QCD transition at finite temperature with Domain Wall fermions

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--- Insight into QCD matter from heavy-ion collisions ---
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sketched QCD phase diagram
QCD phase diagram at $\mu_B = 0$

The fundamental scale of QCD: chiral phase transition $T_c$?

The value of tri-critical point ($m_{s}^{\text{tri}}$) ?

The location of 2$^{\text{nd}}$ order $Z(2)$ line?
scenarios of QCD phase transition at $m_l = 0$

![Diagram showing phase transitions and critical points](image)
QCD phase transition at the physical point

Karsch et al., '03
QCD transition at finite temperature

Recent lattice QCD studies using Highly Improved Staggered Quarks with temporal extent of \( N_t = 6 \) suggest coordinates of the physical point: \((\bar{m}_s/27, \bar{m}_s)\)

coordinates of \( m^c_{\text{max}} \approx (\bar{m}_s/270, \bar{m}_s/270)\)

2nd order \( O(4) \) chiral phase transition seems to be more relevant to the physics at the physical point.
Recent lattice QCD studies using Highly Improved Staggered Quarks with temporal extent of \( N_t=6 \) suggest coordinates of the physical point: \((\bar{m}_s/27, \bar{m}_s)\)

coordinates of \( m^c_{\text{max}} \approx (\bar{m}_s/270, \bar{m}_s/270) \)

2\text{nd order } O(4) \text{ chiral phase transition}?
The symmetries of QCD

At the classical level, the symmetries of QCD with $N_f$ flavors of massless fermions:

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_V \times U(1)_A$$

- Spontaneous $SU(N_f)_L \times SU(N_f)_R$ chiral symmetry breaking gives rise to 8 Goldstone bosons: the $\pi$, $K$, $\eta$

9th Goldstone boson $\eta'$?

- $U(1)_A$ symmetry is violated by axial anomaly at the quantum level and is responsible for the $\eta$-$\eta'$ mass splitting

\[ \chi_{top} \]

\[ \partial_\mu j^\mu_5 = \frac{g^2 N_f}{16\pi^2} tr(\tilde{F}_{\mu\nu} F^{\mu\nu}) \]

\[ m_\eta^2 + m_{\eta'}^2 - 2m_K^2 = \frac{2N_f}{F_\pi^2} \chi_{top} \]
Is $U(1)_A$ symmetry restored at or above $T_{\chi_{SB}}$? — Shuryak '94

$N_f=2$ QCD at $m=0$ can be first order or second order $U(2)_L \times U(2)_R / U(2)_V$ if $U(1)_A$ is effectively restored — Pisarski & Wilczek '84, Butti, Pelissetto & Vicari '03

Possible influence on the particle yield, reduction of $\eta'$ mass? — Shuryak '94, Csorgo, Vertesi and Sziklai, PRL '10

the fate of $U(1)_A$ symmetry at finite $T$ and its underlying mechanism are not yet clear
First principle calculations on the lattice

- Recent studies using non-chiral fermions, e.g. staggered fermions, give some evidence of $O(N)$ scaling in the chiral limit of $N_f=2+1$ QCD

  S. Ejiri et al., PRD '09, HTD, lattice '13

- Not yet conclusive due to the infamous taste symmetry breaking in the staggered discretization scheme with remnant $U(1)_A$ symmetry
First principle calculations on the lattice

Difficulties:

- Chiral fermions that preserve exact chiral symmetry and produce correct axial anomaly are needed

  \[
  \{D^{-1}, \gamma_5\} = a\gamma_5
  \]

- Overlap fermions: the only operator satisfies the Ginsparg-Wilson relation, however, there exists a “freezing” topology problem, more expensive than Domain Wall fermions

- Domain Wall fermions: preserve exact chiral symmetry and produce correct axial anomaly when the fifth dimension is sufficiently large. Residual symmetry breaking is quantified by the additive renormalization factor \( m_{res} \) to the quark mass
studies on $U(1)_{A}$ symmetry at finite $T$ using chiral fermions

$N_f=2$ QCD studies using Overlap fermions
$16^3 \times 8$ lattices
topology fixed to 0

Cossu et al., arXiv:1304.6145

$N_f=2$ QCD studies using Optimized Domain Wall fermions
$16^3 \times 6$ lattices

Chiu et al., Lattice 2013, arXiv:1311.6220
simulations of 2+1 flavor QCD using Domain Wall fermions on Nt=8 lattices with two pion masses:

- $m_\pi=140$ MeV, $N_s=32$
- $m_\pi=200$ MeV, $N_s=32, 24, 16$


results are shown for $m_\pi=200$ MeV if not mentioned explicitly
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signatures of chiral symmetry restoration

- $(\sigma, \pi^i)$ and $(\eta, \delta^i)$ each transform as an irreducible 4-dim. rep. of $SU(2)_L \times SU(2)_R$

- $(\sigma, \eta)$ and $(\pi^i, \delta^i)$ each transform as 2-dim. rep. of $U(1)_A$

Susceptibilities defined as integrated two point correlation functions of the eight local operators, e.g. $\chi_{\pi} = \int d^4x \langle \pi^i(x)\pi^i(0) \rangle$
fate of chiral symmetries at finite $T$

$SU(2)_L \times SU(2)_R$

$SU(2)_L \times SU(2)_R$ symmetry is restored at $T_{\chi_{SB}} \sim 170$ MeV

$U(1)_{A}$ symmetry remains broken up to 195 MeV $\sim 1.16 T_{\chi_{SB}}$

Buchof et al., [HotQCD], arXiv:1309.4149
Some issues

LQCD calculations using Domain Wall fermions

❖ the finite lattice cutoff effects ?
❖ the finite volume effects ? contributions from exact zero modes ?
❖ the residual symmetry breaking effects ?

Signatures of $U(1)_A$ restoration

❖ Can $U(1)_A$ restoration be signaled by two point correlation functions and their susceptibilities ? Aoki, Fukaya and Taniguchi, arXiv:1209.2061

❖ Dirac Eigenvalue spectrum

Symmetry restorations in the chiral limit

❖ explicit chiral symmetry breaking by the finite quark mass?
contributions from exact zero modes

\[
\langle \bar{\psi}\psi \rangle = \int_0^\infty d\lambda \rho(\lambda, \tilde{m}) \frac{2\tilde{m}}{\tilde{m}^2 + \lambda^2} + \frac{\langle |Q_{\text{top}}| \rangle}{\tilde{m}V}
\]

\[
\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \rho(\lambda, \tilde{m}) \left( \frac{2\tilde{m}}{\tilde{m}^2 + \lambda^2} \right)^2 + \frac{2\langle |Q_{\text{top}}| \rangle}{\tilde{m}^2V}
\]

The second terms are from exact zero mode contributions related to the non-zero topological charge and should vanish when \( V \) goes to infinity.

The mild volume dependence of chiral condensates and \( \chi_\pi - \chi_\delta \) at \( T \approx T_c \) indicates negligible exact zero mode contributions.
exact DWF ward Identity

Gell-Mann-Oakes-Renner relation: \( \langle \bar{\psi} \psi \rangle = m \chi_\pi \)

In DWF formalism:
\[
\langle \bar{\psi} \psi \rangle_l = (m_l + m_{res}) \chi_{\pi_l} + R_{5d}^l
\]
\[
= m_l \chi_{\pi_l} + \Delta_{mp}^l
\]
\[
\langle \bar{\psi} \psi \rangle_s = (m_s + m_{res}) \chi_{\pi_s} + R_{5d}^s
\]
\[
= m_s \chi_{\pi_s} + \Delta_{mp}^s
\]

\( \bar{\psi} \) subtracted pbp \( \Delta_{l,s} \) to cancel the linear UV divergence in quark mass
\[
\Delta_{l,s} = \langle \bar{\psi} \psi \rangle_l - \frac{m_l + m_{res}}{m_s + m_{res}} \langle \bar{\psi} \psi \rangle_s,
\]
\[
\Delta_{l,s} = \Delta_{l,s}^{imp} + R_{5d}^l - \frac{m_l + m_{res}}{m_s + m_{res}} R_{5d}^s
\]

\( \bar{\psi} \) improved subtracted pbp \( \Delta_{l,s}^{imp} \) : suitable to DWF, cancel further residual chiral symmetry breaking effects
\[
\Delta_{l,s}^{imp} = (m_l + m_{res}) (\chi_{\pi_l} - \chi_{\pi_s})
\]
reproduction of improved subtracted pbp from $\rho(\lambda)$

Subtracted chiral condensates can be reproduced well from Dirac Eigenvalue spectrum.
evaluation of $\chi_\pi - \chi_\delta$ from 2-pt. corr. and Dirac Eigenvalues

$$(\chi_\pi - \chi_\delta)/T^2$$ calculated from 2-pt. corr. 
$$(\chi_\pi - \chi_\delta)$$ calculated from Dirac Eigenvalues

$$\chi_\pi - \chi_\delta = \int_0^\infty d\lambda \rho(\lambda, m) \left( \frac{2m}{m^2 + \lambda^2} \right)^2$$

$$= \frac{1}{N^3 \sigma^4 \tau} \left( \sum_{n=1}^{100} \frac{2m^2}{\Lambda^4_n} \right)$$

Remarkable agreement between $\chi_\pi - \chi_\delta$ evaluated from 2-point correlation functions and Dirac Eigenvalues

At two lowest $T$ the discrepancy may come from the unphysical fluctuation of $\Lambda$ associated with the residual symmetry breaking
signatures from Dirac eigenvalue spectrum $\rho(\lambda)$

\[
\langle \bar{\psi}\psi \rangle = \int_0^\infty d\lambda \rho(\lambda, \tilde{m}) \frac{2\tilde{m}}{\tilde{m}^2 + \lambda^2}, \quad \chi_\pi - \chi_\delta = \int_0^\infty d\lambda \rho(\lambda, \tilde{m}) \left( \frac{2\tilde{m}}{\tilde{m}^2 + \lambda^2} \right)^2
\]

- the restoration of $SU(2)_L \times SU(2)_R$ symmetry

**Banks-Casher formula:** $\langle \bar{\psi}\psi \rangle = \pi \rho(0)$

- $\rho(0)=0$

- the restoration of $U(1)_A$ symmetry

  - $\rho(\lambda)$ must go to zero faster than linearly
  - a sizable gap from zero, i.e. $\rho(\lambda<\lambda_c)=0$
  - $\rho(\lambda) = c |\lambda|^\alpha$, $\alpha > 2$ if observables are analytic in $m^2$

Cohen, nucl-th/980106

Aoki, Fukaya and Taniguchi, arXiv:1209.2061
Dirac Eigenvalue spectrum with $m_{\pi}=200$ MeV

black lines: $16^3$ results
red histograms: $32^3$ results

$T<T_c$
nonzero $\rho(0)$

$T\sim T_c$
vanishing $\rho(0)$

$T>T_c$
no gap from zero

$U(1)_A$ remains broken up to 195 MeV
possible behaviors for $\rho(\lambda,m)$

Possible forms of $\rho(\lambda,m)$ making $<\bar{\Psi}\Psi>=0$ and $\chi_\pi - \chi_\delta \neq 0$?

$$\rho(\lambda,m) = c_0 + c_1 \lambda + c_2 m^2 \delta(\lambda) + c_3 m + c_4 m^2 + O(\lambda,m)$$

<table>
<thead>
<tr>
<th>Ansatz</th>
<th>$&lt;\bar{\Psi}\Psi&gt;$</th>
<th>$\chi_\pi$</th>
<th>$\chi_\delta$</th>
<th>$\chi_\pi - \chi_\delta$</th>
<th>$2\chi_{disc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>$c\pi$</td>
<td>$c\pi/m$</td>
<td>0</td>
<td>$c\pi/m$</td>
<td>0</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>$-2m \ln(m)$</td>
<td>$-2\ln(m)$</td>
<td>$-2\ln(m)$</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$m^2\delta(\lambda)$</td>
<td>$m$</td>
<td>1</td>
<td>-1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$m$</td>
<td>$\pi m$</td>
<td>$\pi$</td>
<td>0</td>
<td>$\pi$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>$m^2$</td>
<td>$\pi m^2$</td>
<td>$\pi m$</td>
<td>0</td>
<td>$\pi m$</td>
<td>$\pi m$</td>
</tr>
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</table>

Bazavov et al., [HotQCD] arXiv:1205.3535

$n>2$ point correlation functions and their susceptibilities are needed to investigate $U(1)$ breaking only if $\rho(\lambda)$ is analytic in $m^2$

Aoki, Fukaya and Taniguchi, arXiv:1209.2061
fits to the Dirac Eigenvalue spectrum

\[ (\chi_\pi - \chi_\delta) / T^2 \]
calculated from 2-pt. corr.

\[ \text{calculated from Dirac Eigenvalues} \]
\[ \text{calculated from fits to } \rho(\lambda) \]

fractions of different contributions

\[ \chi_\pi - \chi_\delta = a_0 \pi + 2a_1 + 2a_2 \]

fitting ansatz to the Dirac Eigenvalue spectrum

\[ \rho(\lambda, m) = a_0 m + a_1 m^2 \delta(\lambda) + a_2 \lambda \]

- current fitting ansatz to the Dirac Eigenvalue spectrum gives good description of \( \chi_\pi - \chi_\delta \) at three highest temperatures

- SU(2)_L x SU(2)_R symmetry breaking term dominates below \( T_c \) while near zero modes contribution dominates above \( T_c \)

a0: SU(2)_L x SU(2)_R symmetry breaking contribution
a1: near zero modes contribution
a2: linear infra. behavior

\[ \rho(\lambda, m) = a_0 m + a_1 m^2 \delta(\lambda) + a_2 \lambda \]
Underlying mechanism of $U(1)_A$ breaking

Resulting from non-zero global topology

density of exact zero modes $\sim 1/\sqrt{V}$

In a relatively dilute gas of instantons and anti-instantons (DIGA)

density of near zero modes independent of $V$

No evidence of $\rho(\lambda)$ shrinking by a factor of $\sqrt{8}$ from $16^3$ to $32^3$ lattices is found, which favors DIGA
Chirality of near zero modes

Distribution of chirality per configuration:

- obeys bimodal if coming from nonzero topology
- obeys binomial if coming from a dilute gas of instantons
Distribution of chirality

<table>
<thead>
<tr>
<th># of configurations with $N_0$ and $N_+$</th>
<th>$32^3 \times 8, T=177$ MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_+ \setminus N_0$</td>
<td>0 1 2 3 4 5</td>
</tr>
<tr>
<td>$N_0 = 1$</td>
<td>40 29 - - - -</td>
</tr>
<tr>
<td>$N_0 = 2$</td>
<td>11 20 12 - - -</td>
</tr>
<tr>
<td>$N_0 = 3$</td>
<td>3 11 6 2 - -</td>
</tr>
<tr>
<td>$N_0 = 4$</td>
<td>0 1 2 1 0 -</td>
</tr>
<tr>
<td>$N_0 = 5$</td>
<td>0 2 0 0 0 0</td>
</tr>
</tbody>
</table>

$N_0$: total # of near zero modes
$N_+$: # of near zero modes with positive chirality

Data behaviors more like a binomial contribution

A dilute instanton gas model can describe the non-zero $U(1)_A$ breaking above $T_c$!
mass dependence of chiral symmetry restorations

SU(2)\_L \times SU(2)\_R

\begin{align*}
\pi : & \quad \bar{q} \gamma_5 \frac{\tau}{2} q \\
\delta : & \quad \bar{q} \frac{\tau}{2} q \\
\eta : & \quad \bar{q} \gamma_5 q
\end{align*}

SU(2)\_L \times SU(2)\_R

(\chi_\pi - \chi_\sigma): finite quark mass effects negligible at T \gtrsim 158 \text{ MeV}

U(1)\_A certainly does not restore at T_{\chi_{SB}} \sim 170 \text{ MeV},
remains broken up to 195 MeV \sim 1.16 T_{\chi_{SB}}
mass dependence of chiral symmetry restorations

\[ \frac{(\chi_{\pi} - \chi_{\delta})}{T^2} \]

\( m_{\pi} = 140 \text{ MeV} \) and \( m_{\pi} = 200 \text{ MeV} \)

\[ \text{SU(2)}_L \times \text{SU(2)}_R \]

\[ U(1)_A \]

\[ \chi_{\pi} - \chi_{\delta} = \chi_{\pi} - \chi_{\sigma} + \chi_{\sigma} - \chi_{\delta} \]
\[ = \chi_{\pi} - \chi_{\sigma} + 2\chi_{\text{disc}} \]

\[ \pi : \bar{q} \gamma_5 \frac{\tau}{2} q \]
\[ \sigma : \bar{q} q \]

\[ \delta : \bar{q} \frac{\tau}{2} q \]

\[ \eta : \bar{q} \gamma_5 q \]

Differences in quark mass dependences:
disconnected susceptibilities

Non-zero values of $\chi_{\text{disc}}$ suggest the breaking of $U(1)_A$ symmetry in the current $T$ window

$\chi_{\pi} - \chi_{\delta} = (\chi_{\pi} - \chi_{\sigma}) + (\chi_{\sigma} - \chi_{\delta}) = (\chi_{\pi} - \chi_{\sigma}) + 2\chi_{\text{disc}}$

restoration of $U(1)_A$ requires:

$\chi_{\pi} - \chi_{\delta} = 2\chi_{\text{disc}} = 0$

$\chi_{\text{disc}} / T^2$ at the physical quark mass peaks at $T_{pc} = 154$ MeV consistent with the result from staggered fermions
O(N) scaling behavior in the high temperature region

Magnetic Equation of State (MEoS):

\[ M = h^{1/\delta} f_G(z), \quad z = t/h^{1/\beta\delta} \]

external field: \( h = \frac{l}{h_0 m_s} \)
reduced temperature: \( t = \frac{l}{t_0} \frac{T-T_c}{T_c} \)

\( f_G(z) \): universal scaling function, \( O(N) \) etc
\( \beta, \delta \): universal critical exponents

According to \( O(4) \) scaling for large positive values of \( z \), i.e. high \( T \)

\[ f_G(z) \sim R_\chi z^{-\beta(\delta-1)} \]
\[ M = h^{1/\delta} f_G(z) \sim R_\chi t^{-\beta(\delta-1)} h \]
\[ \chi_M \sim R_\chi t^{-\beta(\delta-1)} \]

Engels et al., NPB 675(2003)533
O(N) scaling behavior in the high temperature region

Magnetic Equation of State (MEoS):

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\( \beta, \delta \): universal critical exponents

According to \( O(4) \) scaling for large positive values of \( z \), i.e. high \( T \)

The mass independence of chiral susceptibility observed in the high temperature region indicates the \( O(4) \) scaling

Another evidence of the breaking \( U(1)_A \) symmetry
Summary

• Calculation with DWF at the physical pion mass on Nt=8 lattices
  ✴ Crossover behavior
  ✴ $T_{pc} \approx 154$ MeV
  ✴ agreement with staggered results

• $U(1)_A$ symmetry of 2+1 flavor QCD on Nt=8 lattices
  ✴ remain broken up to 195 MeV
  ✴ breaking weakens rapidly after the restoration of SU(2)_LxSU(2)_R
  ✴ quantitatively explained by near zero modes
  ✴ well described by a dilute gas of instantons and anti-instantons