Sub-leading order heavy quark potential and jet quench parameter from AdS/CFT

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Outlines

* Introductory review of gauge/string duality

* NL Heavy quark potential from AdS/CFT and heavy quarkonium melting

* NL Jet quenching parameter from AdS/CFT

* Summary

Zhang, Hou, Ren, JHEP1301 (2013) 03
Wu, Hou, Ren, PRC 87 (2013), 025203
Zhang, Hou, Ren, Yin, JHEP07:035 (2011)
Hou, Ren, JHEP01:029 (2008)
Chu, Hou, Ren, JHEP08: 004 (2009)
Many interesting phenomena in QCD lie in the strongly coupled region
Robust $v_2$ well described by hydro $\Rightarrow$ sQGP
sQGP seems to be the almost perfect fluid known $\eta/s = .1-.2 \ll 1$
New theoretical techniques needed!

**Lattice QCD**

difficulty with Finite baryon density, Real time dynamics

**Continuum**

(1) Phenomenological models: (p)NJL, (p)QMC…

(2) Field Theory: HD(T)L, pQCD, Chiral Perturbation, Renormalization Group

DS equations ….

(3) AdS/CFT
AdS/CFT Correspondence

4D Large-Nc strongly coupled SU(Nc) N=4 SYM (finite T).

Maldacena '97 \(\leftrightarrow\) conjecture \(\leftrightarrow\) Witten '98

Type II B Super String theory on AdS5-BH\(\times\)S5

(Adopted from S. Brodsky figure)
Maldacena conjecture: Maldacena, Witten

\[ N = 4 \text{ SUSY YM on the boundary } \iff \text{ TypeIIB string theory in the bulk} \]

\[ \lambda \equiv N_c g_{YM}^2 = \frac{1}{\alpha'^2} \quad \text{(string tension } = \frac{1}{2\pi\alpha'}) \]

\[ \frac{\lambda}{N_c} = 4\pi g_s \]

\[ <e^{\int d^4x \phi_0(x)O(x)}> = Z_{\text{string}}[\phi(x,0) = \phi_0(x)] \]

In the limit \( N_c \to \infty \) and \( \lambda \to \infty \)

\[ Z_{\text{string}}[\phi(x,0) = \phi_0(x)] = e^{-I_{\text{sugra}}[\phi]} \bigg|_{\phi(x,0)=\phi_0(x)} \]

\[ I_{\text{sugra}}[\phi] = \text{classical supergravity action} \]
AdS/CFT applied to heavy-ion physics

* Viscosity ratio, $\eta/s$.
  \[ \frac{\eta}{s} = \frac{1}{4\pi} \]
  Policastro, Son and Starinets

* Thermodynamics.
  \[ s = \frac{3}{4} s^{(0)} \]
  Gubser

* Jet quenching
  \[ \hat{Q} = \pi \frac{3}{2} \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \sqrt{\lambda} T^3 \]
  Liu, Rajagopal and Wiederman

* Photon production
  Yaffe et al

* Heavy quarkonium (hard probe)
  Maldacena

* Thermalization, phase transition

* Hardron spectrum (AdS/QCD)

* AdS/CDM
The gravity dual of a Wilson loop at large $N_c$ and large $\lambda$

$$\text{tr} < W(C) > = e^{-\sqrt{\lambda} S_{\text{min}}[C]}$$

$W(C) = Pe^{-i \int dx^\mu A_\mu(x)}$

Heavy quark potential probes confinement, hadronic phase and meson melting in plasma

$$F(r, T) = T\left(S_{\text{min}}[\text{parallel lines}] - 2S_{\text{min}}[\text{single line}]\right)$$
Heavy quark potential at zero temperature

\[
x_3 = \pm \int_{z}^{z_0} d\zeta \frac{\zeta^2}{\sqrt{z_0^4 - \zeta^4}}
\]

The potential

\[
V(r,0) = F(r,0) = -\frac{4\pi^2 \sqrt{2N_c} g_{YM}^2}{\Gamma^4 \left(\frac{1}{4}\right)^r}
\]

No confinement in N=4 SYM!
Heavy quark potential at a nonzero temperature

The world sheet at the minimum

\[ x_3 = \pm \sqrt{z_h^4 - z_0^4} \int_0^{z_0} d\zeta \sqrt{\frac{\zeta^2}{(z_0^4 - \zeta^4)(z_h^4 - \zeta^4)}} \]

\[ \frac{r}{2} = \pm \sqrt{z_h^4 - z_0^4} \int_0^{z_0} d\zeta \sqrt{\frac{\zeta^2}{(z_0^4 - \zeta^4)(z_h^4 - \zeta^4)}} \]

\[ z_h = \frac{1}{\pi T} \]

Rey, Theisen and Yee

Defu Hou @ YITP
Free energy:

\[ F(r,T) = -\frac{4\pi^2 \sqrt{\lambda}}{\Gamma^4 \left(\frac{1}{4}\right)} \phi(\pi Tr) \theta(r_c - r) \]

\[ \phi(\pi Tr_c) = 0 \]

\[ r_c \approx \frac{0.7541}{\pi T} \]

Potential:

F-ansatz

\[ V(r,T) = F(r,T) \]

U-ansatz

\[ V(r,T) = F(r,T) + TS(r,T) = -T^2 \frac{\partial}{\partial T} \left( \frac{F}{T} \right) \]

Non Yukawa screening!
Heavy quarkonium Dissociate Temperature

Hou , Ren, JHEP0801:029

<table>
<thead>
<tr>
<th>ansatz</th>
<th>$J/\psi(1S)$</th>
<th>$J/\psi(2S)$</th>
<th>$J/\psi(1P)$</th>
<th>$\Upsilon(1S)$</th>
<th>$\Upsilon(2S)$</th>
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<tr>
<td>$U$</td>
<td>143-265</td>
<td>27-50</td>
<td>31-58</td>
<td>421-780</td>
<td>80-148</td>
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With deformed metric

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<tr>
<td>$F$</td>
<td>NA</td>
<td>235-385</td>
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<tr>
<td>$U$</td>
<td>219-322</td>
<td>459-780</td>
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<table>
<thead>
<tr>
<th>ansatz</th>
<th>$I_{s/t}/(\text{holo})$</th>
<th>$J/\psi(\text{lattice})$</th>
<th>$\Upsilon(\text{holographic})$</th>
<th>$\Upsilon(\text{lattice})$</th>
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<tbody>
<tr>
<td>$F$</td>
<td>NA</td>
<td>1.1</td>
<td>1.3-2.1</td>
<td>2.3</td>
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<tr>
<td>$U$</td>
<td>1.2-1.7</td>
<td>2.0</td>
<td>2.5-4.2</td>
<td>4.5</td>
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Relativistic correction

Wu, Hou, Ren, PRC 87 (2013), 025203

<table>
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<th>$b\bar{b}$</th>
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<tr>
<td></td>
<td>$\lambda = 5.5$</td>
<td>$\lambda = 6\pi$</td>
<td>$\lambda = 5.5$</td>
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<tr>
<td>$1s$</td>
<td>162.54</td>
<td>387.54</td>
<td>478.76</td>
</tr>
<tr>
<td>$2s$</td>
<td>29.15</td>
<td>62.75</td>
<td>85.67</td>
</tr>
<tr>
<td>$1p$</td>
<td>32.04</td>
<td>62.14</td>
<td>94.18</td>
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This lists the results of $T_0 + \delta_1 T$ in MeV's, that we just considered the correction of the $p^4$ term, which increased the dissociation temperature.
## Relativistic correction

Wu, Hou, Ren, PRC 87 (2013), 025203

We wrote the state as:

\[ \eta J^{2S+1}_L \]

<table>
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<tr>
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<tbody>
<tr>
<td></td>
<td>$\lambda = 5.5$</td>
<td>$\lambda = 6\pi$</td>
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<tr>
<td>$1s_0^1$</td>
<td>130.79</td>
<td>188.65</td>
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<td>$1s_1^3$</td>
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<td>$2s_0^1$</td>
<td>26.71</td>
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<tr>
<td>$2s_1^3$</td>
<td>26.71</td>
<td>48.16</td>
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<tr>
<td>$1p_1^1$</td>
<td>31.53</td>
<td>61.33</td>
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<tr>
<td>$1p_0^3$</td>
<td>32.65</td>
<td>68.48</td>
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<tr>
<td>$1p_1^3$</td>
<td>32.09</td>
<td>64.90</td>
</tr>
<tr>
<td>$1p_2^3$</td>
<td>30.96</td>
<td>57.76</td>
</tr>
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</table>

For J/Psi, the magnitude of the correction ranges from 8% to 30%!
Higher order corrections

Leading orders are strictly valid when $N_c \to \infty$, $\lambda \to \infty$

- **For real QCD. The t’Hooft coupling is not infinity**
  
  $$5.5 < \lambda < 6\pi.$$

- **The super gravity correction to the AdS-Schwarzschild metric is of order**
  
  $$O(\lambda^{-3/2})$$

- **The fluctuation around the minimum world sheet presents at all T, and is of order**
  
  $$O(\lambda^{-1/2})$$ (more important)
Gravity dual of a Wilson loop at finite coupling

\[ W[C] \equiv \exp \left( i \int_C dx^\mu A_\mu \right) = \int [dX] [d\theta] \exp \left[ \frac{i}{2\pi \alpha'} S(X, \theta) \right] \]

**Strong coupling expansion** \( \iff \) **Semi-classical expansion**

\[
\ln W[C] = i \sqrt{\lambda} \left[ S(\bar{X}, 0) + \frac{b[C]}{\sqrt{\lambda}} + \ldots \right]
\]

\( \bar{X} \) = the solution of the classical equation of motion;

\( b[C] \) comes from the fluctuation of the string world sheet around \( \bar{X} \)

more significant than \( \alpha'^3 \) -correction for Wilson loops.
\[
\frac{1}{2\pi\alpha'} S(X, \theta) = \text{the superstring action in } AdS_5 \times S^5
\]

Metsaev and Tseytlin

With fluctuations:

\[
X^\mu = \overline{X}^\mu + \delta X^\mu, \quad \theta \neq 0 \quad \Rightarrow \quad g_{ij} = \overline{g}_{ij} + \delta g_{ij}
\]

\[
S(X, \theta) = S(\overline{X}, 0) + S_B^{(2)}(\delta X) + S_F^{(2)}(\theta) + \ldots
\]

Bosonic and fermionic fluctuations decouple.

\[
W[C] = e^{iS(\overline{X}, 0)} Z \quad Z = Z_B Z_F
\]
Partition function at finite T with fluctuations underlying the potential

Straight line:

\[ Z = Z_B Z_F = \frac{\det^2 \left(-\nabla_+^2 + 1 + \frac{1}{4} R^{(2)} \right) \det^2 \left(-\nabla_-^2 + 1 + \frac{1}{4} R^{(2)} \right)}{\det^\frac{3}{2} \left(-\nabla^2 + \frac{8}{3} + \frac{1}{2} R^{(2)} \right) \det^\frac{5}{2} (-\nabla^2)} \]

Parallel lines:

\[ Z = \frac{\det^2 \left(-\nabla_+^2 + 1 + \frac{1}{4} R^{(2)} \right) \det^2 \left(-\nabla_-^2 + 1 + \frac{1}{4} R^{(2)} \right)}{\det^\frac{1}{2} \left(-\nabla^2 + 4 + R^{(2)} - 2\delta \right) \det(-\nabla^2 + 2 + \delta) \det^{\frac{5}{2}} (-\nabla^2)} \]
Next leading order Results

Chu, Hou, Ren, JHEP 0908, (2009)

\[ V(r) \approx -\frac{4\pi^2}{\Gamma^4(\frac{1}{4})} \frac{\sqrt{\lambda}}{r} \left[ 1 - \frac{1.33460}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right) \right] \quad \text{for } \lambda \gg 1 \]

Confirmed by Forini, JHEP 1011 (2010) 079

\[ a_1 = \frac{5\pi}{12} - 3 \ln 2 + \frac{2K}{\pi} \left( K - \sqrt{2} (\pi + \ln 2) + I^{\text{num}} \right) \]
\[ = -1.33459530528060077364 \ldots , \]

\[ V_{\text{ladder}}(r) = -\frac{\sqrt{\lambda}}{\pi r} \left( 1 - \frac{\pi}{\sqrt{\lambda}} \right). \]

Erickson etc. NPB 582, (2000)

\[-\frac{\lambda}{4\pi r} \left[ 1 - \frac{\lambda}{2\pi^2} \left( \ln \frac{2\pi}{\lambda} - \gamma_E + 1 \right) + O(\lambda^2) \right] \quad \text{for } \lambda \ll 1 \]

Defu Hou @ YITP
Next leading order potential at finite $T$

Zhang, Hou, Ren, Yin  JHEP07:035 (2011)

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4 \left(\frac{1}{4}\right)} \frac{\sqrt{\lambda}}{r} \left[ g_0(rT) - \frac{1.33460 g_1(rT)}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right) \right]$$

**Figure 3.** The left curve represents $g_1(rT)$, while the right represents $g_0(rT)$.

*Defu Hou @ YITP*
Jet quenching in QGP

Energy loss/scattering (Gyulassy and XNW’94)

\[ \Delta E \approx -\frac{\alpha_s}{2\pi} N_C \hat{q} L^2 \]

Baier, Dokshitzer, Mueller, Peigne, Schiff (1996):

\[ \hat{q} = \frac{\mu^2}{\lambda} \] jet transport coefficient

\[ \hat{q} \] reflects the ability of the medium to “quench” jets.

Defu Hou @ YITP
jet quenching parameter from AdS/CFT

Liu, Rajagopal & Wiedemann, PRL, 97, 182301 (2006)

\[ \hat{q}_0 = \frac{\pi^{3/2} \Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \sqrt{2} T^3 \]

Dipole amplitude: two parallel Wilson lines in the light cone:

\[ W^A[C] = \exp\left(-\frac{\hat{q} L L^2}{4\sqrt{2}}\right) \]
NL correction to jet quenching parameter

\[ \hat{q} = \frac{\pi^{3/2}}{\Gamma\left(\frac{3}{4}\right)} \frac{\Gamma\left(\frac{5}{4}\right)}{\Gamma\left(\frac{3}{4}\right)} \sqrt{\lambda T^3} \left[ 1 - 1.97 \lambda^{-1/2} + O(\lambda^{-1}) \right] \]

dominant

\[ 1 - 1.765 \lambda^{-3/2}. \]

Zhang, Hou, Ren, JHEP1301 (2013) 032

N. Armesto et al JHEP09 (06)
Jet quenching in QGP & hadronic phase

Chen, Greiner, Wang, XNW, Xu (2010)

\[ \hat{q}_0 = 0.9^{+0.05}_{-0.04} \text{GeV}^2/\text{fm} \]

perturbative \( \alpha_s = 0.3 \)

\[ \hat{q}_N = 0.02 \ \text{GeV}^2/\text{fm} \]

30% quenching from hadronic phase

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Jet quenching parameter discussion

\[ q_{\text{exp}} = 1 \rightarrow 15 \text{GeV}^2 / \text{fm} \]


Take \( N_c = 3, \ \alpha = 0.3 \quad \longleftrightarrow \quad \lambda = 3.6\pi \)

Choose \( T = 300 \text{MeV} \)

\[ q_0 = \frac{\pi^{3/2} \Gamma(\frac{3}{4})}{\sqrt{2} T^3} \Gamma(\frac{5}{4}) = 3.45 \text{ GeV}^2/\text{fm} \]

\[ \hat{q} = 1.37 \text{ GeV}^2/\text{fm} \]

1. Sub-leading order gives rise to 40% reduction from the leading order.

2. The negative sign of sub-leading order is consistent with a monotonic behavior from strong coupling to weak coupling.
AdS/CFT provides a useful way to address the physics at strong coupling.

The partition function of Wilson loop with fluctuations in strongly coupling N=4 SYM plasma are derived.

We computed the jet quenching parameter and heavy quark potential up to sub-leading orders.

We estimated the melting T with holographic potential and its relativitivitc correction

The applicability of these AdS/QCD results demands phenomenological work to explain them in a way which can be translated to real QCD.
# QCD versus N=4 Super Yang-Mills from gravity dual

<table>
<thead>
<tr>
<th></th>
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<th>Super YM</th>
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<tbody>
<tr>
<td>$N_c$</td>
<td>3</td>
<td>$&gt;&gt;1$</td>
</tr>
<tr>
<td>t’Hooft coupling</td>
<td>5.5-18.8</td>
<td>$&gt;&gt;1$</td>
</tr>
<tr>
<td>Quarks</td>
<td>Fundamental</td>
<td>Adjoint</td>
</tr>
</tbody>
</table>
| Conformal symmetry     | No        | Yes at zero T
                               | No at nonzero T |
| Supersymmetry          | No        | Yes at zero T
                               | No at nonzero T |
Thanks