Vector mesons in a strong magnetic field

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Orders of magnitude for magnetic fields

Typical magnet
- Neodymium magnet (strongest permanent magnet)
- Strongest continuous magnetic field

Produced in a laboratory

Magnetars

- Heavy ion collisions
  \[ \sim 10^4 \text{MeV}^2 \sim 10^{17} \text{G} \]

The early Universe
- (Electroweak transition)
  \[ \sim 10^{22} \text{G} \]
Chiral symmetry breaking is enhanced in \( B \).

Model

Kawati, Konishi, Miyata('83), Klevansky, Lemmer('89), Suganuma, Tatsumi('91), Klimenko('92), Krive, Naftulin('92), Schramm, Muller, Schramm('92), ....

Lattice QCD

Buividovich, Chernodub, Luschevskaya, Polikarpov ('09) Braguta, Buividovich, Kalayzhyan, Kuznetsov, Polikarpov('10) D’Elia, Mukherjee, Sanfilippo('10) D’Elia and Negro('11) Ilgenfritz, Kalinowski, Muller-Preussker, Petersson, and Schreiber('12), ....
QCD + magnetic field at finite T

Decreasing $T_c$!
“Inverse Magnetic Catalysis”

Possible explanations:

Magnetic inhibition,
Fukushima, YH (’12)

Deconfining phase transition,
Fraga, Palhares (’12)

Quark mass gap, ....
Kojo, Su (’12), (’13)
QCD + magnetic fields

Chiral magnetic effect

\[ J^i_V = \sum_f q_f B^i N_c \frac{\mu_A}{2\pi^2} \]

Kharzeev, McLerran, Warringa (07)  Fukushima, Kharzeev, Warringa (08)

Chiral separation effect

\[ J^i_A = \sum_f q_f B^i N_c \frac{\mu}{2\pi^2} \]

Son, Zhitnitsky (04), Metlitski, Zhitnitsky (05)

closely related to chiral anomaly.
Today’s focus:

Hadron spectra in B
in particular,

Vector mesons
Charged particle in a magnetic field

Classical equation of motion

\[
H \ddot{x} = e(E + \dot{x} \times B),
\]

\[
H = \sqrt{p^2 + m^2}
\]

Lorentz force

Closed orbital motion in the transverse plane

(Landau) quantization
Landau quantization

\[ E^2 = p_z^2 + m^2 + (2n + 1)qB - g_s qB \]

Landau quantization

Zeeman splitting
Why is a vector meson interesting?

Vector meson mass

\[ m^2_\rho(B) \approx m^2_\rho - eB \]

\[ m^2_\rho(B = B_c) = 0 \]

Vector meson condensation?

Schramm, Muller, Schramm ('92)
Does a charged vector meson condense in a strong B in QCD?
Model study I

Hadronic model

Extended NJL model


In the seminal paper [17], the effect of including the nontrivial eB-induced dynamics on the value of the naive critical \(eB_c = m^2_r\) was estimated roughly to be about 15%, so that \(eB_c = m^2_r\) appeared to be an educated guess to set the scale at which new QCD effects would appear. Our result supports this, since including the chiral magnetic catalysis 'only' leads to a \(\ll 10\%\) correction on the critical magnetic field. In [22], a quenched \(N_f=2\) lattice simulation was made of the \(r\) condensate, also revealing an estimate for the critical magnetic field, \(eB_c \approx 0.924 \text{ GeV}^2\), which is somewhat larger than our result (109). It is reassuring that two quite distinct non-perturbative approaches, be it our holographic analysis or the lattice output, are in qualitative agreement.
Vacuum superconductivity

Inhomogeneous $\rho^\pm$ condensate

Similar result in holographic approach: Bu, Erdmenger, Shock, Strydom ('13)
Lattice study I

Bragutaa, Buividovichb, Chernodubbd, Kotovb, Polikarpovb (‘12)

Figure 3: The superconducting condensate $\langle \rho \rangle$ of the charged $\pi$ mesons as the function of the magnetic field $B$. The green points correspond to the condensate calculated for small lattice $14^4$, while the blue squares represent the data extrapolated to an infinite volume $L \to \infty$. The dashed blue line is the fit by the linear function (15). The red arrow marks the point of the insulator–superconductor phase transition (16).

Figure 4: The massive parameter $\mu$ corresponding to the best fits (11) and (12), (13).

Lattice Study II (our study)  
YH, Yamamoto ('12)
Model study III

$q\bar{q}$ system in confined potential + magnetic field


![Graph showing the masses of some selected systems as a function of $eB$, GeV$^2$. The graph includes lines and markers for different states and charges.](image-url)
Does the vector meson condensation really occur in QCD?

Our answer is NO. I want to convince you this.
Theoretical analysis

Vafa-Witten theorem
Convexity of effective action

\[ V(\phi) \]

Potential in the Lagrangian

\[ \Gamma[\phi] \]

Effective action
**Convexity**

**Generating functional:**

\[ e^{R[J]} = \int \mathcal{D}\phi e^{-S[\phi] + J\phi} \]

\[ e^{R[J+\Delta J]} = \int \mathcal{D}\phi e^{-S[\phi] + J\phi + \phi\Delta J} = e^{R[J]} \langle e^\phi \Delta J \rangle_J \]

Where \( \langle \mathcal{O} \rangle_J = \int \mathcal{D}\phi e^{-S[\phi] + J\phi - R[J] \mathcal{O}} \)

\[ e^{R[J+\Delta J]} - R[J] = \langle e^\phi \Delta J \rangle_J \geq e^{\langle \phi \rangle} \Delta J \]

\[ \langle e^\mathcal{O} \rangle \geq e^{\langle \mathcal{O} \rangle} \] Jensen’s inequality

\[ R[J + \Delta J] - R[J] \geq \frac{\delta R[J]}{\delta J} \Delta J \] Convex

\[ \Gamma[\phi] = J\phi - R[J] \] is also convex.
Spontaneous symmetry breaking

Explicit breaking term
\( \epsilon \neq 0 \)

\( \mathbb{Z}_2 \) symmetric at \( \epsilon = 0 \)

\[ V \rightarrow \infty \]
\[ \epsilon \rightarrow 0 \]

\( \phi_c = 0 \) symmetric phase

\( \phi_c \neq 0 \) broken phase
Symmetry breaking

Add explicit breaking term to term.

\[ S \rightarrow S + \int d^4x \epsilon \bar{\psi} \Gamma \psi \quad \text{e.g., } \Gamma = \tau^3 \]

Calculate the order parameter

\[ \phi \equiv \frac{1}{N} \int d^4x \bar{\psi}(x) F \psi(x) \quad \text{e.g., } F = \tau^3 \]

\[ \langle \phi \rangle_{\epsilon} = \left\langle \text{Tr} F \frac{1}{D + m + \epsilon \Gamma} \right\rangle_{A,\epsilon} \]

\[ \langle \mathcal{O} \rangle_{A,\epsilon} \equiv \int d\mu \mathcal{O} \quad \text{d}\mu = \prod_{\mu,a,x} dA_{\mu}^a(x) \det(D + m + \epsilon \Gamma)e^{-S[A]} \]

Take \( \epsilon \to 0 \) limit \quad \text{If } \lim_{\epsilon \to 0} \langle \phi \rangle_{\epsilon} \neq 0 \quad \text{SSB!} \]
Vafa-Witten theorem (B=0, T=0)
No SSB occurs in the isospin channel.

\[
\lim_{\epsilon \to 0} \langle \phi \rangle_\epsilon = 0
\]

- Fermion operator has no zero modes. Fermion propagator is well defined.
- Fermion determinant is nonnegative. Schwarz inequality works.
- Order parameter is nonsinglet.Disconnected diagrams do not contribute.
Vafa-Witten theorem for $\theta = 0$ vacuum

**Dirac operator**

\[ D = \gamma_\mu (\partial_\mu + igA_\mu) \]

**Anti-Hermite**

\[ D^\dagger = -D \]

**Chiral symmetry**

\[ \gamma_5 D \gamma_5 = -D \]

**Eigenvalue of** \( D^\dagger + m \) : \( \pm i\lambda_n + m \neq 0 \)

**Positivity:**

\[ \det(D^\dagger + m) = \prod_{\lambda} (i\lambda + m) = m^{n_0} \prod_{\lambda > 0} (\lambda^2 + m^2) > 0 \]

**Upper bound of propagator:**

\[ \left\| \frac{1}{D^\dagger + m} \right\|_{op} = \frac{1}{m} \cdot \]
\[ \langle \phi \rangle_\varepsilon = \langle \text{Tr} F \left( \frac{1}{\mathcal{D} + m + \varepsilon \Gamma} \right) \rangle_{A, \varepsilon} \]

Expanding the order parameter with respect to \( \varepsilon \).

\[
\text{Tr} F \left( \frac{1}{\mathcal{D} + m + \varepsilon \Gamma} \right) = \sum_{n=1}^{\infty} (-1)^n \varepsilon^n \text{Tr} F \left( \frac{1}{\mathcal{D} + m} \right)^n \left( \frac{1}{\mathcal{D} + m} \right) \]

\[
\left| (-1)^n \text{Tr} F \left( \frac{1}{\mathcal{D} + m} \right)^n \left( \frac{1}{\mathcal{D} + m} \right) \right| \leq \| \Gamma \|_{\text{op}}^n \left\| \frac{1}{\mathcal{D} + m} \right\|_{\text{op}}^{n+1} \sqrt{\text{Tr} FF^\dagger} \]

\[
= \frac{C^n}{m^{n+1}}, \quad \text{where} \ C \equiv \| \Gamma \|_{\text{op}}
\]

\[ \langle \phi \rangle_\varepsilon \leq \left\| \text{Tr} F \left( \frac{1}{\mathcal{D} + m + \varepsilon \Gamma} \right) \right\|_{\epsilon, A} \leq \sum_{n=1}^{\infty} \frac{(\varepsilon C)^n \varepsilon \rightarrow 0}{m^{n+1} \rightarrow 0} \]

No Spontaneously Symmetry Breaking!
Finite B

\[ D_\mu = \partial_\mu - igA_\mu \rightarrow \partial_\mu - igA_\mu - iqA^\text{em}_\mu \]

\[ B^z = \partial_x A_y - \partial_y A_x \neq 0 \]

**Symmetry:**

\[ SO(3, 1) \rightarrow SO(1, 1)_{t,z} \times SO(2)_{x,y} \]

\[ SU(2)_I \times U(1)_B \rightarrow U(1)_{I_3} \times U(1)_B = U(1)_{\text{em}} \times U(1)_B \]

**Positivity:** OK
Possibility of inhomogeneous phase:

\[ \phi \equiv \frac{1}{N} \int d^4 x \bar{\psi}(x) F \psi(x) , \]

\[ F = \tau + \gamma + f(x) \text{ space dependent} \]

If \( \langle \bar{\psi}(x) \tau + \gamma + \psi(x) \rangle = g(x) \), we may choose \( f(x) = g^*(x) \)

Positivity, lower bound of quark propagator: OK
Order parameter: nonsinglet.

No symmetry breaking.
Counter arguments

bullet QCD x QED should be considered
  Comment on “Charged vector mesons in a strong magnetic field”
  Chernodub, arXiv:1309.4071

bullet Multivalued generating functional
Counter arguments

QCD x QED should be considered


His claim

Because of gauge symmetry, no NG mode appears (Higgs phase), which is consistent with the Vaffa-Witten theorem.

Our claim

Our situation corresponds to a fixed U(1) gauge, and no dynamical photons.

(Our result does not change in any gauge fixing conditions)

Technically, it corresponds to a fixed $eB$ with $e \rightarrow 0$

In this case, the rho meson condensation is necessary in the Higgs phase.
Counter arguments

Nontrivial generating functional

Their claim
If the generating functional is not single valued, the Vafa-Witten theorem may not hold.

Our claim
The generating functional is convex, so that it is single valued.
Comments
Does VW theorem work at

Finite $T$? OK!

Finite $\mu_B$? NO.
Fermion determinant is complex.
No positivity.

Finite $\mu_I$? NO.
Fermion determinant is nonnegative.
\[
\frac{1}{\slashed{D} + m + \gamma_4 \tau_3 \mu_I}
\]
can be zero.
Generalized NJL model?  NO!

\[ \mathcal{L} = \bar{\psi}(i\Slash{D} + m)\psi + \frac{1}{2G} V_\mu^2 \]

\[ D_\mu = \partial_\mu - i\tau^a V^a_\mu - iqA^{em}_\mu \]

Vector meson carries isospin, so that disconnected diagrams also contribute to the order parameter.

Supersymmetric model?  NO!

Fermion determinant has no positivity.
Summary
Vector meson condensation? Our answer is no.

- Vafa-Witten theorem
- Lattice simulation

Any models based on QCD should satisfy this theorem.

If you find any loop hole, please let us know!