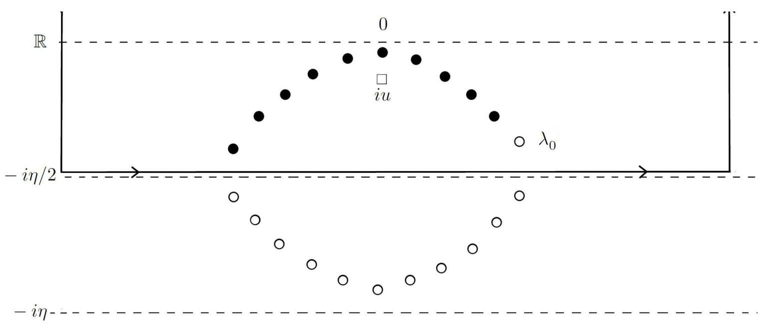


- $\mu < 0$: activated behaviour, $c(T), n(T) \simeq \exp(\mu/T)$,
- $\mu = 0$: critical value, $c(T), n(T) \simeq T^{1/2}$,
- $\mu > 0$: CFT, $c(T) \simeq T$, $n(T) \simeq const.$

1-component Bose gas: Greens function $\langle \Psi^+(x)\Psi(0)\rangle$

Exponential vs. algebraic decay of correlators at $T=0 \Leftrightarrow$ finite vs. divergent $\xi(T)$





- $\mu < 0$: gap, $1/\xi(T)$ finite,
- $\mu = 0$: critical value, $1/\xi(T) \simeq T^{1/2}$,
- $\mu > 0$: CFT, $1/\xi(T) \simeq 2\pi \dfrac{T}{v}\dfrac{1}{4\mathcal{Z}^2}$, no k_F oscillations! (AK, Patu 2013)

1-component Bose gas: Limiting cases

The impenetrable limit, $c \to \infty$: Its, Izergin and Korepin (1993)

Greens function

$$\mu < 0: \qquad \frac{1}{\xi} = \frac{1}{2\pi} \int_{\mathbb{R}} \log \left(\frac{e^{(k^2-\mu)/T} + 1}{e^{(k^2-\mu)/T} - 1} \right) dk + \sqrt{|\mu|},$$

$$\mu > 0: \qquad \frac{1}{\xi} = \frac{1}{2\pi} \int_{\mathbb{R}} \log \left(\frac{e^{(k^2-\mu)/T} + 1}{e^{(k^2-\mu)/T} - 1} \right) dk - i\sqrt{\mu} = \frac{1}{2\pi} \int_{\mathbb{R}} \log \left| \frac{e^{(k^2-\mu)/T} + 1}{e^{(k^2-\mu)/T} - 1} \right| dk,$$

General $c > 0$, $\mu > 0$, low-temperature limit: CFT

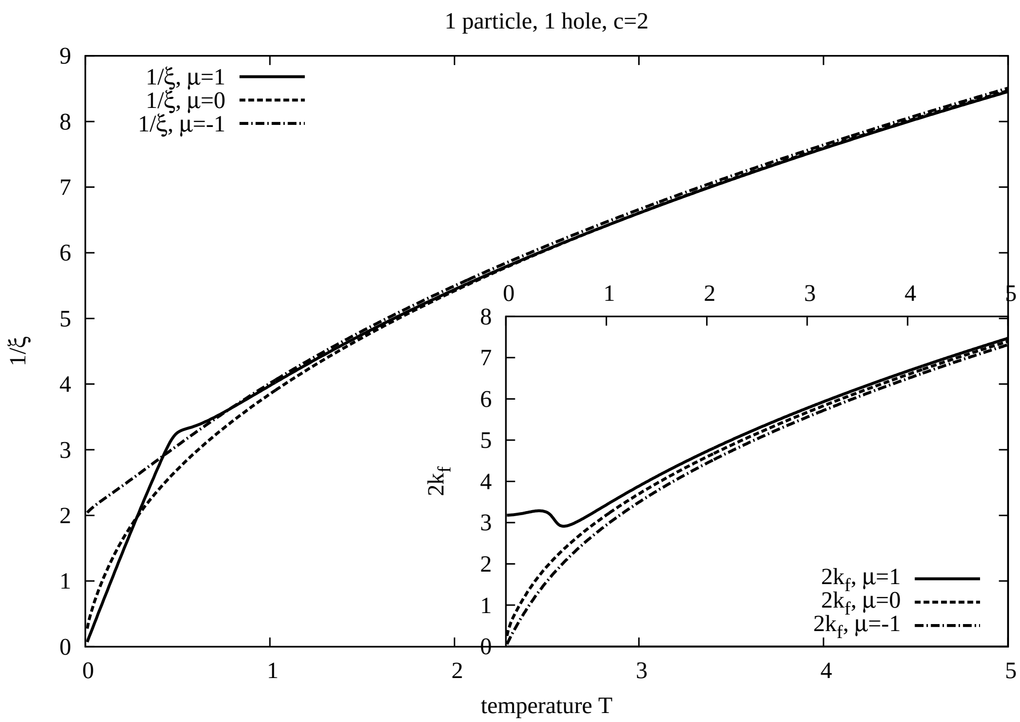
dressed energy and charge formalism, Tomonaga-Luttinger liquid

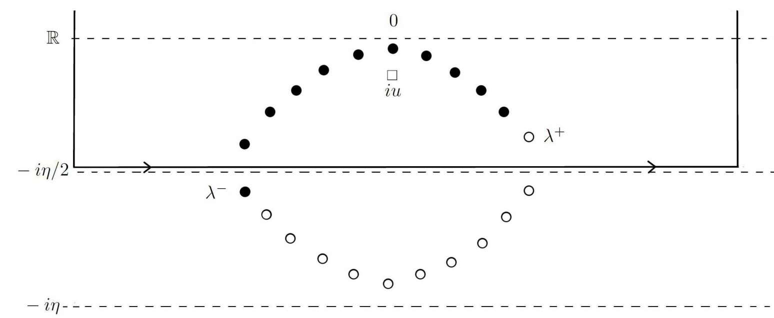
XXZ chain: general, AK, Scheeren 2003

Bose gas, density-density correlation: Kozlowski, Maillet, Slavnov 2011

1-component Bose gas: density-density correlator $\langle n(x)n(0)\rangle$

Exponential vs. algebraic decay of correlators at $T=0 \Leftrightarrow$ finite vs. divergent $\xi(T)$





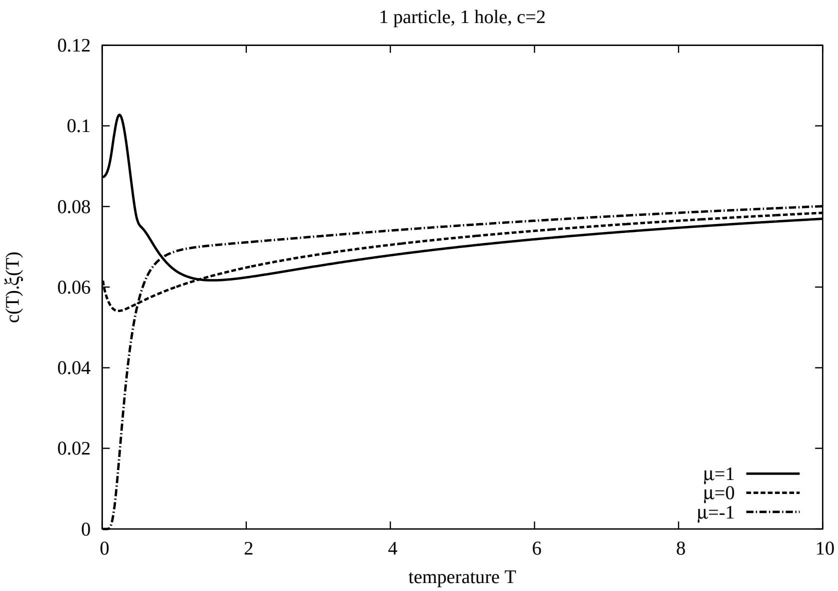
- $\mu < 0$: gap, $1/\xi(T)$ finite,
- $\mu = 0$: critical value, $1/\xi(T) \simeq T^{1/2}$,
- $\mu > 0$: CFT, $1/\xi(T) \simeq 2\pi \frac{T}{v} Z^2$, $2k_F$ oscillations, $Z > 1$

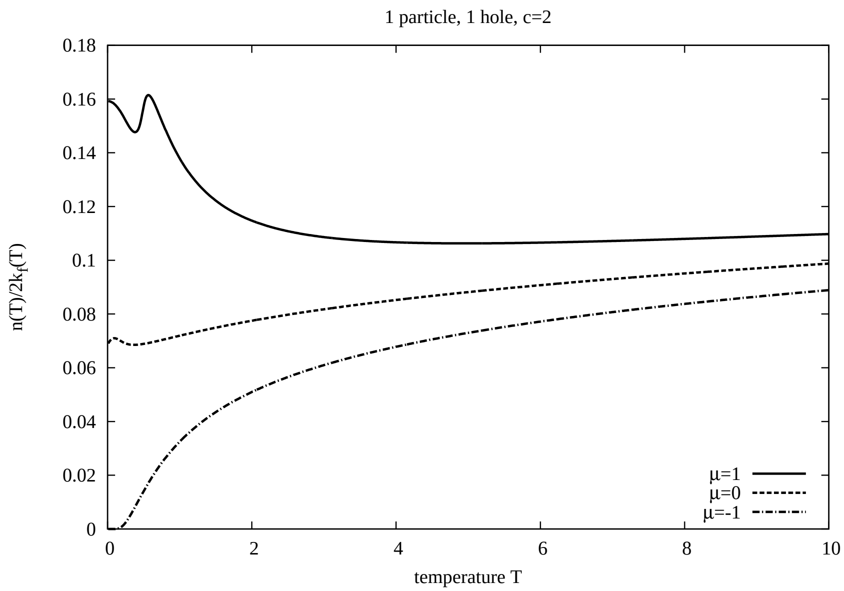
1-component Bose gas: density-density correlator

Low-temperature asymptotics

$$\frac{1}{\xi(T)} \simeq 2\pi \frac{T}{v} x, \quad k_F(T) \simeq \pi n(T), \quad c(T) \simeq \frac{\pi}{3}\frac{T}{v} \cdot 1,$$

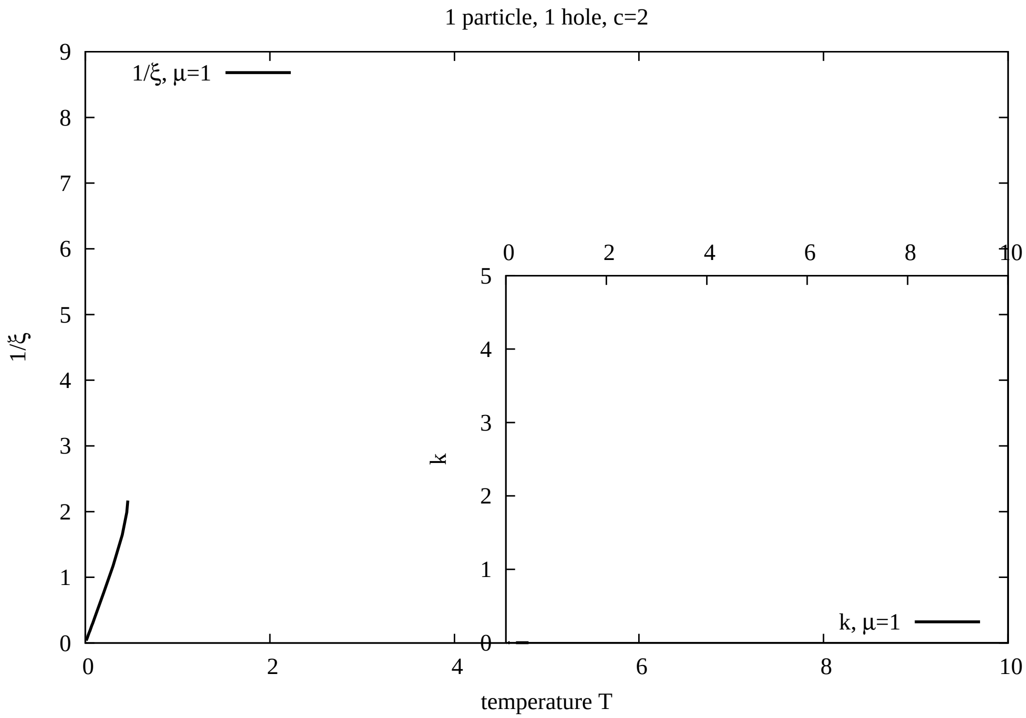
but no smooth behaviour of $c(T) \cdot \xi(T)$ and $n(T)/k_F(T)$:

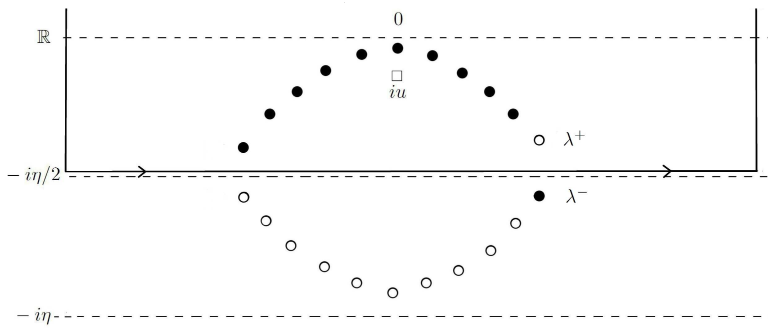




1-component Bose gas: density-density correlator $\langle n(x)n(0)\rangle$ I

Leading contribution: non-oscillating at low T

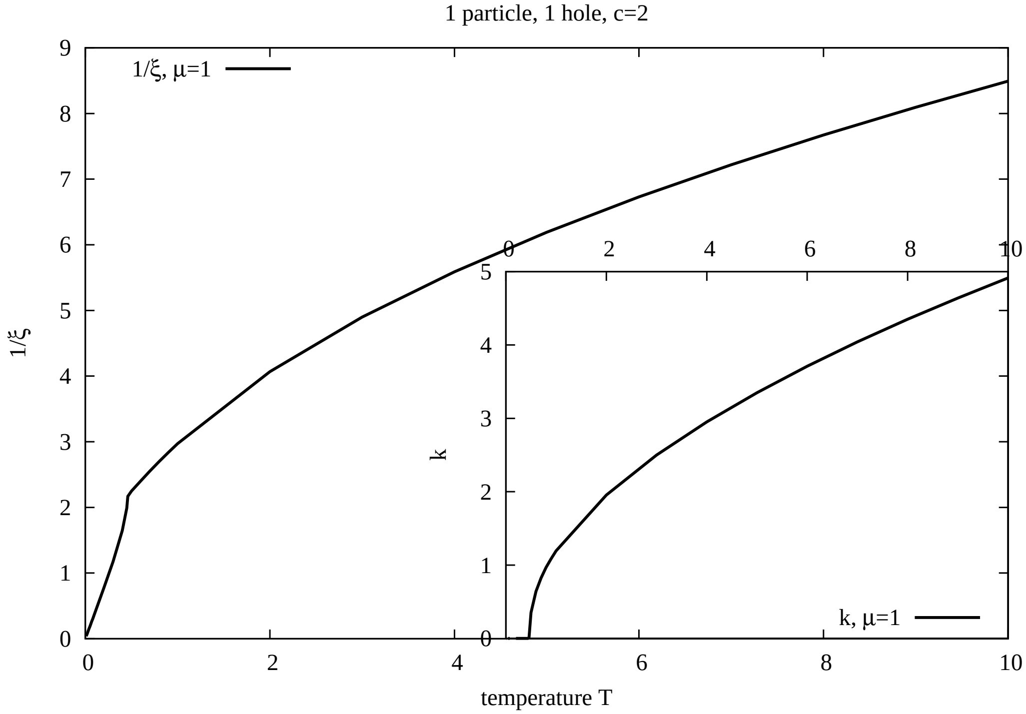


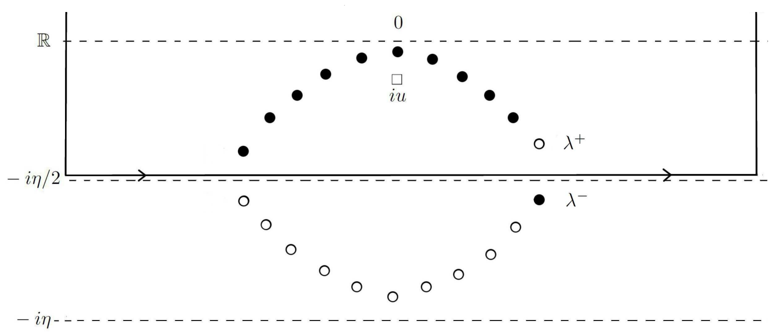


- $\mu > 0$: CFT, $1/\xi(T) \simeq 2\pi \frac{T}{v}$
- no consistent solution at higher T

1-component Bose gas: density-density correlator $\langle n(x)n(0)\rangle$ II

Leading contribution: crossover from non-oscillating to oscillating behaviour at higher T

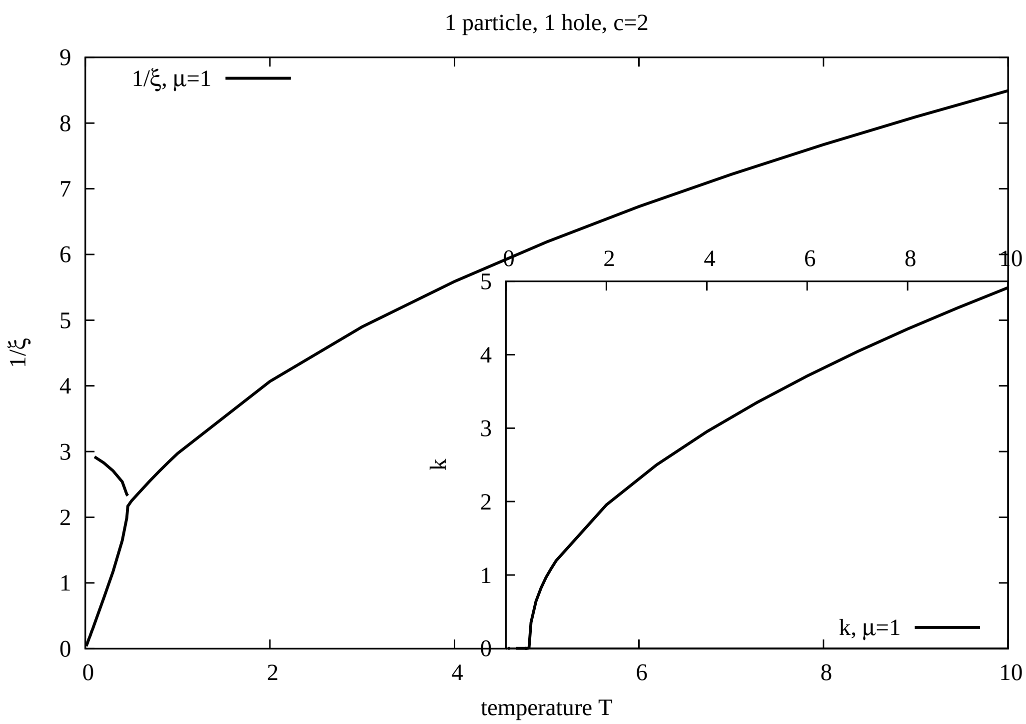


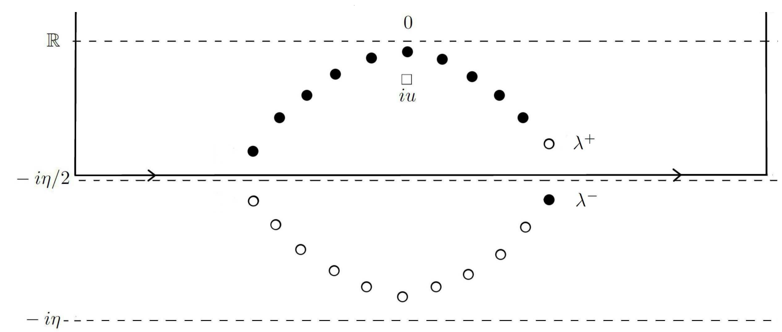


- $\mu > 0$: CFT, $1/\xi(T) \simeq 2\pi \frac{T}{v}$
- similarities to incomm. oscill. in attractive spin-1/2 XXZ, $T \in [T_L, T_U]$, $h = 0$

1-component Bose gas: density-density correlator $\langle n(x)n(0)\rangle$ III

Leading contribution: crossover from non-oscillating to oscillating behaviour at higher T





- $\mu > 0$: CFT, $1/\xi(T) \simeq 2\pi \dfrac{T}{v}$
- similarities to incomm. oscill. in attractive spin-1/2 XXZ, $T \in [T_L, T_U]$, $h = 0$

Temperature dependent spatial oscillations in XXZ I

Crossover behaviour for $-1 < \Delta < 0$ (Fabricius, AK, McCoy, PRL 1999)

$$H = \frac{1}{2}\sum_{j=0}^{L-1}(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y + \Delta\sigma_j^z\sigma_{j+1}^z).$$

Longitudinal correlations for zero temperature

$$S^z(n; T = 0, \Delta) \sim -\frac{1}{\pi^2\theta n^2} + (-1)^n \frac{C(\Delta)}{n^{\frac{1}{\theta}}}, \qquad \cos\pi\theta = -\Delta.$$

At finite temperature

$$S^z(n; T, \Delta) = \sum_j A_j \left(\frac{\Lambda_j}{\Lambda_0}\right)^{n-1}$$

$T < T_L(\Delta)$	leading Λ_j/Λ_0 real > 0, $A_j < 0$
$T_L(\Delta) < T < T_U(\Delta)$	leading Λ_j/Λ_0 and A_j complex
$T_U(\Delta) < T$	leading Λ_j/Λ_0 real > 0, $A_j > 0$
ex.: $\Delta = -0.5$	$T_L(\Delta) = 0.2015$ and $T_U(\Delta) = 0.4328$

Temperature dependent spatial oscillations in XXZ II

Phenomenological picture: **excitations of QTM**

$$\text{`energy'}\ \mathcal{E} \quad \frac{\Lambda_j}{\Lambda_0} = \exp(-\mathcal{E}), \qquad \text{`momentum'}\ \mathcal{K}$$

free states

$$\begin{aligned} \mathcal{E} &= \varepsilon_1 + \varepsilon_2 \\ \mathcal{K} &= \kappa_f(\varepsilon_1) + \kappa_f(\varepsilon_2) \end{aligned}$$

$$\kappa_f(\varepsilon) = \pm v \sinh \varepsilon$$

matrix element/coefficient $A_f > 0$

bound states

$$\begin{aligned} \mathcal{E} &= \varepsilon \\ \mathcal{K} &= \kappa_b(\varepsilon) \end{aligned}$$

$$\kappa_b(\varepsilon) = \mp v\, 2 \sinh\frac{\varepsilon}{2} \sqrt{1 - a^2 \sinh^2\frac{\varepsilon}{2}}$$

matrix element/coefficient $A_b < 0$

What is the meaning of $\mathcal{E}, \mathcal{K}$ / physics at low T?

system is dominated by bound state, calculate ε from $\kappa_b(\varepsilon) = \frac{2\pi}{\beta} = 2\pi T$

small $T \longrightarrow$ real ε, large $T \longrightarrow$ complex ε

change defines lower crossover temperature T_L $\leftrightarrow$ (Sakai, Shiroishi, Suzuki, Umeno 99)

Thermodynamics: two-component Bose gas

$n = 2$ previous results

Eisenberg, Lieb (2002):

'ferromagnetism' for spin-independent interacting multi-component Bose gases

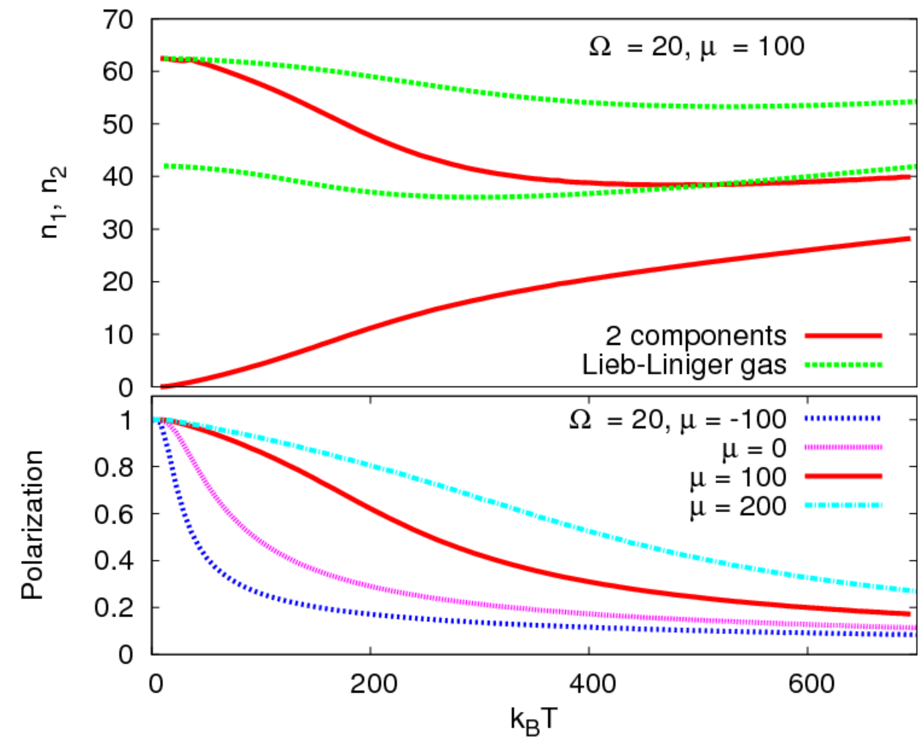
magnetic field $\Omega = (\mu_1 - \mu_2)/2$, magnetization $P = (n_1 - n_2)/2$

Guan, Batchelor, Takahashi (2007): analytical low-temperature asymptotics

Caux, Klauser, van den Brink (2009, 2011):

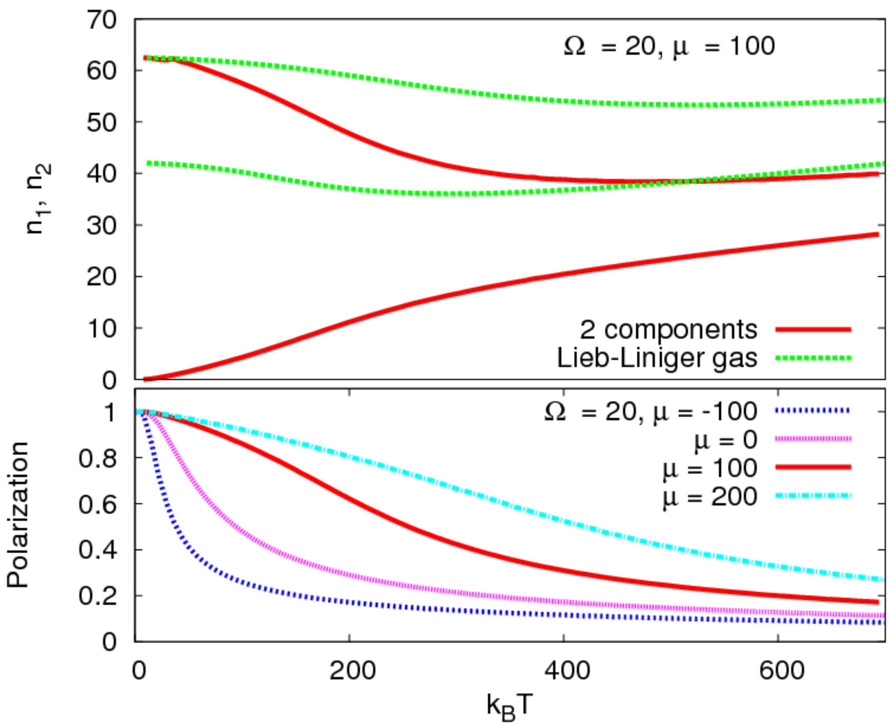
numerical solution of TBA for relatively large Ω

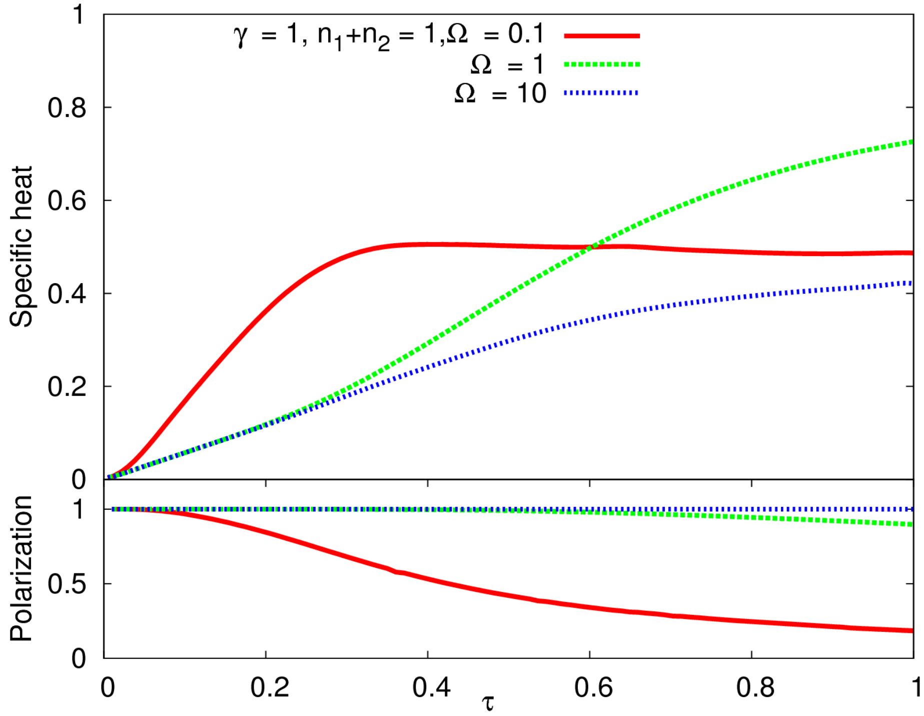
$(\hbar = 1,\ 2m = 1)$



TBA for two-component Bose gas

Caux, Klauser, van den Brink (2009, 2011):





TBA problematic at low T and small Ω:

similar to problems appearing in ferromagnetic spin-1/2 Heisenberg TBA

Continuum limit of the 3-state Uimin-Sutherland model

Quantum chain: Uimin-Sutherland model (1970, 1975)$\leftrightarrow$ classical model: Perk-Schultz model (1981)

$$R_{\alpha\alpha}^{\alpha\alpha}(v) = \frac{\sin(\gamma+\varepsilon_\alpha v)}{\sin\gamma} \qquad R_{\alpha\beta}^{\alpha\beta}(v) = \varepsilon_{\alpha\beta}\frac{\sin v}{\sin\gamma}, \quad (\alpha\neq\beta) \qquad R_{\alpha\beta}^{\beta\alpha}(v) = e^{i\,\mathrm{sign}(\alpha-\beta)v}, \quad (\alpha\neq\beta).$$

The largest eigenvalue of the QTM for the n-state model has the form $\Lambda_{QTM}(v) = \sum_{j=1}^{n}\lambda_j(v)$ with

$$\lambda_j(v) = \phi_-(v)\phi_+(v)\frac{q_{j-1}(v-i\varepsilon_j\gamma)}{q_{j-1}(v)}\frac{q_j(v+i\varepsilon_j\gamma)}{q_j(v)}e^{\beta\mu_j}$$

where $\phi_\pm(v) = \left(\frac{\sinh(v\pm iu)}{\sin\gamma}\right)^{N/2}$, $u=\frac{\beta}{N}$ and $q_j(v) = \begin{cases}\phi_-(v) & j=0\\ \prod_{r=1}^{N/2}\sinh(v-v_r^{(j)}) & j=1,2,\cdots,n-1\\ \phi_+(v) & j=n\end{cases}$

Transformation to non-linear integral equations:

(i) ∞-many of TBA type $\to$ still ∞-many functions in continuum limit,
(ii) finite number based on excitations on physical vacuum $\to$ even fewer functions in continuum limit,
(iii) Takahashi's 1999/2000 approach

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(i) Jüttner, AK, Suzuki 1998, (ii) Fujii, AK 1999

Derivation of integral equations

Two sets of Bethe ansatz equations for 3-state model (with $\gamma = \pi - \varepsilon$) written as two contour integrals

$$\log a_1(v) = \beta(\mu_1 - \mu_3) + \beta\frac{\mathrm{sh}^2(\mathrm{i}\varepsilon)}{\mathrm{sh}v\,\mathrm{sh}(v-\mathrm{i}\varepsilon)} + \frac{1}{2\pi\mathrm{i}}\int_C \frac{\mathrm{sh}(2\mathrm{i}\varepsilon)}{\mathrm{sh}(v-w-\mathrm{i}\varepsilon)\mathrm{sh}(v-w+\mathrm{i}\varepsilon)}\log(1+a_1(w))dw$$

$$-\frac{1}{2\pi\mathrm{i}}\int_C \frac{\mathrm{sh}(\mathrm{i}\varepsilon)}{\mathrm{sh}(v-w-\mathrm{i}\varepsilon)\mathrm{sh}(v-w)}\log(1+a_2(w))dw$$

$$\log a_2(v) = \beta(\mu_2 - \mu_3) + \beta\frac{\mathrm{sh}^2(\mathrm{i}\varepsilon)}{\mathrm{sh}v\,\mathrm{sh}(v+\mathrm{i}\varepsilon)} + \frac{1}{2\pi\mathrm{i}}\int_C \frac{\mathrm{sh}(\mathrm{i}\varepsilon)}{\mathrm{sh}(v-w+\mathrm{i}\varepsilon)\mathrm{sh}(v-w)}\log(1+a_1(w))dw$$

$$-\frac{1}{2\pi\mathrm{i}}\int_C \frac{\mathrm{sh}(2\mathrm{i}\varepsilon)}{\mathrm{sh}(v-w-\mathrm{i}\varepsilon)\mathrm{sh}(v-w+\mathrm{i}\varepsilon)}\log(1+a_2(w))dw$$

where $a_1(v)$ and $a_2(v)$ are ‘lhs/rhs’ of the Bethe equations.

C is closed contour around real axis (in total 4 straight integration paths)

$\rightarrow$ in continuum limit only upper or lower part contributes $\rightarrow$ 2 equations

Integral equations for 2-component Bose gas

Thermodynamical potential density

$$g = -\frac{T}{2\pi}\int_{-\infty}^{\infty} dk \ln[(1+a_1(k))(1+a_2(k))]$$

where a_1 and a_2 satisfy

$$\ln a_1 = -\beta(k^2-\mu-\Omega)+\kappa_2 * \ln(1+a_1)+\kappa_1^+ * \ln(1+a_2),$$
$$\ln a_2 = -\beta(k^2-\mu+\Omega)+\kappa_1^- * \ln(1+a_1)+\kappa_2 * \ln(1+a_2),$$

where

$$\kappa_2(k) = \frac{1}{\pi}\frac{c}{k^2+c^2}. \qquad \kappa_1(k) = \frac{1}{\pi}\frac{c/2}{k^2+(c/2)^2}, \qquad \kappa_1^{\pm}(k) = \kappa_1(k \pm \mathrm{i}c/2)$$

(AK, Patu 2011)

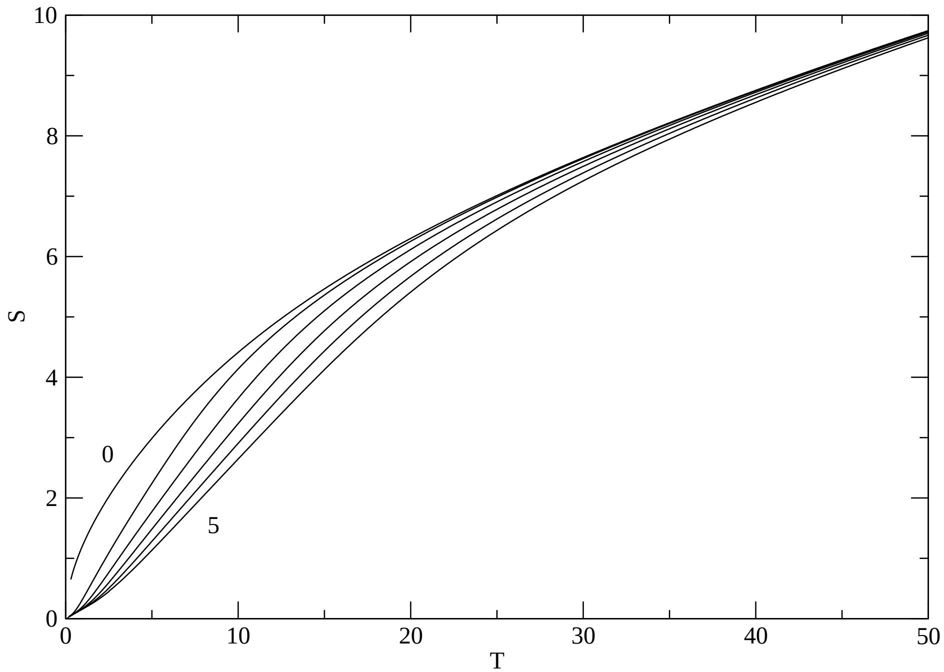
Results: entropy, specific heat

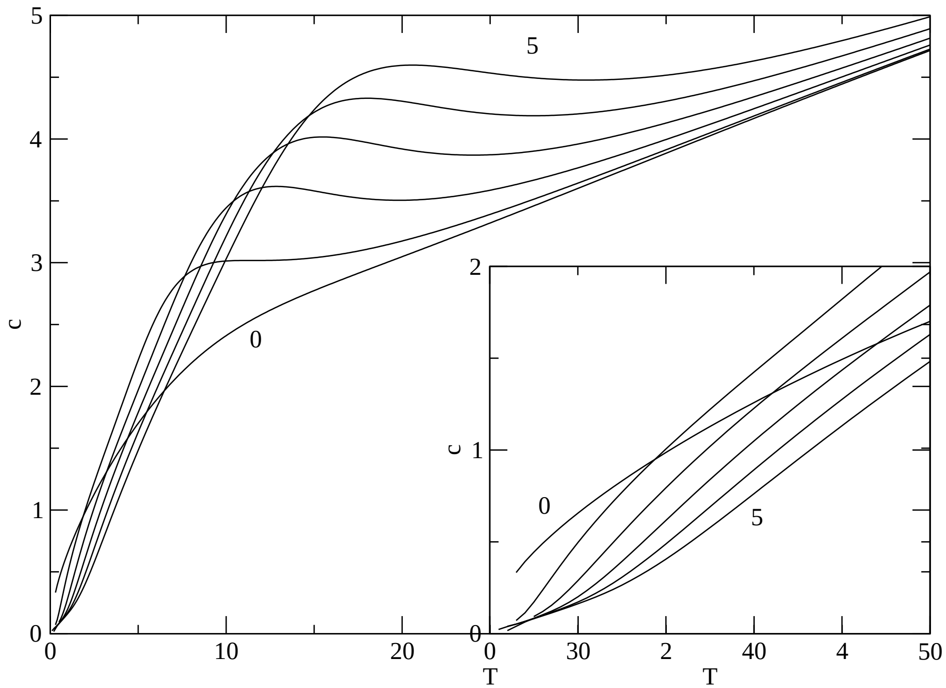
interaction $c = 1$

chemical potentials $\mu_1 = 15 + \Omega$, $\mu_2 = 15 - \Omega$

$\Omega = 0, 1, 2, 3, 4, 5$

entropy and specific heat





Note: square root dependence on T for $\Omega = 0$, linear behaviour for $\Omega \neq 0$

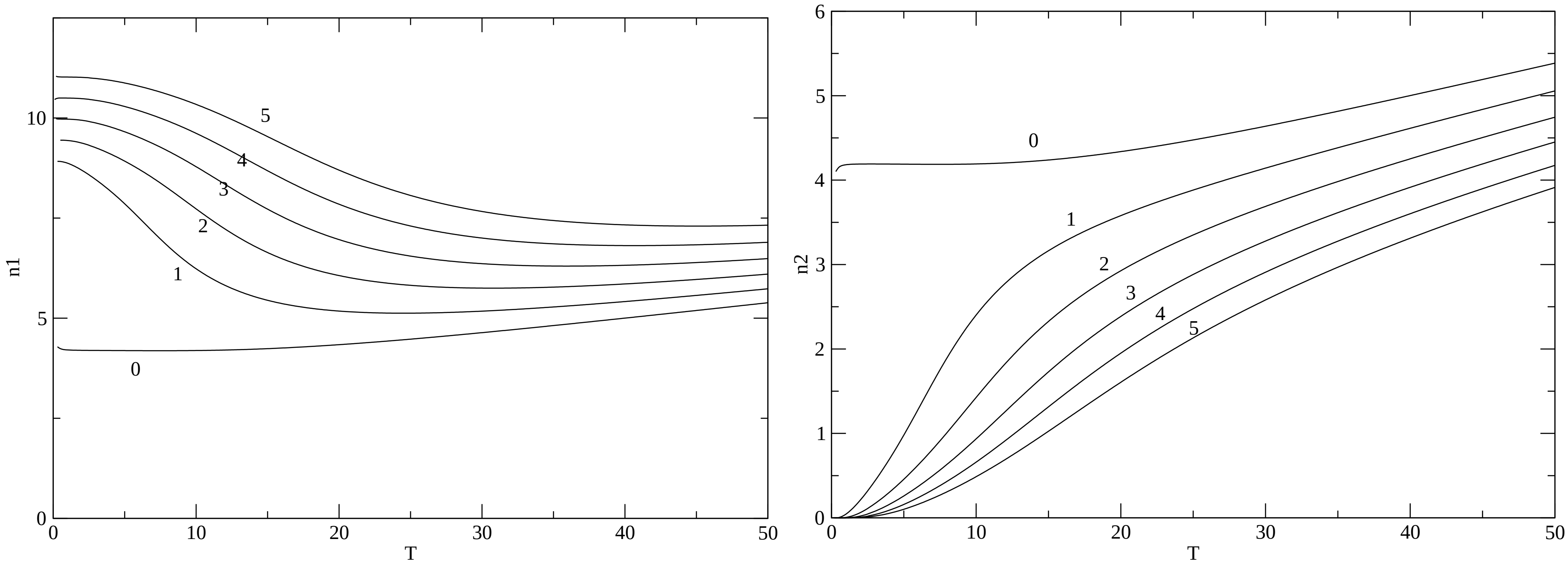
Results: particle densities

interaction $c = 1$

chemical potentials $\mu_1 = 15 + \Omega$, $\mu_2 = 15 - \Omega$

$\Omega = 0, 1, 2, 3, 4, 5$

particle densities n_1, n_2



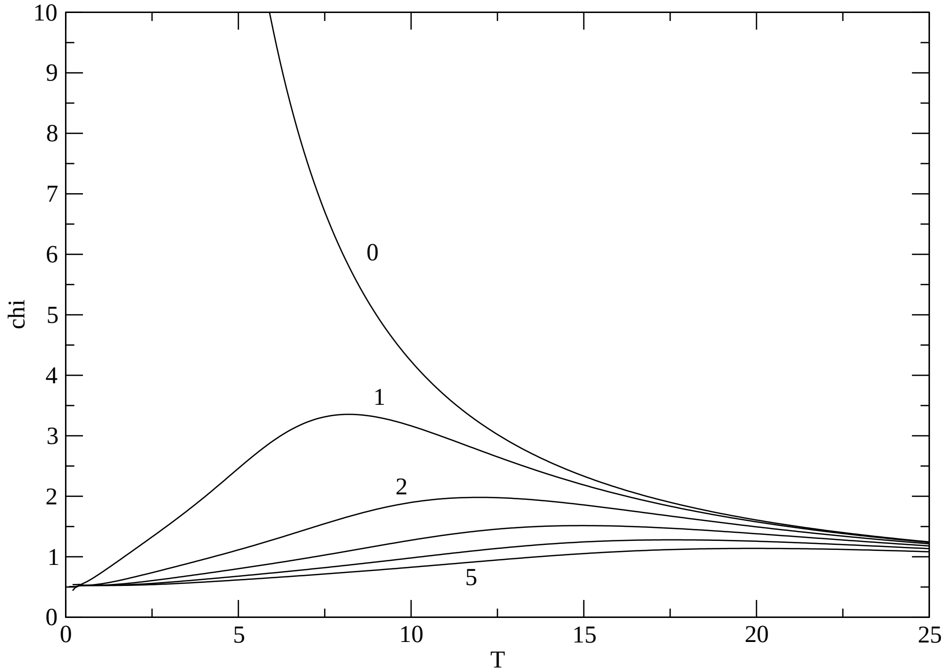
Note: continuous dependence on Ω for $T > 0$, jump at $\Omega = 0$ for $T = 0$

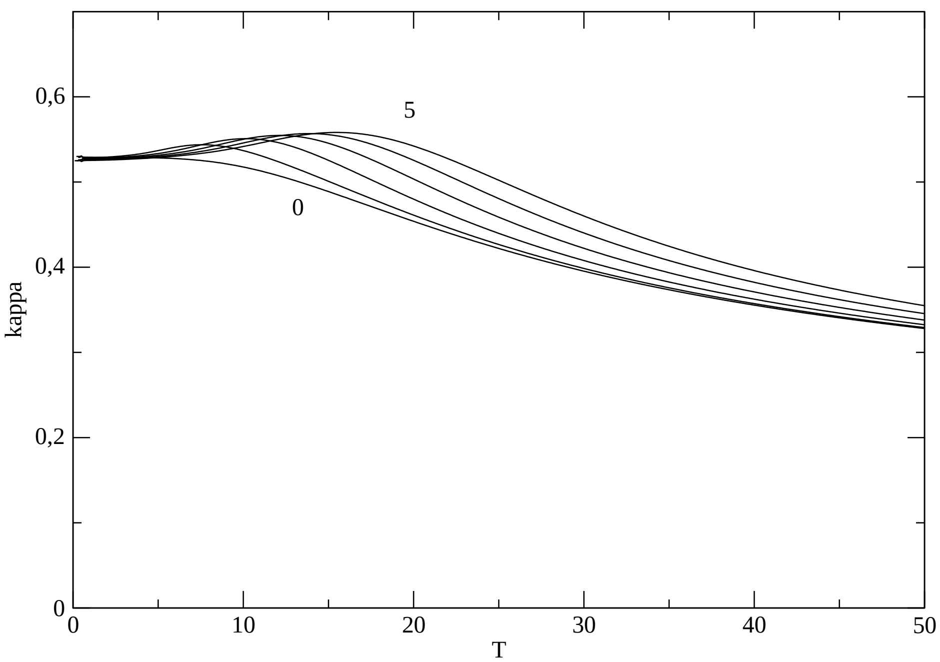
Results: susceptibilities

interaction $c = 1$

chemical potentials $\mu_1 = 15 + \Omega$, $\mu_2 = 15 - \Omega$

$\Omega = 0, 1, 2, 3, 4, 5$ magnetic and particle susceptibilities χ, κ





Note: for $\Omega = 0$: divergence of χ for $\Omega \neq 0$: $\chi(0) = \kappa(0)$

Integral equations for excited states

Correlation length ξ of the field-field correlation function $\langle \Psi_1^\dagger(x)\Psi_1(0)\rangle \sim e^{-x/\xi}$

$$\frac{1}{\xi} = \ln\left(\frac{\Lambda_0}{\Lambda_1}\right)$$

where $k_{0,1}$ are the largest and next-largest eigenvalue of the “continuum" QTM.

We find $\ln k_1 = ik_0 + \frac{1}{2\pi}\int \ln[A_1(k)A_2(k)]\,dk$, where

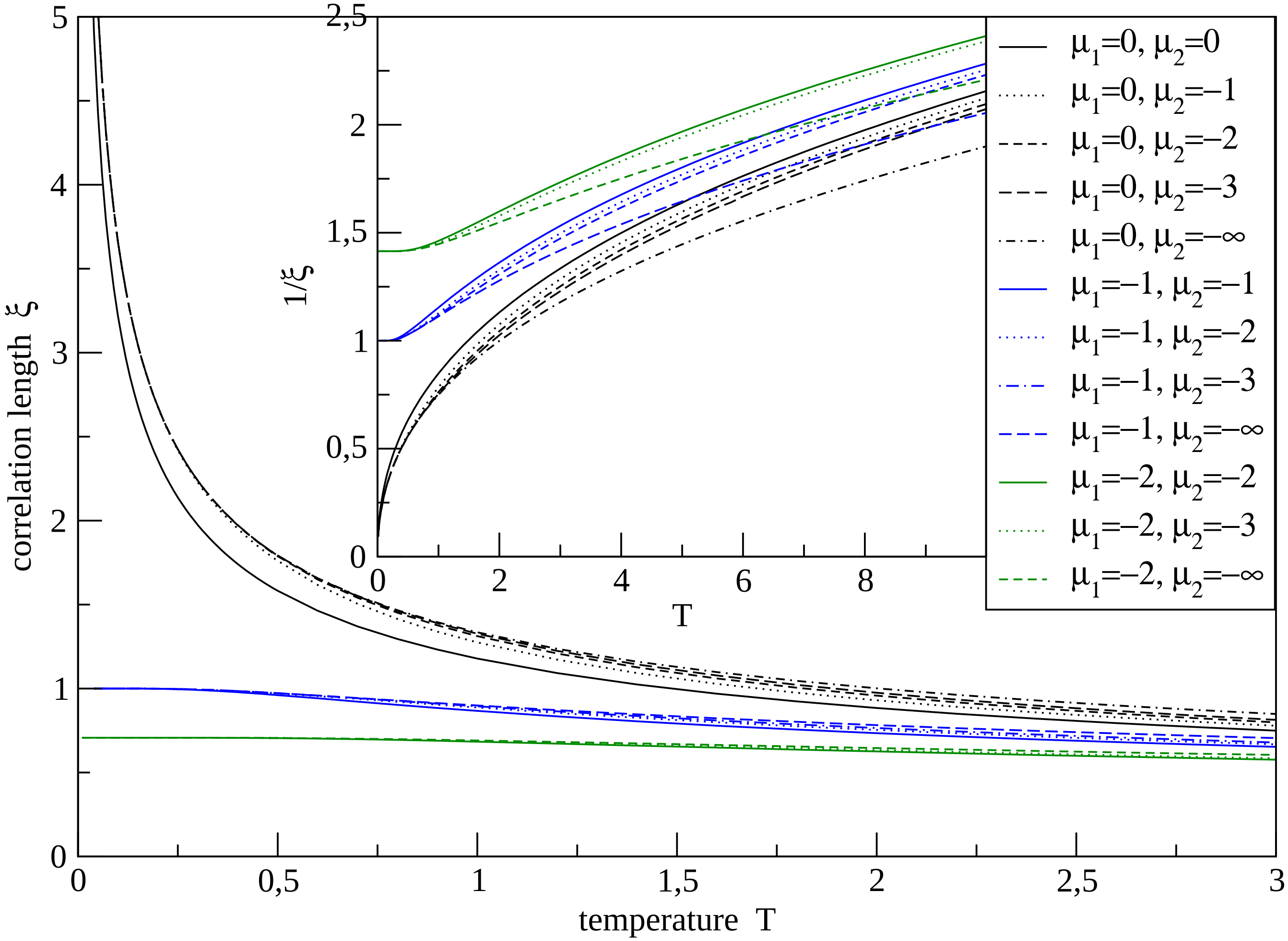
$$\ln a_1 = -\beta(k^2-\mu-\Omega) + \ln\left(\frac{k-k_0+ic}{k-k_0-ic}\right) \quad +\kappa_2 * \ln(1+a_1) + \kappa_1^+ * \ln(1+a_2),$$
$$\ln a_2 = -\beta(k^2-\mu+\Omega) \quad +\kappa_1^- * \ln(1+a_1) + \kappa_2 * \ln(1+a_2),$$

The rapidity k_0 is subject to the condition $1+a_1(k_0)=0$
appears to be purely imaginary in the dilute gas phase (μ_1, $\mu_2 \leq 0$).
(AK, Patu 2012)

These NLIE can be connected to the ∞ many TBA equations

Field-field correlators: correlation lengths

Correlation length ξ of the Green's function for particles of type 1 (interaction $c = 1$)



$T \to 0$ behaviour of ξ: finite for $\mu_1 < 0$ and $T^{-1/2}$ divergent for $\mu_1 = 0$.

Summary and outlook

- (i) study of 1- and 2-component Bose gas with δ-function interaction

  (ii) lattice discretization of the Bose gas by anisotropic vertex models

  (iii) derivation of alternative thermodynamical equations: closed set of 1 resp. 2 equations

  (iv) correlation lengths

  (v) numerical study

- Outlook

  (i) best numerical approach?

  (ii) more than 2 components?

  (iii) other correlation lengths?

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